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Coupling Equivalent Plate and Beam Models at Conceptual Design Level
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Structured Abstract:

Purpose
A formulation that couples equivalent plate model and beam model for aircraft structures analysis is presented, suitable in conceptual design where fast model generation and efficient analysis capability are required.

Design/methodology/approach
Assembling the complete model with common techniques such as Lagrange multipliers or penalty function method would require a solver capable of handling the combined set of linear equation. The alternative approach proposed here is based on a static reduction of the beam model at specified connection points and the subsequent ``embedding'' into the equivalent plate model using a coordinate transformation, translating physical degrees of freedom in Ritz coordinates, i.e. polynomial coefficients. Displacements and forces on beam elements are recovered with the inverse transformation once the solution is computed.

Findings
An aeroelastic trim analysis on a Transonic CRuiser (TCR) civil aircraft conceptual model validates the hybrid model: since the TCR features a slender flexible fuselage and a wide root chord wing, the capability to reduce the beam model for the fuselage at more than one connection point improved aeroelastic corrections to steady longitudinal aerodynamic derivatives.

Originality/value
Although the equivalent model proposed is simpler than others found in literature it offers automatic mesh generation capabilities and it is fully integrated into an aeroelastic framework. The hybrid model represents an enhancement allowing both dynamical and static analyses.

Introduction

Structural reduced order models –beams and plates– are widely used in aircraft optimization studies, especially when aeroelastic constraints and controls need to be included with ease in the problem formulation; they are also used in the conceptual and preliminary design phases to assess static and dynamic behavior before resorting to complete finite element models, thus trading accuracy for speed.

According to (Reddy, 2004) plate theories can be classified in three main categories: Classical Plate Theory (CPT), First Order Shear Deformation Theory (FSDT) and Higher Order Shear Deformation Theory (HSDT).

CPT is based on Kirchhoff–Love (Love, 1888) hypothesis that a straight line normal to plate middle surface, i.e. a section, remains straight and normal after deformation; sections don't extend and rotate remaining perpendicular to middle plane, which means both transverse shear and transverse normal effects are neglected.

FSDT, based on the Reissner–Mindlin model (Reissner, 1945; Mindlin, 1951), relaxes CPT's hypothesis: the sections are allowed to rotate but remain straight, which means shear stress along the thickness is uniform.

HSDT allows sections to deform following a generic polynomial rule: in (Reddy, 2004) is proposed a Third Order Shear Deformation Theory (TSDT) that models a parabolic variation of shear stress, which is closer to actual plate behavior; computationally expensive it offers a slight increase in accuracy compared to FSDT, making it unsuitable for conceptual design applications. CPT works well for thin isotropic plates, but for thick isotropic plates and thin laminated plates tends to overestimate the plate's stiffness because the effects of through–the-thickness shear deformation are neglected. FSDT is used for thick and anisotropic laminated plates owing to its simplicity and its low computational effort: it provides the best compromise of solution accuracy, economy and simplicity.

A further distinction is between linear and nonlinear: all listed theories can account for geometric nonlinear effects using Von Karman's model (moderately large displacements).

Equivalent plate models for aircraft wing studies date back to 1950s, developed by Alex Grzedzielski, see (Gallagher, 1964);starting from late 70s, many mathematical models based on equivalent plate representation combined with global Ritz analysis techniques have been developed. TSO (McCullers and Lynch, 1974) aeroelastic wing optimization code relies on CPT's kinematic assumptions to model low aspect ratio wings as equivalent plates, neglecting shear deformation in spar and rib webs; it was used for the analysis and preliminary design of wings of real airplanes, such as the YF16 and the F15, with quite good results (Lynch *et al*., 1977; McCullers, 1983; Triplett, 1980), but could only handle trapezoidal planforms.

An important enhancement in terms of wing structural modeling capabilities, the equivalent laminated plate solution (ELAPS) computer code, was developed by Giles (Giles, 1986, 1989), setting the basis for modern methods: under CPT's assumptions, composite general planform wing with symmetric/unsymmetric cross sections can be studied. Moreover, Giles showed that polynomial power series can be used to approximate displacements over the wing planform made of several trapezoidal segments, obtaining accurate information on stresses and displacements; the simplicity in handling power series leads to analytical integration instead of numerical, speeding up stiffness and mass matrices generation. ELAPS was used for multidisciplinary structural/aerodynamic optimization and design oriented structural analysis on a real aircraft (Kao *et al*., 1990; Giles, 1994; Chang *et al*., 1993), coupled with CFD codes (Tatum and Giles, 1987) for a static aeroelastic study on a TVC (Thrust to Vector Control) aircraft, coupled with membrane finite element via the penalty function method (Kao, 1992), and with beam finite elements (Giles and Norwood, 1994) using the penalty function, the transition element method and Lagrange multipliers.

