Skin-friction drag reduction by spanwise forcing: the Reynolds-number effect

Maurizio Quadrio[1], Davide Gatti[2]

[1]Politecnico di Milano
[2]Karlsruhe Institute of Technology

EDRFCM 2015, Cambridge, Mar 25, 2015

The starting point

- Spanwise wall forcing (oscillating wall, traveling waves, etc) is very effective in reducing turbulent skin-friction drag:

$$W(x,t) = A\cos(\kappa_x x - \omega t)$$

- Large positive energy budget is possible
- Current knowledge (DNS, experiments) mostly comes from data at low *Re*
- However, envisaged applications are at high *Re* !!

Drag reduction and *Re*

A power law decrease?

- Drag reduction R decreases with Re
- Earlier attempts assumed $R \propto Re_\tau^\gamma$ with $\gamma = -0.2$
- Recent discovery: $\gamma = \gamma(A, \kappa_x, \omega)$

Building a new database

Large, reliable, complete

- DNS of turbulent channel flow at CFR
- $Re_\tau \approx 200$ and $Re_\tau \approx 1000$
- Modest size of the computational domain ($L_x = 1.4h$, $L_z = 0.7h$)

Large!

- More than 4,000 DNS datapoints

Building a new database

Large, reliable, complete

- DNS of turbulent channel flow at CFR
- $Re_\tau \approx 200$ and $Re_\tau \approx 1000$
- Modest size of the computational domain ($L_x = 1.4h$, $L_z = 0.7h$)

Reliable!

- Control simulations (CFR, CPG) with larger domains
- Uncertainty

Building a new database

Large, reliable, complete

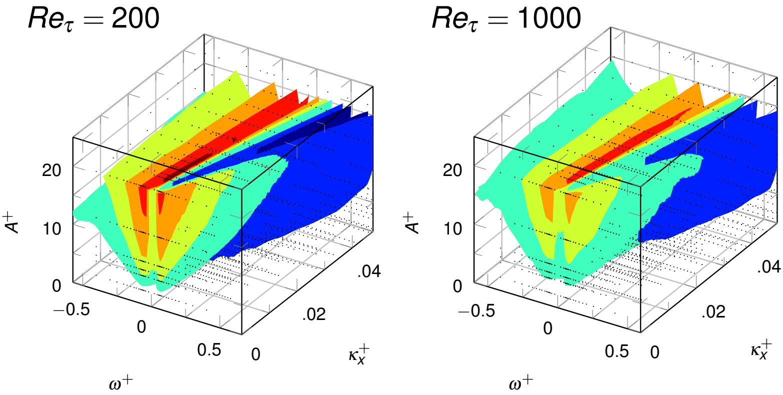
- DNS of turbulent channel flow at CFR
- $Re_\tau \approx 200$ and $Re_\tau \approx 1000$
- Modest size of the computational domain ($L_x = 1.4h$, $L_z = 0.7h$)

Complete!

- 3-parameter study (A considered for the first time)

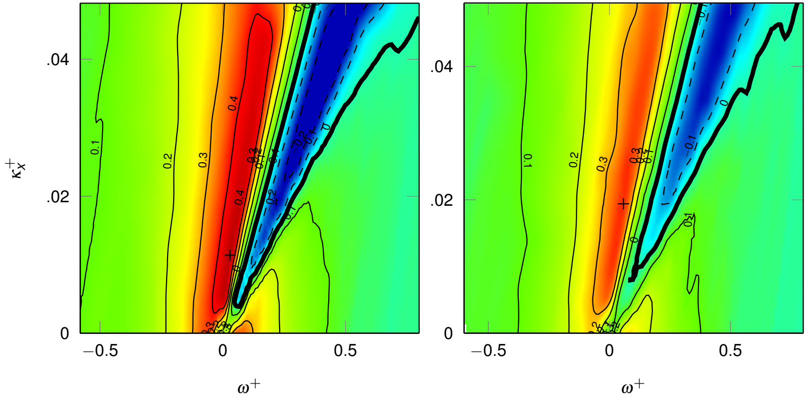
Results: global view

Map of drag reduction R



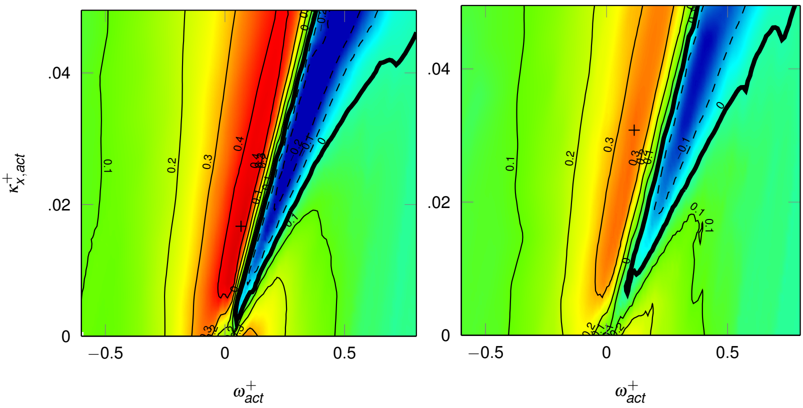
Travelling waves at $A^+ = 12$: outer scaling

