



16th Conference on Water Distribution System Analysis, WDSA 2014

## Probabilistic modeling of rainwater tanks

A.Raimondi<sup>a\*</sup>, G. Becciu<sup>a</sup>

<sup>a</sup>Politecnico di Milano, Piazza L. da Vinci 32, 20133 Milano, Italy

---

### Abstract

The use of rainwater tanks is always more encouraged, to achieve a dual objective: water saving from water distribution systems and reduction of runoff discharged into drainage systems. In last decades, different approaches have been proposed for their modeling, most of which are empirical or based on continuous mass balance simulations. To combine the simplicity of practical approaches and the accuracy of continuous simulations some authors have proposed the use of probabilistic approaches; their applications, in literature, still have some limitations. This paper seeks to improve the modelling of rainwater tanks by mean of an analytical probabilistic approach.

© 2014 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/3.0/>).

Peer-review under responsibility of the Organizing Committee of WDSA 2014

*Keywords:* rainwater tanks; probabilistic modeling

---

### 1. Introduction

The continuous population's growth and the increasingly stringent environmental regulations pose important challenges in the design of modern water infrastructures. Rainwater Harvesting Systems (RWHSs) can play an important role in the development of sustainable urban water systems. In the past decades, RWHSs have become a significant source of water supply in Africa, Asia and South America. Also in countries that have no serious concerns about water supply, thanks to existing extended water infrastructures, rainwater harvesting can represent a good practice in terms of sustainable development [1].

Rainwater collected from roofs are generally not much polluted and can be used with low or no treatment for some uses, such as WC flushes, washing machines and irrigation, fire suppression, street washing, etc.. The contribution to residential and industrial water needs of RWHSs cannot be not so high in most cases, mainly due to

---

\* Corresponding author. Tel.: +039-02-2399-6228; fax: +039-02-2399-6208.

E-mail address: [anita.raimondi@polimi.it](mailto:anita.raimondi@polimi.it)

rainfall variability and to difficulties of long-term water storing. However, there are also benefits from an environmental point of view, because RWHSs also reduce stormwater runoff to urban drainage systems and consequently help in the pollution reduction of downstream receiving water bodies. For all these reasons some countries encourage RWHSs by subsidies or by the introduction of regulations, which make their implementation convenient or, in some cases, mandatory.

The main element of a rainwater harvesting system is the storage tank. In literature, different approaches have been proposed for its modeling and design. They can be classified mainly in four types: simplified methods based on user-defined relationships [2], continuous mass balance simulations [3, 4, 5], non-parametric approaches based on probability matrix methods [6, 7] and statistical methods [8, 9, 10].

Simplified methods generally are very easy to be applied, but their results are not very reliable due to poor modelling of rainfall and/or of water storing processes. Typical examples of these approaches are demand side and supply side approaches. Demand side approach neglects rainfall, assuming that the roof is large enough to capture the required amount of rainwater before the use, and the tank design is based only on the water needs and the duration of the period of water scarcity [3]. Supply side approach, on the contrary, does not take into account water demand and use only water availability for the design of storage tank. In [2], a simplified method to model rainwater tanks, combining both demand and supply side approaches, is also proposed. All these simplified approaches are useful mainly for preliminary design [11].

Continuous mass-balance simulation approach may bring to more reliable results, providing that correct data on rainfall and water demand dynamics are used. This approach is popular also because it can be applied with simple mathematical tools as spreadsheet applications. More, the behavior of the system can accurately mimicked and it is easy to incorporate seasonal changes in rainfall or water demand [3]. Main differences in the models proposed in literature are the finite-difference scheme used in the water storage modelling. The simpler ones are called “Yield Before Spill” (YBS) and “Yield After Spill” (YAS). With the first one (YBS), the water is supposed to be abstracted for use before the inflow at each time step and this leads to underestimation of the storage volume that is needed. The opposite happens with second one (YAS), which is more conservative and then is usually preferred. Other schemes, intermediate between the two cited above, have been proposed [3].

