Comments to "Simple explicit formulae for calculating limit dimensions to avoid undercutting in the rotor of a Cycloid rotor pump"
by Ye, Zhonghe; Zhang, Wei; Huang, Qinghai; Chen, Chuanming;
Mechanism and Machine Theory, Volume: 41, Issue: 4 April, 2006, pp. 405-414.

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I want to express a general appreciation for the deep analysis of our paper [1], which has been made by the authors of paper *"Simple explicit formulae for calculating limit dimensions to avoid undercutting in the rotor of a Cycloid rotor pump"*, but I wish also to observe that unfortunately the formula and the presented methodology based on profile curvature radius are not original and new as Ye et al. claim.

Formula (10) and its consequent formula (11) on page 412, of which the authors claim the originality and the novelty, are tenth of years old. In fact they appeared also in a Colbourne's paper [2] in 1975. Perhaps this reference is difficult to be retrieved by the authors, but Colbourne practically has presented a very similar study in [3] (even without reporting explicitly this formula there, since the considered case was not limited to the circular profile for the lobe of the outer rotor) that should be available more easily.

Anyhow, for their convenience I wish to summarize Colbourne's method presented in [2], using the notation of our paper [1]. An arbitrary curve Γ_2 of parametric equation in the reference system S_2:

$$\Gamma_2^{(2)} : \begin{cases} \xi = \xi(\theta) \\ \eta = \eta(\theta) \end{cases} \tag{1}$$

is used for the outer gear profile, where θ is the parameter. This curve is used to generate the inner rotor profile that is the envelope of Γ_2 during its motion in a planetary manner, while the inner rotor is held fixed. The profile, in the reference system solid with the inner rotor S_1, is given by:

$$\Gamma_1^{(1)} : \begin{cases} x = -e\cos(n_2\psi) + \xi\cos\psi - \eta\sin\psi \\ y = -e\sin(n_2\psi) + \xi\sin\psi + \eta\cos\psi \end{cases} \tag{2}$$

Angle ψ is the counter-clockwise rotation of the outer gear relative to the inner gear, when a general point $\mathrm{M}^{(2)}$ of the outer rotor is in contact with the inner rotor, e is the distance between the centres and $n_2 = n_1 + 1$ the number of lobes of the outer gear. To determine the value of ψ, Colbourne considered Figure 1, in which the outer rotor is held fixed, the line $\mathrm{O_2P}$ connecting the centre $\mathrm{O_2}$ to the pitch point P is rotated counter-clockwise of $n_1\psi$, while the inner rotor is rotated ψ clockwise. The contact point is M and the common normal to both profiles passes through the pitch point P. It is easy to show that:

$$\zeta \sin\gamma = \xi - n_2\, e \cos\left(n_1\psi\right) \tag{3}$$

$$\zeta \cos\gamma = n_2\, e \sin\left(n_1\psi\right) - \eta \tag{4}$$

$$\tan\gamma = \frac{\mathrm{d}\eta/\mathrm{d}\theta}{\mathrm{d}\xi/\mathrm{d}\theta} = \frac{\eta'}{\xi'} \tag{5}$$

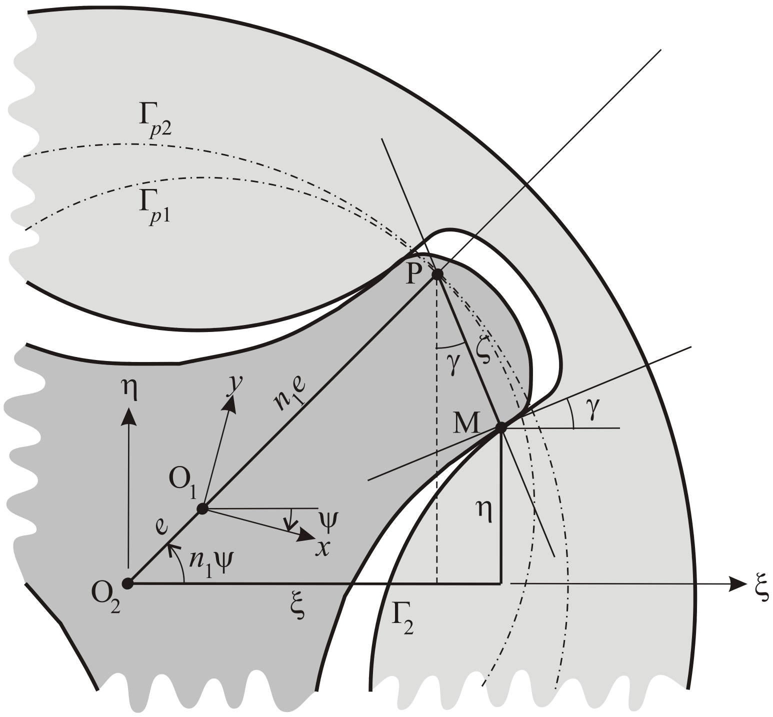

Figure 1. Determination of the inner rotor profile.

