

‘I can – you can’: cooperation in group activities

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We stem from phenomenological stances that each thought originates from a perceived competence (“I can”), which has a strong affective nature. In groupwork activities the “you can”, namely the competence that a student recognizes in the classmates, is also important. We propose a codifying system for inferring the students’ “I can” and “you can” during a groupwork activity. The results are 2D-diagrams that capture the main moments of the activity and help us identifying the leader(s) of the activity and the different roles the students play.

Keywords: group interaction, perceived competence, cooperation.

INTRODUCTION AND FRAMEWORK

Learning in mathematics occurs in and through interaction with others, the learner becomes conversant with new concepts emerging in the activity, as Sfard (2001) points out: “communication may be defined as a person’s attempt to make an interlocutor act, think or feel according to her intentions” (p.13). Activity allows the growth of mutual understanding and coordination between the individual and the rest of the community (Sfard, 2001). Cooperation is one of the main focuses. Different kinds of cooperation have been depicted (Gooding & Stacey, 1993), with respect to the objective of the activity. Also, conditions that foster cooperation have been highlighted: operational, computational requests of the activity enhance cooperation, whilst reflexive ones may impede it (Hertz-Lazarowitz, 1989). But, are there conditions that shape cooperation which can be traced back to the relationships among the students, rather than to the features of the task?

We stem from Merleau-Ponty (2002) claim that every thought originates from an “I can” rather than from an “I think of”. Di Martino and Zan (2011) have identified the students’ “I can” in the narrative accounts of their relationship with mathematics during the school years: they call it *perceived competence*. “I can” may prompt the student to intervene in a conversation, proposing a strategy, rephrasing another student’s utterance, adding a detail, fostering her thinking. “I can’t” may push a student to ask questions, seek for clarification, stay silent and listen to her mates (see Andrà & Liljedahl, 2014). Since each student may (or not) recognize a competence to each one of his classmates, there is another dimension that counts: “you can”. Andrà and Liljedahl (2014) have shown that, during a group activity, a student may care of a certain classmate’s attention and, at the same time, be annoyed by another student’s reaction to his proposal. “I can” and “you can” are seen as interior states, whilst utterances, postures, glances, gestures are the external expressions of such internal states that determine each student’s will to propose, to react, to stay silent.

METHOD OF ANALYSIS

We exploit an idea introduced by Sfard and Kieran (2001) to capture “two types of speaker’s meta-discursive intentions: the wish to react to a previous contribution of a partner or the wish to evoke a response in another interlocutor” (p.58): reactive and proactive utterances, respectively. Proactive and reactive statements are, in our view, indicators for inferring a sense of “I can”, a sense of likelihood of success that provides the basis for thought and communication. A sense of “I can’t”, conversely, can be inferred from a student’s silence, but also from his/her questioning the strategies/results proposed by his/her classmates. Postures, glances, tone of voice can sustain our inferences about a student’s sense of “I can” versus “I can’t”. A sense of “you can’t” is, as well, inferred from absence of reactions to other students’ utterances. A student who does not glance at a classmate, but looks at the paper, or elsewhere, may not recognize a competence to her (see also Andrà & Liljedahl, 2014). Conversely, a student who care about a classmate’s attention, a student who gives her way, who reacts to her, may feel a sense of “you can”.

We analyze two video excerpts. We codify the excerpts as follows. Each example is divided into *moves*, lasting few seconds each and being identified by the placement of the students into a 2D-diagram, representing the internal states: “I can” versus “I can’t”, “you can” versus “you can’t”. If a student change his/her placement in the 2D-diagram, a new move starts. Even if there can be various degrees of “I can” and “you can”, in our first analysis we decided to depict only two possible, dichotomous states. The result of the analysis is a series of 2D-diagrams (4 possible “states”) that capture the subsequent moves of the activity. We added arrows to denote changes (if any) for a student with respect to the previous move. The first episode has no arrows.

The diagrams are the result of: (i) extensive observation made by each author, (ii) intense discussion about possible interpretations of data. After hours of joint work in team, it is not possible to provide indices for inter-rater agreement within us, given that each interpretation is strictly intertwined with each other’s one. This can be seen as a weakness of this work. Hence, in a poster (Andrà, Brunetto, Parolini, & Verani, 2015), we report on several secondary school teachers’ comments, and we specifically seek for agreement/disagreement with respect to our interpretations.

