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A LAGRANGIAN PARTICLE FINITE ELEMENT METHOD FOR FLUID-STRUCTURE INTERACTION PROBLEMS

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Abstract. In this work a Lagrangian finite element approach is developed for the solution of fluid-structure interaction problems. The fluid part is solved with the Particle Finite Element Method, while the structural part is solved with a classical finite element approach. Particular provisions have been implemented for the identification of the interaction boundary and for the stabilization tecnique.

1 INTRODUCTION

In fluid-structure interaction problems the computational treatment of the interfaces between solids and fluids is always critical. Different approaches are possible in Eulerian and ALE approaches to solve this problem, like volume of fluid and level sets. Another possibility to overcome this difficulty is to adopt a Lagrangian approach for both fluid and structure. With this approach the interfaces between the two phases are naturally identified by the node positions avoiding the necessity of a method to track the interfaces.

In the present work a fluid-structure interaction algorithm based on a staggered approach is presented. The fluid part is treated in a Lagrangian framework using a version [3] of the Particle Finite Element Method [1] [2] and the structural part using a classical finite element method.

The Particle Finite Element Method (PFEM) is a method to solve fluid problems based on a Lagrangian approach, particularly suitable for the solution of flow problems characterized by free-surfaces and breaking waves. The difficulty of a Lagrangian approach is the necessity to regenerate the mesh whenever the distortion of the elements becomes excessive. A remedy which alleviates the burden of a complete remeshing consists of regenerating the connectivity of the elements by means of efficient implementation of a Delaunay triangulation keeping fixed the node (called particle in this approach) positions. This has the additional advantage that the information stored at the nodes need not to be convected to new nodes. The Delaunay triangulation generates the convex hull of the particle distribution, so that, to define the integration domain and to correctly impose the boundary conditions, a method to identify the external boundary is necessary. This is achieved using a criterion based on the mesh distortion called *alpha shape method* [1] [2]. The same criterion allows to identify the particles which separate from the rest of the domain. These separeted particles are treated as concentrated masses moving under the action of the external forces and their initial velocity.

When the connectivity between elements is regenerated, data have to be transmitted from the old mesh to the new one. To avoid interpolation from mesh to mesh, only nodal degrees of freedom are used, so that only linear shape functions can be used for the discretization of both velocity and pressure. However, it is well known that this type of discretization does not satisfy the *inf-sup compatibility condition* so a stabilization method is required. To this purpose a *pressure-stabilizing Petrov-Galerkin* (PSPG) stabilization is introduced in the formulation [4].

In a Lagrangian approach, the nodes of the mesh move as a consequence of the fluid flow and it may happen that the fluid particles concentrate in some regions of the domain while they space out, so that in others the number of points becomes too low to obtain an accurate solution. To overcome these difficulties, in the proposed implementation the possibility of adding and removing points has also been introduced.

2 FLUID-STRUCTURE INTERACTIONS

The Particle Finite Element is particularly suitable for the solution of fluid-structure interaction problems. To this purpose a staggered scheme is developed in which the PFEM is used to solve the fluid part of the problem and a classical finite element method is used for the solid part. The coupling algorithm is based on the superposition of a set of fictitious fluid particles to the nodes of the solid domain, which can come in contact with the fluid domain. When the Delaunay triangulation is performed, the alpha-shape criterion selects those parts of the interface where the fluid particles are actually in contact with the structure. Figure 1 shows the superposition of fictitious particles and the generation of two distinct domains (Figure (a), (b) and (c)) and of a coupled domain (figure (d) and (e)). If the two discretized domains are not in contact, the fluid and the solid analyses are performed separately without any interaction. If instead the two discretized domains are in contact, a coupled analysis is performed using a Dirichlet-Neumann iterative approach, as explained below.

For every time step:

1. solve the structural-dynamics problem considering the fluid-solid interfaces as Neumann boundaries for the structure;

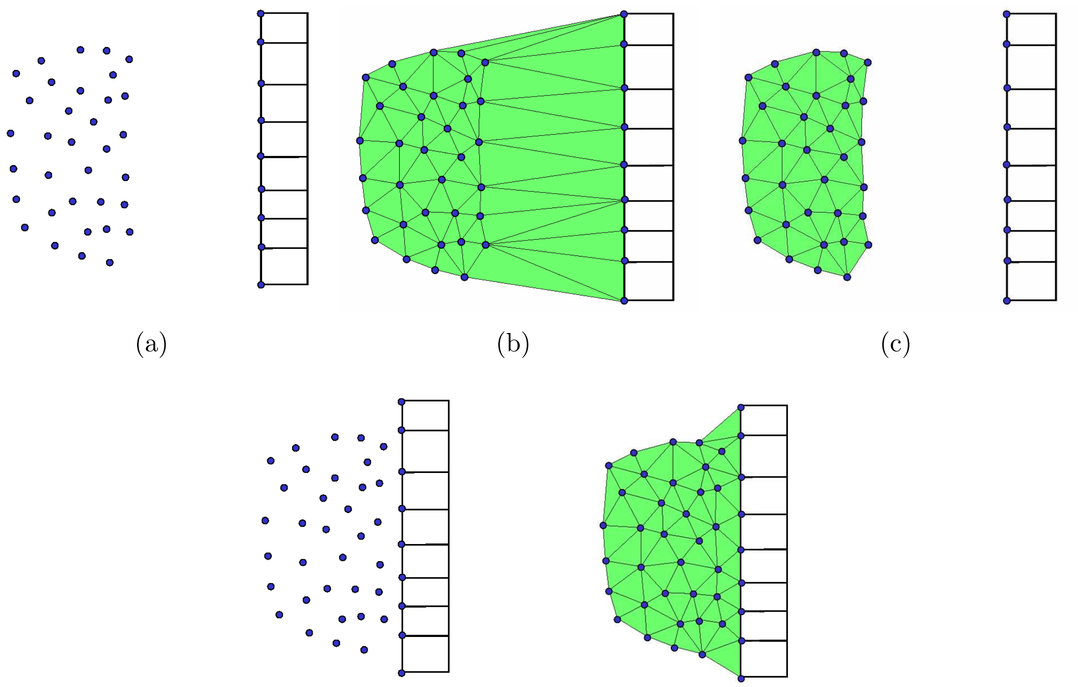
2. update the discretization of the fluid domain according to the new configuration of the boundary;