By dividing eq. (3) by eq. (4), thus obtaining $\tan\gamma$ and replacing eq. (5), it results:

$$n_2\, e\left(\xi' \cos(n_1\psi) + \eta' \sin(n_1\psi)\right) = \xi\,\xi' + \eta\,\eta' \tag{6}$$

Previous equation can be transformed in a quadratic form in $\cos(n_1\psi)$ and results:

$$n_1\psi_{(1,2)} = \arctan \frac{\eta'\left(\xi\,\xi' + \eta\,\eta'\right) \mp \xi' \sqrt{n_2^2 e^2\left(\xi'^2 + \eta'^2\right) - \left(\xi\,\xi' + \eta\,\eta'\right)^2}}{\xi'\left(\xi\,\xi' + \eta\,\eta'\right) \pm \eta' \sqrt{n_2^2 e^2\left(\xi'^2 + \eta'^2\right) - \left(\xi\,\xi' + \eta\,\eta'\right)^2}} \tag{7}$$

Then Colbourne passed to consider what he defined as a special case, i.e. that of Γ_2 represented by a circular arc (see Figure 2). This is also the case considered by Ye et al. The radius is equal to r_{l2} and the circle centre C is distant d from O_2. He considered that if the inner rotor is held fixed, the line $\mathrm{O}_2\mathrm{P}$ turns through $n_2\psi$, while the outer rotor turns through ψ, then centre C describes an epitrochoid, the equations of which are:

$$\begin{cases} x = -e\cos(n_2\psi) + d\cos\psi \\ y = -e\sin(n_2\psi) + d\sin\psi \end{cases} \tag{8}$$

The curvature of the locus described by C is:

$$\kappa = \frac{x'\,y'' - x''\,y'}{\left(x'^2 + y'^2\right)^{3/2}} = \frac{d^2 + e^2 n_2^3 - e\,d\,n_2(n_2+1)\cos(n_1\psi)}{\left(d^2 + e^2 n_2^2 - 2e\,d\,n_2\cos(n_1\psi)\right)^{3/2}} \tag{9}$$

The curve Γ_1, generated by the circular arc, is equidistant from the locus of point C and its radius of curvature differs from that of the locus by the radius r_{l2} of the circular arc. Therefore the radius of curvature of Γ_1 is given by:

$$\rho_1 = \frac{\left(d^2 + e^2 n_2^2 - 2e\,d\,n_2\cos(n_1\psi)\right)^{3/2}}{d^2 + e^2 n_2^3 - e\,d\,n_2(n_2+1)\cos(n_1\psi)} - r_{l2} \tag{10}$$

Eq. (10) is the same of formula (8) of the paper of Ye et al. apart from the different notation used here. Then Colbourne calculated the minimum positive radius of curvature in correspondence of

$$n_1\psi = \arccos \frac{e^2 n_2^2(2n_2 - 1) - d^2(n_2 - 2)}{e\,d\,n_2(n_2+1)} \tag{11}$$

with the value:

$$\rho_{1\min} = \frac{\sqrt{3^3}\sqrt{n_1\left(d^2 - e^2 r_2^2\right)}}{\sqrt{\left(n_2+1\right)^3}} - r_{l2} \tag{12}$$

that coincides, except for the notation, with formula (10) of the paper of Ye et al. By putting $\rho_{1\min} = 0$ we have finally formula (11) of the paper of Ye et al.

Therefore Colbourne's idea about undercut was the determination of the curvature of the inner profile, as Ye et al. did. He noted also that his analytical derivation led to a formula for the minimum radius of curvature, which coincided to one proposed by Hall in 1968. Hall, in [6], claimed that this was the first time the formula of the minimum curvature radius is presented. Therefore formulas (10) and (11) are at least 38 years old. Finally, Hall observed that Hill invented the first practical pump of this type in 1921 [6].

Actually Colbourne used the same identical approach of Ye et al. in [2], but noted also that in Buckingham's well-known book *"Analytical Mechanics of Gears"* the profiles of internal gear pumps are briefly presented. In my edition of 1943 [4] this is on page 43 and the profiles are determined without the use of differential geometry.