With this type of approaches, the reliability of results is strongly related to the use of long continuous records of rainfall [4] and often these records are difficult to find or are not available at all. In order to overcome this kind of difficulties, probabilistic approaches have been proposed [8, 9, 10 and 12]. With these methods, based on analytical derivation of probability distribution functions of design parameters, probabilistic modelling of the storage process is possible without the need of continuous simulations. Methods proposed by [8] and [9] are analytically robust and locally useful, but are difficult to adapt for general use in other locations because the statistical characteristics of the precipitation record are typically hardwired into results. In [10] a parametric rainfall simulation approach is used to adapt probabilistic models to different locations. The main limit of this method is that it considers a maximum of two isolated rainfalls instead of the whole chain of events, assuming the tank full at the end of the first one.

This paper seeks to overcome to these two limits and to improve the use of analytical probabilistic approaches to the modeling of rainwater tanks, also considering the number of rainfall events in the reference period. Proposed equations result function of the stochastic rainfall process, water demand volume and system characteristics. To test their reliability, an application to a case study is presented.

## 2. Probabilistic modeling

Generally, a RWHS can be schematized as shown in Fig. 1, where  $h$ : rainfall depth;  $Y$ : water yield;  $V$ : water storage;  $D$ : water demand;  $Q$ : tank inflow;  $S$ : tank capacity;  $A$ : roof area;  $M$ : water from distribution system; and  $O$ : tank overflow

In engineering practice, the feasibility of a rainwater system is often evaluated by comparing the water demand  $D$  with the mean rainfall volume harvested by the roof, evaluated by the product of the roof area  $A$  by the mean number  $M$  and the mean depth  $\mu_h$  of rainfall events in the regulation time  $T_R$ . Although this approach is very simple and effective in many cases, especially when hydrological data are scarce, if the reliability of the harvesting system has to be evaluated a probabilistic analysis is needed.

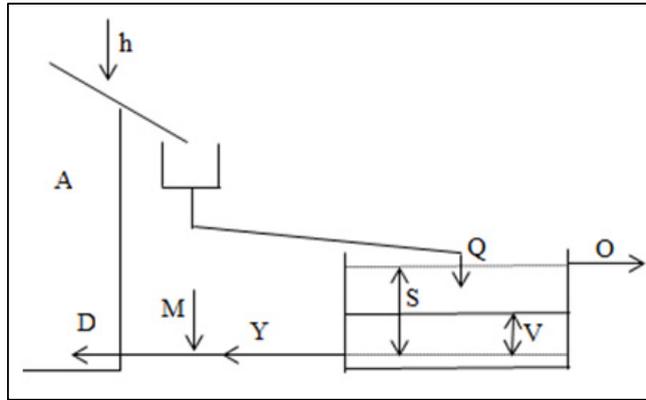


Fig 1. Typical configuration of a RWHS.

For small catchments, as roofs are, inflow is approximately proportional to rainfall intensity and runoff to rainwater tank  $Q$  can be expressed by:

$$Q = A \cdot (\varphi \cdot h - w_f) \tag{1}$$

where  $\varphi$  is the roof runoff coefficient and  $w_f$  is the first flush rainfall volume (specific for unit of area) that is diverted from the tank to avoid pollution from roof washing. Obviously, tank inflow and consequently mean water yield depend mainly on rainfall depth  $h$ . Although, in many cases, the Weibull PDF fits better the cumulated frequencies curves of rainfall records [16], an exponential distribution is mathematically more convenient and so it is often considered [14, 15]:

$$f_h = \xi \cdot e^{-\xi h} \tag{2}$$

where  $\xi = 1/\mu_h$  is the scale parameter and  $\mu_h$  is the average rainfall depth. A good trade-off alternative to exponential distribution may be the double-exponential, which in many cases fits better to observed hydrological data [17, 18].

If the stochastic process of rainfall storms is supposed to be Poissonian, the number  $N$  of independent events in the regulation time  $T_R$  is a random variable with the following PDF:

$$f_N = \frac{M^N \cdot e^{-M}}{N!} \tag{3}$$

where  $M$  is the mean of  $N$ . The asymptotic upper limit of  $N$  is:

$$N_{max} = \max(N) = 1 + T_R / IETD \tag{4}$$

Assumed a random number  $N$  of storm events, the probability  $P_R$  that the rainfall runoff is sufficient to satisfy the water demand  $D$  in the regulation time  $T_R$  can then be expressed by the relation:

