# AN ANALYTICAL PROBABILISTIC APPROACH FOR SIZING RAINWATER TANKS

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ABSTRACT

In the past decades Rooftop RainWater Harvesting Systems (RRWHS) have become a significant source of water supply in Africa, Asia and South America. According to predictions regarding global warming, water scarcity would intensify in many regions, with a significant increase in water demand, especially in great cities. Moreover, the continuous over-exploitation and pollution of many existing water sources is leading to a growing interest in alternatives such as rainwater tank systems as supplementary water sources with multi-purpose functions.

Rainwater tanks not only save water which would otherwise be supplied from municipal water distribution system, but also reduce rainwater runoff which would otherwise be handled through urban drainage system. Sometimes, detention storage facilities are used also as rainwater tanks. However, a different design approach should be considered. While detention storages must be emptied as quickly as possible, so that the entire capacity can be available at the begin of each rainfall event, rainwater tanks must guarantee as much as possible a carryover from prior event to satisfy the desired water reuse. This paper proposes an analytical probabilistic approach for sizing rainwater tanks according to this goal. Resulting formula is expressed in terms of reliability and is only function of the water volume required for reuse and of the moments of random hydrological variables (rainfall height, rainfall duration and interevent time). An application to a case study, based on rainfall intensities recorded in Milano is presented. Results are influenced not only by the assumed level of reliability, but also by the type of water reuse, i.e. only irrigation or a combination with toilet flushing and clothes washing.

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## 1. INTRODUCTION

Traditionally rainwater harvesting has been most commonly associated with remote rural areas, with lacking alternative water sources (Lee et al., 2000). It has long been known that domestic water consumption is highly influenced by the 'ease' with which it can be obtained (White et al., 1972). Thus a household to which water is hand-carried from a distant spring will typically consume under 10 liters per capita per day whereas a household in the same country will consume over 100 liters per capita per day if it has a reliable piped supply to taps within the building. So it is not surprising that the strongest current interest in domestic rainwater harvesting can be found in poorer (developing) countries (Terry, 1998). However, in the longer term we can expect clean-water scarcity to intensify globally, so that water autonomy within buildings, coupled with a reduction in the water intensity of human activities, will become an attractive option even for some living in richer countries (Kilbert, 2005, Postel, 1992). Moreover the continuing over-exploitation and pollution of many existing water sources is leading to a growing interest in alternative systems as supplementary water sources. The most important issue in this regard is the growing number of potential catchment surfaces available globally due to the use of modern impervious roofing materials and paved surfaces (Liav et al. 2004; Worm et al., 2006). The roof collection system is the simplest method for rainwater harvesting and is relatively convenient to set up. A typical rooftop rain water harvesting system (RRWHS) comprises three basic subsystems: a catchment system (roof), a delivery system (filter sand gutters) and a storage system. The first must be large enough to intercept in a year not less than the building occupants' annual water need; the second element, guttering, is usually the cheapest of the three but often rather neglected; the third poses the greatest cost burden. The storage capacity must be large enough to buffer both the short-term fluctuations in water usage and the longer-term fluctuations in rainfall.

RRWHS is attractive to householders from a several of points of view. First, for an existing dwelling the catchment area is available at no additional cost; second, contamination of rainwater runoff from a well constructed and properly maintained roof is small compared with that from aground catchment system; third, roof catchments provide a water supply at the point of consumption .The reasons for collecting and using rainwater in an urban context are more plentiful and various (Gould 1999):

- flood control: by greatly reducing urban runoff;
- stormwater drainage: by reducing the size and scale of infrastructure requirements;
- fire fighting and disaster relief: by providing independent household reservoirs;
- domestic water supply: in periods of water shortage;
- water conservation: as less water is required from municipal water distribution system;
- reduced groundwater exploitation and subsidence: as less groundwater is required;
- financial savings: where rainwater can be used in place of water purchased from water vendors often charging up to 10-50 times the official water tariff.

Atmospheric pollution remains a major constraint as it contaminates both the rainwater and catchment surfaces making rainwater unsuitable for drinking in many cities around the world. Nevertheless, rainwater can still be used for non-potable uses such as toilet flushing, clothes washing and gardening (Konig 1994). Increased climatic variability and the greater frequencies of droughts and floods possible in many areas will also make the role of rainwater harvesting systems even more important as sources of supplementary, back-up, or emergency water supply: much actual or potential water shortages can be relieved if rainwater harvesting is practiced more widely.