It is worth also to note that the formula (11) was also already reported in our paper [7], referred also in [1], surely more recent, being only 9 years old, and of easy retrieval. In this paper it is reported that Colbourne had undoubtedly obtained the formula:

$$r_{l2\ limit} = \frac{\sqrt{3^3}\sqrt{n_1\left(d^2 - e^2 r_2^2\right)}}{\sqrt{\left(n_2+1\right)^3}} \qquad (13)$$

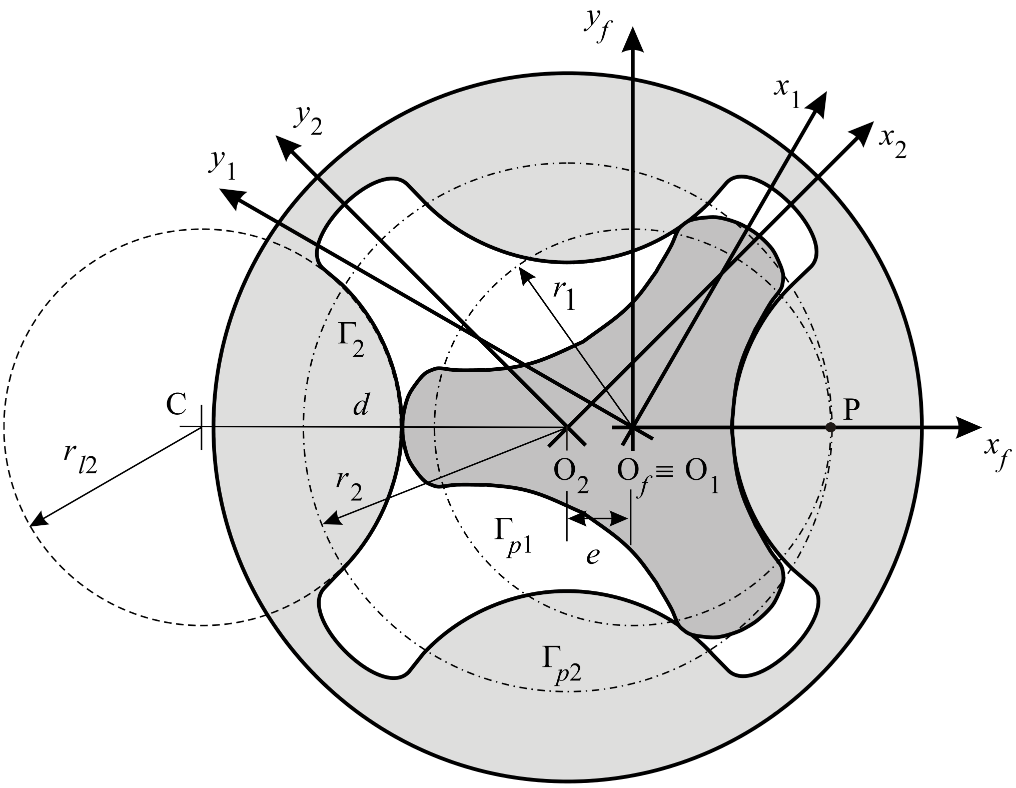

Figure 2. Circular arc for the outer rotor.

Moreover, in our papers [7] [8] [9] we already presented methods, based on both geometrical considerations and on differential geometry, to define the profile of the internal rotor in internal lobe pumps. The topics were in some cases more general than that of Ye et al., since the profile of the outer rotor had not to be necessarily circular. The geometrical method did not require the numerical solution of any equation.

I wish also to make two other observations that have evidently passed unnoticed to Ye et al.

First, the approach of the limit curve is of general validity for every type of conjugated surfaces (see for instance Litvin's book [10]), while that proposed by the authors is related to the considered type of pump and to the circular profile. In other terms, the approach based on geometrical considerations is the starting point, that on differential geometry the finishing point. For Ye et al. this process appears to be reversed.

Second, by using the (re)proposed method for the undercut it is not immediate to obtain a meaningful geometrical representation of which part of the outer gear lobe that determines undercut on the inner gear. On the contrary, this is easy to obtain with the method of the limit curve.

I am convinced that Ye et al. have determined independently the formula for the minimum radius, but a better documentation would probably have allowed the authors to avoid to redo a study already appeared many years ago and to attribute themselves credits of originality. Undoubtedly the retrieving of documentation has not been done by the authors, so that in their references there are only our paper [1] and a book of the same authors. Their references are (numerically) very poor for a journal paper.

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