DATA

We present two excerpts taken from a course in probability aimed at preventing gambling abuse. These excerpts come from 22 hours of video recording in 13 different schools of different types in Northern Italy. These two excerpts have been selected because they capture some interesting dynamics. True, the small sample is a limitation of this study. Another limitation is the focus only on the perceived competence: Di Martino and Zan (2011) identify another important dimension, ‘I like’. Motivation is third dimension that plays a crucial role in framing students’ interaction. As a first step, we decided to focus only on the ‘I can’-‘you can’.

The first excerpt comes from a group activity engaging 4 grade-12 (17 years old) students: Enrico (E), Federico (F), Giovanni (G), and Michele (M). They are attending a technical high school program and are asked to invent a fair game using two dices (possible outcomes are all the sums from 2 to 12). In a fair game, the expected winning equals the ticket price. They have computed the probability of each sum, reporting it on the paper (Figure 1A). They now have to assign the prize to each sum from 2 to 12: G wants to start from 7, and F from 12. The excerpt lasts almost 1 minute. Out-of-school knowledge emerge, as well as school mathematics.

The second excerpt comes from a group activity engaging 4 grade-10 (15 years old) students: Alice (A), Barbara (B), Carola (C), and Dora (D). They are attending a scientific high school program and are analyzing a slot machine, which has three rolls with 9 different symbols each. The number of different sequences is $9^3=729$. There is only one winning symbol: the gold bar. The students have computed the probabilities of one, two and three gold bars, and reported them on the paper (Figure 1B). They now compute the expected winning. To do so, they have to report the probabilities in the table, to multiply each of them by the corresponding prize, and to sum up the results (weighted prizes). The excerpt lasts 8 minutes.

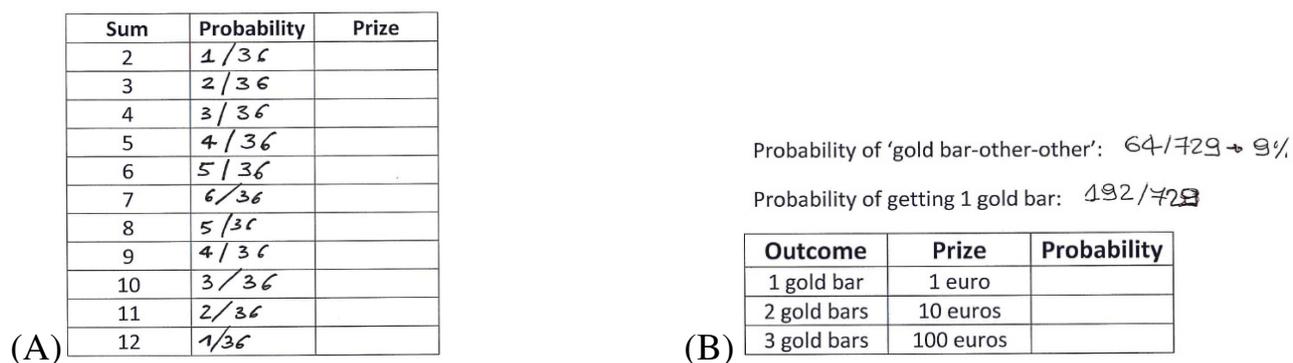


Figure 1: parts of tasks completed before the first (A) and second (B) excerpts begin.
DATA ANALYSIS

Excerpt 1—move 1. We now present the excerpts, inferring the students’ “I can” and “you can” from their utterances, their postures, their gazes and the tone of their voice. Data analysis begins with the first 13 seconds of the first excerpt.

- 1 G: We should start with 7, which has the highest prize. ...No, the lowest prize.
- 2 F: No, let’s do, let’s bet 1 euro.
- 3 M: Easy.
- 4 F: If e.g. you bet on 12, it comes out, you win...
- 5 M: But bet 2 euros.
- 6 F: ...you win 36 euros. Let’s do 36 to 1.

