AN ANALYTICAL PROBABILISTIC APPROACH TO SIZE CISTERNS AND STORAGE UNITS IN GREEN BUILDINGS

Anita Raimondi [1], **Gianfranco Becciu**[2]
[1,2] Politecnico di Milano, Milano, Italy
[1]anita.raimondi@polimi.it, [2]gianfranco.becciu@polimi.it

Abstract

In the last decades the importance of stormwater control has increased considerably, especially in urban areas, due to both urbanization extension and to a greater concern for environment pollution. Downstream runoff control by detention storages has proved to be very effective in reducing the risk of drainage network overload and of polluted spills in receiving water bodies. In recent years, source control with storage units for green buildings has gained considerable recognition and acceptance. They not only reduce rainwater runoff which would otherwise be handled through urban drainage system but also save water which would otherwise be supplied from municipal water distribution system. While detention storages must be emptied as quickly as possible so that the entire capacity can be available at the begin of each rainfall event, storage units must guarantee as much as possible a residual volume to satisfy the desired water reuse. This paper proposes an analytical probabilistic approach to size these stormwater utilities. Resulting formulas are only function of the desired water use rate, reliability and of the probability distribution of the three fundamental hydrological parameters (rainfall height, rainfall duration and inter event time). An application to a case study, based on rainfall intensities recorded in Milano (Italy), is presented.

Keywords

Stormwater reuse, analytical probabilistic approach, cisterns, storage units.

1. INTRODUCTION

The concept of rainwater collection system can be traced to the early centuries of Roman domination in Sardinia [1] where people built crude collection system to collect rainwater for water supply. In past several decades, due to population increases and concentration, rainwater cistern systems have become a significant source of water supply in regions of Africa, Asia and South America [2] so much so that in many areas of the world, water in rainwater cisterns constitutes the only source of water, and consequently serves all potable and non-potable uses. The roof collection system is the simplest method for rainwater harvesting and is relatively convenient to set up.

Water stored in rainwater cisterns can serve a wide range of reuse applications. Modern green buildings make use of storage unitts to store roof runoff for irrigation, hardscape cleaning and maintenance purposes [3]. In addition they can also supply water for toilet flushing purposes and laundry. Rainwater reuse not only saves water which would otherwise be supplied from municipal water distribution system but also reduces stormwater runoff which would otherwise be handled through urban stormwater management system [4].

Storage units can be used on an individual household basis, or alternatively larger storages can be constructed to collect roof stormwater from several houses. For single buildings oversize or undersize of these facilities may not result in significant economical or environmental losses; however they can become significant as the number of green building increases.

While stormwater detention facilities are designed to capture and detain the more polluted first flush, storage units can bypass first flush and store only cleaner rainfall water. Moreover while studies on stormwater detention facilities focus on overflows risk and receiving water impacts, in sizing cisterns for water reuse the interest is on the rate of use that can be supplied during dry periods and its reliability. Depending on the aim of water reuse different water shortage can be accepted.

This paper proposes a method, based on an analytical probabilistic approach, for sizing storage units. Water reuse for irrigation has been here investigated: roof water is collected in the storage unit during the rainfall event and released during interevent time. Moreover a fire volume has to be constantly stored in the cistern, that is a carryover from prior storm must be stored at the beginning of each rainfall event.

An explicit relationship for storage unit volume has been derived; it is a function of the use daily rate, of the probability of failure for water supply and of the distribution of the three fundamental hydrological parameters (rainfall height, rainfall duration and interevent time).

An application to a case study, based on rainfall intensities recorded in Milano (Italy), has been presented; anyway, the universality of the proposed method allows its applicability to different climates all over the world.

2. ESTIMATION OF THE VOLUME

Let consider a storage unit that supplies water during dry periods and must constantly store a fixed water volume (for example a fire volume for building safety). To achieve this goal, a carryover volume from prior events must be stored in the cistern at the beginning of each rainfall event. Depending on the characteristics of water reuse, which can sustain water shortage to a certain degree, different possibility of failure can be taken into account.

