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Energy Efficient Control Strategy for Machine Tools with Stochastic Arrivals and Time Dependent Warm-Up

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Abstract

Energy efficiency in manufacturing is becoming a challenging goal due to the demand of this sector in the worldwide scenario. One of the measures for saving energy is the implementation of control strategies that reduce machine energy consumption during the machine idle periods. This paper extends a threshold policy, that switches off the machine during interruptions of part flow, by modelling explicitly the warm-up time as dependent on the time period the machine stays in low power consumption state. The optimal policy parameter is provided numerically for general distributions of the part arrival time and general functions modelling the warm-up time. Numerical results are based on data acquired with dedicated experimental measurements on a real machining center.

1. INTRODUCTION

1.1. Motivation

Energy efficiency in manufacturing is becoming a challenging goal due to the demand of this sector in the worldwide scenario. The amount of energy consumed by the industrial sector accounts for more than 50% of the world energy consumption. Particularly in 2008, the 21.2% of the total industrial consumption belonged to metal industry [1]. In the last years there has been a growing interest on technical solutions to reduce the energy consumption in manufacturing. This trend is mainly driven by governments which are conceiving new regulations to reduce the environmental impact of manufacturing, e.g. [2].

The most relevant measures for reducing energy consumption at machine level are the eco-design of components aiming at minimizing power demand, the kinetic energy recovery systems (KERS) to reuse and recover energy, and the implementation of control strategies for the efficient usage of components by minimizing processing time and non-adding-value tasks [3]. Actually, a machine tool keeps consuming energy even if the production is interrupted because a fixed power is required for the operational readiness of the machine. Frigerio and Matta showed that the benefits achievable by implementing control policies oriented to reduce machine power consumption when production is not needed, i.e. during machine idle periods, are meaningful [4].

1.2. Related literature

The power requirement of a machine tool can be divided into two main components. A *Fixed Power*, demanded for the operational readiness of the machine and independent from the process, and a *Load Dependent Power*, demanded to distinctively operate components enabling and executing the main process [5–9]. The state control for energy saving aims at reducing the fixed energy consumption, which is required

even if the production is not requested. Other research efforts have focused on the problem of scheduling startup and shutdown of machines to minimize total energy consumption.

A first group of researchers have not considered any warm-up transitory when the machine tool is triggered in a low consumption state. In order to give some example, Prabhu et al. developed an analytical model by combining an M/M/1 model with an energy control policy [10]. Firstly for a station and then for a production line, they calculate the time interval for switching the machine off during idle period with respect to a target energy waste limit. In this study, the machine switch-off accounts for a certain idling power, but the switch-on is instantaneous once the part arrives. Chang et al. analyzed several real-time machine switching strategies using energy saving opportunities windows in a machining line under stochastic failures [11].

A second group of studies considered a warm-up with non-negligible duration in their modeling. Almost all have considered a deterministic and constant warm-up time whenever the machine is switched off. Mouzon et al. presented several switch-off dispatching rules for a non-bottleneck machine in a job shop. They introduced the possibility to include different warm-up times according to the machine off-state considered [12]. Chen et al. formulated a constrained optimization problem for scheduling machines on-off modes in a production line based on Markov chain modeling, considering machines having Bernoulli reliability model [13]. Sun et al. proposed an algorithm to estimate opportunity windows for real-time energy control in a machining line [14]. Stochastic failures are considered, whereas the cycle time and warm-up time are deterministic and constant. Mashahei et al. proposed a control policy to switch-off machine tools in a pallet constrained flow shop. The policy aims to minimize energy consumption under design constraints and considering two idle modes with deterministic warm-up durations [15]. Frigerio and Matta studied analytically four policies to control a machine during production [4]. The policies are assessed in terms of expected energy consumed by the machine under the assumption of stochastic arrivals.

In [10–15], several numerical cases have been investigated through simulation to assess the benefit of energy control policies. However these results, being based on simulation, are case dependent and they do not provide a complete analysis on where the policies perform efficiently.

In the industrial market, there are only few energy saving control systems available. Most of them have been developed by machine tool builders in order to support the final users with new devices for the shutdown of the machine tool, or some functional modules, once the machine idle period exceeds a user-defined limit [16–18].

1.3. Contribution

The literature analysis points out a lack of theoretical modeling concerning the machine energy efficiency control problem for systems under uncertainty. In addition, most of these studies do not deal with warm-up duration, or consider the warm-up as constant and deterministic. However, warm-up duration is often dependent on the amount of time the machine is switched off.

