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A physically based methodology  
for regional flood frequency analysis

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Abstract

The problem of flood frequency estimation in ungauged catchments is approached by using statistical procedures jointly with derived distribution techniques. The application of this methodology to a large region in Northern Italy is presented. It is shown that the conjunctive use of statistical and deterministic concepts can improve the reliability of flood predictions.

Introduction

Statistical methods are usually the natural choice for regional flood frequency analysis. However, some major problems can arise in their practical application. The most critical point of this approach is the estimation of the index flood for ungauged sites. Multiregressive techniques are generally used for this purpose (see, e.g., NERC, 1975); however, the results achievable by this techniques are generally unsatisfactory. As shown by Hebson & Cunnane (1987), estimates of the index flood obtained by regional regression are even less precise than those which can be obtained from only one year data.

These difficulties can be overcome by incorporating in the estimation procedure some knowledge of the physical processes involved in the rainfall-runoff transformation. This is generally undertaken within the derived distribution framework, as indicated by Eagleson (1972).

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After his pioneer paper, many researchers devoted their attention to this problem. However, the results of some applications to real cases showed that these distributions are very sensitive to parameter estimation and that they generally could not adequately represent the entire peak discharge frequency distribution (e.g. Moughamiam et al., 1987). Nevertheless, their capability to reproduce its central characteristics has been shown to be quite satisfactory in several cases (Becciu et al., 1992).

Moving from these results, we report in the present paper a method for flood frequency estimation, which uses statistical procedures jointly with derived distribution techniques. The derived distribution approach is used to estimate the index flood, whereas the estimation of the higher order moments of the distribution at regional scale is performed by using the statistical approach. The aim of the method is to take advantage of the main feature of the derived distribution approach, i.e. of its capability to relate flood frequency with climatic, geomorphological and pedological factors within a parsimonious synthesis.

#### The model for index flood estimation

Let us denote with  $Q_p$  the peak discharge, which is originated from a generic storm, with  $Z$  its annual maximum. Let us further denote with  $F_{Qp}(\cdot)$  and  $F_Z(\cdot)$  respectively their cumulative distribution functions (CDF). By assuming that the annual number of occurrences of  $Q_p$  is a Poisson random deviate with mean  $\lambda$ ,  $F_Z(\cdot)$  is given by

$$F_Z(x) = \exp\{-\lambda[1-F_{Qp}(x)]\} \quad (1)$$

and it can be straightforwardly determined from  $F_{Qp}(\cdot)$  (Todorovic, 1978). Accordingly, the mean value  $\mu_Z$  of the annual maximum, that is the index flood, can be related to the mean value of  $Q_p$ ,  $\mu_{Qp}$ , as

$$\mu_Z = \mu_{Qp} \cdot g(\underline{\Theta}) \quad (2)$$

where  $\underline{\Theta}$  is the vector of parameters of  $F_{Qp}(\cdot)$  and  $g(\underline{\Theta})$  is a function of  $\underline{\Theta}$ , the form of which descends from the probability model assumed for  $F_{Qp}(\cdot)$ . Therefore, once this model has been assessed, the problem of index flood estimation can be solved by estimating the value of  $\mu_{Qp}$ .

For this purpose, the approach proposed by Adom et al. (1989) can be followed. Infact, the goal of this approach is to estimate the moments of  $Q_p$  independently from the probability model used for  $F_{Qp}(\cdot)$ . Accordingly, the form

of  $F_Z(\cdot)$  can be fixed a priori, based on its capability to reproduce the frequency behaviour of observed data at the regional scale.