Rainwater tanks can be used on an individual household basis, or alternatively larger storages can be constructed to collect roof stormwater from several houses. For single buildings oversize or undersize of these facilities may not result in significant economical or environmental losses; however they can become significant as the number of green building increases.

While stormwater detention facilities are designed to capture and detain the more polluted first flush, rainwater tanks, depending on the purpose for which they are used, can bypass first flush and store only cleaner rainfall water. Moreover while studies on stormwater detention facilities focus on overflows risk and receiving water impacts, in sizing cisterns for water reuse the interest is on the rate of use that can be supplied and its reliability. Depending on the aim of water reuse different water shortage can be accepted.

In literature, several methodologies have been presented for sizing of storage tanks (McMahon and Mein, 1978; Heggen, 1993; Gould, 1993; Lo and Fok, 1981; Schiller and Latham, 1987; Waller, 1989; Chu and Fok, 1991, Dixon et al., 1999, Guo et al. 2007). However, results are not completely satisfactory from an engineering point of view because they don't take into account that rainwater tanks reliability depends on its ability to cope with drought.

This paper proposes a method, based on an analytical probabilistic approach, for sizing rainwater tanks. Water reuse for irrigation and domestic use (toilet flushing and clothes washing) has been here investigated: roof water is collected in the rainwater tank during the rainfall event and released during interevent time. A carryover from prior storm (depending on days of autonomy that the rainwater tank is able to manage) is taken into account at the beginning of each rainfall event. An explicit relationship for rainwater tank volume has been derived; it is a function of the use daily rate, of the probability of failure for water supply and of the nature of the storm stochastic process, particularly on the probability distribution of the three fundamental hydrological parameters (rainfall height, rainfall duration and interevent time).

An application to a case study, based on rainfall intensities recorded in Milano (Italy), has been presented; results focus on different volumes obtained considering water reuse for irrigation, toilet flushing and clothes washing in combination and for irrigation only and on different return time periods that can be achieved to ensure the required water supply varying outflows.

## 2. RAINWATER TANK VOLUME

Let consider a tank that collects rainwater from roofs and supplies water for toilet flushing and clothes washing and for irrigation during dry periods. To guarantee some days of autonomy in case of water shortage, a carryover volume from prior event must be stored in the rainwater tank at the beginning of each rainfall event. Based on the quality of rainwater collected from the roof and on the water reuse, first flush can be bypassed and discharged in the urban drainage system. Considering also an Initial Abstraction (IA) that includes all losses before runoff begins, a generic load/use cycle can be schematized as shown in Figure 1.

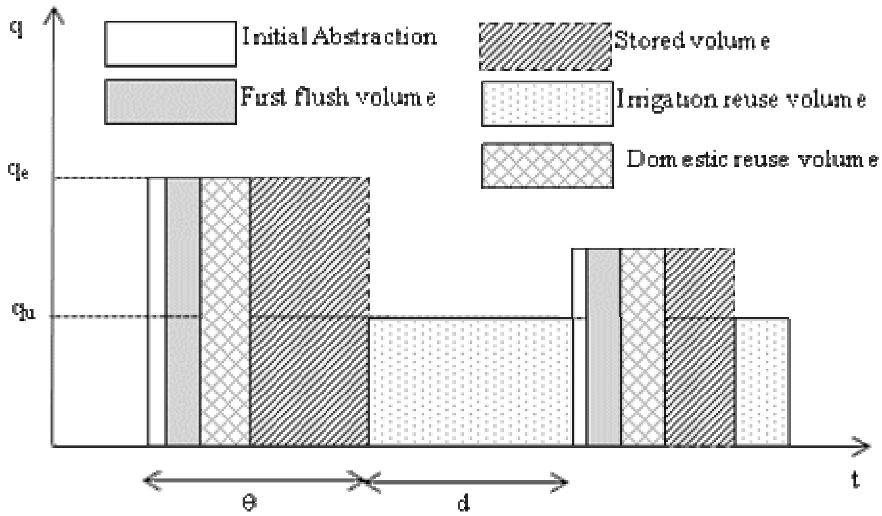


*Figure 1 - Schematization of a load/use cycle*

To analyze the dynamics of stored volume in a rainwater tank, a simplified scheme has been adopted, based on the following hypothesis:

- rainfall-runoff transformation has been neglected, considering rainfall intensities instead of water discharges (as a roof is a small surface, discharges may be considered approximately proportional to rainfall intensities);
- rainfall intensities have been considered constant in time (if inflow are always greater than the constant outflow for all the storm duration, the final stored volumes are independent from hydrograph pattern and rectangular events may be used);
- carryover has been assumed to be due to the previous storm only, so that a couple of storms at a time has been considered;