Based on the quality of rainwater collected from the roof and on the water reuse, first flush can be bypassed and discharged in the urban drainage system. Considering also an Initial Abstraction (IA) that includes all losses before runoff begins, a generic load/use cycle has been schematized as shown in Figure 1.

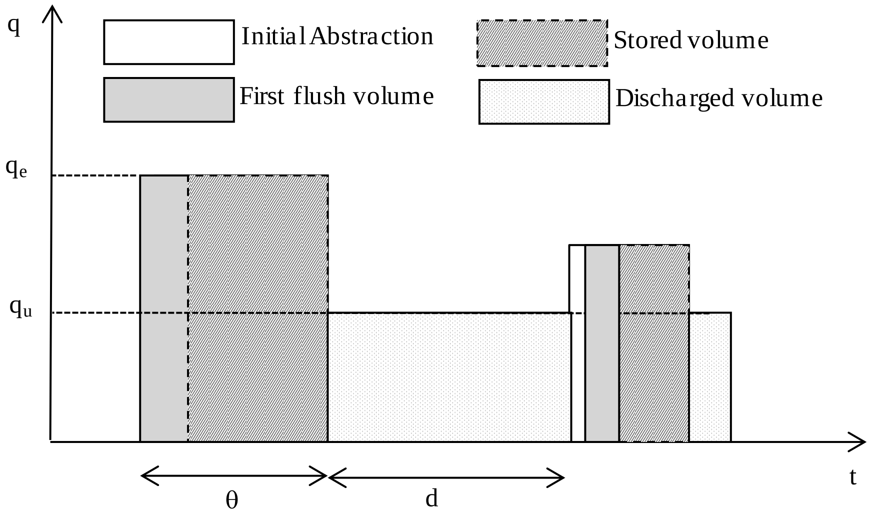

Figure 1. Schematization of a load/use cycle.

To analyze the dynamics of stored volume in a storage unit, a simplified scheme has been adopted, based on the following hypothesis:

- outflow from storage unit has been considered constant and equal to the water demand for the specific water reuse.
- rainfall-runoff transformation has been neglected, considering rainfall intensities instead of water discharges (as a roof is a small surface, discharges may be considered approximately proportional to rainfall intensities);
- rainfall intensities have been considered constant in time (if inflow are always greater than the constant outflow for all the storm duration, the final stored volumes are independent from hydrograph pattern and rectangular events may be used);
- carryover have been assumed to be due to the previous storm only, so that a couple of storms at a time has been considered.

To identify independent events from the storm stochastic process a minimum dry period, the so called InterEvent Time Definition (IETD) [5] has to been defined.

Several earlier studies have shown that exponential probability density functions often fit the hydrological rainfall characteristics histograms satisfactory [6], [7], [8], [9]. Rainfall height h, rainfall duration θ and interevent time d distributions are:

$$f_h = \xi e^{-\xi h}$$

$$f_\theta = \lambda e^{-\lambda\theta}$$

$$f_d = \psi e^{-\psi(d - IETD)}$$

where $\xi = 1/\mu_h$, $\lambda = 1/\mu_\theta$ and $\psi = 1/(\mu_d - IETD)$.
μ_x : expected value of random variable x.

If storage unit is full before the end of the storm event, spills are discharged in the urban drainage system or infiltrated in the soil. The probability of carryover from prior event w_{pr} is expressed as the sum of two probabilities (1): the probability of carryover from prior event when the stored rainwater volume is lower than the storage unit volume and the probability of carryover from prior event when the cistern is full and an overflow occurs.

$$P(w_{pr} > \overline{w}) =$$
$$P(h - IA - w_f - q_u d > \overline{w}) \cdot P(h - IA - w_f < w_0) + P(w_0 - q_u d > \overline{w}) \cdot P(h - IA - w_f \geq w_0) \quad (1)$$

where :

w_f : first flush volume;

w_0 : specific storage unit volume;

$\overline{w}$: fire volume;

q_u : specific outflow.