Both G and F make proactive statements. G looks at the paper but addresses F. F, in fact, reacts immediately to G (“No”), and makes a proactive statement (“let’s do, let’s bet 1 euro”), which involves everyone: in fact, M reacts to it in 3 (“Easy”), and in 5

("bet 2 euros"), and E will react later, in 13 ("Else, bet 50 cents"). G opens the conversation, proposing to start with the prize to be assigned to 7. A proactive statement is an indicator for G to have a sense of "I can". We claim that such an "I can" is grounded on the intuition that 7 should be assigned the lowest prize, and this intuition comes from his experience with betting games: the higher the probability to win, the lower the prize. F makes three proactive statements, showing a clear sense of "I can": in 2, 4 and 6. In his proactive statement in 4 F proposes to start from the prize to assign to 12. At 6 he intuits that one should win 36 euros, and this intuition comes from his vivid experience with sport betting practice. The language he uses is the language of sport betters. In the first two proactive statements, F looks at the paper, in the last one he looks at G. To look at the paper is an indicator of "you can't", and in fact F speaks over his classmates both in 2 and 5. In 2 F contradicts G with a "No", without providing any justification to the group concerning why should they do differently from what G has proposed. In 5 M reacts to F, but F does not react to M. F discards M's proposal to bet 2 euros instead of 1.

In 1, indeed, G feels a sense of "you can", using the first plural person ("we"). He makes his proposal, but he remains silent to listen to the others, without trying to impose his standpoint. As well, also M shows a sense of both "I can" and "you can": the adjective "Easy" he uses at 3 to comment on F's proposal, clearly refers to his perceived competence, and his proposal in 5 to bet 2 euros is both involving and bearing a sense of competence with respect to the activity. E is silent in this move of the activity and we conjecture that E is listening ("you can"), but he is still making sense of the activity and hence his perceived competence is low ("I can't"). We infer this by looking also at his posture: from 1 to 5 he stares at F, but in 6 he takes the paper and brings it closer to him, starting reading the task silently.

Excerpt 1—move 2. Also this move starts with a statement from G, who reacts to F.

- 7 G: Wait: 7, how much is it? We should compute the average prize and...
- 8 F: It's enough to do this (points to 36, the denominator) divided by this (points to 6, the numerator of the probability to get a sum of 7). If you do 36 divided by 7, what do you get? (makes computations with the calculator) 5. If you bet 1 euro on 7, you win 5 euros.

In 7 G reacts to F, recalling his will to start with 7 and the mathematical activity on the mean prize during previous lessons at school. In this moment, G feels a sense of "I can't" and asks for F's help in computing such a mean. F reacts to G, showing a sense of "I can" and "you can't". The sense of "I can" can be inferred from his prompt, procedural reaction, and "you can't" can be inferred from his proposal that differs from the one recalled by G. In fact, F again refers back to sport betting practice rather than the mathematical activity. A conflict starts to emerge, the conflict between the experience at school (the average) personified by G, and the betting experience personified by F. Since E and M are silent, but they are listening to F and G, we conjecture that they feel a sense of "I can't" that hinders their will to speak,

and a sense of “you can” since they give F and G their way. We further observe that F and G occupy two mirroring positions: F is the “I can”-“you can’t” area, whilst G is in the “I can’t”-“you can” area (table 1, column 2). This area is occupied also by E and M and it seems that the group has reached a sort of frozen situation: there is F, who has a strong sense of “I can” and “you can’t”, whilst his mates feel that they “can’t”. How can the group sort this situation out? A possibility is that E, G or M start to question the sense of “you can” with respect to F. If also F “can’t”, the group activity can go on *as a group activity*. This is happening in *Excerpt 1—move 3*.

- 9 G: Hence, the minimum you can win is 5 euros.
10 E: (inaudible)
11 G: It’s too much.
12 F: The highest prize is 36 euros.
13 E: Else, let’s bet 50 cents.
14 F: It’s the same, finally. If we bet 1 euro at least we have (inaudible)
15 M: (inaudible)

In 9 G reacts to F’s result and tries to give sense to it. This confirms that, even if he has a low sense of perceived competence in the second move of the activity (“I can’t”), he trusts F and he listens to him (“you can”). In 11 G adds: “it’s too much”. His emotions provide G with a sense of unlikelihood of success, a sense of both “I can’t” (as in the previous phase of the group activity), and “you can’t”: F now can’t solve the problem, the solution he is proposing is “too much”. G looks at the paper, but E and F, conversely, look at him. F, specifically, recognizes his role (“you can”). Even if they are almost inaudible, also E and M contribute to the activity in its third move: their perceived competence has increased (“I can”), and—as we have already commented—we infer a sense of “you can” towards G and F.

In 12 F makes his fourth proactive statement to the whole group, and this is taken as an indicator of “I can”. In 13 E reacts to F’s proactive statement, and F in 14 makes his first (and only one) reactive statement, reacting to E. F looks at E, differently from his usual practice. Hence, F not only recognizes G’s role (“you can”), but also sees E as an interlocutor (again, “you can”).

