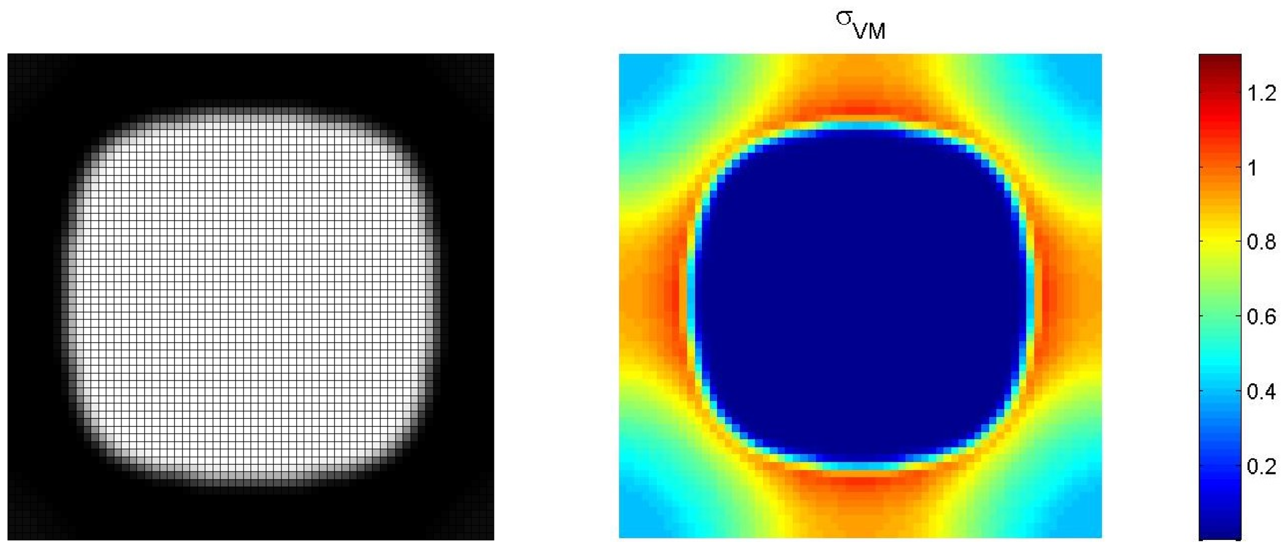
# Design of microstructures using stress–based topology optimization

Maxime Collet[1], Matteo Bruggi[2], Lise Noël[1], Simon Bauduin[1], Pierre Duysinx[1]
[1] University of Liège, Aerospace and Mechanical Engineering Department, Liège, Belgium
[2]Politecnico di Milano, Civil and Environmental Engineering Department,Milan, Italy

## Motivation

Microstructural design (MD) using topology optimization along with homogenization to compute equivalent elastic properties is used to conceive optimal microstructures addressing specific needs:




max $\mathcal{K}$ s.t. $V \leq 0,5$: $\mathcal{K}_{optimized} = 0,6734$

Accounting for the stress level within the optimized layouts ensures the structural integrity of the microstructure.

## Sensitivity analysis of the stresses

Sensitivity along with the qp-relaxation (Bruggi,2008)[a]:

$$\frac{\partial \sigma_{VM,e}}{\partial x_k} = \delta_{ek}(p-q)x_e^{p-q-1}\,\bar{\sigma}_{VM,e} + \frac{\partial \bar{\sigma}_{VM,e}}{\partial x_k} x_e^{p-q}$$

Adjoint approach (active set : $N_a$ selected constraints $< N$ total constraints)

$$\frac{\partial \bar{\sigma}_{VM,e}}{\partial x_k} = \lambda^T \frac{\partial \mathbf{K}}{\partial x_k}\mathbf{u} - \lambda^T \left( p x_k^{p-1} f^i_{e,0} \right)$$