Another equivalent plate analysis code, LS–CLASS (Lifting Surface Control Augmented Structural Synthesis), was developed at UCLA for aeroservoelastic applications and used to carry out structural/aerodynamic/control optimization for fiber composite actively controlled wings in an integrated approach (Livne, 1990; Livne *et al*., 1993): initially based on CPT model, trasverse shear effects (Livne, 1994b) – later added into ELAPS too (Giles, 1995) with a slightly different formulation – were included to study wings with few ribs attached to a long flexible fuselage (Livne *et al*., 1994); CPT-based equivalent plate approach leads to stiffer models, especially with high aspect ratio thick wing, and overestimates frequencies (largest errors on torsional modes).

Tizzi's formulation (Tizzi, 1997, 2000) allows the treatment of several non coplanar trapezoidal segments, but the internal parts of the wing's structure is not considered. Livne added to his FSDT model geometrically nonlinear effects (moderate deformations) (Livne and Navarro, 1999) and assessed model's limits on a joined wing aircraft (Demasi and Livne, 2005): the Von Karman Ritz plate formulation developed is effective on relatively simple structure, but loses accuracy and computational speed on three-dimensional complex configurations.

Interesting studies involving equivalent plate models encompass analytical sensitivities computation for shape optimization purposes (Livne, 1994a), probabilistic assessment of the design space of an innovative configuration (Mavris and Hayden, 1997), fuselage modeling (Giles, 1999), damage detection (Krishnamurthy and Mason, 2006), morphing applications (Gern *et al*., 2002, 2005; Ameri *et al*., 2008, 2009) and analysis of a composite wing with a control surface (Na and Shin, 2013)

Beam models are still popular in aeroelastic studies, but in actual wings with cutouts, wheel bay etc... defining the elastic axis it's not an easy task; moreover, when designing a composite wing, they can lead to inaccurate results if the effect of root boundary conditions is not taken into account. On the other hand, finite element analysis can be time–consuming, and if coarse meshes are adopted to speed up calculations, the results could be sensitive to the number and type of elements.

The equivalent plate is a reduced order structural model, that can represent a simple plate or a wing box, including spars, stringers and ribs. The wing planform and the structural layout depend analytically on a small set of shape and sizing design variables: model generation is fast compared to detailed finite elements and the (aeroelastic) optimization's problem is formulated on a small number of variables improving efficiency. Using the global Ritz solution technique, displacements are approximated with series of functions continuous over large portions of the wing, simplifying stress recovering, concentrated force/moment application, structural and aerodynamic grid interpolation; besides the Ritz coordinates, i.e. polynomial coefficients, don't have the same physical meaning of finite element nodal degrees of freedom, which represent three displacements and three rotations: a space coordinate transformation is needed, especially when coupling the model with FEM codes.

Other known drawbacks are the intrinsic matrix ill–conditioning – particularly true when using simple polynomials instead of an orthogonal basis (Giles, 1989) - and the lack of accuracy in reproducing steep stress gradients, i.e. near concentrated forces or constraints.

However, the equivalent plate method is suitable for the conceptual design phase, where many different aircraft layouts should be examined and the primary concern is a reasonable structural weight estimate

The aim of the present work is twofold: combining a beam model with the equivalent plate model in a so-called “hybrid” model, and assessing its static aeroelastic response (trim). As seen in the literature, the idea of coupling the equivalent plate model with other finite element models is not new, but in this work the two models are mixed together in the same variables’ space, i.e. Ritz polynomial space, overcoming the limitation of the penalty function method and transition element method, since it is only required to know the location of the connection points, and Lagrange multipliers, since it doesn’t lead to an augmented system. Not covered in literature, a static aeroelastic analysis of a free-flying aircraft is now mandatory even in the conceptual design phase, and the hybrid model meets these needs, in particular when dealing with unconventional configuration.

In the following section the computer-based conceptual design framework NeoCASS – developed at Department of Aerospace Science and Technology (Politecnico di Milano) - is presented, then the equivalent plate model and the beam model formulations are briefly detailed. After a section dedicated to the coupling strategies investigated, results on a transonic cruiser aircraft are critically discussed and finally conclusions are drawn.