- outflow from rainfall tank has been considered constant and equal to the water demand for irrigation (it has been assumed constant during the year neglecting the monthly fluctuations in water demand);
- a domestic volume for toilet flushing and clothes washing is subtracted from each rainfall event (total annual domestic volume/number of rainfall events per year);

To identify independent events from the storm stochastic process a minimum dry period, the so called InterEvent Time Definition, IETD (USEPA, 1986) has to be defined. Several earlier studies have shown that exponential probability density functions often fit the hydrological rainfall characteristics histograms satisfactory (Chow 1964; Eagleson 1972, Howard 1976, Adams et al. 1984, Wanielista and Yousef 1993). Rainfall height h, rainfall duration θ and interevent time d distributions are:

$$f_h = \xi e^{-\xi h}$$
$$f_\theta = \lambda e^{-\lambda \theta}$$
$$f_d = \psi e^{-\psi(d - IETD)}$$

where $\xi = 1/\mu_h$, $\lambda = 1/\mu_\theta$ and $\psi = 1/(\mu_d - IETD)$, $\mu_x$ = expected value of random variable x.

If water tank is full before the end of the storm event, spills are discharged in the urban drainage system or infiltrated in the soil. The probability of a pre - filling $w_{pr}$ from the previous event is expressed as the sum of two probabilities: the probability of having a pre - filling when the stored volume at the end of the first event is smaller than rainwater tank storage capacity $W_0$ and the probability of having a pre - filling when rainwater tank is full at the end on the first event and an overflow occurred.

$$w_{pr} = \begin{cases} h - IA - w_f - w_d - q \cdot d & h - IA - w_f < w_0 \\ w_0 - w_d - q \cdot d & h - IA - w_f \geq w_0 \\ 0 & otherwise \end{cases} \tag{1}$$

where :

$w_f$ = first flush volume;
$w_0$ = storage volume;
$q$ = irrigation rate;
$q \cdot d$ = volume for irrigation;

$w_d$ = volume for domestic use (toilet flushing and clothes washing);

Volumes are expressed in the same units of the rainfall depth, that is are volumes divided by the effective drainage area $\varphi S$. Combining the above relationship with the associated conditional probabilities, the probability of having at the beginning of an event a water volume $w_{pr}$ from the previous event, stored in the tank and available for reuse greater than an assumed value $w$, results:

$$P\left(w_{pr} > w\right) = \int_{d=IETD}^{(w_0 - w - w_d)/q} f_d\, dd \int_{h=IA+w_f+w_d+w+qd}^{w_0+IA+w_f} f_h\, dh + \int_{h=w_0+w_f+IA}^{\infty} f_h\, dh \cdot \int_{d=IETD}^{(w_0 - w - w_d)/q} f_d\, dd \tag{2}$$

$$= e^{-\xi\left(IA+w_f\right)}\left[(1-\beta)\left(e^{-\xi\left(w_d + qIETD + \overline{w}\right)} - e^{\frac{\psi}{q}\left(w_d + \overline{w}\right) + \psi IETD - \frac{\xi w_0}{\beta}}\right)\right] = 1 - \frac{1}{T}$$

The general condition $\left(w_0 - w - w_d\right)/q > IETD$ for the possibility of pre-filling has also to be taken into account. Solving equation (2) by the storage unit volume $w_0$ results:

$$w_0 = \frac{\beta}{\xi} \cdot \left\{ \psi IETD + \frac{\psi}{q}\left(w + w_d\right) - \ln\left[e^{-\xi\left(qIETD + w + w_d\right)} - \frac{e^{\xi\left(IA + w_f\right)}}{(1-\beta)}\left(1 - \frac{1}{T}\right)\right]\right\} \tag{3}$$

where $\beta = \dfrac{q\xi}{q\xi + \psi}$ , $T$ = return period = $1/(1 - P(w_{pr} > w))$

Depending on the type of water reuse (toilet flushing and clothes washing and irrigation in combination or irrigation only), different levels of water shortage can be considered acceptable, so different levels of associated risk can be assumed.

The above formula represents a rainwater tank volume per effective drainage area, so the resulting value must be multiplied by the roof area $S$ and the runoff coefficient $\varphi$ (for impermeable surfaces such as roofs may be taken equal to 1). It is noteworthy that rainwater tank volume it is not function of rainfall duration, since it is assumed that the tank supplies water for irrigation during dry periods and water for toilet flushing and clothes washing is independent of rainfall duration and interevent time.

