Ant Colony Optimization algorithm and managerial insights for the Car Sequencing and Inventory Routing problem in a Car Assembly line.

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Abstract

This paper presents an Ant Colony Optimization (ACO) system to deal jointly with sequencing, routing and line-side storing problems in a mixed model car assembly line. In today's market, the customers demand an even wider range of car models. Consequently, the majority of car manufacturers have changed from a single model assembly line to mixed model assembly lines, generating enormous challenges for the operation of the assembly line. Taking a holistic look at car production (i.e. considering sequencing, routing and line-side storing) allows us to study performance optimization of the production process. We started with a review of the existing algorithms. We could not find any papersaddressing this problem jointly. We developed a Mixed Integer Programing (MIP) formulation for the problem. Unfortunately, only small instances of the problem can be solved up to optimality, and thus we developed an ACO.

The mathematical model of the operation of the assembly line was based on the approach of sequence rules, whereby the assembly line can handle a pre-determined production ratio (rate?) for each option. The approach for the inventory and sequence part was an extension of the Inventory Routing Problem, whereby the inventory and the transportation costs are minimized. Two approaches of ownership of the material handling vehicles are examined. In our tests, we prove that the benefits of joint decision are larger when the value of the space is higher than in a low-cost facility. The holding cost acts as an amplifier of the possible savings. The main thrust of this work consists in the development of the MIP and ACO system and discussion of the managerial insights for the production and replenishment. Additionally, the change of the modelling approach from single problems to a jointly approach.

Keywords: Sequencing, replenishment, MIP, ACO.

1. Introduction

The increase in car manufacturing complexity due to the globalization of business, and immense variety of the models being produced, makes it necessary to take the decision to consider more than one part of the production. Car manufacturers have evolved from selling one model of one car, as Ford did with his Ford-T, without offering many choices. Nowadays, a representative case is BMW, which theoretically offers 10^{32} configurations of their cars, out of which millions have been actually demanded (Meyr 2009).

Effective scheduling of the final assembly line could allow good control of the entire system (Miltenburg et al. 1990). The assembly line is the "drum" that sets the rhythm for this orchestra, and the suppliers and the entire related activities should follow them. Some parameters such as production capacity and production ratio are "almost" fixed but the scheduling, the inventory and ways to replenish are not fixed.

The scheduling should pursue the smoothing of requirements of components to facilitate the entire operation of the supply chain (Drexl and Kimms 2001). A related problem is the inventory necessary for this operation. A high inventory level in the assembly line is a big cost contributor. The car manufacturer's objective is to keep low stock levels, performing the replenishment of the production line and providing the required components at the right time

(Monden 1983). If the shipment arrives too early there may be no place to store it, if the shipment arrives too late the car assembly line has to be stopped.
The problem of scheduling the car assembly line is not new, although using a holistic look at car production (i.e. considering sequencing, replenishment and routing) allows us to study performance optimization of the production process. There is a growing interest in solving multi-objective problems, which has led the researcher to combine algorithm and create an extension of the classical algorithms to achieve their objective (López-Ibáñez and Stützle 2010).

The novel contribution of this paper consists in the proposal and test of a joint model to decide the sequencing of the assembly line and to obtain routes that optimize the replenishment and the line-side storage of the automotive assembly line. We developed a Mixed Integer Programing (MIP) formulation and Ant Colony Optimization (ACO) to deal with bigger instances. The idea behind those algorithms is that instead of addressing the scheduling, routing and inventory problems separately, we could obtain a better solution with a joint approach

Authors, after a preliminary analysis that involved factory tours of as many as a dozen of cars assembling plants around Europe and Japan, believe that the problem presented in this paper is still relevant in today's manufacturing environment. The material handling to provide components to the workstations is done using forklifts, tow (tugger) trains or any other transportation vehicles and the proper routing for the replenishment vehicles is still necessary.

The present work is a continuation of an earlier study of car assembly lines to explore the advantage of the joint decision in planning and scheduling. In the earlier study (Pulido et al. 2013), we developed a MIP for the routing and inventory problem. In this work, we deal jointly with the sequencing, routing and inventory problems: since only very small instances can be solved using MIP, we developed an Ant Colony Optimization algorithm to deal with larger instances.

Dincbas et al. (1998) defines the sequencing problem as the selection of the appropriate order in which cars are produced. Sequencing problems have been discussed in the literature for many years. They are NP problems with a high complexity. It is necessary to find proper sequences, since it is unreasonable to require that the assembly line moves slowly enough to allow every option to be put on every car. A set of consecutive cars is subject to sequencing rules that restrict the maximum number of occurrences of certain characteristics in a sequence. The line can handle a predetermined quota of cars for each option. The algorithm searches for a sequence of models that meets the demand without violating any rule (Boysen 2009). The sequencing rules are typically of type $H_0 : N_0$, which means that out of N_0 successive models, only H_0 may contain the option 0 (Drexl and Kimms 2001). Giard and Jeunet (2010) present a model that offers the option of hiring utility workers to allow the violation of spacing constraints which results in more colour grouping (??? Not clear).

Inventory Routing Problem (literature review should be restructured dividing it into subsections. This is clearly one). The Inventory Routing Problem (IRP) is defined as "a starting point for studying the integration of different components of the logistics value chain" (check missing "") i.e. inventory management and transportation. Traditionally, production and transportation issues have been dealt with separately. It is expected that improvements may be obtained by coordinating production and transportation. It is less obvious how to do it (Campbell et al. 1998)(this should be right after the end of the quotation). The replenishment of the production line is critical for the proper operation of the assembly line. An excess of inventory creates an increase in the cost of working capital, space cost, and the risk of material obsolescence. On the other hand, a lack of components will probably result in rework costs or even in the stoppage of the line.