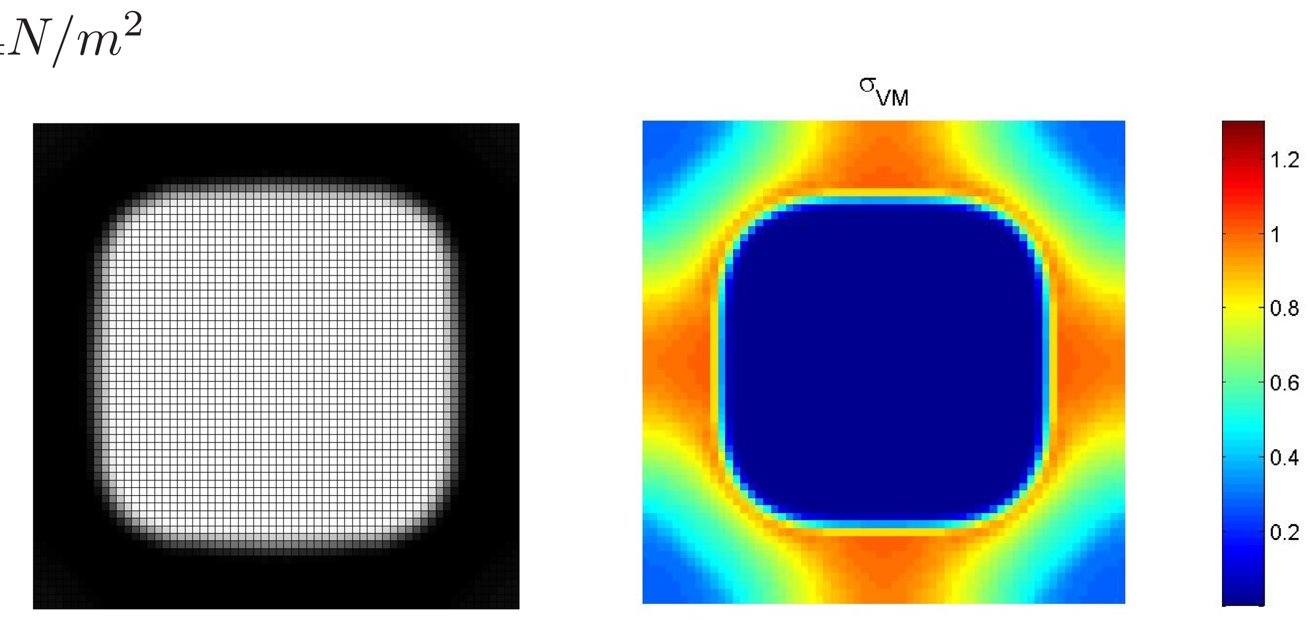
with the adjoint problem and force vector:

$$\begin{aligned} \mathbf{K}\lambda &= -\left[ \left( \mathbf{u_e^T M_e^0 u_e} \right)^{\mathbf{-1/2}} \mathbf{M_e^0 u_e} \right]^{\mathbf{T}} \\ f^i_{e,0} &= \sum_{i=1}^{n1} \sum_{j=1}^{n2} w_i w_j \mathbf{B_e}^T \mathbf{H_e^0} \varepsilon_0^i detJ \end{aligned}$$

[a]Bruggi M (2008) On an alternative approach to stress constraints relaxation in topology optimization.Struct Multidisc Optim 36:125-141

## Example 1: Classical Bulk maximization

The maximization of the bulk modulus under stress constraints with $\sigma_y^0 = 1,4N/m^2$




max $\mathcal{K}$ s.t. $V \leq 0.5$ **AND** $\sigma \leq \sigma_y^0$: $\mathcal{K}_{optimized} = 0,6719$