## 3. APPLICATION

Relationship (3) to estimate rainwater tank volume has been applied to a case study, using the rainfall series recorded in Milano, Italy, from 1971 to 1991 (series with a time resolution of 1 minute and a depth resolution of 0.2 mm). An IETD= 6 hours has been selected to identify independent rainfall events (Adams et al., 1986). The total number of storms in the series for IETD=6 hours is equal to 979. Mean and standard deviation of rainfall characteristics are shown in table 1, while the correlation coefficients among them are shown in table 2.

*Table 1 - Mean and variance of rainfall characteristics in the Milano series.*

| | |
|---|---|
| $\mu_h$ [m] | 0.013 |
| $\mu_\theta$ [hour] | 8.712 |
| $\mu_d$ [hour] | 104.968 |

*Table 2 - Correlation coefficient among rainfall characteristics in the Milano series.*

| | |
|---|---|
| $\rho_{h,\theta}$ | 0,74 |
| $\rho_{h,d}$ | 0,04 |
| $\rho_{d,\theta}$ | 0,03 |

As can be observed, while the other characteristics are only weakly correlated, rainfall depths and durations are not independent random variables as assumed. More, also the hypothesis of exponential CDFs for all three rainfall characteristics is not well suited to the samples of recorded values. Both results are not unexpected, since they were observed for rainfalls in the Alpine catchments in previous studies (Bacchi et al., 1994, 2004, 2008), but these hypotheses have been held anyway to avoid a considerable increase in the complexity of relationships.

Rainwater tank volume is estimated considering water reuse for irrigation, toilet flushing and clothes washing in combination and for irrigation only. A roof surface of S=1000 $m^2$ has been considered; since it has been assumed to be an impermeable surface runoff coefficient $\varphi$ has been set to 1. Initial Abstraction (IA) for evaporation and water retained in surface depression has been assumed equal to 1,0 mm. For irrigation purpose there are no particular restrictions on the use of first flush, so it has been assumed equal to zero; on the contrary for domestic use (toilet flushing and clothes washing) it has been assumed equal to 3,0 mm. Rainwater volume has been estimated varying the daily irrigation rate $Q=q\varphi S$ from 100 to 500 [l/day] and considering different return time periods. Carryover volume has been assumed equal to one, two and three times the daily domestic volume plus the daily irrigation volume, for domestic use and irrigation purpose in combination, and equal to one, two and three times the daily average irrigation volume for irrigation use only. These conditions correspond to the hypothesis that rainwater tank is able to manage one, two or three days of water scarcity.

$$W = x(Q_d + Q) \qquad [1]$$

with:
x : number of days;
Q : daily irrigation rate;
$Q_d$ : daily domestic rate;

Daily volumes per capita for toilet flushing (WC) and clothes washing (Wm) are shown in Table 3 (Dixon et al., 1999) as function of occupants (O) and number of houses in category (N):

*Table 3 - Daily per capita volume use according to appliance (Dixon et al., 1999).*

| O | N | $Q_{WC}$ [l/day*capita] | $Q_{Wm}$ [l/day*capita] | $Q_d$ [l/day*capita] |
|---|---|---|---|---|
| 1 | 2 | 30.9 | 7.8 | 38.7 |
| 2 | 2 | 39.7 | 8.9 | 48.6 |
| 3 | 2 | 30.1 | 8.6 | 38.7 |
| 4 | 4 | 25.8 | 14.5 | 40.3 |
| 5 | 1 | 23.5 | 10.8 | 34.3 |

Daily domestic volume $W_d=w_d \varphi S$ has been calculated as:

$$W_d = \sum_{i=1}^{5} Q_{d_i} N_i O_i = 1.32 \cdot 10^3 [l/day]$$

It has been reported (Edwards and Martin, 1995) that increasing occupancy is linked with decreasing per capita consumption. Domestic volume that can be supplied by each rainfall event has been estimated dividing the total annual domestic volume by the mean number of rainfall events. Results in the case of water reuse for a combination of irrigation, toilet flushing and clothes washing are shown in Figures 2 to 4 while results for irrigation only are shown in Figures 5 to 7.