This paper studies a switch-off policy for energy oriented control of machine tools in manufacturing. The policy is characterized considering time-dependent warm-up durations and random arrival of parts. Under quite general assumptions, the paper shows that an optimal switch-off time always exists and the equations for its numerical calculation are also provided. The policy and its optimal conditions are studied on the basis of a set of numerical cases built to provide useful guidelines for practical implementation of energy saving control policies. All of the numerical cases refer to a real CNC machining center that was experimentally characterized to estimate its power demand.

2. ASSUMPTIONS

A single machine with deterministic processing time t_p is considered. The arrival of parts is stochastic with the probability density function $f(t)$ modeling the time T between a part departure from the machine and the next arrival. It is assumed the machine has no input buffer, that is the same to assume there is an input mechanism that controls the release of parts to the machine. In more detail, a part is sent to the machine only during its idle or not productive periods. Then, the part immediately starts its processing if the machine is ready, otherwise it has to wait until the machine is warmed up. After the completion of the process the part leaves the system. An infinite buffer is assumed downstream of the machine. The machine is assumed to be perfectly reliable, thus failures cannot occur; this assumption can easily be relaxed without any consequence on the developed analysis.

The machine can be in one of the following states: out-of-service, on-service, warm-up and working. In the out-of-service state, some of the machine modules are not ready, indeed only the emergency services of the machine are active while all the others are deactivated. In this state, the machine cannot process a part being in a kind of "sleeping" mode. The power consumption of the machine when out-of-service is denoted with x_{out}, generally lower compared to the other machine states. In the on-service state the machine is ready to process a part upon its arrival. The machine power consumption when on-service, denoted with x_{on}, is due to the activation of all its modules that have to be ready for processing a part. From the out-of-service to the on-service state the machine must pass through the warm-up state, where a procedure is executed to make the modules suitable for processing. The warm-up procedure has power consumption equal to x_{wu}, generally greater compared to the other machine states. The duration T_{wu} of the warm-up is assumed to be time dependent, the more the machine stays in the out-of-service state, the more it requests time to reach the proper physical condition for work (e.g., thermal conditions). As a consequence, T_{wu} is a random value that is correlated with the arrival of parts. In the working state the machine is processing a part and the power requested varies according to the process.

The transition between two states can be triggered by the occurrence of an uncontrollable event (e.g., the part arrival) or

a controllable event. During the idle periods of the machine it is not necessary to keep all the machine modules active, and the machine can be moved, with a proper control, into the out-of-service state characterized by a low power consumption. Nevertheless, if a part arrives when the machine is not on-service – this can happen when the machine is in out-of-service or executing the warm-up procedure – there is a penalty. The penalty is expressed by the power consumption x_q necessary for keeping the part waiting until the on-service state is reached. However, once in out-of-service, the machine can be warmed up in advance in order to avoid x_q.

The following section describes a strategy that can be used to control the state of the machine by activating a transition from the on-service to the out-of-service state.

3. SWITCH OFF POLICY

In the common practice most of the machine tools do not have "green" functionalities and they are kept on-service even if the production is not needed. This *Always On* policy is apt to maintain the machine in the proper condition to work avoiding x_q. In all the situations in which there is not a clear advantage of keeping the machine always on, a *Switch Off* policy must be applied with a properly selected control, in order to be effective.

The Switch Off policy is presented describing the machine behavior in terms of states visited and transitions triggered.

Switch Off Policy: Switch off the machine after a time interval τ has elapsed from the last departure.

After a part departure, the machine remains in the on-service state in order to immediately process parts coming in the short time. The time interval $τ$ is properly set for avoiding too frequent warm-up procedures. Once in out-of-service, the machine is warmed up only after an arrival (Fig. 1), thus the part must always wait consuming x_q for a period equal to the warm-up duration T_{wu}.
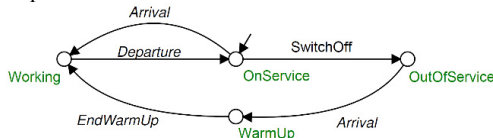

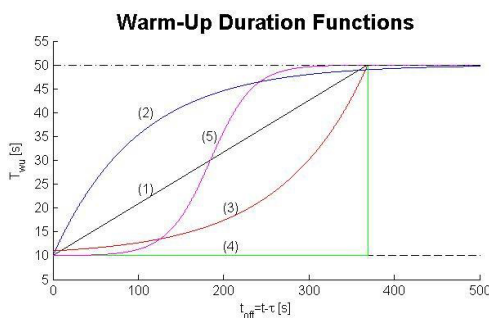Fig. 1. Machine transition graph for the Switch Off policy.