NeoCASS environment

NeoCASS (Cavagna *et al*., 2011a) is a suite of modules that combines state of the art computational, analytical and semi–empirical methods to tackle all the aspects of the aero–structural analysis of a design layout at conceptual design stage. It gives a global understanding of the problem at hand without neglecting any aspect of it: weight estimation, initial structural sizing, aerodynamic performances, structural and aeroelastic analysis from low to high speed regimes, divergence, flutter analysis and determination of trimmed condition and aerodynamic derivatives both for the rigid and deformable aircraft. More specifically, NeoCASS includes two main modules, respectively named GUESS *(Generic Unknowns Estimator in Structural Sizing)* and SMARTCAD *(Simplified Models for Aeroelasticity in Conceptual Aircraft Design)*. A third module, W&B *(Weight and Balance)*, is indeed necessary in order to have a first estimate of the non–structural masses and their location for the estimation of inertial loads. The semi-analytical module named GUESS , based on a modified version of the AFaWWE code (*Analytical Fuselage and Wing Weight Estimation*) (Ardema *et al*., 1996), is run to produce for the whole airframe a first–try stiffness distribution on the basis of user–defined sizing maneuvers. The structural sizing of GUESS produces a reasonable stiffness distribution and a better estimate of the primary structural masses, starting from

W&B results. Moreover, a structural and aerodynamic mesh is automatically generated to allow for the successive numeric aeroelastic assessment and optimization, run in SMARTCAD , which is the numerical module dedicated to aero–structural analysis, where static and dynamic analyses are carried out. The time for the preparation of the aeroelastic model is eased by a CAD–centric concept: a change in a geometric parameter is automatically reflected to the numerical model by means of automatic grid generation, allowing the whole process to be restarted for a successive analysis on the updated model, avoiding the need of detailed structural data. Within NeoCASS framework two reduced order models are available: a linear/non--linear finite--volume beam and a linear equivalent plate model, which will be briefly presented in the following section.

Equivalent plate model

In equivalent plate models the lifting surfaces planform geometry can be decomposed in trapezoidal regions, as shown in Figure 1: all quantities are referred to a local coordinate system, with a generic orientation in the space with respect the global frame and the origin in O' vertex; in the present work the reference plane, attached to the local frame, coincides with plate middle plane (a cambered mid-surface referred to a user–specified reference plane can be found in Giles, (1989)).

Tipically, the global coordinate system lies on aircraft symmetry plane (x–z) with the x axis in the stream–wise direction and the y axis in the span–wise direction (right hand rule), and related quantities are denoted by the subscript “g” (global).

Aiming at integrating the equivalent plate model into the NeoCASS environment, the structural mesh is built starting from the already available aerodynamic mesh (VLM, Vortex Lattice Method, or DLM, Doublet Lattice Method): the designer specifies wing box extents, front and rear spars positions, for each aerodynamic box. The result can be seen in Figure 2: on the left NeoCASS vortex lattice mesh with superimposed (dashed line) the wing box planform, which is also drawn on the right.

Figure 1 Global and local reference frames

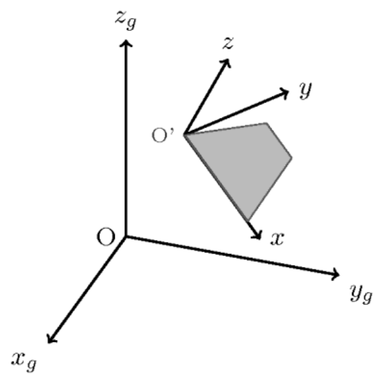
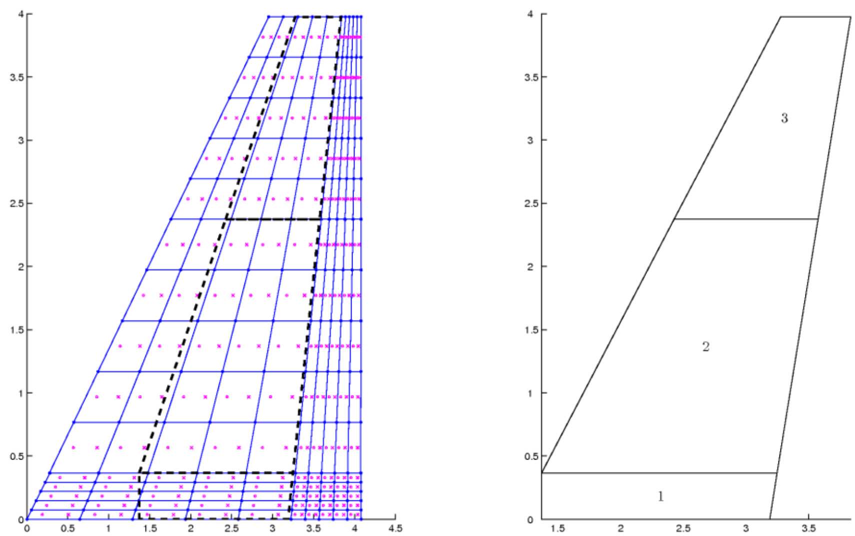