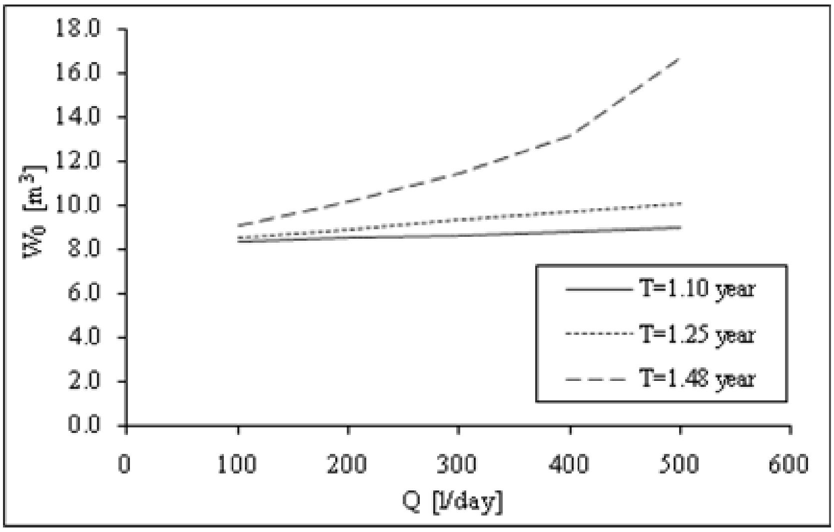


*Figure 2* - *Rainwater tank volume $W_0$ as a function of irrigation rate Q and of return period for a combination of domestic and irrigation water reuse, assuming x=1 day and $Q_d$=1.32*$10^3$ l/day.*

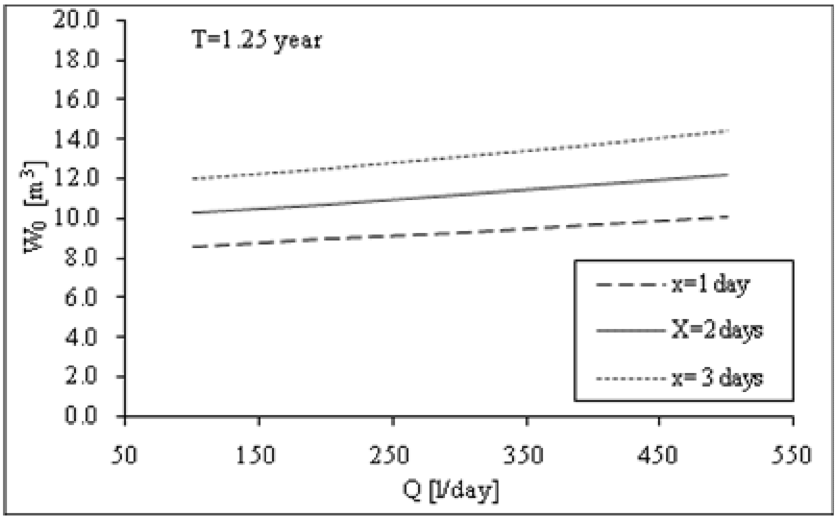


*Figure 3* - *Rainwater tank volume as a function of irrigation rate and of carryover volume for domestic and irrigation water reuse, assuming T=1.25 year and $Q_d$=1.32*$10^3$ l/day.*

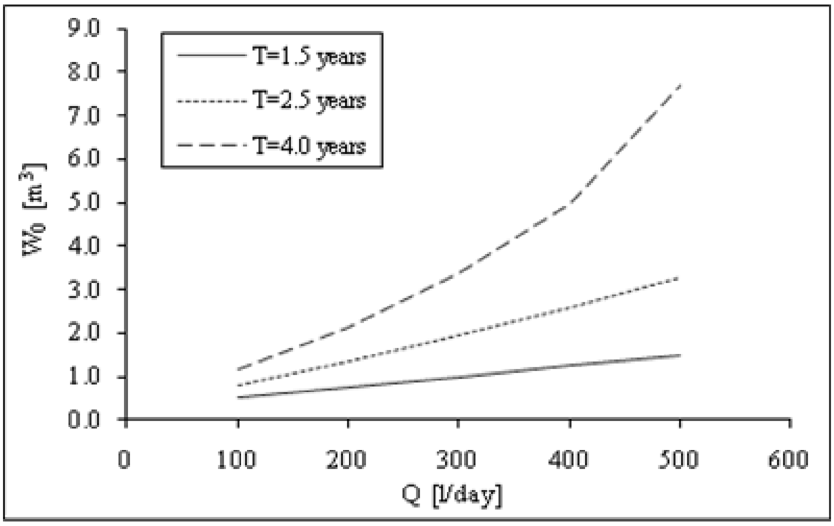


*Figure 4 - Rainwater tank volume as a function of irrigation rate and of return time period for irrigation water reuse only, assuming x= 1 day and W=x*Q*