G’s move towards “you can’t” is mirrored by F’s move towards “you can”: F has recognized to G a role, F is recognizing G as an interlocutor (table 1, column 3). Furthermore, E and M have moved from “I can’t” to “I can”, since they feel they can follow G’s argument and contribute to the group activity. This is mirrored in the next move of the activity, where E nods at F and F stares at E. *Excerpt 1 – move 4*.

- 16 F: E.g. 12 is given ... like the SNAI[1] (all the students laugh)
17 F: 12 is given 36 to 1. If I bet 1 euro I win 36 euros.
18 E: (nods)
19 G: mmm it’s too much, because, then, the 7...?

In 16-17 F makes another proactive statement: “12 is given 36 to 1. If I bet 1 euro, I win 36 euros”. F looks at the paper in 16, and at E in 17. We read a sense of “I can” for F, but a sense of “you can’t”, since he looks at the paper and he does not involve his mates, he does not invite them to react to him. G reacts to F, and this reaction speaks to his sense of both “I can’t” (in fact, he is asking again how to deal with the case of 7) and “you can’t” (in fact, he feels that the result got by F “it’s too much”). E has moved towards “I can”, continuing to feel a sense of “you can” with respect to both F and G. Since M does not intervene, we infer that he moves again to “I can’t”.

We observe that the situation at this point is very balanced (Table 1, column 4). There is a leader, F, who feels a sense of “I can” as well as a sense of “you can’t”, but there is another student, E, who has been almost silent during the activity, who attracts F’s gazes at this point, since he has a sense of “I can” and “you can”. Also M feels a “you can”, while G feels both an “I can’t” and a “you can’t”. At this point, something really interesting happens (*Excerpt 1 – move 5*): F ignores G’s doubt (“the 7?”), and he goes on with the prize to be assigned to 11. Then, he feels a sense of “I can’t” saying “Eh, no, because...”. His emotions provide F with a sense of unlikelihood of success, a sense of “I can’t”, which freezes him. Also M and E, despite their sense of “you can”, feel a sense of “I can’t”. Table 1 summarizes the five subsequent moves that have been analyzed.

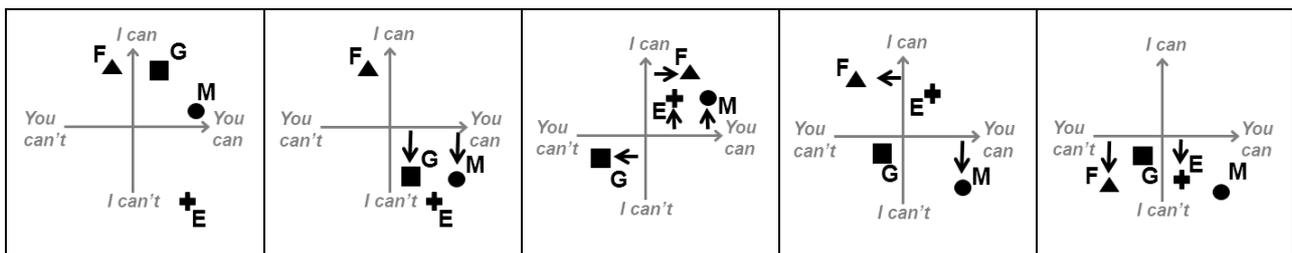


Table 1: the five subsequent moves of the activity involving E, F, G and M.

Excerpt 2 – move 1. In the first 15 seconds B reads the task, then

- 5 B: We should use combinatorial mathematics.
- 6 D: No, we have computed that the probability of finding one gold bar and two other symbols (she takes the sheet of paper to be able to read it) was...
- 7 B: ...was it 192?
- 8 D: No. No. This one maybe (she points to the probability of getting the sequence ‘gold bar–other–other’)
- 9 B: (reads) ‘probability of gold bar-other-other’ (nods). 9%
- 10 D: 9%