3. solve the fluid-dynamics problem considering the fluid-solid interfaces as Dirichlet

boundaries for the fluid;

4. compute the fluid stress resolved on the current fluid-solid interface;

If convergence is not reached, go back to step 1 solving again the structural problem under the updated value of boundary tractions on the fluid-solid interface.



(a) (b) (c)

(d) (e)

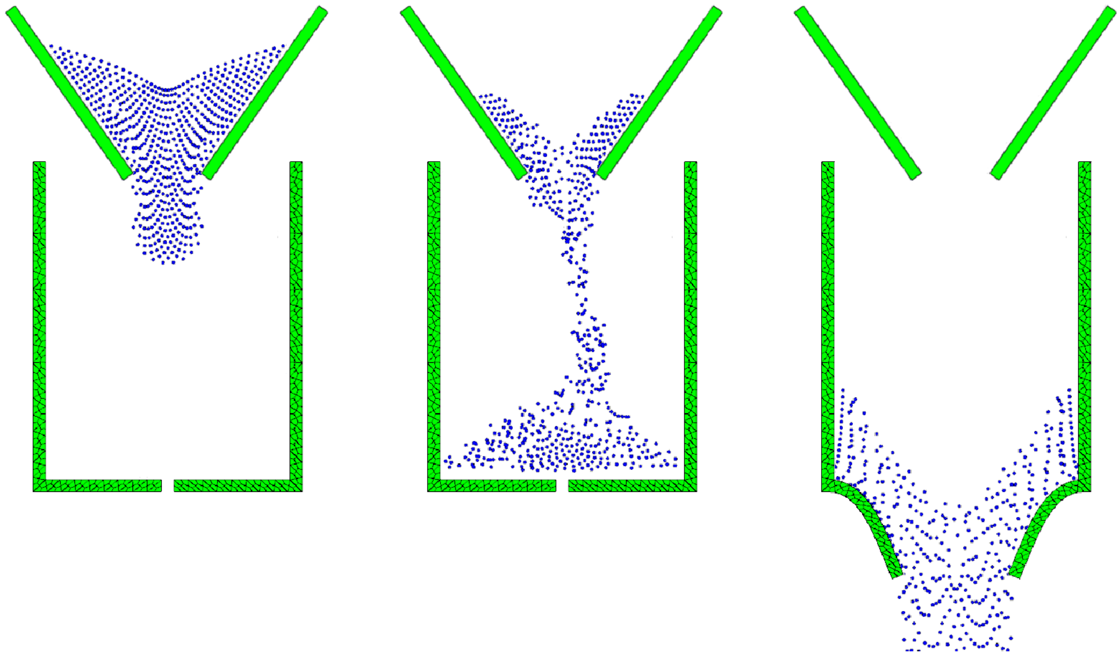
Figure 1: Sketch of fluid structure interaction: (a) superposition of the fictitious fluid particles. (b) Delaunay triangulation; (c) Delaunay triangulation with α-shape correction; (d) same as (a) but with the two domains in contact; (e) same as (c) but with the two domains in contact.

3 NUMERICAL EXAMPLE

The proposed Lagrangian approach is particularly suited to model fluid-structure interactions involving large fluid motions with large structure displacements. To give a qualitative example of the potential of the method, the following idealized problem has been modeled.

A fluid flow hitting an elastic valve is considered. Under the action of the gravity force the fluid drops down from a funnel-shaped rigid container into another rigid container. When all the fluid particles are fallen down, the rigid bottom is suddenly removed, so that the fluid, due to its weight pushes on the elastic valve which deforms and let the fluid pass through the valve.

In this example, all the capabilities of the implemented code are required: a flow with free surface and fluid-structure interaction, in which the solid undergoes large displacements, are considered. Figure 2 shows snapshots of the problem configuration at different time steps, in which the dots represent the nodes of the fluid mesh (here identified as fluid particles) while the structure is discretized by means of quadrilateral finite elements.



(a) (b) (c)

Figure 2: Fluid flow through an elastic valve: snapshots at different time steps

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