$$P_R = \frac{\sum_{N=1}^{N_{max}} \left\{ \frac{e^{-M} \cdot M^N}{N!} \cdot \text{Prob}[\sum_{i=1}^N A \cdot (\varphi \cdot h_i - w_f) > D] \right\}}{\sum_{N=1}^{N_{max}} \frac{e^{-M} \cdot M^N}{N!}} = \frac{\sum_{N=1}^{N_{max}} \left[ \frac{M^N}{N!} \cdot \text{Prob}\left(h_N > \frac{D}{A \cdot \varphi} + \frac{N \cdot w_f}{\varphi}\right) \right]}{\sum_{N=1}^{N_{max}} \frac{M^N}{N!}} \tag{5}$$

where:

$$h_N = \sum_{i=1}^N h_i \tag{6}$$

is a random variable expressing the sum of rainfall depths in  $N$  consecutive storm events. It is to be noted that, according to equation (3), the probability of a number  $N$  of storm events decreases rapidly as its value moves away from the mean  $M$  and the contribution of terms in the sums of equation (5) becomes less significant (see Fig. 2). So, a value  $N^* = 2 \cdot M$  in spite of  $N_{max}$  may be assumed, as it is largely acceptable for practical applications. More, considering that in temperate climatic zones  $M$  of the order of ten, with  $N^* = 2 \cdot M$  the approximation  $\sum_{N=1}^{N^*} (M^N / N!) \cong (e^M - 1)$  can be also applied with errors smaller than 0.1%. It has to be highlighted that  $N^*$  is then a parameter of the probabilistic model.

If the PDF of the rainfall depth is assumed to be exponential, the sum  $h_N$  from  $N$  consecutive independent events  $f_{h_N}$  has a Gamma PDF, that is:

$$f_{h_N} = \frac{\xi^N \cdot h_N^{N-1} \cdot e^{-\xi \cdot h_N}}{(N-1)!} \tag{7}$$

So, the probability in the right part of equation (5) can be written as:

$$Prob\left(h_N > \frac{D}{\varphi \cdot A} + \frac{N \cdot w_f}{\varphi}\right) = \int_{h_N = \frac{D}{\varphi \cdot A} + \frac{N \cdot w_f}{\varphi}}^{\infty} f_{h_N} \cdot dh_N = e^{-\xi \cdot \left(\frac{D}{\varphi \cdot A} + \frac{N \cdot w_f}{\varphi}\right)} \cdot \sum_{i=0}^{N-1} \frac{\left[\xi \cdot \left(\frac{D}{\varphi \cdot A} + \frac{N \cdot w_f}{\varphi}\right)\right]^i}{i!} \tag{8}$$

and the probability  $P_R$  can be expressed as:

$$P_R = \frac{\sum_{N=1}^{N^*} \left\{ \frac{M^N}{N!} e^{-\xi \cdot \left(\frac{D}{\varphi \cdot A} + \frac{N \cdot w_f}{\varphi}\right)} \cdot \sum_{i=0}^{N-1} \frac{\left[\xi \cdot \left(\frac{D}{\varphi \cdot A} + \frac{N \cdot w_f}{\varphi}\right)\right]^i}{i!} \right\}}{e^{M-1}} \tag{9}$$

If the mean rainfall yield  $R = \varphi \cdot A \cdot M \cdot \mu_h$  is introduced, equation (9) becomes:

$$P_R = \frac{\sum_{N=1}^{N^*} \left\{ \frac{M^N}{N!} e^{-M \cdot \left(\frac{D}{R} + \frac{N \cdot A \cdot w_f}{R}\right)} \cdot \sum_{i=0}^{N-1} \frac{\left[M \cdot \left(\frac{D}{R} + \frac{N \cdot A \cdot w_f}{R}\right)\right]^i}{i!} \right\}}{e^{M-1}} \tag{9'}$$

### 3. Case study

Equation (9') was applied for validation to a case study in Milano, Italy. Rainfall data continuously recorded at Milano-Monviso station during the period 1971-2005 have been considered. An IETD of 1 hour was considered, identifying  $N_{tot} = 4161$  events from the whole records series, whose average rainfall depth and average rainfall duration result respectively equal to  $\mu_h = 1 / \xi = 7,74$  mm and  $\mu_\theta = 4,34$  hours. An IETD = 1 hour is usually considered adequate to isolate independent rainfall storms in small catchment with short concentration times [19]. Roof surface was assumed equal to  $A = 250$  m<sup>2</sup> with a runoff coefficient of  $\varphi = 1,0$ . First flush volume has been neglected, that is  $w_f = 0$  mm was assumed.