The other (here we describe a “second” system. Which is the first??) system used in the car-manufacturing environment for the replenishment of components is the Set pallet system (SPS) that is used in some Toyota plants, which consist in change the line-side storage or flow racks for a moving pallet or dollies traveling with the cars being assembled. As the size of the dolly is not enough to carry all the components need to assemble one car, the dolly needs to be changed in the different parts of the assembly line, and the transport of the dollies from the warehouse to the connection points also requires routing techniques. Albeit being conceived to work in plants adopting traditional material handling systems, the model presented in this paper could be adapted for dealing with this material handling approach.

Following a classical approach, we started with the MIP and we followed with a heuristic. The election of the heuristic was Ant Colony Optimization introduced by Dorigo et al (1996) since it has offered good results for this kind of problems (Gottlieb et al. 2006, Silvia et al. 2008), and the representation of the pheromones make the interaction process more didactic (?? Why didactic? I would avoid this comment).

The remainder of the paper is organized as follows: Section 2 describes the problem. Section 3 provides a MIP mathematical formulation of the problem. Section 4 provides an ACO algorithm. The computational study is in Section 5 and in Section 6 and 7 some managerial insights, conclusions and further directions for research are given.

2. Problem Statement

we should be mentioning here that – when defining the problem (in the following) – we will clarify the 3 main aspects characterizing an assembly line: namely assembling process (and takt time), traveling time, CSP. The process to assemble a car requires the car’s chassis (Body in White) to pass through several workstations. We define that the car is assembled when it has passed through all workstations which install the different components. Some car configurations require more than a takt time (we must first define takt time!) at the workstation to carry out the tasks assigned to that very workstation. Other configurations require less time. Additionally, all the components required in the workstation have to be replenished from the warehouse. Each model has a set of characteristics, such as types of wheels and tires, radio, sunroof, car seat, etc. In every workstation, a kit of components is installed; these components can have different trim levels. The combination of components and trims gives us the characteristics. To make it clearer; in the case of radio, High trim could mean radio/MP3, and Low trim could mean radio/CD.
(I would suggest to rewrite: first assembly process split in tasks, then takt time, only then you could introduce the concept of high/low trim)

The holding cost is a figurative (fictitious?) cost or penalty for have an excess of line side storage for the following reason: the probability of damage or loss of a component increases along the time that the component itself remains in the line-side storage. The holding cost of the high trim components will be higher than that of the low trim components. The space of the line-side storage is limited, and an excess of inventory could obstruct the proper operation of the line. When there is excess of components the operator spends more time searching and selecting components.

When considering the forklift – or the towing train (trailer), we have to consider the so-called displacement time. The displacement time is the sum of the acceleration time, the travel distance, the braking time, and the time to unload components. The displacement times between stations are similar since the only time that is distance dependent is the traveling time, (I avoided the sentence because "slower" = "more relevant") then only impact the result if the distance is considerable.

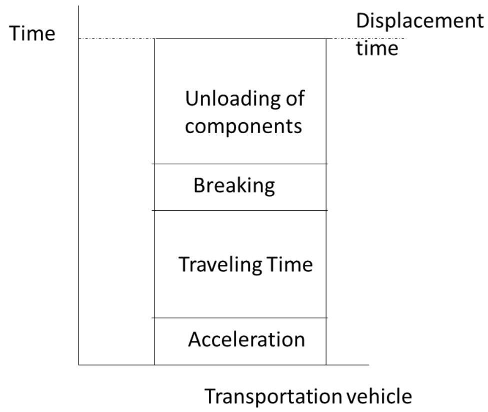

Figure 1 Displacement time of transportation vehicle

Here change in topic is abrupt. Find the right connection.
Then it is necessary to find sequences that keep the ratio in each workstation. Each time the production ratio is above this number, it is considered as a rule violation. The present problem is an extension to the Car Sequencing Problem (CSP) proposed for the ROADEF challenge in 2005, where we also consider the inventory level, and the replenishment of these components.

3. MIP Formulation

In this section, we begin by introducing the notation needed to formulate the problem. Afterwards, we present the mixed integer linear program for the joint solution of the problem.

Any chance to make this table a little bit more compact? Shall we divide the parameters in groups or categories? Eg. Cost parameters; parameters to model other constrains; decision variables; dummy variables.

	Description
R	Homogenous transportation vehicles that could perform a route;
L	locations (workstations and warehouse);
M	car model configurations;
C	car components;
A	trim levels;
J	characteristic J ⊆ all car components × trim levels;
τ	discretized production time;
D_m	total demand of model m ∈ M;
R_{mj}	1 if the model m ∈ M requires characteristic j∈ J;
AVH	amortization cost (per period) per transportation vehicle;
$TDIS_{ll'}$	displacement time from l ∈ L to l' ∈ L;

TC	traveling cost per distance unit;
MC	moving cost of component;
CAP	maximum capacity of kits in a transportation vehicle;
LEN	maximum length of the route;
HC_j	holding cost of component corresponding to characteristic j ∈ J per time unit;
ST_{jl}	safety stock with characteristic $j \in J$ in location l ∈ L;
STo_{jl}	initial stock for characteristic $j \in J$ in location l ∈ L;
$H_j : N_j$	at most H_j of N_j successively sequenced cars may have characteristic $j \in J$;
VC_j	violation rule cost per characteristic j ∈ J;
M	a large scalar value;
w_{rl}	1 if route r ∈ R attends l ∈ L; 0 otherwise;
$x_{rll'}$	1 if l ∈ L immediately precedes l' ∈ L, on route r ∈ R; 0 otherwise;
$y_{m\tau l}$	1 if model m ∈ M is processed on cycle τ ∈ T in location l ∈ L, 0 otherwise;
t_{rl}	discrete time in which the route r ∈ R arrives to the location $l \in L$;
$dem_{j\tau l}$	demand for component of characteristic $j \in J$ in cycle τ ∈ T, in location l ∈ L;
$dem_{j\tau l}^{ac}$	accumulated demand for the component with $j \in J$ at cycle τ ∈ T in l ∈ L;
$c_{jlr\tau}$	amount of component replenished with $j \in J$ required in l ∈ L, in ∈ R in τ ∈ T;
$c_{jsr\tau}^{ac}$	accumulated amount of component with $j \in J$ in l ∈ L, in r ∈ R in cycle τ ∈ T;
$q_{jl'r}$	amount of component required with characteristic $j \in J$ in l'∈ L in route $r \in R$;
$st_{j\tau l}$	Stock of component to characteristic $j \in J$ in l ∈ L at cycle τ ∈ T;
α_r	1 if the route $r \in R$ is used for the replenishment; 0 otherwise;
$\beta_{\tau rl}$	1 if $t_{rl} = ord(\tau)$, 0 otherwise;
$f_{jll'r}$	flow of component with characteristic $j \in J$ between l and l'' ∈ L in $r \in R$;