Fig. 2. Warm-up duration functions over the time the machine stays out-of-service (t_{max} = 50 s; $β$ = 10 s; $α$ = 100).

3.1. Time dependent warm-up

Frigerio and Matta [4] considered the warm-up duration T_{wu} as deterministic, thus independent from the control parameter $τ$. However, a warm-up time that is dependent from the time interval that the machine spends in the out-of-service state is an important extension that should be investigated. In common practice, the warm-up consists in some secondary processes that are necessary to allow the machine executing the manufacturing process, e.g., increasing the pressure in machine fluidic systems, reaching the target temperature in the machine cooling circuit.

Mashaei et al. [14] considered two cases: the *hot idle mode*, when the machine tool can directly handle an operation without demanding a warm-up, and the *cold idle mode*, if an amount of time is needed to prepare the machine. However, it is possible to assume that the passage from the hot idle mode to the cold one is not immediate because the warming up processes concern time-dependent physical phenomena. Thus, the machine warm-up duration should be bounded between a minimum time, generally not *zero*, e.g., required to restart the PLC (Programmable Logic Controller) and to check the availability of the machine sub-systems, and a maximum time, needed to reach the proper machine configuration from the environmental conditions (i.e., after a long stop).

Therefore, a warm-up function can reasonably be assumed to be monotonically increasing over the out-of-service sojourn time (equal to t-$τ$, where t is the realization of T). In order to represent several situations, some alternative functions are proposed to model the warm-up – linear (1), negative exponential (2), positive exponential (3), step (4), sigmoid (5) – and are expressed in their general forms:

$$h(t-\tau) = \begin{cases} \dfrac{\beta}{\delta}(t-\tau)+t_{min} & (t-\tau)\in[0;\delta) \\ t_{max} & (t-\tau)\in[\delta;\infty) \end{cases} \qquad (1)$$

$$h(t-\tau) = t_{max} - \beta e^{-\frac{(t-\tau)}{\alpha}} \qquad (2)$$

$$h(t-\tau) = \begin{cases} t_{min}+e^{\frac{(t-\tau)}{\alpha}} & (t-\tau)\in[0;\delta) \\ t_{max} & (t-\tau)\in[\delta;\infty) \end{cases} \qquad (3)$$

$$h(t-\tau) = \begin{cases} t_{min} & (t-\tau)\in[0;\delta) \\ t_{max} & (t-\tau)\in[\delta;\infty) \end{cases} \qquad (4)$$

$$h(t-\tau) = \frac{\beta}{1+e^{-\lambda\frac{(t-\tau)}{\alpha}+\gamma}} + t_{min} \qquad (5)$$

where $α$, $λ$ are constant coefficients, $β$ is the time range of the possible warm-up durations ($β = t_{max} - t_{min}$), $γ$ is a translation coefficient ($γ = 2\ln β$), and $δ$ is the instant at which the warm-up duration reaches the maximum value ($δ = α\ln β$). For all of the proposed functions, it is possible to conclude that:
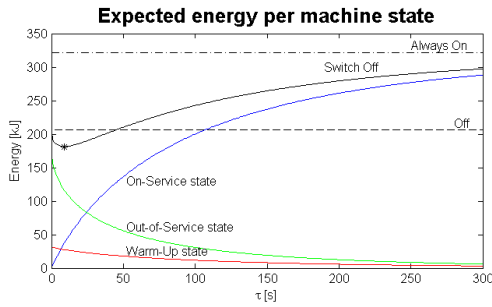
Fig.3. Expected energy adsorbed by the machine with the Switch Off applied. The contribution of each machine state is represented, together with energy constant values of Always On and Off policies.

- A high value of t_{min} represents machine tools without a real *hot idle mode*, i.e., machines requiring long warm-up;
- A high value of t_{max} refers to big-size machine tools that need time to reach thermal stability;
- Large α values mean that the transition from hot idle mode to cold idle mode requires a long time, e.g., the thermal inertia is high. High α values are often related to high t_{max} values;
- Large β values mean that the variability of the warm-up duration is high, probably due to machine size or process requirements, e.g. high quality. High β values are often related to high t_{max} values and α values.