Left: $Re_\tau = 200$. Right: $Re_\tau = 1000$

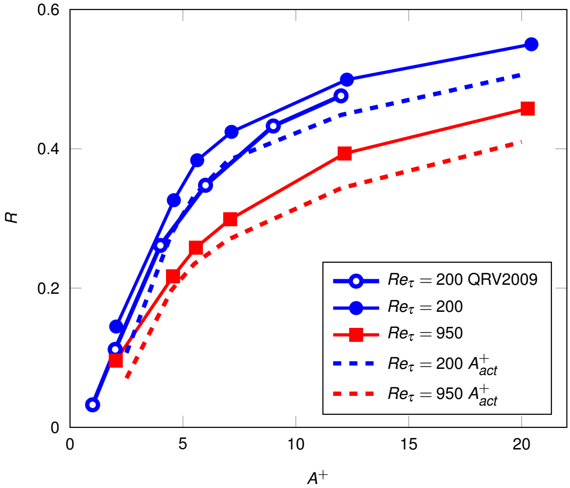


Travelling waves at $A^+ = 12$: inner scaling

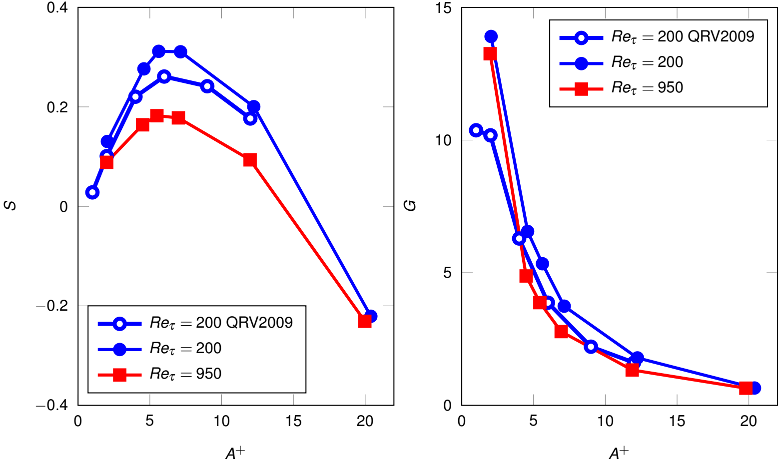
Left: $Re_\tau = 200$. Right: $Re_\tau = 1000$