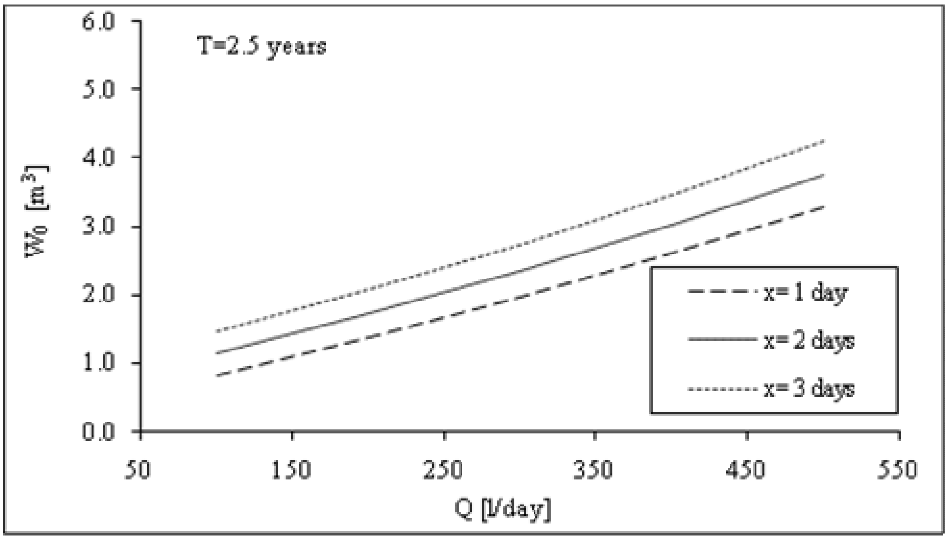


*Figure 5 - Rainwater tank volume as a function of irrigation rate and of carryover volume for irrigation water reuse only, assuming T=2.5 years*

## 4. CONCLUSIONS

A simple relationship for the estimation of rainwater tank volume has been obtained using an analytical probabilistic approach. Design volume results function of the daily rate of reuse, of the probability of failure for water supply and of the distribution of the hydrological parameters (rainfall height, rainfall duration and inter event time). In addition the Initial Abstraction (IA) and the InterEvent Time Definition (IETD) are taken into account in the formula as the possibility of bypassing first flush and retaining a carryover from prior event.

Looking at results obtained in the case study application, it can be observed that rainwater tank volume increases with outflow and return period. It has to be noted that this return period is associated to the risk of having a stored water volume smaller the needed one to meet the demand for reuse. In the case of a combination of domestic and irrigation water reuse this volume is about ten times higher than in the case of irrigation only water reuse (Figures 2 to 4).

Moreover, because in equation (3) the condition $e^{-\xi(qIETD+w+w_d)} - \frac{e^{\xi(IA+w_f)}}{1-\beta}\left(1-\frac{1}{T}\right) > 0$ is to be satisfied, the following limit on return period has also to be taken into account.

$$T < \frac{e^{\xi(IA+qIETD+w+w_d+w_f)}}{e^{\xi(IA+qIETD+w+w_d+w_f)}+\beta-1} = T_{max}$$

Considering water reuse for irrigation only, $T_{max}$=4.26 years, while considering a combination of domestic and irrigation water reuse and the same range of outflows $T_{max}$=1.48 years

For a combination of domestic and irrigation reuse, rainwater tank volume has a minimum of about 8 $m^3$, while for irrigation reuse only it is about 0.5 $m^3$; rainwater tank volume increases with outflow and with return period up to $T_{max}$.

Finally it is noteworthy that that for a combination of domestic and irrigation water reuse, rainwater tank volume grows more increasing carryover from prior event, than increasing outflow unlike in the case of irrigation water reuse only (Figures 3 and Figure 5). This is because carryover from prior event in case of domestic and irrigation water reuse in combination must be much higher than carryover from prior event in case of irrigation water reuse only.

The resulting formula provides information to designers for the estimation of suitable rainwater tank volumes. Further improvements of the proposed methodology will be aimed to better represent the real functioning of this kind of tanks, e.g. taking into account the monthly fluctuations of irrigation rate and considering the possibility of using also greywater, as integration of rainwater, for filling the tank.

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