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ANALYTICAL COMPUTATION OF THE ELECTROMAGNETIC TORQUE IN DOUBLY SLOTTED SYNCHRONOUS MACHINES WITH DISTRIBUTED WINDINGS

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Abstract **–** An accurate, analytical computation of the electromagnetic torque waveform in isotropic synchronous machines is developed, by employing a suited analytical field model. To this aim, the stored co-energy in the air-gap volume, due to all the m.m.f.s is evaluated, as a function of time and rotor position, subsequently obtaining the torque components, by performing the co-energy derivative: the method, for now limited to unsaturated conditions, takes into account the actual winding features (field N° of slots/pole; armature N° of layers, N° of slots/(pole·phase), coil pitch), as like as the air-gap core geometry (air-gap and slot opening widths). Some comparisons among analytical and FEM simulations have been performed, obtaining very good agreement.

Air-gap effective flux density, energy and torque components

As known, the torque evaluation is based on the rotor position derivative of the co-energy W_c, performed at constant currents: adopting the hypothesis of zero ferromagnetic voltage drops, the co-energy W_c is non-zero just in the non-ferromagnetic volume among stator and rotor, where it equals the magnetic energy W: thus, this energy will be directly considered in the following, performing its position derivative at constant currents.

Called x and y the linear peripheral coordinates along the stator and rotor surfaces respectively, and called z the rotor origin position with respect to the stator origin, the coordinate along the rotor can be expressed as follows:

$$y = x - z. \qquad (1)$$

In [1], an expression of the air-gap flux density b has been obtained, giving the b value in each position x along the stator, as a function of the rotor location z and of the time dependent stator and rotor m.m.f.s m_S and m_R:

$$b(x, x-z, t) = (\mu_0/g)\cdot\beta_S(x)\cdot\beta_R(x-z)\cdot\left[m_S(x,t)+m_R(x-z,t)\right] \quad : \qquad (2)$$

g is the air-gap, $\beta_S(x)$ and $\beta_R(x-z)$ the "notch" field functions (giving the stator and rotor slotting effects) [1].

In case of a stator three-phase winding (p = 1, 2, 3, phase index), with N_t = N° turns/coil, a = N° of parallel paths, and with a rotor winding equipped with N_f = N° turns/(field coil), the following expressions are valid:

$$m_S(x,t) = (N_t/a)\cdot\sum_{p=1,2,3} M_{fS}\left[x-(p-1)\cdot 2\tau/3\right]\cdot i_p(t) \quad ; \qquad m_R(x-z,t) = M_{fR}(x-z)\cdot N_f\cdot i_f(t) \ , \qquad (3)$$

where: τ is the pole pitch; $M_{fS}(x)$ and $M_{fR}(x\text{-}z)$ model the m.m.f. space distribution [1], due to the winding structures, while the time dependence is due to the phase and field currents waveforms ($i_p(t)$, $i_f(t)$).

Equation (2) allows to correctly calculate the stator winding flux linkages (based on an integration over the cylindrical surface positioned at the middle air-gap diameter), and the corresponding e.m.f.s waveforms [2].

On the other hand, eq. (2) does not allow to correctly calculate the torque effective energy (i.e. the amount of the machine stored energy which varies with the rotor position, thus affecting the torque); in fact:

- it does not model the radial variation of the flux density, particularly important near the slot openings;
- it neglects the tangential component of the flux density, which contributes to the torque effective energy;
- it does not consider the flux paths in slots: in fact, even if mostly related to leakage paths, also the slot volume energy, near the slot openings, gives some torque contribution.

Thus, in order to improve the energy and torque calculation, equation (2) has been modified, by substituting the functions $\beta_S(x)$ and $\beta_R(x-z)$ with suited "effective" notch field functions $\beta_{eS}(x)$ and $\beta_{eR}(x-z)$:

$$b_e(x, x-z, t) = (\mu_0/g)\cdot\beta_{eS}(x)\cdot\beta_{eR}(x-z)\cdot\left[m_S(x,t)+m_R(x-z,t)\right] \quad ; \qquad (4)$$

b_e is the equivalent flux density, whose b_e^2/μ_0 volume integral in the machine gives the correct stored energy.
As regards $\beta_{eS}(x)$ and $\beta_{eR}(x-z)$, starting from the lost flux densities $\beta_{LS}(x)$, $\beta_{LR}(x-z)$ [1], they are defined as:

$$\beta_{eS}(x)=\left(1-\eta_{\beta S}\cdot\beta_{LS}(x)\right) \quad ; \quad \beta_{eR}(x-z)=\left(1-\eta_{\beta R}\cdot\beta_{LR}(x-z)\right) \quad , \tag{5}$$

in which $\eta_{\beta S}$ and $\eta_{\beta R}$ are field constants; they can be estimated as described in the following.
In order to obtain $\eta_{\beta S}$, consider the slotted stator faced to a smoothed rotor; if U is a constant voltage drop difference applied at the air-gap g, the energy stored within one slot pitch, per unit stack length, is expressed by:

$$W_{sS}=2\cdot g\cdot\int_0^{\tau_{sS}/2}\left\{\left[\left(1-\eta_{\beta S}\cdot\beta_{LS}(x)\right)\cdot\mu_0\cdot U/g\right]^2\Big/(2\cdot\mu_0)\right\}\cdot dx \quad ; \tag{6}$$