The students look mostly at the paper, only B in 5 looks at D. B in 5 makes a proactive statement, but D reacts proposing to use the probabilities they have already computed. In 8 B reacts to D’s proposal and she reads on the sheet of paper the correct answer: the probability of getting one gold bar is 192 (over 729), but D contradicts B and leads the group to use the probability of getting “gold bar–other–

other”, that is 9%. Both B and D feel a sense of “I can”, whilst A and C—being silent—yields us thinking that they are feeling a sense of “I can’t” and “you can”, giving B and D their way. Also B feels a sense of “you can”, but D feels a “you can’t”: in 6 and 8 she contradicts B (“no”), proposing something different from what B has proposed. There is a conflict that is emerging, a conflict between two strategies: B would like to resort to the formulas learned in previous lessons, while D would like to use the computations already made. The group follows D’s proposal, as we will see in the next move, but both B and D are right: B (as well as the rest of the group) is not aware that they have already used combinatory. We add a local comment to this segment of data analysis: this lack of awareness is quite usual when the students deal with strategies and concepts that are “new” for them, since they are still not conversant with them. Learning is, in fact, a progressive becoming aware of mathematical meanings.

In the subsequent 2 minutes, the students make computations and report the results on the paper, then they ask the teacher if their work is correct, and the teacher confirms their doubt: it is wrong. *Excerpt 2 – move 2*. B recalls her first proposal:

- 42 B: That is why we should use combinatorial mathematics. Otherwise, they would not have given it to us.
- 43 C: Let’s use our ingenuity!
- 44 B: For sure it’s with combinatory, then let’s invent something. Well, in combinatory one needs to multiply many numbers, so let’s have a look...

D is silent, we infer that she is feeling a sense of “I can’t”, but C intervenes and say “let’s use our ingenuity” looking at A, the classmate that has remained silent until now. We see a sense of “I can” in C, who looks at the student that has been silent like her, in a sense encouraging her to intervene. B is proactive (“I can”). A does not intervene for the present. The teacher provides a feedback about the mistake the group has made: the probability of getting one gold bar is not 9%. A takes the floor:

- 67 A: This one (she points to $192/729$) is the probability to get one gold bar.
- 68 B: Hence, I have said it correctly at the beginning! It is 192 divided by 729.
- 69 C: (speaks over B) 192 divided by 729.
- 70 B: (addressing D, pleased) Ah! Ah!

B and D look at each other in this sequence, and B expresses a sense of revenge with respect to D: we read it in terms of “I can” and “you can’t” on B’s side. Moreover, the conflict between B and D is solved: D was right in having proposed to use their own computations, and B was right in using $192/729$.

Excerpt 2 – move 3. The students copy the probabilities on the sheet, then they stop.

- 110 C: (looks at B) But what should we compute?
- 111 B: The total average prize. The... mm... namely...

- 112 D: All those prizes times the probability that you win them, namely that you win the prizes, divided by... all the cases?
- 113 C: Eh?
- 114 D: 729? Because there are many cases in which you win nothing.
- 115 A: Exactly. There are many cases where you get nothing, where you get 'other'.
- 116 B: Hence we should do the prize..? No, the cases in which you win, divided...
- 117 D: The cases in which you win, the total prize divided by all.
- 118 B: Why divided by all?

D makes two proactive statements in 112 and 117, and all her classmates look at her. A, B and C react to her proactive statements: C ("Eh?") at 113, A echoes D's comment at 114, and B at 116 makes sure she has understood, but at 118 asks for explanation. D feels a sense of "I can" and "you can": she is proactive, but she also reacts to her classmates. Also A intervenes, showing a sense of "I can" and "you can". B and C, instead, feel a sense of "you can" with respect to D, but they are not understanding D's proposal, and ask for clarifications: we infer a sense of "I can". *Excerpt 2 – move 4.* The group remains silent for a while, then A (the student that feels "I can"), tries to reformulate D's proposal:

- 119 A: You are saying: to sum all the average prizes. Is it what you're saying?
- 120 D: I do not know (smiles).
- 121 C: Eh, (echoing D) 'I don't know what I am doing'! (smiles)

We see a sense of "I can't" in D's utterance "I do not know", and of "you can't". *Excerpt 2 – move 5.* C, after having echoed D, addresses B and asks:

- 122 C: Wait. Is it asked (A pulls the paper close to her) the mean of the prizes?
- 123 B: But, but I would have done this: I would have multiplied the mean prizes for the probability, then summing the probabilities, divided for all the cases.
- 124 C: Like the last time!
- 125 B: Yes, it is the weighted average.