Figure 2 From aerodynamic mesh to wing box middle plane



Each plate segment has an upper and a lower cover skin which may have multiple layers of composite material or a single isotropic layer. A typical plate cross–sectional view is shown in Figure 3: wing thickness distributions, h_u and h_l, as well as the equivalent thickness – equal for upper and lower skin – have polynomial expressions defined in the local coordinate system.

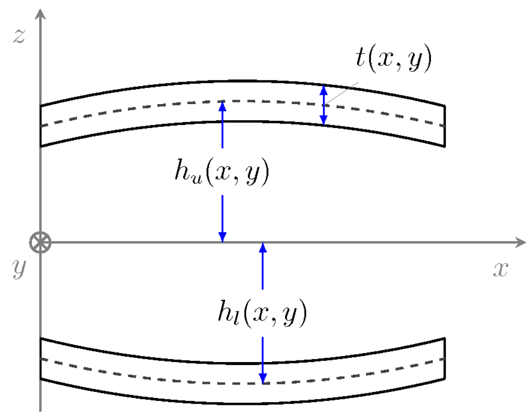
Thus, for the k–th layer these distributions can be expressed as:

$$t_k(x, y) = \sum_{i=1}^{Nt} T_{i,k}\, x^i\, y^i \qquad (1)$$

$$h_{u,k}(x,y) = \sum_{i=1}^{Nhu} Hu_{i,k} x^i y^i \quad (2)$$

$$h_{l,k}(x,y) = \sum_{i=1}^{Nhl} Hl_{i,k} x^i y^i \quad (3)$$

Figure 3 Plate section variables



Since laminate thickness is usually small compared to wing thickness, it is acceptable to consider a unique distribution which represents the height from the middle plane to the laminate mid-surface; this means $h_{u,k}(x,y) \approx h_u(x,y)$ and $h_{l,k}(x,y) \approx h_l(x,y)$.

Given wing characteristics (span, root and tip chords, sweep, taper and dihedral) and airfoil data at each end section of the trapezoid, wing depth distributions are recovered with a least square surface fitting, while thickness distribution can be either user–specified or automatically inferred from the stick model structural mass. Associating each lifting surface to the underlying stick model generated by GUESS , it is at least possible to recover a uniform equivalent thickness $t(x,y)$ for each trapezoid, stating structural mass equality:

$$M_{plate} = M_{stick}$$

$$\rho \cdot (2 t_{eq} A) = M_{stick}$$

$$t_{eq} = \frac{M_{struct}}{2 A \rho} \quad (4)$$

where A is the planform area, M_{stick} is the stick element structural mass and ρ is the material density, equal in the two models. Further details on the equivalent plate model developed can be found in Riccobene Doctoral dissertation, (2011).

Beam model

NeoCASS adopts a three–node linear/non linear finite–volume beam, whose formulation was originally proposed in Ghiringhelli *et al*., (2000), which proved to be intrinsically shear-lock free. The finite–volume approach leads to the collocated evaluation of internal forces and moments, as opposed to usual variational principles which require numerical integration on a one-dimensional domain. Each beam element is divided in three parts, each part related to a reference point G_i: the mid- and the two endpoints. They are referred to geometrical nodes N_i by means of offsets f_i. This allows the elastic axis of the beam to be offset from the center of mass. Every node is characterized by a position vector and a rotation matrix. A reference line p describes the position of an arbitrary point $\mathbf{p}(\xi)$ on the beam section. Parabolic shape functions are used to interpolate displacements and rotation parameters of the generic point $\mathbf{p}(\xi)$ as functions of those of the reference nodes. The derivatives of the displacements and the rotation parameters at the two collocation points C_j (laid at $\xi = \pm 1/\sqrt{3}$ to recover the exact static solution for a beam loaded at the end points as shown in Ref. [30]) are used to evaluate the strains and the curvatures. The latter are used to compute the internal forces and moments, which must balance the external forces m_i and moments t_i. The formulation leads to a loss of symmetry of the stiffness matrix. This is not a major issue considering that usually the models are quite small and the computational cost is very limited, thus the adoption of algorithms specifically suited for symmetric problems are not of primary importance, despite algorithms working with sparse matrices are recommended. Moreover, when dealing with linearized aeroelastic problems the symmetry is always lost since the resulting aerodynamic matrices summing up to structural terms are never symmetric.