Maximum R: outer vs inner scaling

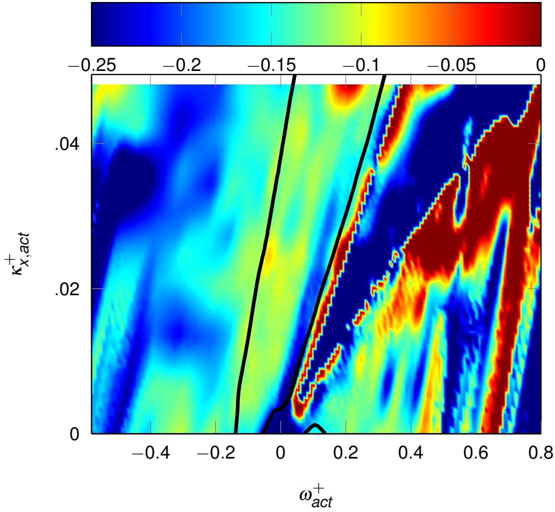


Maximum S and G



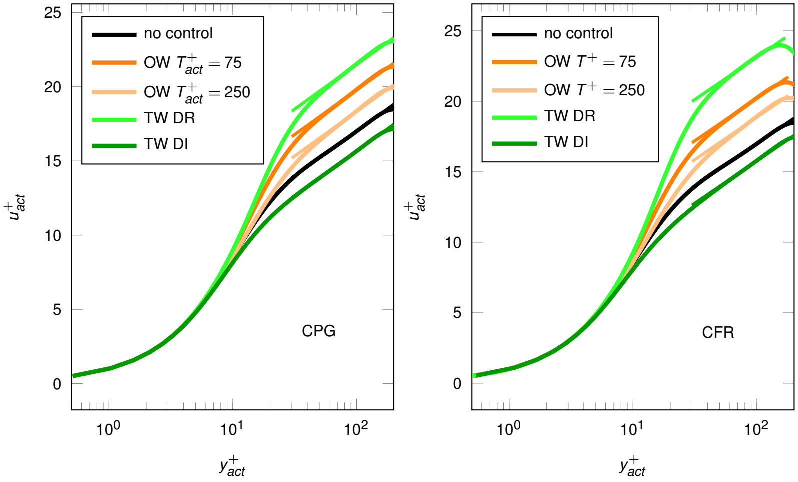
What about the *Re* effect?

γ is not the best quantity to describe it



Vertical shift of the mean velocity profile

Large-scale simulations at $Re_\tau = 200$



The Prandtl – von Kármán friction law

Log law:

$$u^+ = \frac{1}{\kappa} \ln\left(\frac{y}{\delta_v}\right) + B$$

Defect law:

$$U_c^+ - u^+ = \frac{1}{\kappa} \ln\left(\frac{y}{\delta}\right) + B_1$$

Adding together and using $U_c^+ = U_b^+ + 1/\kappa$:

$$\sqrt{\frac{2}{C_f}} = \frac{1}{\kappa} \ln Re_\tau + B + B_1 - \frac{1}{\kappa}$$

A simple subtraction

Writing P-vK for flow with / without control

$Re_{\tau,0}$ and $C_{f,0}$ without control, Re_τ and C_f with control

$$\sqrt{\frac{2}{C_f}} - \sqrt{\frac{2}{C_{f,0}}} = \frac{1}{\kappa} \ln \frac{Re_\tau}{Re_{\tau,0}} + \Delta B + \cancel{\Delta B_1}$$

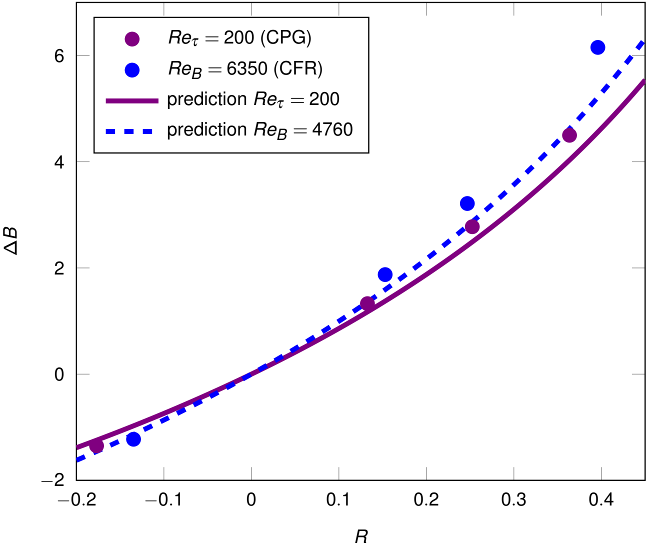
CFR: $C_f = C_{f,0}(1-R)$ and $Re_\tau = Re_{\tau,0}\sqrt{1-R}$:

$$\boxed{\Delta B = \sqrt{\frac{2}{C_{f,0}}} \left[(1-R)^{-1/2} - 1 \right] - \frac{1}{\kappa} \ln (1-R)^{1/2}}$$

CPG: $Re_\tau = Re_{\tau,0}$:

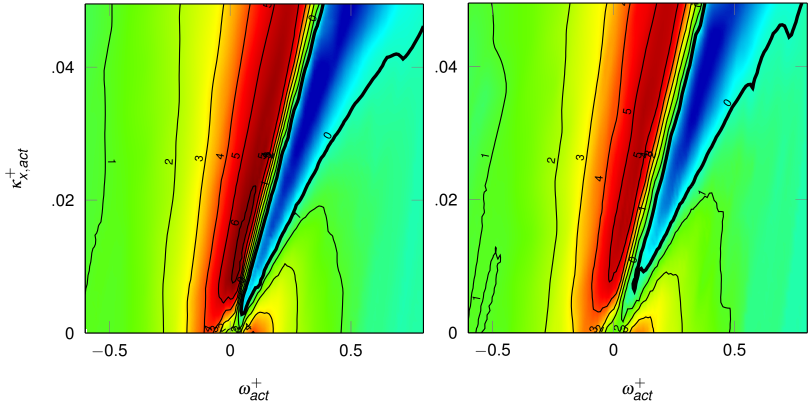
$$\boxed{\Delta B = \sqrt{\frac{2}{C_{f,0}}} \left[(1-R)^{-1/2} - 1 \right]}$$