By means of analytical functions, the carryover probability is calculated for a couple of chained storm events from the probability distribution function of the three rainfall parameters, rainfall depth h, rainfall duration θ, interevent time d (2).

$$P(w_{pr} > \overline{w}) = \int_{h=q_u IETD + IA + w_f + \overline{w}}^{w_0 + IA + w_f} f_h dh \cdot \int_{d=IETD}^{(h - IA - w_f - \overline{w})/q_u} f_d dd + \int_{h=w_0 + w_f + IA}^{\infty} f_h dh \cdot \int_{d=IETD}^{(w_0 - \overline{w})/q_u} f_d dd \quad (2)$$

with T : return time period.

The general condition $(w_0 - \overline{w})/q_u > IETD$ for the possibility of pre-filling has also to be taken into account.

Solving expression (2) and by means of the following auxiliary dimensionless variable $\beta = \dfrac{q_u \xi}{q_u \xi + \psi}$, storage unit volume results (3):

$$w_0 = \frac{\beta}{\xi} \cdot \left\{ \psi IETD + \frac{\psi}{q_u} \overline{w} - \ln \left[e^{-\xi q_u IETD - \xi \overline{w}} - \frac{e^{\xi \left(IA + w_f \right)}}{T(1-\beta)} \right] \right\} \tag{3}$$

The above formula represents a specific storage unit volume, so the resulting value must be multiply by the roof area and the runoff coefficient φ (for impermeable surface such as roof is equal to 1).

Note that storage unit volume it is not function of the rainfall duration since stored rainwater supplies water demand only during dry period.

3. CASE STUDY

Relationship to estimate storage unit volume has been applied to the rainfall series recorded in Milano, Italy, from 1971 to 1991, recorded with a time resolution of 1 minute and a depth resolution of 0.2 mm. An IETD= 6 hours has been selected to identify independent rainfall events [10]. The number of storms in the series for IETD=6 hours is equal to 979. Mean and standard deviation of rainfall characteristics are shown in table 1, while the correlation coefficients among them are shown in table 2.

Table 2. Mean and variance of rainfall characteristics in the Milano series.

μ_h [m]	0.013
μ_θ [hour]	8.712
μ_d [hour]	104.968

Table 2. Correlation coefficient among rainfall characteristics in the Milano series.

$\rho_{h,\theta}$	0,74
$\rho_{h,d}$	0,04
$\rho_{d,\theta}$	0,03

As can be observed, while the other characteristics are only weakly correlated, rainfall depths and durations are not independent random variables as assumed. More, also the hypothesis of exponential CDFs for all three rainfall characteristic is not well suited to the samples of recorded values. Both results are not unexpected, since they were observed for rainfall in the Alpine catchments in previous studies [11] and [12], but these hypotheses has been held anyway to avoid a considerable increase in the complexity of relationships.

Storage unit volume is calculated with reference to a roof surface of 250 m^2 and to a fire volume of 1 m^3.Since a roof can be considered as an impermeable surface runoff coefficient φ is assumed equal to 1 and IA = 1,0 mm. For irrigation purpose there are no particular restrictions on the use of first flush, so it has been assumed equal to zero in the formula.

Storage unit volume has been estimated varying the daily use rate from 10 to 100 [l/day] and considering three different return time periods of 2 – 3 – 5 years.