Coupling strategies for hybrid model generation

The equivalent plate model is suitable to study particular configurations like flying wings (Ameri *et al*., 2008), but actual aircraft have a fuselage or tailbooms connecting lifting surfaces. In literature the wing is, with good reason, preeminent with respect the whole aircraft, but a formally correct assessment of aeroelastic performances should cope with a flexible fuselage, not simply treated as a rigid body. NeoCASS linear aeroelastic solver takes into account the free flying deformable aircraft, thus the fuselage can't be modeled with linear springs as in Livne *et al*., (1994).

An in–house solution is thus combining the two available reduced order structural models described, following the idea in Giles and Norwood, (1994), where conventional FEM beam is mixed with an equivalent plate model: the present work extends the coupling capability to dynamic, linear static aeroelastic and flutter analyses.

This *multiple–method* approach is stated as a *partitioned analysis* problem, schematically written as:

$$\begin{bmatrix} \mathbf{K}_{eqpl} & \mathbf{K}_{interface} \\ \mathbf{K}_{interface}^{T} & \mathbf{K}_{beam} \end{bmatrix} \begin{Bmatrix} \mathbf{q} \\ \mathbf{u}_{joint} \end{Bmatrix} = \begin{Bmatrix} \mathbf{Q} \\ \mathbf{F}_{joint} \end{Bmatrix} \tag{5}$$

The stiffness matrices of the two models, assembled separately, are on the diagonal, while the off–diagonal terms are interface matrices, which realize the coupling. It should be noted the *mixed* nature of the formulation, being the degrees of freedom both polynomial coefficients and physical displacements.

Three methodologies were tested to couple beam and plate models:

- transition element method (Giles and Norwood, 1994)
- penalty function method (Kao, 1992)
- Lagrange multipliers (Giles and Norwood, 1994)

Details of each methodology are widely known and application to equivalent plate can be found in cited references, so a brief critical review based on the authors' experience is presented below.

Transition Element Method

The transition element method substantially implies the generation of an element with an intermediate constitutive law between the two models it joins; if using a beam element, one end degrees of freedom are expressed in terms of plate coefficients and the beam elemental stiffness matrix thus depends on two different sets of coordinates. The four sub–matrices composing the transition stiffness matrix produce the combined system: two terms add to $\mathbf{K}_{eqpl}$ and $\mathbf{K}_{beam}$, while the other two forms the interface $K_{interface}$.

The method modifies original beam and plate matrices and its validity is strictly connected to the transition element chosen: a membrane instead of a beam implies revisiting the procedure; moreover the additional beam properties must be tuned to obtain positive definite stiffness matrix. On the opposite of Giles and Norwood, (1994), no valuable results could be obtained and the method was discarded.

Penalty Function Method

Penalty function method produces a penalty matrix topologically similar to the transition element method. Numerically more reliable, it modifies the original matrices and can add either one or six different penalty term values for each connection point.

Lagrange Multipliers

Lagrange multipliers method leaves $\mathbf{K}_{eqpl}$ and $\mathbf{K}_{beam}$ undisturbed and they can be assembled separately, because the coupling is enforced through constraint equations, but the resulting system is larger and it needs a solver that can handle zeros on the diagonal. The main advantage is the generality: the two models can be assembled in different codes and then coupled in a third one; moreover, until connection points are fixed, the two models can be updated without regenerating the whole system.

Guyan reduction

However, since the partitioned system of Eqn (5) requires programming new solvers, a different approach is taken. The constraint equations at the interface between beam and plate model can be written as:

$$\mathbf{u}_I - \mathbf{G}\mathbf{q} = 0 \qquad (6)$$

which simply states deflection equivalence for the two models at the same points; $\mathbf{u}_I$ refers to beam nodes d.o.f. at interface, $\mathbf{q}$ stores all plates generalized d.o.f. and $\mathbf{G}$ is a coordinate transformation matrix, $6 \cdot N_I$ by nq where N_I is the connection node number.

