**James Clerk Maxwell, a precursor of system identification and control science**

by Sergio Bittanti

*Dipartimento di Elettronica, Informazione e Bioingegneria
Politecnico di Milano, Italy*

and

*Istituto Lombardo – Accademia di Scienze e Lettere
(Academy of Science and Literature of Lombardy)*

E mail sergio.bittanti@polimi.it

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Fig. 1 Maxwell’s portrait

Fig. 2 Frontispiece of Maxwell’s essay on Saturn’s rings.

**Foreword**

150 years ago James Clerk Maxwell published his celebrated paper “Dynamical theory of electromagnetic field”, where the interaction between electricity and magnetism eventually found an explanation. However, Maxwell was also a precursor of model identification and control ideas. Indeed, with the paper “On Governors” of 1869, he introduced the concept of feedback control system; and moreover, with his essay on Saturn’s rings of 1856 he set the basic principle of system identification. This paper is a tutorial exposition having the aim to enlighten these latter aspects of Maxwell work.

**1 Introduction**

While still a student at Cambridge University, James Clerk Maxwell began to study Saturn’s rings [1], in order to discover the nature of their composition. His research was not based on astronomical observations, but rather on pure mathematical modeling. Following the law of universal gravitation, Maxwell constructed various models which explained the rings’ nature, each deduced according to a given working hypothesis. For instance, one model was constructed on the hypothesis that the ring was a single solid crown; another was based on the hypothesis that they were formed by a gaseous crown, and so forth. Once Maxwell had built the models, he set about to determine which one was correct by comparing the hypotheses with the established laws. The models with characteristics that contradicted reality were discarded, and with them the hypotheses upon which they were based. As his basic tool for validation, Maxwell used the notion of stability, starting from the assumption that, since the rings had existed in that configuration for millennia, the model obtained had necessarily to be stable; therefore, instable models were to be rejected, along with their underlying hypotheses. This approach, which anticipated today’s techniques of model identification [2], is magisterially summarized by the author as follows: *… by rejecting every hypothesis which leads to conclusions at variance with the facts, we may learn more on the nature of these distant bodies than the telescope can yet ascertain*. As reported in his essay of 1856, Maxwell came to the conclusion that the rings had to be composed of many solid elements, unattached to one another, in rotation around the planet, a deduction which, over a century later, has been confirmed by modern space missions.

Some years later, he addressed another topic of major importance for systems and control, the notion of feedback control system. This was indeed the subject of his paper “On Governors” of 1968 [2]. Underlying both contributions of 1856 and 1868 there is a common *leit motiv*, the concept of *stability*.

In this paper, we will trace in a tutorial manner the history of these major contributions by Maxwell, with special emphasis on the systems and control aspects.

**2. The Discovery of Neptune and the Adams Prize of 1855**

In the middle of the 19th century, John Adams, a professor at Cambridge University, had devoted himself to the study of the orbit of Uranus, the last planet to have been discovered at that time. Adams reached the conclusion that the irregularities in its trajectory could not be explained except by the existence of another celestial body, an *external planet*. The same conclusion was reached, independently, by Urbain Le Verrier, who was working in Paris. Their calculations made it possible to estimate the position of the “new” celestial body, which was observed by Johan G. Galle, with the telescope of the observatory of Berlin. It was 23 September 1846. The solar system was thus enriched with a new planet, which was given the name Neptune.

To celebrate the discovery, in 1848 Cambridge University instituted an award called the *Adams Prize*, announcing it as follows:

The University has accepted a fund raised by several members of St John’s College, for the purpose of founding a Prize to be called the ADAMS PRIZE, for the best essay on some subject of Pure Mathematics, Astronomy, or other branch of Natural Philosophy.

The prize was bi-annual and was open to anyone who had been admitted – at any time – to Cambridge University, therefore to all alumni past and present.

Herein we will be focusing on two Adams Prizes, that of 1855 and that of 1876. We shall begin with the first, and discuss the second later on.

The announcement of 1855 ran as follows:

The Examiners give Notice, that the following is the subject for the prize to be adjudged in 1857: - “The Motions of Saturn’s Rings”.

This topic was part of the great interest for the study of the solar system by many scholars of the 18th- and 19th centuries, [3]. The prize went to a young student, James Clerk Maxwell (Fig.1), originally from Edinburgh, where he had been born on 13 June 1831, for the essay “The Stability of Saturn’s Rings”, the cover of which is reproduced in in Fig. 2. Note that the essay’s title does not coincide exactly with the theme assigned by the prize announcement for that year.