3.2. Optimal switch-off time

The total energy consumed in a cycle is the sum of the product *power×time* for each state visited by the machine. The considered cycle starts from the departure of a part ($t = 0$) and finishes when the machine starts processing the next part. The part processing (i.e. machine working state) is not considered in the cycle because it does not affect the selection of the policy parameter τ. Since the time spent in a certain state by the machine during a cycle is the output of a stochastic process (indeed the arrivals are random), the expected value of the energy consumed in a cycle by the machine is the objective function to be minimized [4].

Let the part arrival time T (as defined in Section 2) be a continuous random variable, two different events may occur:

- *The part arrives before the machine is switched off*: $A_1 = \{T | T < \tau\}$. In this case the machine spends the whole cycle in the on-service state, and the expected value of adsorbed energy is:

$$g_1(T) = \mathrm{E}[x_{on}T | A_1] \tag{6}$$

- *The part arrives when the machine is out-of-service*: $A_2 = \{T | T \geq \tau\}$. This situation represents the fact that the machine has been switched off after τ time from the cycle starting, and it is waiting a part arrival to be switched on. During the time in which the part has not arrived yet the machine consumes x_{on} before being switched off, and it requires x_{out} once in the out-of-service state. When the part arrives the machine starts executing the warm-up during which the part has to wait, and the related power requested

is $x_{wu}+x_q$. Since the part arrival time T is a random variable, the out-of-service sojourn time (equal to $T - \tau$) is stochastic too as well as the warm-up time T_{wu}:

$$\begin{aligned} g_2(T) = x_{on}\tau &+ \mathrm{E}\big[x_{out}(T - \tau) | A_2\big] \\ &+ \mathrm{E}\big[(x_{wu} + x_q)T_{wu} | A_2\big] \end{aligned} \tag{7}$$

Let A=$\{A_1;A_2\}$ be the sample space composed of events A_i (with i=1, 2) mutually exclusive and collectively exhaustive, and $\mathrm{Pr}(A_i)$ be the probability of occurrence of event A_i, the total expectation theorem (c.f. [19, 20]) helps to calculate the expected energy adsorbed by the machine in a cycle:

$$\mathrm{E}[energy] = \sum_{A_i \in A} \mathrm{E}[g(T) | A_i]\mathrm{Pr}(A_i) \tag{8}$$

Let $f(t)$ be the PDF—probability density function— of the arrival time T, the expected value of the energy consumed by the machine in an average cycle is:

$$\begin{aligned} J = \mathrm{E}[energy] = x_{on}\int_0^\tau tf(t)dt &+ x_{out}\int_\tau^\infty (t-\tau)f(t)dt \\ + x_{on}\tau\int_\tau^\infty f(t)dt &+ (x_{wu} + x_q)\int_\tau^\infty h(t-\tau)f(t)dt \end{aligned} \tag{9}$$

If $h(t - \tau) \in C^1$, the expression in equation (9) is continuous and differentiable on the right-bounded interval $[0, \infty)$ and presents finite limits. As a consequence, equation (9) has both a maximum and a minimum on this interval, and the extremum (∞) occurs at a critical point (Weierstrass Extreme Value Theorem). Further, the minimum of J in equation (9) can be found by the proper selection of τ^* such that $J(\tau^*)$ is minimum over the parameter set $[0,\infty)$.

If the minimum of the function J in equation (9) occurs at the extremum (i.e., $\tau^* \to \infty$), it means that the machine is never switched off, or formally this can happen only after an infinite time, that is the *Always On policy* [4]. This policy is apt to maintain the machine in the proper condition to work, thus it is used for high utilized machines.

If the minimum of the function J in equation (9) occurs at *zero*, it means that the machine is triggered in the out-of-service state immediately after the part departure, resulting in the *Off policy* [4]. After the departure of a part the machine moves from working to the out-of-service state, and the warm-up state is visited after every arrival. This policy is adopted for low utilized machines or with negligible warm-up duration.

An example of optimal solution is shown in Fig. 3, together with the energy consumed in each state, and the energy consumed according to *Always On* and *Off* policies.