Rainwater uses only for WC flushing and washing machines were considered. Water demand for modern WC has been assumed equal to  $W_{WC} = 40$  l/(person·day) while water demand for washing machines has been assumed equal to  $W_{WM} = 15$  l/(person·day), [20]. The total rainwater demand  $D$  in the regulation time  $T_R$  is then expressed as:

$$D = W_D \cdot P \cdot T_R \tag{10}$$

where  $W_D = W_{WC} + W_{WM} = 55$  l/(person·day) is the average daily water demand per person and  $P$  is the number of persons that use water.

A monthly regulation was considered, that is  $T_R = 720$  hours. The maximum number of rainfall events results  $N_{max} = 721$ . The average number of rainfall events  $M$  can be calculated by:

$$M = \frac{N_{tot} \cdot T_R}{N_A \cdot 365 \cdot 24} \tag{11}$$

with  $N_A = 35$  number of years in the data record series. For a monthly regulation  $M = 9,77$  events/month and  $N^* = 2 \cdot M = 19.54 \approx 20$  events/month.

Fig. 2 shows the probability  $P_R$  to satisfy water demand with rainwater collected from roofs as a function of the number  $N^*$  of consecutive independent storm events considered in the regulation time  $T_R$  and the number  $P$  of persons who demand water.

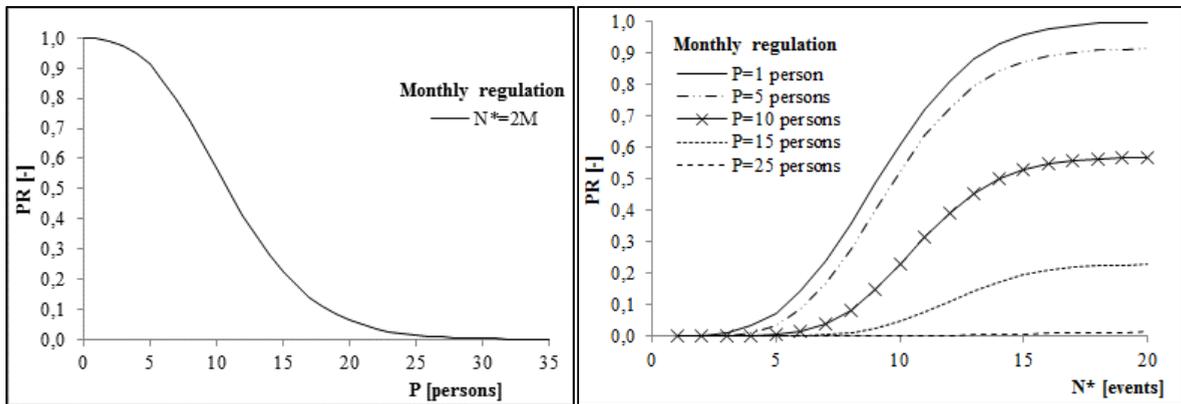


Fig. 2. Probability  $P_R$  : as a function of  $N^*$ , for different values of  $P$  (left); as a function of  $P$ , for  $N^* = 2 \cdot M$  (right).

Obviously, the probability of a complete water fulfillment decreases with the demand extent and increases with the number  $N^*$  of considered storm events. This relationship, however, is not linear, as can be seen by equation (9’).

Particularly, if the water demand coincides with the mean rainfall yield ( $D = R = \varphi \cdot A \cdot M \cdot \mu_h$ ), the probability of its complete fulfillment is of about 45%, that is less than 50% (see Fig. 2). In this case study, the mean rainfall yield is  $R = \varphi \cdot A \cdot M \cdot \mu_h = 18,91$  m<sup>3</sup>/month, corresponding to the water demand of about 12 persons.

Table 1. Monthly rainfall data in the period of records.