Table 1. Set, Parameters and Variables

This part could be maybe postponed. Consider this please.
Equation (1) is the objective function. (shall we invest 3 lines to explain the component of the Objetive Function?) Eq. (2) warrants that the demand is satisfied. Eq. (3) allows only one car is at one station. Eq. (4) makes sure that the car pass to the next station. Eq. (5) is the production ratio rule. Eq. (6), Eq. (7) and Eq. (8) ensure that each location is served by one route. Eq. (9) obeys route to have a predecessor except for the warehouse. Eq. (10) forces that if a route reaches a location, the route departs from that location. Eq. (11, 12) set the number of routes equal to the number of vehicles. Eq. (13) accounts a route if the vehicle visits at least one location. Eq. (14) sets the maximum length of the route. Eq. (15) limits the number of vehicles used to the available ones. Eq. (16, 17) define the time that arrival time for each location. Equation (18) sets the maximum capacity of the route. Eq. (19) sets the demand of certain characteristic only when the car required this characteristic. Eq. (20) defines the amount of components that is left at the station. Eq. (21) sets the accumulated demand. Eq. (22, 23) set that the accumulated components required. Eq. (24) defines the stock. Eq. (25) establishes the safety stock. Eq. (26, 27, 28) establish that the required amount of components will be equal

only to the replenished components when the replenishment occurs. Finally, equations (29, 30) define the time of the replenishment.

The problem:

$$min. \sum_{rll'} TC \times TDIS_{ll'} \times x_{ll'} + \mathrm{MC} \sum_{jll'r} f_{jll'r} + AVH \times \sum_r \alpha_r + \sum_j HC_j \times st_{j\tau l} + \sum_{j\tau} VC_j \times z_{j\tau} \quad (1)$$

subject to

$$\sum_\tau y_{m\tau l} = D_m \quad \forall\, m \in M, \forall\, l \in L \quad (2)$$

$$\sum_m y_{m\tau l} \le 1 \quad \forall\, \tau \in T, \forall\, l \in L \setminus \{WH\} \quad (3)$$

$$y_{m\tau l} = y_{m\,\tau-1\,l-1} \quad \forall\, m \in M, \forall\, \tau \in T, \forall\, l \in L \setminus \{WH\} \quad (4)$$

$$\sum_m \sum_{\tau'-\tau}^{\tau+N_{j-1}} R_{mj}\, y_{m\tau' l} \le H_j + M z_{j\tau} \quad \forall\, j \in J, \forall\, \tau \in T, \forall\, l \in L \setminus \{WH\} \quad (5)$$

$$\sum_{rl \mid l \ne l'} x_{rll'} = 1 \quad \forall\, l' \in L \ \{WH\} \quad (6)$$

$$\sum_{rl' \mid l' \ne l} x_{rll'} = 1 \quad \forall\, l \in L \ \setminus \{WH\} \quad (7)$$

$$\sum_r w_{rl} = 1 \quad \forall\, l \in L \setminus \{WH\} \quad (8)$$

$$w_{rl} = \sum_{l' \mid l' \ne l} x_{rll'} \quad \forall\, r \in R, \forall\, l \in L \setminus \{WH\} \quad (9)$$

$$\sum_l x_{rll'} = \sum_l x_{rl'l} \quad \forall\, l' \in L,\ \forall\, r \in R \quad (10)$$

$$\sum_{rl} x_{r\,l\,wh} = \sum_r \alpha_r \quad (11)$$

$$\sum_{rl'} x_{r\,wh\,l'} = \sum_r \alpha_r \quad (12)$$

$$\sum_{ll'} x_{rll'} \le M \times \alpha_r \quad \forall\, r \in R \quad (13)$$

$$\sum_{ll'} x_{rll'} = LEN \ \forall\, r \in R \quad (14)$$

$$\sum_r \alpha_r \le |R| \quad (15)$$

$$\boldsymbol{if\ l = wh} \quad t_{rl'} \ge TDIS_{ll'} - M(1 - x_{rll'}) - M(2 - w_{rl} - w_{rl'}) \quad \forall\, r \in R, \forall\, l, l' \in L \quad (16)$$

$$\boldsymbol{else} \quad t_{rl'} \ge t_{rl} + TDIS_{ll'} - M(1 - x_{rll'}) - M(2 - w_{rl} - w_{rl'}) \quad \forall\, r \in R, \forall\, l, l' \in L \quad (17)$$

$$\sum_{jl} f_{j\,wh\,lr} = CAP \quad \forall\, r \in R \quad (18)$$

$$dem_{j\tau l} = \sum_m R_{mj} y_{m\tau l} \quad \forall j \in J, \forall \tau \in T, \forall\, l \in L \quad (19)$$

$$f_{jll'r} - f_{jl'l''r} \ge q_{jl'r} - M(1 - x_{rll'}) - M(1 - x_{rl'l''}) - M(3 - w_{rl} - w_{rl'} - w_{rll''}) \quad \forall j \in J,\ \forall\, l, l', l'' \in L, \forall\, r \in R \quad (20)$$