**3. First Studies of the Solar System.**

Newton’s book “*Philosophiae Naturalis Principia Mathematica”*, published in 1687, offered the world the methods for understanding the laws that govern celestial phenomena, which had always been the subject of investigation for humans.

This led to a fervor of studies on the solar system. One of the questions that was asked was what the effect could be of a perturbation due to the intrusion of an external body, such as a comet or an asteroid. By the way, events of this kind are not so remote. In 1994 for example the comet Shoemaker-Levy crashed into Jupiter. More recently, on 19 October 2014, the comet Siding Spring just missed Mars, passing 140,000 kilometers from the red planet (a third of the distance between the Earth and the moon), with heavy consequences for its atmosphere. The Earth has also been threatened by an event of this kind, when the comet Lexel passed just 2 million kilometers from us. This occurred 1770. The intrusion of an external body into the solar system could have various effects on the paths of the planets, such as a) a temporary deformation, after which the planets would return to their usual paths, b) a permanent deformation, c) the deflagration of the solar system. It was thus a question regarding the *stability* of the solar system. Numbering among the pioneers of these studies of *mathematical astronomy* were great scientists such as Joseph-Louis Lagrange (1736-1813), Pierre Simon Laplace (1749 – 1827), Simon-Denis Poisson (1781-1840) and Carl Jacobi (1804-1851). Laplace in particular made two fundamental contributions, the essay *Exposition du système du monde* (1796), [4], and the monumental *Mécanique Céleste*, a work in five volumes, the first of which was published in 1799, [5]. Laplace is also remembered for his celebrated motto: *what we know is not much; what we do not know is immense*. The study of the solar system was of such passionate interest in public opinion that, in 1887, King Oscar of Sweden instituted a prize(1) for studies on the three-body system (for example, Sun, Earth, Moon). The task was to determine the paths of the system, knowing the initial conditions. The prize (one is tempted to say the “Oscar”) went to Henry Poincaré (1854-1912), who demonstrated the impossibility of writing the solution explicitly, and also proved that two solutions associated with only slightly different initial conditions could diverge in over the long term.

**4. The Birth of the Concept of Dynamic System and the Rise of System Theory.**

Since force determines acceleration, which is the variation of velocity, and, in turn, velocity is the variation of position, the famous law of universal gravitation translates mathematically into a dynamic link between force and position (via acceleration), which today we call a differential equation. In the scientific world of the time, this type of modeling was an important innovation, as previously scholars had for the most part focused on algebraic equations. In dynamical system, the effect depends not only upon the cause but also on the initial state: the same cause may have quite different effects. Later on, the same concept was extended to other mathematical descriptions, such as difference equations, automata and so on.

While differential equations, and more in general, dynamical systems had to become a main research subject in mathematics, they soon became *the* modeling tool in many fields, far beyond celestial mechanics, something like a flowing tide permeating all prescriptive sciences. An example is electrical networks analysis, as developed in the fifties of past century, largely based on systems of linear differential equations.

While the tide extended, it became apparent that some concepts emerged as a common *leit motiv* notwithstanding the different frameworks in which they were conceived. This was especially clear from the mid of the 20th century, when the diffusion of mathematical descriptions as means for the comprehension of the real world and the design of engineering tools was accompanied by the recognition that many problems could be treated in a unified conceptual frame thanks to models. This led to the concept of *system* and the rise of *system theory* [6], [7].

An emblematic story is related to the concept of stability. As we have seen, studies on the stability of the solar system had the aim of establishing the effect of a disturbance produced by a comet of by another celestial body travelling through the system. It is a very interesting problem, which understandably drew the interest of not only scholars but of the public at large. The concept of stability also however has a general value that goes beyond the solar system, and indeed can be applied to any dynamic system. This notion plays a particularly crucial role in control systems. In order for such systems to work properly, the effect of unforeseen perturbations (disturbances) must be neutralized, and this can happen only if the system is stable. The credit for the general definition of stability, universally accepted today, goes to Alexander M. Lyapunov (1857-1918), who gave it for the first time in his doctoral thesis, defended at the University of St. Petersburg in 1884. Nevertheless, the notion of stability had already appeared and been put to use in scientific literature some decades before. It had for instance played a particularly important role in the works of Maxwell of interest in this article.