MONTH	1	2	3	4	5	6	7	8	9	10	11	12
$\mu_h$ [mm]	7,5	5,9	7,2	5,9	6,6	5,7	8,6	9,8	10,6	7,92	7,24	11,06
M [events]	11	10	12	16	19	13	8	10	8	13	12	9
$NR_{max}$ [events]	26	33	21	45	36	21	16	20	23	27	27	21

From Fig. 2 it is clear that higher levels of reliability can be achieved only for water demands  $D$  that are significantly lower than the mean rainfall yield  $R$ . If, for example, a return period of  $T = 10$  years (that is a probability  $P_R = 0.9$  of complete fulfillment of water demand  $D$  with rainwater) is considered for system design, a water demand of about 5 persons and a corresponding volume of the storage tank should be considered.

Monthly variation of  $P_R$  have been analyzed using rainfall data recorded in the period 1991-2005 at Milano-Monviso gauge station. Table 1 contains the average depths of monthly rainfall  $\mu_h$  and the average  $M$  and maximum  $NR_{max}$  numbers of recorded storm events in the series.

From data in Table 1, the recorded maximum monthly rainfall volume  $R_{max}$  can be calculated:

$$R_{max} = NR_{max} \cdot \varphi \cdot A \cdot \mu_h \tag{12}$$

as well as the maximum number of persons  $P_{max}$  whose water needs could be satisfied:

$$P_{max} = \frac{R_{max}}{W_D \cdot T_R} \tag{13}$$

Results of the application of equations (12) and (13) are shown in Table 2:

Table 2. Maximum monthly rainfall volume and number of persons whose water need can be satisfied.

MONTH	1	2	3	4	5	6	7	8	9	10	11	12
$T_R$ [days]	31	28	31	30	31	30	31	31	30	31	30	31
$R_{max}$ [m <sup>3</sup> ]	48,7	48,6	37,7	66,2	59,1	30,1	34,4	48,8	60,9	53,4	48,9	58,0
$R$ [m <sup>3</sup> ]	20,80	14,74	22,15	24,12	30,56	18,76	16,89	24,76	22,12	25,82	22,54	25,15
$P_{max}$ [persons]	29	32	22	40	35	18	20	29	37	31	30	34

Fig. 3 shows the variation of probability  $P_R$  during the year, for different hypothesis of water demand expressed in terms of persons, when sample monthly values of  $M$  are used.

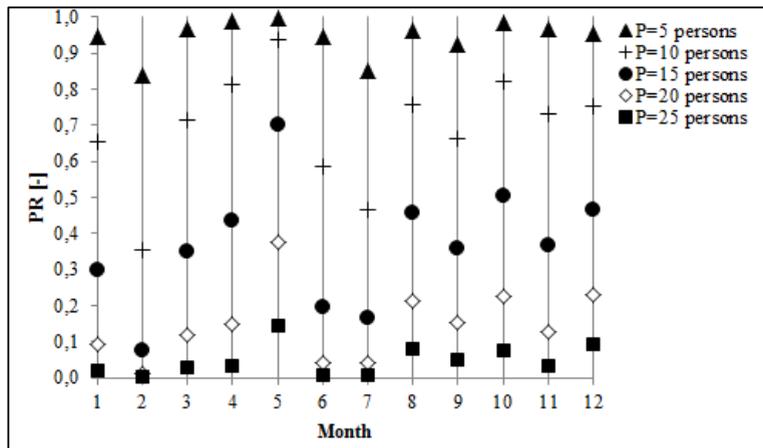


Fig. 3. Monthly probability of supplying water demand with rainwater for  $N^* = 2 \cdot M$ .

As can be seen, the influence of  $M$  on  $P_R$  is more significant for intermediate values of  $P$ , which is when the water demand  $D$  is around the mean rainfall yield  $R$ . For higher values of  $P$  (and so of  $D$ ), the relative small variations of  $M$  have a small influence on  $P_R$  values that are “flattened” on the extremes of its domain.

#### 4. Conclusions

The use of a probabilistic approach to the modelling of rainwater tanks allow to couple the simplicity of practical approaches to the accuracy of continuous simulations, also when only average rainfall data, instead of the continuous series of record, are available.