$$dem^{ac}_{j\tau l} = dem^{ac}_{j\tau-1l} - dem_{j\tau l} \quad \forall\, j \in J, \forall l \in L, \forall\, \tau \in T \setminus \{1\} \quad (21)$$

$$\boldsymbol{if\ \tau = 1} \quad c^{ac}_{jl'r\tau} = c_{jl'r\tau} \quad \forall\, j \in J, \forall l' \in L, \forall\, r \in R, \forall\, \tau \in T = \{1\} \quad (22)$$

$$\boldsymbol{else} \quad c^{ac}_{jl'r\tau} = c^{ac}_{jl'r\tau-1} + c_{jl'r\tau} \quad \forall\, j \in J, \forall l' \in L, \forall\, r \in R, \forall\, \tau \in T \setminus \{1\} \quad (23)$$

$$st_{j\tau l} = STo_{jl} - dem^{ac}_{j\tau l} + \sum_r c^{ac}_{jlr\tau} \quad \forall\, j \in J, \forall\, \tau \in T \setminus \{1\}, \forall l \in L \quad (24)$$

$$st_{j\tau l} \ge ST_{jl} \ \forall\, j \in J, \forall\, \tau \in T, \forall l \in L \quad (25)$$

$$c_{jl'r\tau} \ge q_{jl'r} - M(1 - \beta_{\tau rl}) - M(1 - \textstyle\sum_l x_{rll'}) \ \forall\, j \in J, \forall l' \in L, \forall r \in R, \forall\, \tau \in T \quad (26)$$

$$c_{jl'r\tau} \le q_{jl'r} \ \forall\, j \in J, \forall l' \in L, \forall r \in R, \forall\, \tau \in T \quad (27)$$

$$c_{jl'r\tau} \le M \times \beta_{\tau rl} \ \forall\, j \in J, \forall l' \in L, \forall r \in R, \forall\, \tau \in T \quad (28)$$

$$t_{rl'} \le \tau + M(1 - \beta_{\tau rl}) + M(1 - \textstyle\sum_l x_{rll'}) \quad \forall r \in R, \forall l' \in L, \forall\, \tau \in T \quad (29)$$

$$t_{rl'} \ge \tau - M(1 - \beta_{\tau rl}) - M(1 - \textstyle\sum_l x_{rll'}) \quad \forall r \in R, \forall l' \in L, \forall\, \tau \in T \quad (30)$$

4. Ant Colony Optimization

This should be completely rewritten (more precise)

For the ACO problem, we use the following notation. The problem is defined by a 12-tuple (C, Class, O, J, A, S, h, n, r, V, T DIS) such that:

- C $O = \{c_1, \ldots, c_m\}$ is the set of cars to be produced;
- Class = the set of all different cars sharing the trim level for all components;
- $O = \{o_1, \ldots, o_m\}$ is the set of different components;
- A: trim levels; (follow order: J comes first)
- J : characteristic $J \subseteq O \times A$;

- $S = \{s_1, \dots, s_m\}$ is the set of stations to install the different components;
- H_j: N_j at most H_j of N_jNj successively cars may have characteristic j;
- r_{ij} : C × O →{0, 1}, r_{ij} =1 if in station s_i the component with the characteristic H_j is installed, $r_{ij} = 0$ otherwise;
- V : defines a maximum number of transportation vehicles; and
- T $(missing?)$
- $TDIS_{ss'}$: defines displacement times from station S_i to station S_j.

The following notation is used to denote the change of sequences:

- a sequence is noted $\pi = \{c_{i1}, c_{i2}, \dots, c_{ik}\}$;
- unique set of options required by a car is a class $classOf(c_i) = \{h_j \epsilon H | r_{ij} = 1\}$;
- a route is defined as a nonempty subset of stations attended by each vehicle, $R_{v_i} = \{s_0, s_{1,} \dots, s_{m+1}\}$ where $s_0 = s_{m+1}$ denotes the depot;
- the set of all sequences that may be built is π_c;
- the concatenation ⊕ of two sequences is the first followed by the second;
- a sequence π_1 is a subsequence of another π_2 , $\pi_1 \subseteq \pi_2$, if there exist another sequence that can be concatenated to π_1 to create π_2;
- τ cycle (takt) time; and
- the cost of the sequence π and the route R depend upon the number of violated constraints, the vehicles used and the distance traveled by each vehicle, and the amount of stock in the assembly line (see equation 32).

The problem is solved when a production plan is found that violates the !!minimum number of!! (if there is a cost for the violation, then the solution is the one minimizing costs, not violations!) sequence rules and the routes for replenishing all the components are identified, so that the capacity and constraints are met. Each route will be attended by only one vehicle. A production plan will be defined as the set of production sequences and the routes for the vehicles that permit replenishment of the components for the given production requirements. The solution is driven by four main costs: the cost to the use utility workers for overloading of the station, due to violations of the sequence rule (33), the use of the transportation vehicles (34), the distance travelled by the transportation vehicles (35), and the inventory cost of the components (36).

$$cost(\pi, R) = \sum_{o_i \in o} \sum_{\pi_k \square \pi} \text{violation}(\pi_k, O_i) \times \text{vioCost} + \sum_n (\text{travelCost(Vn)} + vehicles\ Used(V_n) \times Cost) + \sum_m \text{holdingCost}(s_m) \quad (32)$$

where

$$\text{violation}(\pi_k, O_i) = 0\ if \sum_{cl \square \pi_k} r_{lj} \leq H_j \text{ ; 0 otherwise} \quad (33)$$

$$vehicles\ Used(V_n) = 0\ of\ distanceTraveled(V_n) = 0\text{, 1 otherwise} \quad (34)$$

$$\text{travelCost(Vn)} = \sum_{l \in R} \text{TDIS}_{ll'} \times unitCOstKm \quad (35)$$

$$\text{holdingCost}(s_m) = \sum_{j\tau} \text{stock}_{j\tau} \times unitCostStock \quad (36)$$