*Observation (of laws and models)*

The law of gravitation cites the force of attraction between two bodies. As it is normally stated, it is implied that the force is one that is “exerted from afar”, instantaneously. Laplace, during his studies on the movement of the moon, had already doubted whether this was correct; certain anomalies could not be explained if the force of gravity were indeed transmitted at infinite speeds. For complete clarity on this fundamental question, we would have to wait until the first decades of the 20th century, when, thanks to Henry Poincaré and Albert Einstein the theory of relativity was developed. It then became clear that the force of gravity did not spread from one body to another instantaneously, but at a finite speed, namely that of light. Thus, if in this moment the sun stopped its activity, the force of gravity to which the Earth is subject would continue to be present for another eight minutes, the time it takes for light to reach us. This observation, although of fundamental importance, does not of course invalidate Newton’s Law, which, for bodies moving at speeds far from that of light, has brought about great results for humanity. This is why in recent years, we tend to speak of *models* rather than *laws*.

This observation reminds us what statistician George Box (1919 – 2013) was used to say: *all models are wrong, but some are useful*.

**5. Maxwell’s Essay on the Rings of Saturn**

Now we come to the essay on Saturn’s Rings. We shall first analyze the method of study adopted, and then go on to describe the organization of the contribution itself.

In his essay, Maxwell makes various conjectures on the composition of the rings and maps out the corresponding mathematical model for each. The model obtained is then validated by comparing its salient features with “the facts”, that is to say known reality. If the model contradicts reality, the underlying hypothesis is rejected. If it is not contradictory then the conjecture is not falsified and could be correct. The essay contains a splendid passage in which Maxwell masterfully sums up his approach: … *by rejecting every hypothesis which leads to conclusions at variance with the facts, we may learn more on the nature of these distant bodies than the telescope can yet ascertain*.

 Note that this line of thinking is typical of the discipline that today is known as *model identification and data analysis*, whose goal is to provide a valid mathematical description of a given system from experimental data, for the purpose of gathering knowledge, making predictions or control. One starts by postulating a group of models, from which the “best model” is chosen, which is normally the one that provides predictions that most closely align with the experimental observations. Then the result obtained is validated: if the model is not satisfying, for example because one of its features is not in line with the experimental evidence, one moves on to another group of models and begins the procedure anew.

And herein lies Maxwell’s winning idea: the validation of a given model is to be done by means of the notion of stability: since the rings have been around since time immemorial, a valid model must be a stable model. If it does not meet this criterion it is to be rejected, along with the hypothesis on which it is based.

The essay on Saturn’s Rings contains 68 pages, followed by an appendix and by a final table with the figures, 13 in total. The bibliographic references are cited directly in the text when they come up.

The work is divided into two parts. The first (pages 6-17) bears the title *On the Motion of a Rigid Body of Any Form about a Sphere*. Here the model is based on the hypothesis that the rings are made up of a single rigid block. As Maxwell himself points out in the general introduction (pages 1-5), this hypothesis had already been proposed by Pierre Simon Laplace in the *Mécanique Céleste*, namely in the sixth chapter of the third volume and in the third chapter of the fifth volume. Maxwell comments as follows: *[Laplace] proves most distinctly that a solid uniform ring cannot revolve around a central body in a permanent manner, for the slightest displacement of the centre of the ring from the centre of the planet would originate a motion which would never be checked, and would inevitably precipitate the ring upon the planet… We may draw the conclusion more formally as follows: If the rings were solid and uniform, their motion would be unstable, and they would be destroyed. But they are not destroyed, and their motion is stable; therefore they are either not uniform or not solid.*

The second part of the essay, entitled *On the Motion of a Ring, the Parts of Which Are Not Rigidly Connected*, studies the case of multiple elements that are not rigidly connected, as though the ring were made up of various independent satellites. Naturally, in this case the modeling is more complex because it is necessary to take into consideration, in addition to the force of attraction exercised by Saturn and the centrifugal force, also the mutual effect of one satellite on the others. The study is conducted based on several simplifying hypotheses: a) the trajectory of each satellite is circular; b) their speed of rotation is uniform; c) the various satellites of the ring are all identical to one another d) the transversal dimension is negligible. The family of model is parametrized in the number of satellites, indicated by the letter $μ$. Maxwell calculated that in order for the movement to be stable the mass of Saturn must be sufficiently high in relation to that of the ring: $S>0.4352μ^{2}R$ , where R and S indicate the mass of the ring and mass of Saturn, respectively. As Maxwell himself observes, if the number of satellites were so high as to violate this condition, the movement would be unstable, and would lead to destructive oscillations that would cause the satellites collide and therefore –presumably – join together in a single body, reducing the number of satellites to around the maximum number that Saturn can sustain (or, as the author puts it, *keep in discipline*). When to the contrary if there were a disparity, the satellites would be unable to remain at an appropriate distance from Saturn’s center of gravity, but, given the planet’s preponderant mass, they avoid interfering with on another.