Proposed approach allows estimating the probability of supplying water demand for domestic use with rainwater

harvested by roofs. Resulting formulas are function of the average rainfall depth, the average and maximum number of rainfall events, water demand and catchment characteristics.

The estimation of this probability under different conditions can give valuable indications on whether to use a rainwater harvesting system, on its efficiency and on the opportunity of integration with water supplied from the water distribution system. This approach can be a useful and simple tool for preliminary studies for the design of rainwater tanks and to verify their proper functioning if water demand changes or/and with different climatic regimes.

## References

- [1] A. Palla, I. Gnecco, L.G. Lanza, Non-dimensional design parameters and performance assessment of rainwater harvesting systems, *Journal of Hydrology* 40 (2011) 65-76.
- [2] S. Ward, F.A. Memon, D. Butler, Rainwater harvesting model-based design evaluation, *Water Science and Technology* 61 (2010) 85-96.
- [3] A. Fewkes, D. Butler, Simulating the performance of rainwater collection and reuse systems using behavioural models, *Building Services Engineering Research and Technology*, 21 (2000) 99-106.
- [4] V.G. Mitchell, How important is the selection of computational analysis method to the accuracy of rainwater tank behaviour modelling?, *Hydrological Processes*, 21 (2007) 2850-2861.
- [5] C. Liaw, Y. Tsai, Optimum storage volume of rooftop rainwater harvesting systems for domestic use, *Journal of the American Water Resources Association*, 40 (2004) 901-012.
- [6] J.R. Cowden, D.W. Watkins, J.R. Mihelcic, Stochastic rainfall modeling in West Africa: parsimonious approaches for domestic rainwater harvesting assessment, *Journal of Hydrology*, 361 (2008) 64-77.
- [7] M. Basinger, F. Montalto, L. Upmanu, A rainwater harvesting system reliability model based on nonparametric stochastic rainfall generator, *Journal of Hydrology* 392 (2010) 105-118.
- [8] K.W., Lee, C.H. Lee, M.S. Yang, C.C. Yu, C.C., Probabilistic design of storage capacity for rainwater cistern systems, *Journal of Agricultural Engineering Systems* 77 (2000) 343-348.
- [9] M. Su, A probabilistic approach to rainwater harvesting systems design and evaluation, *Resources, Conservation and Recycling*, 53 (2009) 393-399.
- [10] Y. Guo, B.W. Baetz, Sizing of rainwater storage units for green building applications, *Journal of Hydrologic Engineering*, 12 (2007) 197-205.
- [11] E. Ghisi, Parameters influencing the sizing of rainwater tanks for use in houses, *Water Resources Management*, 24, (2010) 2381-2403.
- [12] B.J. Adams, F. Papa, *Urban Stormwater Management Planning with analytical probabilistic models*, John Wiley & Sons, New York, USA, 2000.
- [13] U.S. Environmental Protection Agency (USEPA), *Methodology for analysis of detention basins for control of urban runoff quality*, EPA440/5-87-001, Washington, D.C., 1986.
- [14] B.J. Adams, H.G. Fraser, H.G., C.D.D. Howard, M.S. Hanafy, Meteorological data analysis for drainage system design, *Journal of Environmental Engineering*, 112 (1986) 827-848.
- [15] P.B. Bedient, W.C. Huber, *Hydrology and floodplain analysis*, 2nd Ed., Addison Wesley, New York, USA, 1992.
- [16] B. Bacchi, M. Balistricchi, G. Grossi, Proposal of a semi-probabilistic approach for storage facility design, *Urban Water Journal*, 5 (2008) 195-208.
- [17] G. Becciu, A. Raimondi, Factors affecting the pre-filling probability of water storage tanks, *WIT Transactions: Ecology and the Environment*, 164 (2012) 12.
- [18] A. Raimondi, G. Becciu, On pre-filling probability of flood control detention facilities, *Urban Water Journal*, published on line DOI:10.1080/1573062X.2014.901398.
- [19] A. Raimondi, G. Becciu, Probabilistic design of multi-use rainwater tanks, *Procedia Engineering* (2014) 1391-1400.
- [20] UNI 11445, *Impianti per la raccolta e utilizzo dell'acqua piovana per usi diversi dal consumo umano: progettazione, installazione e manutenzione*, 2012.