4.1 *ACO Algorithm*. From the literature review, we found out that the majority of algorithms for the CSP has a single objective of Minimize the violations, and CSPLib and ROADEF are the reference instances. Recently the use of multiples pheromones has given good results. For the IRP or vehicle routing problem with extensions (VRP) more approaches that are multi-objective appear and multiple ant colonies (?? Is it relevant??). The test instances for VRP problems is out of our context (WHY?) (see Table 2). In general I wouldn't put this paragraph here. Better in the literature review…

Paper	Type of prob-lem	Data	ACO Type	Multiple Objective	Multiple Pheromone
Gottlieb et al., 2003	CSP	CSPLib	classic	No	No
Gravel et al. 2005	CSP	CSPLib,	classic	No	No
Gagné et al. 2006	CSP	Roadef	classic	No	No
Solnon 2008	CSP	Roadef	classical	No	YES
Morin et al. 2009	CSP	CSPLib,	ACS-3D	No	YES
Solomon 1987	VRP	Own	ACS	No	No
Gambardella et al. 1990	VRP	Solomon	ACS	No	No
Barán and Schaerer 2003	VRP	Solomon	multiple	YES	YES
Bell et al. 2004	VRP	Christofides	multiple	YES	No
Huang and Lin 2010	IRP	Solomon,	modified	YES	No

Table 2. Review of the CSP and IRP problems using ACO (is a short explanation of the "ACO TYPE" required? Here or in section about literature?)

Following the ACO scheme, where each part of the problem is modeled as the search for a best Hamiltonian (define) path in the graph, solutions are constructed using a pheromone model, then the solutions are used to modify the pheromone values. As we use utility workers for the sequence part, all the sequences are feasible by definition. But sequences not respecting the takt-time or workload balncing will have to bear extra costs). A big enough set of transportation vehicles is defined to ensure that all the routes are feasible and capable of delivering the components when needed.
The algorithm uses two types of ants: the sequencing ants and the routing ants. A routing ant will have terminated its path when all the stations are visited.

The vehicles depart from the depot with a load of components equal ling (on average) the number of vehicles v divided by the number of stations s times the number of cars produced n, always respecting the vehicle capacity (see Eq. 37).
Here there is confusion (also in the way concepts are preented) between the physical structure of the problem and the logical structure of the solving algorithm. The various parts should be re-ordered to make reading smoother.

$$\frac{\mathrm{v} \times \mathrm{n}}{\mathrm{s}} \leq capacityOfVehicles \qquad (37)$$

When describing the problem you mix some parts describing the "physical" structure of the problem and the logic behind the algorithm. It would be better to separate them and proceed in order.
Each vehicle from the replenishment route departs from the warehouse ($s_i = 0$), visits the stations and goes back to the warehouse again at the end of the route. The transportation time includes traveling from station $s_i$to station s_i and unloading the components. In order to promote the exploration of different solutions each routing ant starts the exploration from a different point, we multiply the probability matrix for the Eq. 38 as a factor of the selection of the $candS$ in the algorithm 1. This last sentence is not clear – explain by introducing probability matrix first

$$\frac{(\text{ord(ant)} + 1)}{\text{totalNnumberOfAnts}} \tag{38}$$

We give a detailed formulation of the construction algorithm in the appendix (ok but explain this at the very beginning: we are going to give a brief outline of how the algorithm works, and in the appendix you will find full details…). First, pheromone trails are initialized, and then at each cycle sequence ants construct a full sequence and a full route from an empty sequence and empty route. Cars are iteratively added until the sequence is completed. At every step, candidate cars are restricted to the ones that generate the minimum cost; this means that the election is restricted only to cars which create minimum extra cost. With this set of candidates ($cand$),the next car is chosen using transition probability Eq. (39 or 40). the sequencing ants keep doing this, until all the cars are sequenced. Then the demand over the time is calculated and the replenishment route is built. In order to build the route (the routing ant?) start from an empty route. The depot is duplicated a number of times to equal the number of transportation vehicles. We start to add stations from the non-attended locations ($candS$) between the ones that generate the minimum cost and we choose for each vehicle the one that adds the minimum cost; the probability Eq. (41) will depend on τ_3 and η values. Once all stations have been attended, we decrease the number of vehicles and repeat the creation of routes, unless that number of vehicles cannot attend all the stations on time. We should keep the best solution to lay pheromones. Finally, we repeat the entire process. The algorithm stops iterating after a maximum number of cycles have been performed.

(the following sentence is not very clear – could you explain better?) The probability of building the car sequence is inspired by the one described by Solnon (2006) for combining two pheromones in section 6. The vehicle routing is inspired by the approach of Baran (2003). The first colony minimizes the number of vehicles, while the second colony minimizes the inventory cost. Both colonies use independent pheromones and collaborate in sharing a global best solution. This solution is used to update the pheromones.

$$\text{p}(\text{c}_i, \text{candCar}, \pi) = \frac{[\tau_1(c_j,c_i)]^{\alpha 1}[\tau_2 classOf(c_i)]^{\alpha 2}}{\sum_{c_k \in cand}[\tau_1(c_j,c_i)]^{\alpha 1}[\tau_2 classOf(c_i)]^{\alpha 2}} \text{ if the last car of } \pi \text{ is } \text{c}_\text{j} \tag{39}$$

$$\text{p}(\text{c}_i, \text{candCar}, \pi) = \frac{[\tau_2 classOf(c_i)]^{\alpha 2}}{\sum_{c_k \in cand}[\tau_2 classOf(c_i)]^{\alpha 2}} \text{ if } \pi \text{ is empty} \tag{40}$$

$$\text{p}(\text{s}_i, \text{candS}, R_{v_i}) = \frac{[\tau_3(s_i,s_j)]^{\alpha 3}[\square(s_i,s_j)]^{\beta}}{\sum_{s_k \in candS}\left[[\tau_3(s_i,s_j)]^{\alpha 3}[\square(s_i,s_j)]^{\beta}\right]} \text{ if } \text{s}_\text{j} \subseteq \text{candS, 0 otherwise} \tag{41}$$