The beam model is then reduced at these connection points using the Guyan reduction[7, chap.11]. The beam stiffness matrix can be partitioned as follows, separating interface d.o.f, $(.)_I$, and slaves d.o.f., $(.)_S$:

$$\mathbf{K}_b\mathbf{u} = \mathbf{F} \rightarrow \begin{bmatrix} \mathbf{K}_{b,SS} & \mathbf{K}_{b,SI} \\ \mathbf{K}_{b,IS} & \mathbf{K}_{b,II} \end{bmatrix} \mathbf{u}_S\mathbf{u}_I = \mathbf{0F}_I \tag{7}$$

Solving the first row for slaves d.o.f:

$$\mathbf{u}_S = -\mathbf{K}_{b,SS}^{-1}\mathbf{K}_{b,SI}\mathbf{u}_I = \mathbf{\Psi}\mathbf{u}_I \tag{8}$$

and expressing slaves and interface degrees of freedom as function of the interface ones:

$$\mathbf{u}_S\mathbf{u}_I = \begin{bmatrix} \mathbf{\Psi} \\ \mathbf{I} \end{bmatrix} \mathbf{u}_I = \mathbf{T}_G\mathbf{u}_I \tag{9}$$

Combining Eqn (6) and Eqn (9), the beam model degrees of freedom become function of generalized degrees of freedom:

$$\begin{Bmatrix} \mathbf{u}_S \\ \mathbf{u}_I \end{Bmatrix} = \mathbf{T}_G\mathbf{G}\mathbf{q} \tag{10}$$

Beam stiffness and mass matrices, and load vector if any external force is applied on beam model, are firstly reduced at connection points:

$$\begin{aligned} \mathbf{K}_{b,red} &= \mathbf{T}_G^T\mathbf{K}_b\mathbf{T}_G \\ \mathbf{M}_{b,red} &= \mathbf{T}_G^T\mathbf{M}_b\mathbf{T}_G \\ \mathbf{F}_{b,red} &= \mathbf{T}_G^T\mathbf{F} \end{aligned} \tag{11}$$

and subsequently “embedded” into plate model using the coordinate transformation matrix $\mathbf{G}$:

$$\begin{aligned} \mathbf{K}_{Hybrid} &= \mathbf{K}_{eqpl} + \mathbf{G}^T\mathbf{K}_{b,red}\mathbf{G} \\ \mathbf{M}_{Hybrid} &= \mathbf{M}_{eqpl} + \mathbf{T}_G^T\mathbf{M}_{b,red}\mathbf{G} \\ \mathbf{Q}_{Hybrid} &= \mathbf{Q}_{eqpl} + \mathbf{G}^T\mathbf{F}_{b,red} \end{aligned} \tag{12}$$

The procedure outlined allows to keep plate solvers unchanged: once the hybrid system is solved and polynomial coefficients are known, $\mathbf{q} = \bar{\mathbf{q}}$, the beam model displacement vector is recovered through backward substitution from Eqn (10).

The method showed the same accuracy of Lagrange multipliers and penalty function when applied to the complete aircraft.

A likely weakness of this method is that Guyan reduction transformation relies only on stiffness matrix; resorting to Component Mode Synthesis, Craig–Bampton for instance, which recovers normal modes interaction with constraint modes for the mass matrix, could be an enhancement but since it works in modal coordinates, it requires an appropriate solver.

Results

A target aircraft named TransCRuiser (TCR), selected as a test–case during the FP6 SimSAC project to test the CEASIOM framework (Rizzi *et al*., 2009), is used here to validate the hybrid formulation capabilities.

The TCR is meant to show the difficulties in using handbook methodology when designing aircraft in the transonic speed region. The design specifications are briefly outlined in Table 1.

Table 1 Design specifications for TCR

Cruise Mach	0.97 at altitude ≥ 37.000 ft
Range	5.500 nm + 250 nm to alternate airport + 0.5 hour loiter at 1.500 ft
Max payload	22.000 Kg
Passengers	200
Crew	2 pilots, 6 cabin attendants
Take-off distance	2.700 m at max W_{TO} altitude 2.000 ft
Landing distance	2.000 m at max W_L altitude 2.000 ft

	max payload and normal reserves
Powerplant	2 turbofans
Certification	JAR25
Maneuvering load factors	2.5, -1
Max load factors	3.1, -1.7