Maxwell used a not insignificant number of symbols in the paper, introducing them discursively, one by one as they come up, which makes the reading somewhat difficult. The study does however conclude with a useful summary of the first and second parts, a concise summing up of the cases considered and the results obtained.

**6. Maxwell’s contribution to the study of control systems**

Consider now another Maxwell’s important contributions, the article published in 1868 in the *Proceedings of the Royal Society*, entitled simply *On Governors*. With the goal of imposing a determined behavior onto a given system, recourse can be made to specially designed device that, connected to the system itself, ensures that said system functions properly. In article this device is called a *governor*; today we use the term *controller*.

The author considers several operating systems in different physical realities, in particular the well-known *centrifugal governor*. Conceived for the control of windmills and then adapted to the problem of the control of the speed in steam engines, this device contains in itself the basic principle of feedback enabling the regulation of the driveshaft speed without the need for any human operator. As such, it has become something of an icon, and has been reproduced on the covers of many a book.

In his 1868 article, in addition to the centrifugal governor, Maxwell discusses several other regulation systems, belonging to a wide range of different fields. It is worth noting that all the devices discussed are described with linear dynamic systems, albeit with varying degrees of complexity. What they all have in common is being feedback systems, in addition to the common concept of the governor:

*''A governor is a part of a machine by means of which the velocity of the machine [...] is kept nearly uniform, not with standing variations in the driving power or the resistance. Most governors depend on the centrifugal force of a piece connected with a shaft of the machine. When the velocity increases, this force increases, and either increases the pressure of the piece against a surface or moves the piece, and so acts on a break or a valve.''.*

Further on:

*''I propose [...] to direct the attention of engineers and mathematicians to the dynamical theory of such governors. It will be seen that the motion of a machine with its governor consists in general of a uniform motion, combined with a disturbance which may be expressed as the sum of several component motion. These components may be of four different kinds:*

*The disturbance may continually increase
It may continually diminish
It may be an oscillation of continually increasing amplitude
It may be an oscillating continually decreasing amplitude*

*The first and the third cases are evidently inconsistent with the stability of the motion, and the second and fourth alone are admissible in a good governor. This condition is mathematically equivalent to the condition that all the possible roots, and all the possible parts, of the impossible roots of a certain equation shall be negative. I have not been able completely to determine these conditions for equation of a higher degree than the third; but I hope that the subject will obtain the attention of mathematicians''.*

In final analysis, the study of stability leads to the problem of resolving a polynomial equation with real coefficients, what today is known as a *characteristic equation*. The basic result, well known to all control students around the world, is that a linear system is stable if all of the solutions of its characteristic equation have a negative real part. One of Maxwell’s merits was to have understood that so-called *impossible solutions*, that is to say complex solutions, are equally important as real solution and also must be analyzed in the study of stability. Note also that, according to the particular system under consideration, the characteristic equation reached by the author could have a different degree. Maxwell concludes by admitting that he had been unable to reach a condition of stability for polynomials with a degree above three.

For more information on the historical development of control science we refer the reader to [8] - [10].

**7. Solving an Algebraic Equation – a Long Story Made Short**

We have thus come to a celebrated problem, a challenge that has vexed many great minds: the solution to algebraic equations of various degrees.

There is of course an explicit formula to solve equation of the second degree. But also third degree equations admit solutions in a closed form. The history of this latter formula is quite curious. We owe its discovery to two great Italian mathematicians, Niccolò Fontana of Brescia, nicknamed Tartaglia (1499-1557), and Scipione del Ferro of Bologna (1465 –1526), who reached it independently of one another. Neither Tartaglia nor del Ferro, however, went public with the formula. Tartaglia revealed it in strict confidence to Gerolamo Cardano (1501–1576); while del Ferro confided it to his son-in-law Annibale della Nave, who in turn showed it to Cardano during one of the latter’s visits to Bologna. The formula was finally published in Cardano’s book *Ars Magna*, universally acclaimed as the greatest scientific treatise of the Renaissance. In the book, Cardano admits that he is not the author of the formula, and credits it to Tartaglia and to del Ferro. Nonetheless, Tartaglia did not take it well, and the episode led to a feud between the two that would last for years.