Check at $Re_\tau = 200$

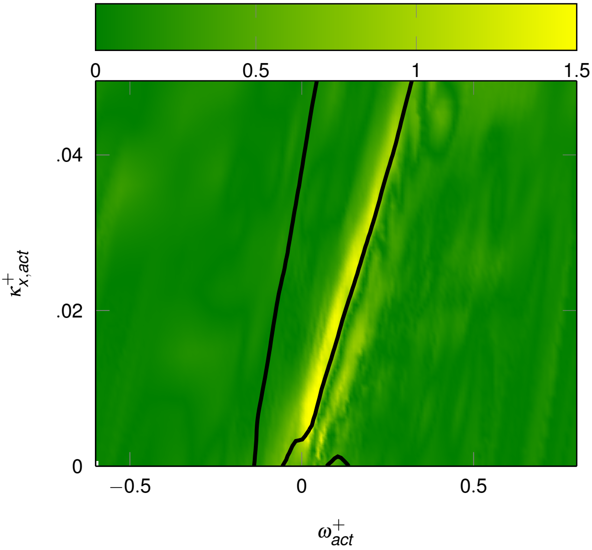


Map of ΔB

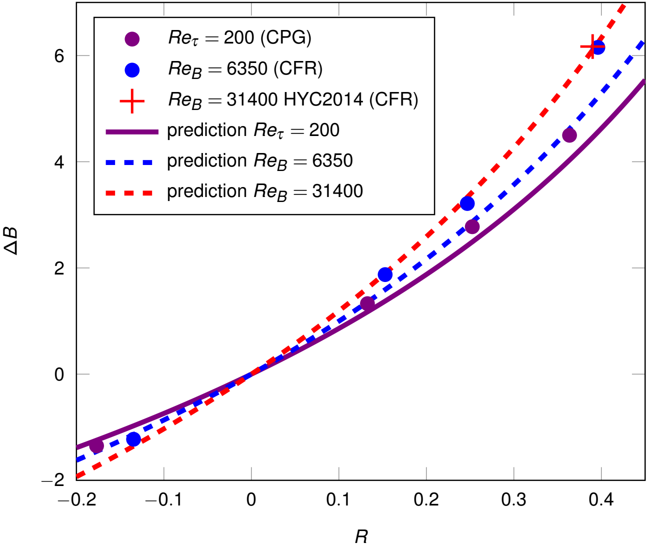
$A^+_{act} = 12$ at $Re_\tau = 200$ (left) and $Re_\tau = 1000$ (right)