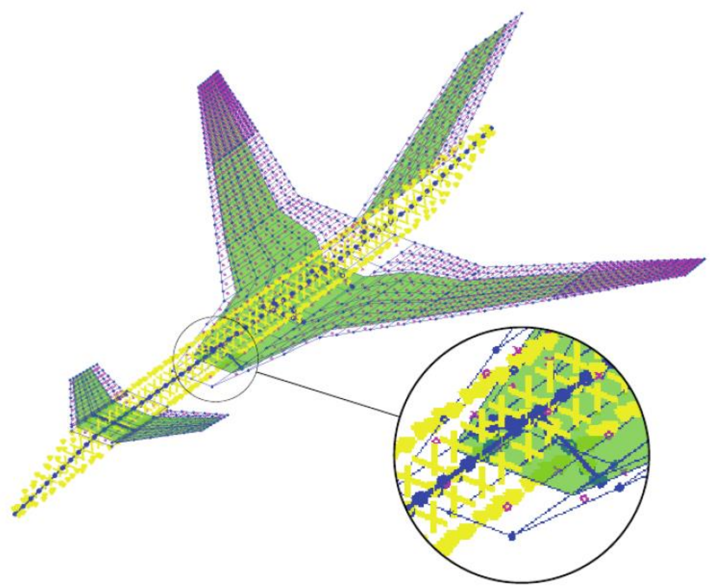
The baseline TCR T-tail design showed trim problems during flight dynamical analysis: too large deflections are needed to trim the aircraft at the design point, so a new approach with a canard configuration was designed. This configuration was judged to be better than the T-tail configuration and was built as a model and tested in the wind tunnel to verify the entire design functionality.

To assess hybrid modeling capability and conduct linear aeroelastic analysis, a TCR model is generated, see Figure 4 (green areas mark equivalent plate geometry). The test case is interesting for the wide root chord: fuselage is joined to the wing box at three stations – front, middle and rear – in six points by triangle-shaped rigid linkage. The usual beam model generated in NeoCASS automatically joins fuselage and wing at a single station losing the chord effect, but flexibility can significantly alter the flight dynamics, as can be seen computing aerodynamic stability derivatives and their aeroelastic corrections.

To prove this effect, trim angles and stability derivatives computed on the hybrid model are compared with those of a NeoCASS full-beam model and, as a further comparison, with a NASTRAN FEM beam model. In (Cavagna *et al,* 2010), the NeoCASS full-beam model was coupled with a medium fidelity CFD Euler code, Edge, http://www.foi.se/FOI/Templates/ProjectPage____4690.aspx (accessed 26 January 2011), to have a better estimate of the aerodynamic stability derivatives and to validate them against wind tunnel measurements (only for the rigid case); results for the low-fidelity lattice methods of NeoCASS and NASTRAN are then compared with these refined ones.

.The flight condition imposed is cruise at $z = 0$ and $Mach = 0.65$.

Figure 4 TCR hybrid model (in evidence the front rigid linkage)



Trim solutions either for rigid and flexible aircraft are reported in Table 2. Results for the hybrid model are overestimated for the rigid case, but at least the derivative trend is caught, while NASTRAN results display a greater error since the aerodynamic model has no built-in geometry effect (flat plate). Comparing longitudinal steady derivatives for the rigid aircraft, Table 3, it can be seen a fairly good agreement between the three numerical methods on force coefficient slope CN_{α}, which however results overestimated with respect the wind tunnel data, the hybrid model showing the greatest error (45%). Only Edge obtains a value closer to wind tunnel for the pitching moment slope Cm_{α}, though underestimated.

It is now interesting to compare aeroelastic corrections to steady longitudinal derivatives, where the correction is expressed as $k = (.)_D/(.)_R$.

In Table 4 is reported the NeoCASS solution using the full beam model-based aircraft (marked with B). While in the rigid case the stability derivatives have the same values either for hybrid and beam model since they depend only on the aerodynamic model, in the flexible case some differences arise due to fuselage flexibility.

The fuselage indeed is a long slender element and, in the beam case, the stiffness contribution due to the large chordwise extension of the wing–box is not easily captured: this causes an inversion of Cm_α sign, leading to static instability about pitch axis. On the contrary, NeoCASS hybrid model, attached in three points along the fuselage, has a greater margin on instability and the correction value is closer to Edge result (8.4% error). Finally, in Figure 5 the deformed shape of the TCR hybrid model is shown: the lifting surfaces structural boxes are displayed, while deformed aerodynamic mesh is omitted for clarity.