Where $\alpha 1, \alpha 2, \alpha 3, \beta$ (take care - the formatting of the indexes is gone) are relative weights for the pheromone and heuristic values respectively. A full solution is defined as the sequence and the route of vehicles to replenish the components. After each iteration, we obtain a full feasible solution to the problem which is improved after each iteration. A decrease in the use of the vehicle is given after several iterations where vehicles select the 'nil' route.Rewrite this last sentence, it is not clear

4.2. *Pheromones.* The three proposed (ok so open stating clearly that we decided to use 3 kinds of pheromone. Also the concept of pheromone itself shall be defined BEFORE. otherwise the following paragraph will not be clear) pheromone structures achieve complementary goals; the first aims to identify a good sequence; the second aims to identify critical cars; the third aims to identify vehicle routes that could deliver the components on time.

- *pheromone* τ_1. Ants lay an amount of pheromone $\tau_1(c_i,c_j)$ on a couple of cars $(c_i,c_j) \in C\times C$. τ_1 represents the past experience of sequence car c_j after c_i. This pheromone is bounded with [Tmin, Tmax] and it is initialized at Tmax for every couple.
- *pheromone* τ_2. Ants lay pheromones on car classes cc ∈ Classes(C) and the amount of pheromone τ_2(cc) represents the past experience with the car sequence of this class without violating constraints. This pheromone is bounded with [Tmin, Tmax] and it is initialized at Tmin.
- *pheromone* τ_3. Ants lay pheromones on the path between the current location and the possible location $(s_i,s_j) \in S$ and the amount of pheromone levels of $\tau_3(s_i,s_j)$, indicating how proficient it has been in visiting station j after i. This pheromone is bounded with [Tmin, Tmax] and it is initialized at Tmin.
- *heuristic* $\eta(s_i\ s_j)$. The dynamic attractiveness (once again – before you should have explained the concept of attractiveness depending on the pheromone level) of the arc (i,j) will be: $\eta(s_i\ s_j) = 1/stock_j$, it will be computed dynamically depending on the inverse of the stock level in each station at each time.

4.3. *Pheromones update.* Each pheromone will be laid and updated according to its characteristics.

Updating Pheromone τ_1 Once every ant has constructed a sequence, the quantity of pheromone in all pheromone trails is decreased (take care of the many typos – read over if you have time) in order to simulate evaporation multiplying every arch by $(1-\rho1)$. Then the best ant deposits along its path a trail of pheromone inversely proportional to the total cost generated by the violated constraints. If the resulting pheromone value is lower or higher than the range, it will be adjusted to the closest boundary.

Updating Pheromone τ_2 Ants lay pheromones on car classes during the construction; when no more cars can be scheduled without new (why new? Better saying "without breaking constraints"???) constraints, some pheromone is laid in the classes of the cars that have not been scheduled. The pheromone update occurs during the construction step. Every ant adds pheromone, not just the best ant. In order to simulate evaporation each class is multiplied by $(1 - \rho2)$.

Updating Pheromone τ_3 First, local updating is conducted by reducing the amount of pheromone on all visited arcs by multiplying current pheromone levels by $(1 - \rho3)$. Global trail updating is performed for all the arcs included in the best route found by one of the ants.

Title of the subsection here is "ACO parameters tuning"
The ACO was tuned, using as starting values those suggested by Dorigo (1996) and Solnon (2008). Then we keep the values fixed and run the algorithm for the different instances (this is not clear – explain how many times you moved the parameters and by how much, and how many instances you used to test the different settings), and we select the best values. (explain that in the following table we will present the values of the parameters which performed better/best)

	β	α_n	ρ_n	τ_{min}	τ_{max}	Note
Pheromone 1		3	1%	0.01	4	experience of car_j after car_i
Pheromone 2		6	2%	1	10	Critical Models

Pheromone 3		2	0.5	0.1	5	experience of loc_2 after loc_1
Heuristic η	5					Heuristic info

Table 3. ACO Parameter settings

5. Computational study

The MIP was modelled in AIMMS 3.13 and the standard solver Gurobi 5.5 was used to obtain the solution to the problem. The ACO was programmed in C++ using Code::Blocks. The computational experience was performed in a machine with a processor Intel Core i3-2350 M 2.30 GHz 6 Gb RAM running under Windows 7.

As there are no public instance in the literature (for which problem exactly? Instance of the "right" scale probably – I mean, some are existing but not large enough, correct?), we based all the experimentation in the instances used on Regin & Puget (1997) instances #1, #2, #3 and #4, which has been widely, used in other articles These instances are public at car sequencing problem lib (www.csplib.org). From this sequence we make up the missing data in order to obtain some smaller and some larger instances. We create a reduced instance (R#No.r), which contains the first 50 cars of the instances. We create an extended instance duplicating the number of stations (R#No.e) keeping the same production ratio. For the instances of 200 or more cars found in the literature, the MIP is not able to build the model. As no comparison point for this problem exists, no experimentation was performed with bigger instances. A terminating criterion of 3600 seconds was set for all the instances.

There are two typical ownership options for the transportation vehicles that will be experimented. The first one is when the car manufacturer is the owner of the fleet and each transportation vehicle generate an amortization, traveling and moving cost. The second one is used a material handling company, in this case only traveling, and moving cost exist.