With regarding to solving algebraic equations, Paolo Ruffini (1765-1822) and Niels H. Abel (1802 - 1829) proved that, from the fifth degree on, there is no explicit formula.

Following the above considerations, it would thus seem that the problem of analyzing stability is difficult indeed, especially for complex systems, of a high degree. However, upon further reflection it turns out the analysis of stability does not require the explicit solution of the characteristic equation, indeed it requires a great deal less: establishing that all the solutions have a real negative part.

Maxwell unsuccessfully tried to obtain a necessary and sufficient condition, as he writes in the final passage of the article cited above.

**8. The 1877 Adams Prize and the Routh Criterion**

Following the Maxwell’s suggestion at the end of his 1868 article, an Adams Prize was announced for someone who could establish a criterion for stability. The prize, awarded in 1877, went to Edward J. Routh (1831 – 1907) for his essay *A Treatise in the Stability of a Given State of Motion*, [11]. Routh had worked out a (necessary and sufficient) condition thanks to which it was possible to establish whether all of the solutions of an algebraic equation had a negative real part, without the need to find the solutions of the equation itself. The condition only required the construction of a table whose elements could be obtained with elementary calculations based on the coefficients of the initial polynomial. Thus the famous Routh Criterion(2) was born, which is still taught today in every course on Control the world over.

One might say that Maxwell and Routh were destined to cross paths. Both were born in 1831, and both studied at Cambridge in the same period. At the time, students at that university had to undergo a very selective test, a sort of “competition of the best of the best”, called the *Mathematical Tripos*, which was held at the end of the first three years of study. The Triposconsisted in a series of complex mathematical tests, which could take candidates several days of work to complete. The etymology of the term Tripos is unclear; it may have to do with the word tripod, referring to the three-legged stool which students used during their oral exams. The top ranking students in the competition were placed on a list known as the *Wrangler List*. This was a very desirable distinction, and a prestigious calling card for future occupation (and not necessarily in only academic or even scientific fields, but also for example to become a judge). The ranking obtained in these tests were indicated with an abbreviation, for instance 7W 1893 indicated seventh place in the Tripos of 1893, while 12W 1905 referred to twelfth place in 1905 (the scores of Bertrand Russell and John Maynard Keynes, respectively). Maxwell and Routh both participated in the Tripos of 1854, coming in first and second (1W 1854 e 2W 1854). Routh took first place and Maxwell second.

They were also both Adams Prize winners; as we have seen Maxwell won for his essay on Saturn’s rings, and, about twenty years later, Routh was awarded for his study on the general condition of stability.

**8 Epilogue**

In the outstanding set of contributions by J. Clerk Maxwell, [??!!], we have focused here on two of them, where he discussed fundamental ideas pertaining to the field we today call systems and control: the essay on the rings of Saturn of 1856 and the paper “on Governors” of 1868. To these contributions we have to add his fundamental work on electromagnetism, for which we celebrate the 150th anniversary this year, and further work to other fields. It is therefore understandable why many hail Maxwell as the greatest scientist of 19th century. At the commemoration for centenary of his birth Albert Einstein said of Maxwell that he had been *"the most profound and the most fruitful scientist that physics has experienced since the time of Newton*". I myself, a more humble admirer, have often wondered what other contributions this illustrious scientist might have left us if a tumor had not killed him at just 48 years of age.

Footnotes

(1)

King Oscar established the prize to celebrate his sixtieth birthday. For the same occasion, together with Michel Gevers, the author of this footnote had the idea to organize a special issue of the *European Journal of Control*, of which he was the Editor-in-Chief from 2003 to 2013, entirely devoted to the dawn and evolution of control science, [10].

 (2)

In 1894, Adolf Hurwitz (1859-1916), a mathematician working in the ETH Zurich University, worked out a necessary and sufficient condition that was different but equivalent to Routh’s. Thus the criterion is also known as the Routh-Hurwitz Criterion.

List of figures of captions

FIGURE 1 Portrait of James Clerk Maxwell.

FIGURE 2 Frontispiece of Maxwell’s essay on Saturn’s rings.

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