Table 2 Trim solution for rigid and flexible aircraft

RIGID			
	Edge	NASTRAN	NeoCASS (H)
α_T [°]	-0.08	2.38	-0.58
δ_T [°]	13.93	7.82	15.43
FLEXIBLE			
	Edge	NASTRAN	NeoCASS (H)
α_T [°]	0.84	3.53	0.59
δ_T [°]	9.84	1.34	7.59

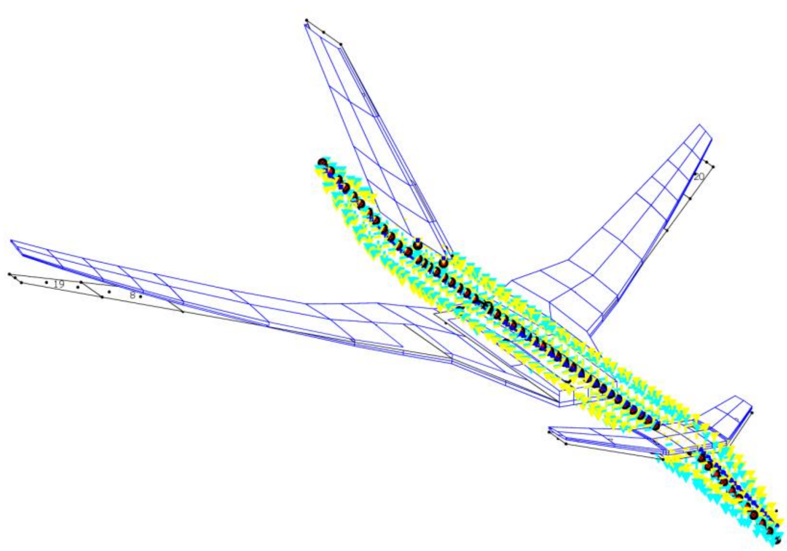
Table 3 Longitudinal steady aerodynamic derivatives (rigid aircraft)

	Edge	NeoCASS (H)	NASTRAN	WT
CN_α	3.57	3.94	3.46	2.71
Cm_α	-1.16	-0.72	-0.8	-1.55

Table 4 Aeroelastic corrections to steady longitudinal derivatives

	Edge ($\alpha = \alpha_T$)	NeoCASS (B)	NeoCASS (H)
$\mathbf{k}(C_N)_\alpha$	0.797	0.715	0.867
$\mathbf{k}(C_m)_\alpha$	0.31	-0.205	0.284

Figure 5 TCR deformed shape at cruise (magnification factor equal to 5)



Conclusions

An equivalent plate model has been presented which has the capability to embed a finite–volume beam model to give an hybrid model for a complete aircraft within the NeoCASS framework: static analysis, vibration modes calculation, linearized flutter analysis, steady and unsteady aerodynamic analysis to extract derivatives for flight mechanics applications, linear static aeroelastic analysis and trimmed calculation for a free–flying rigid or deformable aircraft can be conducted without modifying the equivalent plate solvers.

Moreover exporting results in physical space is only a point–wise evaluation of the displacement polynomials: static displacements as well as modal shapes and related stresses can be easily converted in the usual finite element domain.

The equivalent plate trapezoids are joined together using a penalty function method and an automated boundary springs tuning is used to select the optimal value to keep matrix ill-conditioning low.

The adaptation of the point–in–polygon test – borrowed from computational geometry – to the three–dimensional space allowed to automatically link adjacent plates or assign loads and masses to a trapezoidal domain easing the meshing process.

To assembly the equivalent plate with beam model different coupling strategies –Lagrange multipliers, penalty function method, transition element method– have been tested and a Guyan reduction with a coordinate transformation is chosen, offering results comparable with the other methods but without programming a new solver and allowing to embed the beam model into the equivalent plate.

The hybrid structural model leads to the generation of a simplified model for the complete airframe, retaining an acceptable level of accuracy to reproduce the global structural response characteristics at least at conceptual level.

The aeroelastic trim analysis demonstrated a good correlation with an analysis which coupled the NeoCASS beam model with a medium fidelity aerodynamic code: flexible fuselage effects on aeroelastic correction to longitudinal stability derivatives were correctly computed.

Recent developments (Cavagna *et al.,* 2011b) involve the integration of structural morphing concepts, such as active camber variation, in NeoCASS: many solutions impose deformable sections, thus violating classical beam assumption of rigid rotation about elastic axis, while the equivalent plate model allows to account for chord–wise deformation; moreover the mesh updating process is fast thanks to the analytical displacements formulation over the trapezoidal domain.

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