Instance	No mod	No car	No loc	Car seq (a)	VRP (b)	HC (c)	Total (d)= a+b+c	Obj Joint (e)	Δ % d-e	aco μ (f)	aco σ	Δf % d-f	Δf % e-f
R#1.r	9	50	5	500	458	910	1868	1740	6.8	1803	37.8	3.5	-3.6
R#2.r	7	50	5	0	458	912	1370	1235	9.8	1307	43	4.6	-5.8
R#3.r	8	50	5	300	458	1001	1759	1564	11.1	1565	58.1	11	-0.1
R#4.r	10	50	5	600	458	893	1951	1841	5.6	1899	29.4	2.6	-3.2
R#1	22	100	5	0	828	3192	4020	3632	9.6	3625	125	9.8	0.2
R#2	22	100	5	600	828	2887	4315	4050	6.14	3954	42.3	8.4	2.4
R#3	25	100	5	400	828	3324	4552	4072	10.5	4099	154	9.9	-0.7
R#4.	23	100	5	200	828	3213	4241	3963	6.5	3876	76.2	8.6	2.2
R#1.e	22	100	10	0	1650	6214	7864	7759	1.34	7464	194	5.0	3.8
R#2.e	22	100	10	1200	1650	6332	9182	9038	1.57	8785	179	4.3	2.8
R#3.e	25	100	10	800	1783	6427	9010	8810	2.22	8442	139	6.3	4.18
R#4.e	23	100	10	400	1650	6174	8224	8113	1.35	7808	168	5.1	3.8

Table 4. Computational Results of car manufacturing ownership of transportation vehicles.

In the Table 4 the computational results are presented. The first four columns present the instances and their characteristics. The following columns present the solution for the car sequencing (a), vehicle routing problem (b) and the holding cost (c). Column (d) presents the sum of these costs. Column (e) presents the total cost of the Joint Approach solved with the

MIP. The next column (f) is the difference between the MIP joint approach and the separate approach. The column (f) shows the mean of the 50 ACO runs, the next column is the standard deviation of 50 ACO runs. The last column presents the difference between the ACO and the separate approach. The solution of the car sequencing R#1, R#2, R#4 are the same of the best known in literature, while in the case of R#3 our algorithm could not achieve the best known solution (of 3 failures???) in the given time. Despite the fact that the ACO cannot achieve the optimal solution, with exception of small instances, within all the instances obtains better results than the MIP using the one-hour stop criterion; in all the cases, in less than 2 minutes, especially for the extended instance where the GAP is bigger.

Instance	No mod	No car	No loc	Car seq (a)	VRP (b)	HC (c)	Total (d)= a+b+c	Obj Joint (e)	Δ % d-e	aco μ (f)	aco σ	Δf % d-f	Δf % e-f
R#1.r	9	50	5	500	510	866	1876	1781	5.06	1812	43.8	3.4	-1.7
R#2.r	7	50	5	0	514	926	1440	1295	10.07	1342	62.7	6.8	-3.6
R#3.r	8	50	5	300	522	961	1783	1613	9.53	1708	75.1	4.2	-5.9
R#4.r	10	50	5	600	505	995	2100	1855	11.67	1955	61.5	6.9	-5.4
R#1	22	100	5	0	1189	2918	4107	3887	5.36	3932	89.6	4.3	-1.2
R#2	22	100	5	600	1192	3002	4794	4399	8.24	4410	102.5	8.0	-0.3
R#3	25	100	5	400	1194	3153	4747	4460	6.05	4443	80.7	6.4	0.4
R#4.	23	100	5	200	1194	3134	4528	4339	4.17	4278	55.3	5.5	1.4
R#1.e	22	100	10	0	5120	6373	11493	11491	0.02	11074	126.1	3.6	3.6
R#2.e	22	100	10	1200	4530	5836	11566	11485	0.70	11135	138.2	3.7	3.0
R#3.e	25	100	10	800	4522	6642	11964	11841	-0.65	11381	107.5	3.3	3.9
R#4.e	23	100	10	400	3992	6170	10562	10489	0.69	10148	149.2	3.9	3.3

Table 5. Computational Results of using a material handling company.

In the Table 5, the computational results of using a material handling company are presented. The joint approach obtains better results, or in the bigger instances similar results. When the bigger instances are solved, the size of the model do not allow the solver to explore all the branches of the tree. In one instance (R#3.e) the Joint approach was 0.6% below the traditional approach, but in the rest of the case achieve at least the same results but in the majority better results. In this approach the routing cost in the big instances become more relevant than in the first approach because of the intense use of material handling and no discount for intensive use is modeled.

The aim of this paper is not to discuss whether or not to outsource material hanling, but to discuss the benefits of joint approach and ACO in the two cases. The outsourcing decision will depend on the strategy of the company. The purpose of the experimentation of the two approaches is to highlight that make sense a joint approach in both cases. Therefore, the resultant cost of Table 4 and Table 5 are not comparable since the transportation cost is highly dependent on the negotiation of the contractual terms with suppliers and workers.

Allowing (this sentence is not clear) the decision model to decide from among several options increases the possibility of achieving a better solution. Unfortunately, as we expect for the MIP the computational cost in some cases is excessive for the savings. (not clear at all – please explain better – with examples)

6. Managerial insights

The “engine” of car supply chain is the assembly line, it keeps the rhythm of the orchestra. Using the traditional car assembly line sequencing approach, the production-planning department obtains the better car sequence, then the better routing and the better inventory could be calculated (see Figure 2). The joint approach searches among all combinations to obtain better solutions. This is possible since, we could have many similar cost car sequences, replenishment routes and inventory levels. If we play with the combination, we could obtain better solution.