Change of ΔB from $Re_\tau = 200$ to $Re_\tau = 1000$

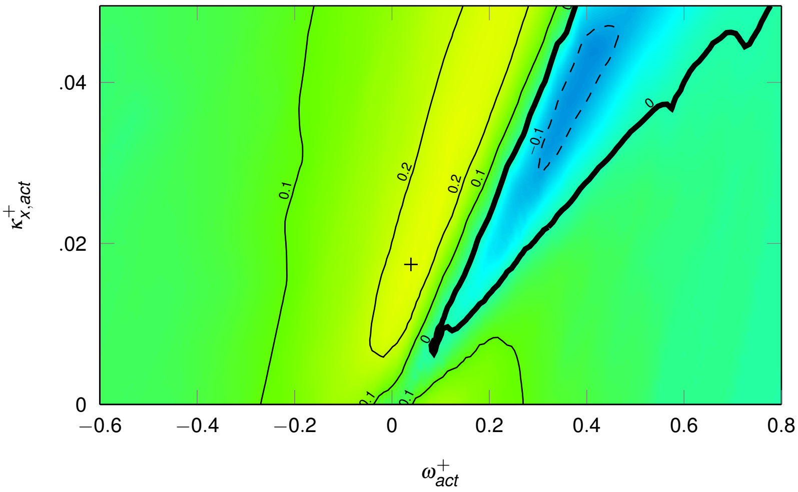


An indipendent check



Extrapolation to $Re_\tau = 10^5$

Assumption: ΔB at $Re_\tau = 1000$ remains constant

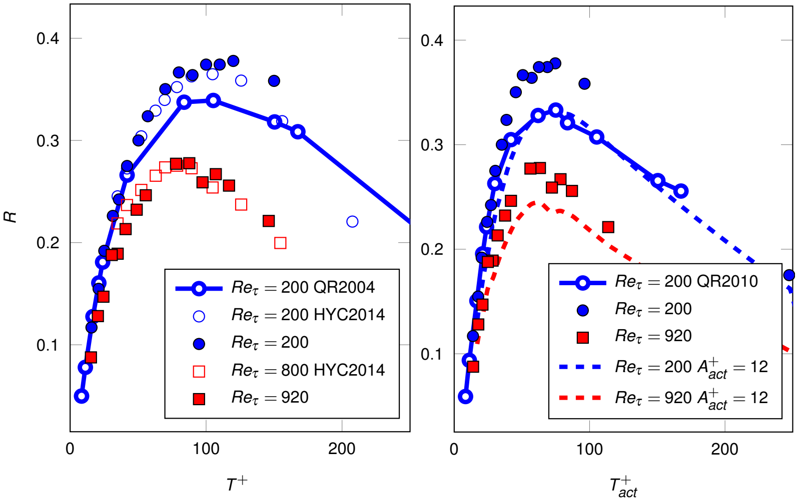


Conclusions

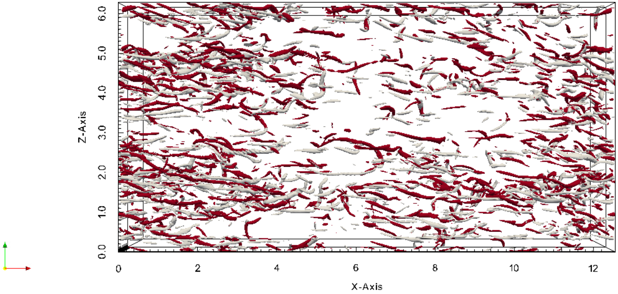
- Drag reduction is not as simple as "percentage change of C_f"
- A wall-based control scheme (like riblets, etc) is characterized by its ΔB
- ΔB is constant with (not too low) Re
- Extrapolation to flight-level Re is possible

Oscillating wall at $A^+ = 12$

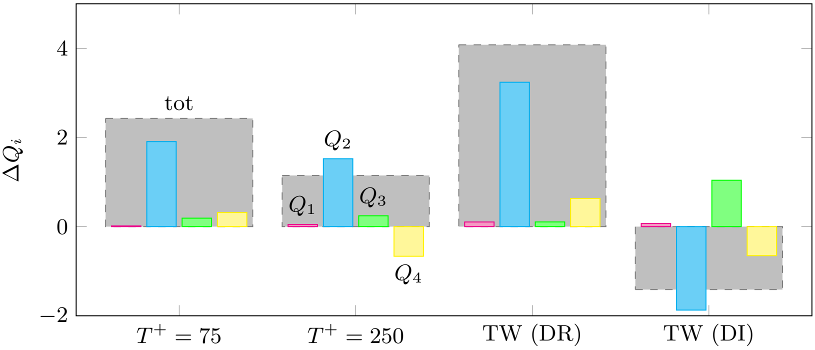
Left: outer scaling. Right: inner scaling.



Eduction and conditional analysis



Quadrant analysis



The Prandtl – von Kármán friction law

$$\frac{U_c}{u_\tau} = \frac{1}{\kappa} \ln \left[Re_c \frac{u_\tau}{U_c} \right] + B + B_1$$

$$\sqrt{\frac{2}{c_f}} = \frac{1}{\kappa} \ln Re_\tau + B + B_1$$

$$\frac{U_c - U_b}{u_\tau} = \frac{1}{\kappa}$$

$$\sqrt{\frac{2}{C_f}} = \frac{1}{\kappa} \ln Re_\tau + B + B_1 - \frac{1}{\kappa}$$

Prandtl - von Karman (2)

$Re_{\tau,0}$ and $C_{f,0}$ without control, Re_τ and C_f with control

$$\sqrt{\frac{2}{C_f}} - \sqrt{\frac{2}{C_{f,0}}} = \frac{1}{\kappa} \ln \frac{Re_\tau}{Re_{\tau,0}} + \Delta B$$

With CFR $C_f = C_{f,0}(1-R)$ and $Re_\tau = Re_{\tau,0}\sqrt{1-R}$:

$$\Delta B = \sqrt{\frac{2}{C_{f,0}}} \left[(1-R)^{-1/2} - 1 \right] - \frac{1}{\kappa} \ln (1-R)^{1/2}$$

With CPG $Re_\tau = Re_{\tau,0}$:

$$\Delta B = \sqrt{\frac{2}{C_{f,0}}} \left[(1-R)^{-1/2} - 1 \right]$$