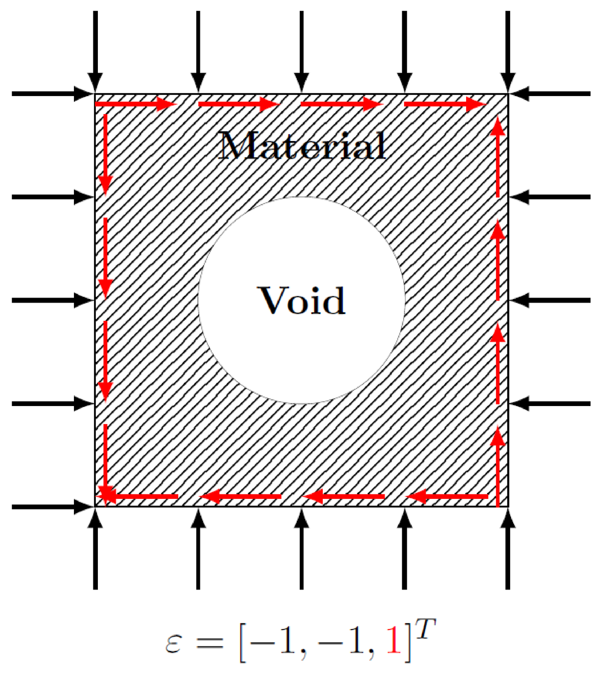
- Similar topology compared to the classical solution but with a controlled stress level ($\frac{\sigma}{\sigma_y^0} \leq 1$)
- More square shaped compared to the classical solution
- Lower optimized bulk modulus

## Example 2: *Bronzi di Riace* seismic insulator

- *Bronzi di Riace* statue with its marble seismic insulator:

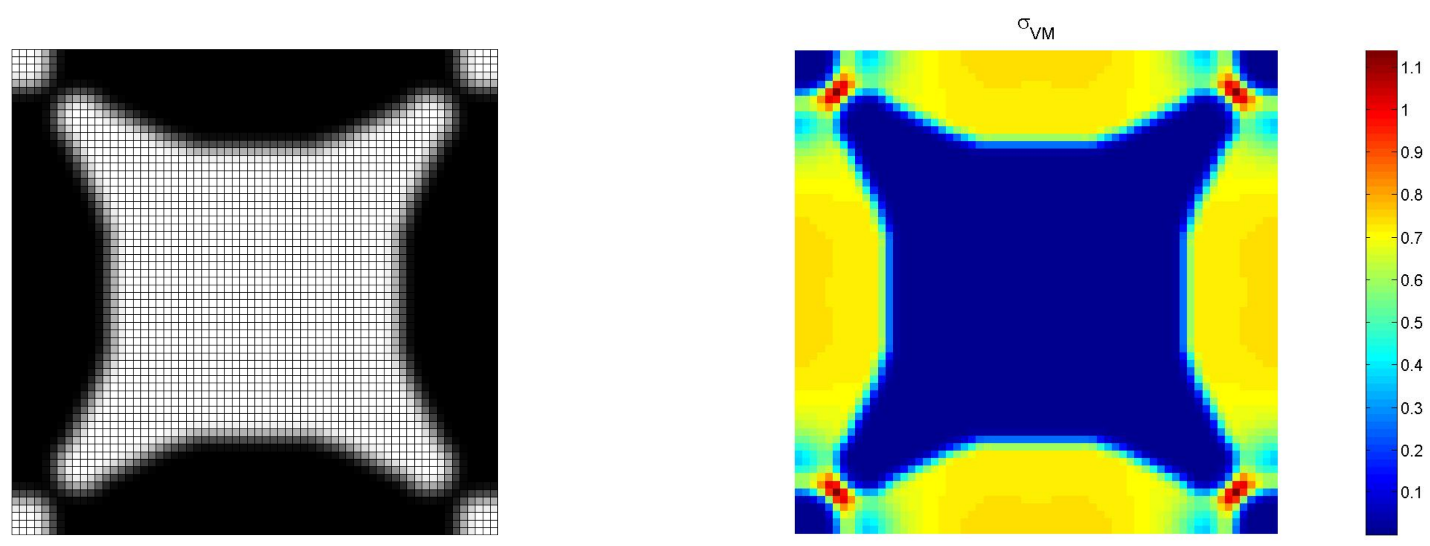
Currently: Four marble balls used to isolate the statues in case of earthquakes. Design requirements are high bulk and low shear (under prescribed value). A topology optimization can be formulated:

$$\begin{aligned} \max_{0<x_{min}\leq x_e \leq 1} \quad & \mathcal{K} \\ s.t. \quad & \mathbf{K}(\mathbf{x})\mathbf{u}^i = \mathbf{f}(\mathbf{x})^i; \quad i = 1,2 \\ & \mathcal{G} \leq \mathcal{G}^* \\ & \left( \langle \sigma_{VM,e}(x) \rangle \leq \sigma_y^0 \right) \end{aligned}$$