4. NUMERICAL ANALYSIS

A real CNC machining center with 392 dm³ of workspace, five linear axes, horizontal synchronous spindle, and local chiller cooling both spindle and axes is considered. The machine executes machining operations on an aluminum cylinder head for automotive purpose. The machine requires *5.35 kW* when on-service, and *0.52 kW* when out-of-service;

whereas the warm-up is characterize by a power consumption of *6 kW*. The penalty for part waiting is $x_q = 1\ kW$. The data reported has been acquired with dedicated experimental measurements. As an example, the power signals acquired in a measurement test are represented in Fig. 4. In order to model the machine operating in different situations and environments, the arrival of parts *T* is assumed to follow a Weibull distribution with mean t_a and shape parameter *k,* and several warm-up functions and durations are considered.
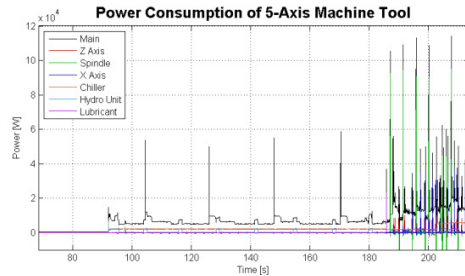

Fig.4. Power required by the analyzed machine tool during a progressive switch-on: out-of-service, warm-up, on-service, and working states.

4.1. Constant Warm Up Duration

A deterministic and constant warm-up duration is initially considered in the numerical analysis. A change of the arrival distribution affects the optimal solution τ^*. Fig. 5 shows that as the frequency of part arrivals decreases (from $t_a=10\ s$ to $t_a=100\ s$) the expected energy consumed increases. Moreover, it becomes more profitable to quickly switch-off the machine. This behavior also appears with other hazard rates *k*.

In Fig. 6 the optimal control parameter is represented over increasing t_a and warm-up duration T_{wu}. When the part is supposed to arrive in a short time ($t_a \rightarrow 0$), the optimal control is highly dependent on the warm-up duration. Indeed, for negligible warm-up time ($T_{wu} \rightarrow 0$) the machine should be switched off after few seconds. Whereas, for long warm-up time it is better to keep the machine on-service ($\tau^* \rightarrow \infty$) because there is no advantage in switching the machine off due to the warm-up energy request. By increasing arrival frequency, the importance of warm-up request on the energy consumed is decreasing, and the policy will optimally switch the machine as soon as the probability of an arrival decreases.

4.2. Exponential Warm-Up Duration

Let represent the warm-up duration with a negative exponential function, as the equation (2) in Section 3.1, where the parameters *α* and *β* can vary. In the first case (Fig. 7a; *α = 1*), parameter *β* does not affect the optimal point τ^*, that is the same as considering a constant warm-up time equal to the maximum t_{max}. The warm-up duration in such a case goes from the minimum value t_{min} to the maximum t_{max} in less than *5 s*. As a consequence, the situation is close to consider a constant value. In the second case (Fig. 7b; *α = 100*), parameter *β* is significant: when *β* approaches *zero* the effect over the optimal solution τ^* becomes negligible with respect to the constant case t_{max}, whereas increasing *β* leads to reduce the optimal point – i.e., to switch the machine earlier. For high values of *β* coupled with high values of *α,* the warm-up goes

slowly from t_{min} to t_{max}, and the optimal parameter τ^* is closer to the case of a constant warm-up time equal to the minimum t_{min}. This interaction existing between the parameters *α* and *β* is significant also for increasing hazard rates ($k \geq 1$).

4.3. Other functions

According to the previous considerations on *α* and *β*, the warm-up functions (1–5) proposed in Section 3.1 are analyzed considering *β = 40 s* and $t_a = 60\ s$. Constant cases are also included in the analysis. The energy consumed by the machine in a cycle assuming different warm-up time functions is represented in Fig. 8 for increasing values of *α*. In Fig. 8a the optimal parameter τ^* is almost constant with respect to function (1–5), and it is close to the solution for deterministic maximum time t_{max}. Whereas, increasing *α* (Fig. 8b), the different shapes of warm-up functions affect the results according to their derivative. Furthermore, if the maximum value of warm-up time is reached very slowly, i.e., for higher values of *α*, the solution tends to the deterministic minimum warm-up time t_{min} (Fig. 8c).

5. CONCLUDING REMARKS

A control policy has been studied analytically in this paper, considering a time-dependent warm-up duration, together with stochastic arrival times. The influence of arrival time distribution over the optimal solution has been discussed for different functions representing the warm-up time. Assuming a Weibull distribution for arrivals it is possible to remark that:

- Increasing the mean arrival time originates decreasing optimal control parameter independently from the shape parameter;
- The shorter is the warm-up time T_{wu}, the faster the machine should be switched off. The effect of warm-up time on the optimal control is reduced as arrival times increase;
- If T_{wu} changes within a narrow range ($\beta \rightarrow 0$), there is no difference in assuming variable or constant warm-up time;
- If T_{wu} reaches the maximum value in a time comparable to the arrival time, the warm-up time function affects the optimal control τ^*;
- f T_{wu} reaches the maximum value in a short time (slowly), the energy consumed is similar to the deterministic case associated with t_{max} and $\alpha \rightarrow 1$ (t_{min} and $\alpha \rightarrow \infty$).