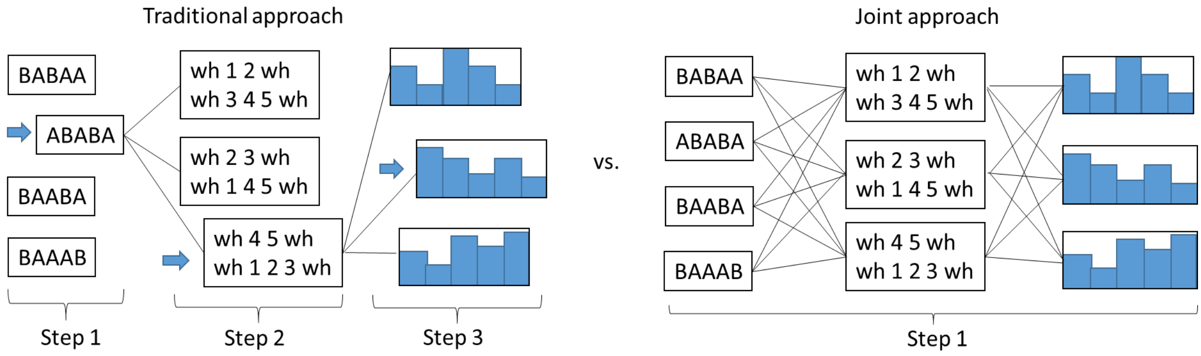

Figure 2. Comparison of Traditional (model single problems) vs Joint approach

This (the picture above) was to be explained BEFORE presenting results in table 10. otherwise the reader will NOT understand how you obtained the values in columns a, b, c.!!!

According to our expectations, the integration of the decision has resulted in the achievement of better results: the fact that one part of the organization achieves the best possible performance might not be beneficial for the entire organization. The decision making process should be done in conjunction with the other stakeholders of the process. Unfortunately, this is not always possible since real-life problems are far more complicated than this model. However, as can be seen in Table 4, the goal of the decision maker of the scheduling, routing and inventory should be to reduce the total cost, not only the cost associated with the process he/she is responsible of, and this cannot be achieved without a compound approach.

Comparing the Afshin et al. (2012) review of the paper that deals with more than one part of the supply chain against the number of papers for each echelon of the supply chain there is a lack of compound approaches from the researchers. On the other hand, the industry is using decision systems such as Oracle E-Business SCM, SAP SCM, i2, IBM, or LogicTools which, despite their inability to give the global optimal (why? Do they use simplified heuristics?), pursue an integrated optimization.

This last few lines are not clear and not connected with the previous paragraphs. Explain betterThe stock level decreases until it reaches the level of the safety stock before the replenishment; this opens the opportunity to manage the risk to work without safety stock in order to realize the possible saving, and also the possibility to incur in cost if there is any delay in the transportation.

7. Conclusions

The advantage (this part in my opinion should still be part of section 6) of joint decision making becomes more important when the cost of the space is higher than in a low-cost facility.

The production space is a limited resource; the space has to be used in an activity that adds value to the product and decreases the holding space. This becomes a key factor in factories where the inventory is limited and there is no possibility to store more than one or two hours' inventory.

Ok from now on these are actually "Conclusions".
The first contribution of this paper is the development of a mixed integer programming model for solving an inventory routing problem to satisfy the sequence requirements. This kind of model is not reported in the literature and the authors believe that this could be an interesting research area.

This paper uses the natural cooperative behavior of the ants to obtain a solution to combined problems. The second contribution of this work consists in the development of a collaborative ant colony optimization system to obtain a high-quality solution for problems that cannot be solved to optimality, and the joint solution to the problem using ACO, to the best of the authors' knowledge, has not been described throughout the literature.

The results yield savings of around 7% on all costs in the instance tested with respect to the solution obtained…. It is expected that in larger instances the same performances (or better) in terms of savings will be maintained, since the decisions are taken independently (of any feedback) (this "on any feedback" is not clear to me. Shall we remove it?). Factories with a reduced production space could be more interested in this kind of approach. This would justify the investment in more computational power or the design of other solution methods.

We believe that the results tested in the small and medium-sized cases are promising. The decrease in the number and use of transportation vehicles, reduction in inventory next to the assembly line, and minimization of the number of utility workers to handle violation of the sequencing rules could be interesting for future research. Therefore, making an industry-sized model could be justified.

For a future research as this is an NP hard problem and since the sub problem of routing is NP hard, the overall problem is NP hard. From this point, many research directions could be followed. The first one is to try to add cutting planes or decomposition methods to handle real-life problems. The other option is to implement other metaheuristics that could provide a good solution in a short period of time with an average computational power. In the ACO line, the focus could be placed on larger problems, combining different techniques like additional types of pheromones, ranking methods, different construction strategies for the route, such as the local exchange, or candidate list. For the modeling part, we could also add some "ad-hoc" features to customize the model to represent better the client reality, with discounts for excess of use, or non linear holding cost.

The final section should contain: benefits; limits; further developments. Make sure you clearly discussed the limits of our approach also.
Limits could be:

- instances too small
- not correct balance between costs (?) – could depend on the specific real case
- inability to suggest structural changes in management policies (eg outsourcing MH)

8. Acknowledgements

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10. Appendix

Algorithm 1

```
Initialize pheromones
Repeat
  π←empty {Start to sequence the cars}
  while |π|≤|C| do let C−π denote the set of cars of C that are not sequenced
  cand←the minimum cost generated by {c_k∈C−π}|∀c_j∈C−π, cost(π<c_k>)≤cost(π<c_j>)
  if ∀c_i∈cand, cost(π<c_k>)≤cost(π<c_j>) then
    for every car class cc ∈{classOf(c_i)}∈ C − π do
      T_2(cc) ←T_2(cc)+ cost(π<c_j >) − cost(π)
    end for
  end if
  Choose c_i ∈ cand with the probability p(c_i, candCar, π)
  π ←<c_i >
  end while
  keep the best sequence
  calculates the instant demand for the best sequence
  repeat
    R← empty {Routes for transportation vehicles}
    for v_n = maxVehicles to v_n =1 do
      while |R|≤|S| do
        let (R–S) denote the set of non-attended stations
        duplicate the depot a number=V_n
        the ants select a candS←the min cost generated by {sk ∈ R − S} with p(s_i, candS, R)
      end while
      if stock at station ≤ safety stock then
        break
      end if
      decrease one replenishment vehicle v_n = v_n − 1
      keep the best route for the transportation vehicles
    end for
  until stop criteria
```

calculate the cost
keep the best solution and update the pheromones
until stop criteria