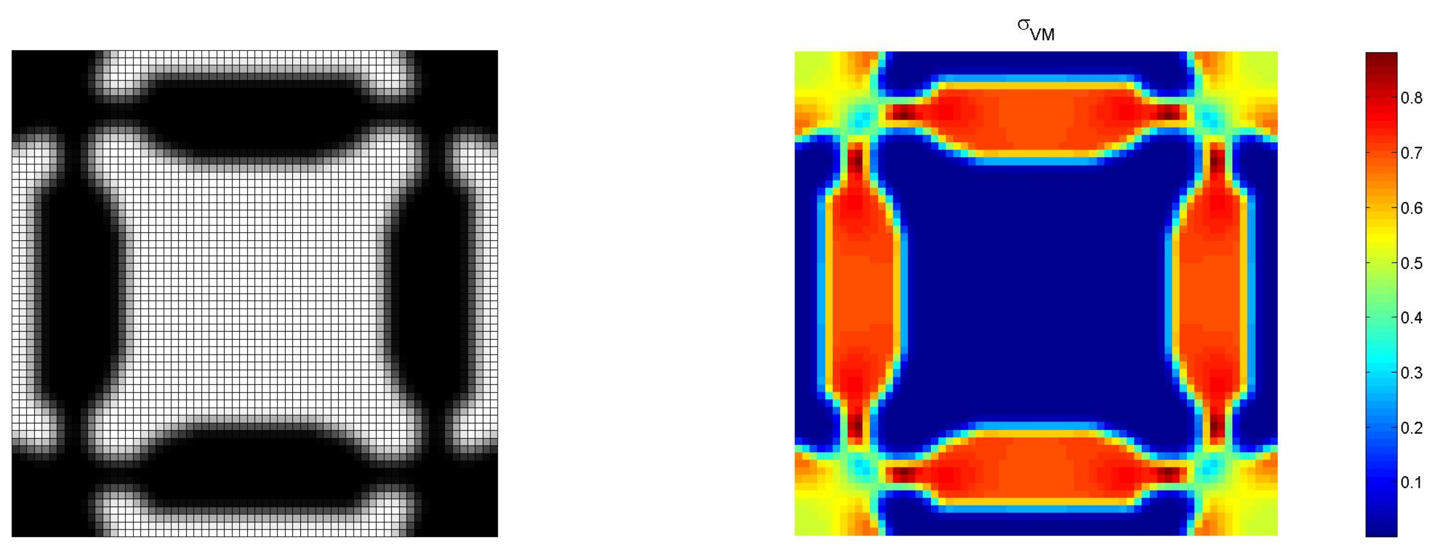



- Two load cases !
- Starting point: Hole with void at the center of the RVE
- $\sigma_y^0 = 1.8N/m^2$ and $\mathcal{G} \leq 10^{-3}N/m^2$

- **No** stress constraints : $\mathcal{K}_{optimized} = 0,3665$



- **With** stress constraints : $\mathcal{K}_{optimized} = 0,4376$



- Control of the local stress constraints ($\frac{\sigma}{\sigma_y^0} \leq 1$)
- Change in the topology of the optimized layouts
- Existence of local optima !

## Conclusion and Perspectives

- Control of the stress level introduced in a topology optimization framework for microstructural design (sensitivity analysis)
- The optimization might be steered toward better optimum $\Longrightarrow$ Presence of local minima
- Speed up the computation ! CPU requirement is still to high !
- Extension to 3D manufacturable microstructures

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