The remarks above hold also for other distributions different from the Weibull; the only condition that must be satisfied is that the arrival probability distribution is unimodal.
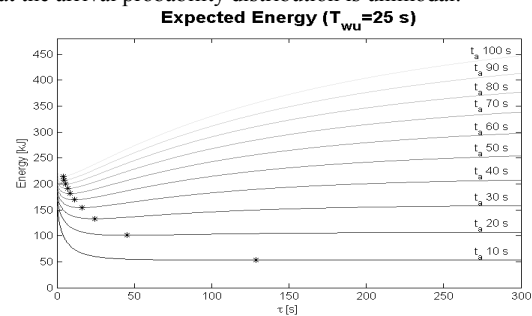

Fig. 5. Expected consumption over τ varying the mean arrival time. Weibull arrival distribution with *k* = 0.6
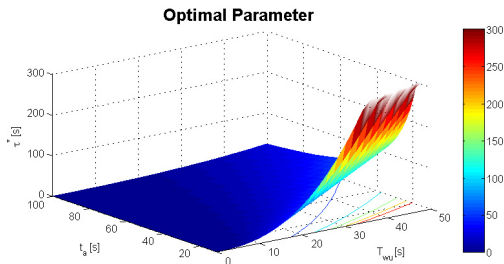
Fig. 6. Optimal control parameter τ varying the mean arrival time (Weibull distribution with k=0.6) and considering different warm-up times.
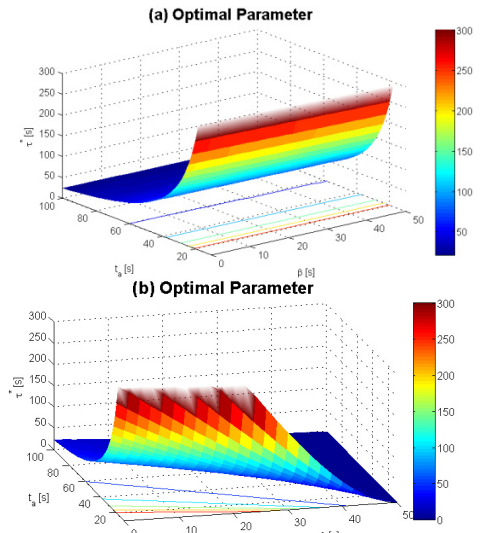

Fig. 7. Optimal control parameter varying the mean arrival time (Weibull distribution with *k*=0.6) and variable warm-up duration. (a) *α*=1 ; (b) *α*=100.

6. ACKNOWLEDGEMENTS

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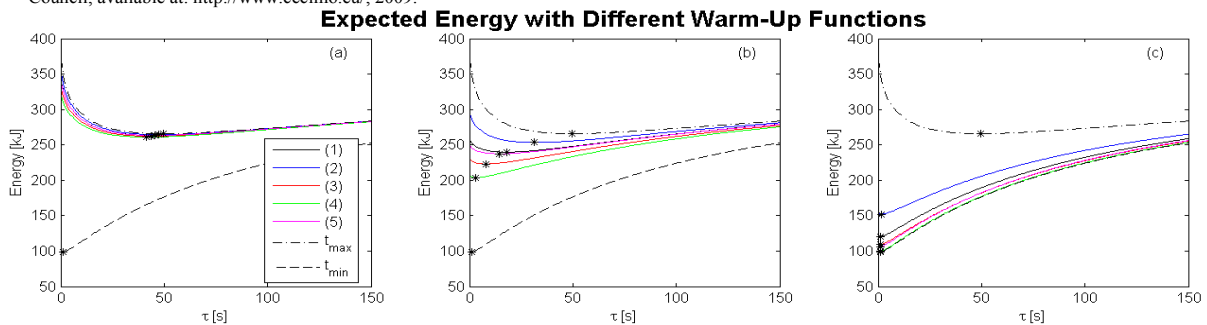
Fig.8. Expected energy required over *τ* (Weibull with *t*_a=60 s, k=0.6) and variable warm-up duration (*t*_{max}=50 s, *t*_{min}=10 s). (a) *α=1* ; (b) *α=10* ; (c) *α=200*.