D remains silent and stares at the paper, but C in 122, even if she is addressing B, looks at D, and also B looks at D in 123. D's silence leads us inferring a feeling of "I can't" and "you can": she is listening to her mates. A shares the same state of D, being silent. B and C, indeed, share a sense of "you can", accompanied by a sense of "I can". They have found the correct strategy, and they are confirming it to each other trying to involve the rest of the group (looking at D, for example). They are recognizing the distinguishing features of a procedure they have already used, and their positive emotions provide them with a sense of likelihood of success. They are intuiting the analogy with respect to a procedure previously used, that can work.

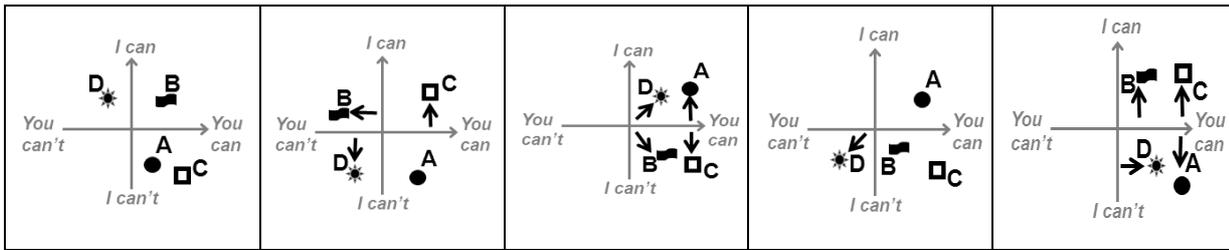


Table 2: the six subsequent moves of the activity involving A, B, C and D.

DISCUSSION

Answering to the research question (are there conditions that shape cooperation which can be traced back to the relationships among the students?), we can see that: (a) proactive utterances and *vertical arrows* allow us to identify *the leader(s)*; (b) *horizontal arrows* allow us to identify *collaboration*. In excerpt 1, F provides the majority of proactive utterances, whilst G is proactive only at the beginning, then he reacts to F's proposals. In excerpt 2, B and D are proactive throughout the excerpt; B and D are the leaders, but also A and C make 2 and 3 proactive utterances, respectively. B and D are collaborative leaders.

E and M show only vertical movements: they occupy always the “you can” area, and they *oscillate* between the “I can”-“I can't” positions.

We may also notice that vertical arrows characterize the moves of students who are not leaders: E and M are clearly not leaders in excerpt 1, like A and C in excerpt 2. But, there are downward arrows also for F (table 1, last move) and D (table 2, move 2). These two downward arrows, indeed, depict two different situations. In F's case, it is F alone that stops speaking and starts to doubt about his strategy: his emotions provide him with a sense of unlikelihood of success. F, in fact, is a leader that is not collaborative. In D's case, D goes down because it is B that is leading in that move. Hence, the leaders' “I can” can be diminished either by his/her emotional sense of likelihood of success, or by the feedback of the classmates. This may open the possibility for the others to intervene (Andrà & Liljedahl, 2014). Negative feedback determine a decreasing of “I can”. The other way around, to increase one's “I can” by providing positive feedbacks, does not work likewise: in excerpt 2, D receives the glances of her mates (that is to say “you can” to D), but her “I can” does not improve.

We have also seen that *conflicts* emerge: between G and F, between B and D. Conflicts can be read as contrasting views that can impede true dialogue between students. Each student's view of mathematics shape cooperation in groupwork activities, but cooperation is driven also by “you can”, the competence recognized by the others. Horizontal arrows correspond to movements on the “you can”-“you can't” axis. In table 1, we see that F shows only movements of this sort, and moreover he is quite stable in the “I can”-“you can't” position. F is a leader, which *does not recognize* an ability to his mates. In excerpt 2, also B and D occupy the “I can”-“you can't” area, but for few seconds. We sense that such a position is (or can be)

necessary for the development of the activity, it is when a student occupies it *for the most of time* that cooperation between the students may be hindered.

One can question whether there are configurations, in the 2D-diagram, identifying situations where the group activity gets. We observe that each excerpt ends with either all the students in the “I can’t” area (excerpt 1), or all of them in the “you can” one (excerpt 2). Interestingly, these two configurations are followed by an intervention from the teacher. This issue needs further reflection, however.

This paper reports a qualitative study aimed at defining a descriptive schema to analyze group activities, and the focus of this work is mainly methodological: stronger theoretical foundations are needed in future works.

NOTES

1. SNAI is an Italian acronym: “Sindacato Nazionale Agenzie Ippiche” [National Consortium of horse-race Agencies].

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