The derivation of the moments is performed by approximate evaluation of the moments of derived random variables, based on Taylor's series expansions (Mood et al., 1974). In the following, we give a brief outline of the conceptual framework adopted for moment derivation. Point rainfall is described by means of the Poisson Rectangular Pulses model, according to Eagleson (1972). Intensity and duration are assumed to be mutually independent random variables with exponential distributions, with mean  $\mu_i$  and  $\mu_t$ , respectively. The effects due to the spatial reduction of precipitation over the basin area are accounted for by means of an areal reduction factor  $\kappa$ . The absorption at the basin scale is described by the SCS-CN method. This formulation has been chosen because of its parsimonious structure, involving only one parameter, namely CN ( $0 < CN \leq 100$ ), to be estimated. Finally, the transformation of rainfall excess into surface runoff at the basin outlet is modelled according to the linear reservoir cascade analogy. Adom et al. (1989) used this approach to get

$$\mu_{Qp} = \mu_i \cdot \eta \cdot A \cdot \{ [(1 - e^{-\chi}) \cdot (1 + \kappa^2) - \kappa^2 \cdot \chi \cdot e^{-\chi} \cdot (1 + \chi/2)] \cdot [1 + 3 \cdot \kappa^2 \cdot (1 - \eta)^2] + \kappa^2 \cdot (2 - \eta) \cdot [e^{-\chi} \cdot (1 + \chi) - 1] \} \quad (3)$$

with  $A$  denoting the basin area, and with  $\eta$  and  $\chi$  two dimensionless factors, i.e.  $\eta = \mu_i \cdot \mu_t / (\mu_i \cdot \mu_t + S)$  and  $\chi = \mu_t / t_L$ , where  $t_L$  is the time lag of the basin and  $S$  the saturated infiltration capacity of the soil, dependent on CN.

#### The model for regional flood frequency analysis

The distribution of annual maximum,  $Z$ , adopted in the present study is the TCEV (Two Component Extreme Value, see e.g. Fiorentino et al., 1987). It is written as

$$F_Z(x) = \exp(-\lambda_1 \cdot e^{-x/\theta_1} - \lambda_2 \cdot e^{-x/\theta_2}) \quad (4)$$

and its mean,  $\mu_Z$ , is given by

$$\mu_Z = \theta_1 \cdot \left( \ln(\lambda_1) + \gamma - \sum_{i=1}^{\infty} (-1)^i \cdot \lambda_1^i / i! \cdot \Gamma(i/\theta^*) \right) = \theta_1 \cdot f_2(\lambda_1, \lambda^*, \theta^*) \quad (5)$$

where  $\theta^* = \theta_2/\theta_1$ ,  $\lambda^* = \lambda_2/\lambda_1^{1/\theta^*}$  and  $\gamma = 0.5772$ .

The TCEV model descends from the assumption that the CDF of the peak discharge,  $Q_p$ , is a mixture of two exponential distributions, that is

$$F_{Q_p}(x) = \frac{\lambda_1}{\lambda} \cdot (1 - e^{-x/\theta_1}) + \frac{\lambda_2}{\lambda} \cdot (1 - e^{-x/\theta_2}) \quad (6)$$

with  $\lambda = \lambda_1 + \lambda_2$ . One can observe that the expected value of  $Q_p$  can be obtained as

$$\mu_{Q_p} = \theta_1 \cdot \left( \frac{\lambda_1}{\lambda} + \frac{\lambda_2}{\lambda} \cdot \theta^* \right) = \theta_1 \cdot f_1(\lambda_1, \lambda^*, \theta^*) \quad (7)$$

By using (5) and (7), the form of the function  $g(\Theta)$  can be derived as

$$g(\Theta) = f_2(\lambda_1, \lambda^*, \theta^*) \cdot [f_1(\lambda_1, \lambda^*, \theta^*)]^{-1} \quad (8)$$

Parameters  $\theta^*$ ,  $\lambda^*$  and  $\lambda_1$  only depend on the skewness and the coefficient of variation of the annual maximum  $Z$  and they can be estimated from a regional analysis of observed AFS. Hence, the function  $g(\Theta)$  can be regarded as invariant in each homogenous region. Finally, by combining (2) and (3), the index flood  $\mu_Z$  can be estimated for each site in the region, either gauged or not.



Fig.1-Sketch of the study area (dashed).