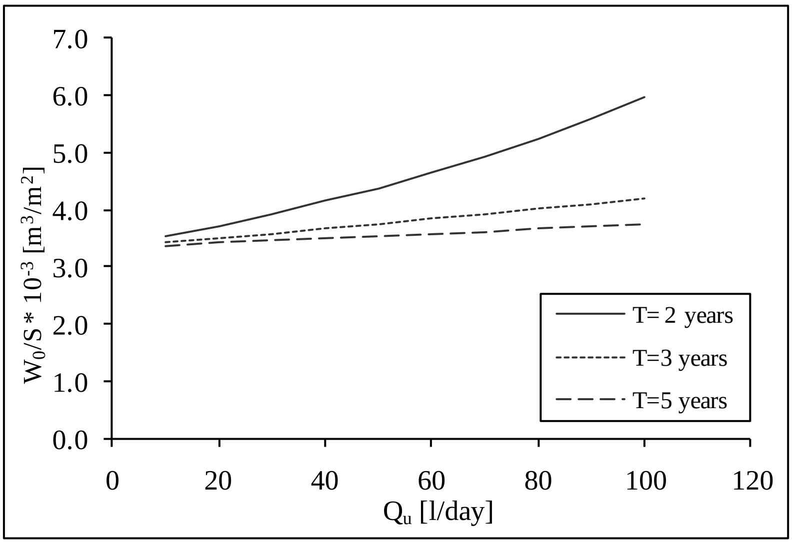

Figure 2. Storage unit volume as a function of daily use rate for return time periods of 2 – 3 – 5 years.

Figure 2 points out how only for a return time period of two years storage unit volume significantly increases with the use daily rate; with a return period of five years or more volume tends to stabilize around the asymptotic value of about 3,5 m^3/m^2. This aspect is best shown in Figure 3: for a storage unit volume lower than about 1 m^3 (corresponding to a roof surface of 250 m^2), the probability of carryover from prior event greater than or equal to the established fire volume is nil. Carryover probability increases diminishing the daily use rate and tends to the asymptotic value of 0,58 increasing the storage unit volume that corresponds to a return period of 1,7 years. In the application ranging storage unit volume from one to two cubic meters the return period varies from 1,7 years to hundreds of years.

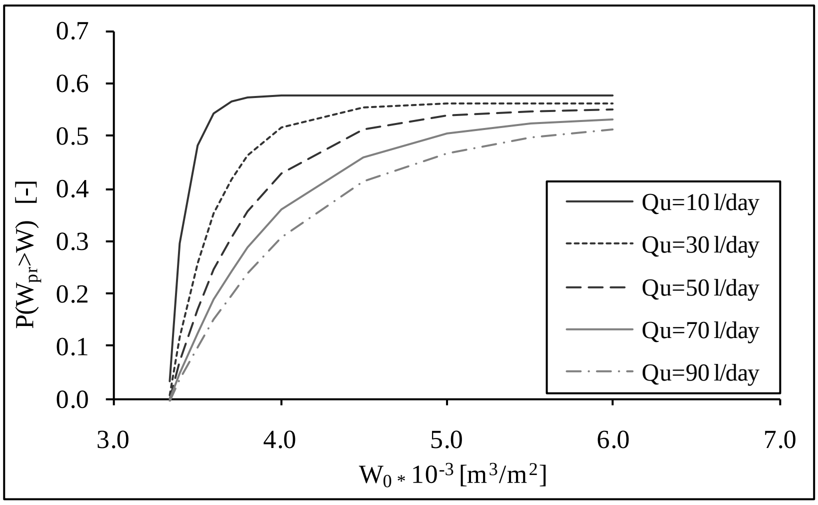

Figure 3. Carryover probability CDFs as a function of the storage unit volume for different use daily rate.

4. CONCLUSIONS

The procedure described herein offers a probabilistic approach for sizing storage units and rainwater system. Design volume results function of the use daily rate, of the probability of failure for water supply and of the distribution of the three fundamental hydrological parameters (rainfall height, rainfall duration and inter event time). In addition the Initial Abstraction (IA) and the InterEvent Time Definition (IETD) are taken into account in the formula as the possibility of bypassing first flush and retaining a certain fixed volume. For convenience in practical applications the resulting expression is expressed in cubic meters per square meter.

To meet a certain water supply carryover from prior event must be at least equal to the water demand in dry periods. Lower carryover storage indicates a higher water supply risk. Depending on water reuse different water shortages can be sustained. For case study, increasing the return time period the storage unit volume tends to an asymptotic value and it does not vary increasing use daily rate. Similarly a return time period of a year (corresponding to a probability of one) can't be achieved. However for the suitable return time period of two years, storage unit volume doubles from an use daily rate of 10 l/day to an use daily rate if 100 l/day.

The universality of the proposed method allows its applicability to different climates all over the world. Work in progress will be the application of the proposed method for other water reuses and the use in combination of greywater and rainwater for water supply in green buildings.

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