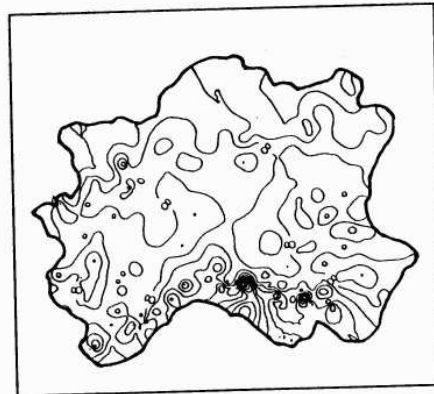


Fig.2-Map of the estimated values of  $\mu_i$ .



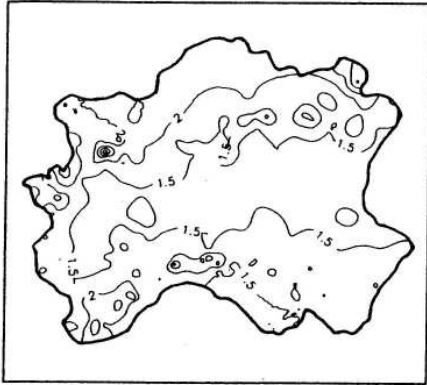


Fig.3-Map of the estimated values of  $\mu_t$ .

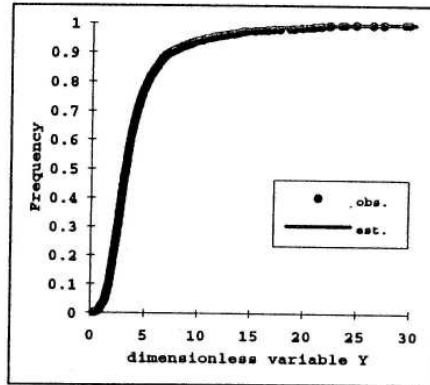


Fig.4-Observed and estimated CDF of the dimensionless ratio  $Y=Z/\mu_Z$ .

The model has been applied to analyse the flood frequency distribution for a large region in Northern Italy, which includes 75 hydrometric stations in the Po basin and in the Liguria region. A sketch of the study area is reported in Fig.1. The parameters of the TCEV distribution ( $\theta^*$ ,  $\lambda^*$  and  $\lambda_1$ ) were estimated following the hierarchical approach suggested by Fiorentino et al. (1987). The estimation of the geomorphoclimatic parameters in (3) (i.e.  $CN$ ,  $t_L$ ,  $\mu_i$ ,  $\mu_t$ , and  $\kappa$ ) has been performed according to the methods described in Becciu et al. (1992). In Figs.2 and 3, we map the estimated values of  $\mu_i$  and  $\mu_t$ . In Fig.4, the theoretical CDF of the dimensionless ratio  $Y = Z/\mu_Z$  is compared with the observed one.

Finally, the values of index flood, estimated with the proposed methodology, are plotted against the corresponding observed ones; the results show that the predictive ability of the methodology is quite satisfactory (see Fig.5).

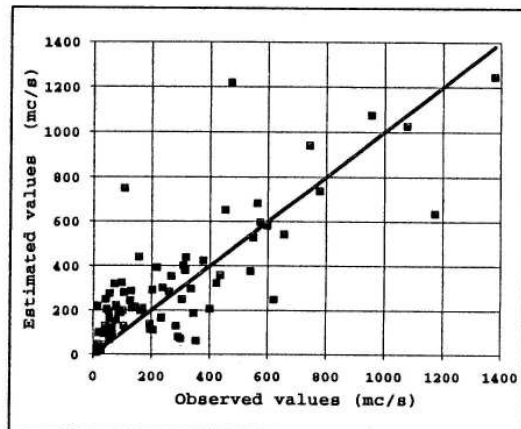


Fig.5-Comparison between observed and estimated values of the index flood.

### Conclusions

A method for flood frequency estimation, which uses a statistical procedure jointly with a derived distribution technique has been presented. The application of the proposed methodology to a large region in Northern Italy showed its capability to satisfactorily reproduce the frequency distribution of the observed data. Because of the explicit appearance of physically meaningful parameters, this methodology could provide a basis for the estimation of flood frequency in the absence of streamflow records, i.e. for ungauged basins.

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