$\beta_{LS}(x)$ must be considered as known [1], while the energy W_{sS} can be numerically evaluated by means of a FEM simulation; thus, (6) allows to obtain the value of $\eta_{\beta S}$ (lower than unity); a similar procedure can be used for the evaluation of $\eta_{\beta R}$, by introducing W_{sR}, τ_{sR}, $\eta_{\beta R}$, $\beta_{LR}(y)$ and $\int(\cdot)dy$ in (6), instead of W_{sS}, τ_{sS}, $\eta_{\beta S}$, $\beta_{LS}(x)$, $\int(\cdot)dx$.
On the basis of eq. (4), and called ℓ the stack length and ζ the rotor angular position, the torque equals:

$$T(z,t)=\frac{\partial W(z,t)}{\partial\zeta}=\frac{D}{2}\cdot\frac{\partial W(z,t)}{\partial z}=\frac{D}{2}\cdot\ell\cdot g\cdot\frac{\partial}{\partial z}\left[\int_0^{\pi\cdot D}\left(\frac{b_e^2(x,x-z,t)}{2\cdot\mu_0}\right)\cdot dx\right] \quad ; \tag{7}$$

carrying the $\partial/\partial z$ operator under the integral $\int(\cdot)dx$, and observing that $\partial f(x-z)/\partial z = -\,\partial f(x-z)/\partial x$, eq. (7) becomes:

$$T(z,t)=\frac{D}{2}\cdot\ell\cdot g\cdot\frac{\partial}{\partial z}\left[\int_0^{\pi\cdot D}\left(\frac{b_e^2(x,x-z,t)}{2\cdot\mu_0}\right)\cdot dx\right]=-\frac{D\cdot\ell\cdot g}{2\cdot\mu_0}\cdot\int_0^{\pi\cdot D}\left[b_e(x,x-z,t)\cdot\frac{\partial b_e(x,x-z,t)}{\partial x}\right]\cdot dx\,. \tag{8}$$

Called Λ_g the following quantity:

$$\Lambda_g=\mu_0\cdot\ell\cdot(D/2)/g\,, \tag{9}$$

by developing eq. (8), and putting $\partial(\cdot)/\partial x = D_x(\cdot)$, the following three torque contributions derive:
- mutual torque, due to the simultaneous existence of stator and rotor m.m.f.s:

$$T_m(z,t)=-\Lambda_g\cdot\int_0^{\pi\cdot D}\left\{\begin{array}{l}m_S(x,t)\cdot\beta_{eS}^2(x)\cdot\beta_{eR}(x-z)\cdot\\ \cdot\left[2\cdot m_R(x-z,t)\cdot D_x\beta_{eR}(x-z)+\beta_{eR}(x-z)\cdot D_x m_R(x-z,t)\right]\end{array}\right\}\cdot dx \quad ; \tag{10}$$

- stator slotting reluctance torque, to which the torque reduces in case of zero rotor m.m.f.:

$$T_S(z,t)=-\Lambda_g\cdot\int_0^{\pi\cdot D}\left\{m_S^2(x,t)\cdot\beta_{eS}^2(x)\cdot\beta_{eR}(x-z)\cdot D_x\beta_{eR}(x-z)\right\}\cdot dx \quad ; \tag{11}$$

- rotor slotting reluctance torque, to which the torque reduces in case of zero stator m.m.f.:

$$T_R(z,t)=-\Lambda_g\cdot\int_0^{\pi\cdot D}\left\{\begin{array}{l}m_R(x-z,t)\cdot\beta_{eS}^2(x)\cdot\beta_{eR}(x-z)\cdot\\ \cdot\left[m_R(x-z,t)\cdot D_x\beta_{eR}(x-z)+\beta_{eR}(x-z)\cdot D_x m_R(x-z,t)\right]\end{array}\right\}\cdot dx \quad . \tag{12}$$

The integration of eq.s (10), (11), (12) appears cumbersome, because of the heavy expressions of the involved quantities, and because a different integration solution seems to be required for each instantaneous rotor position $z(t)$; moreover, the time dependence of the m.m.f.s (3) seems to complicate the evaluation; actually, it is possible to extract the time dependent factors out of the integrals, leaving inside just the space dependent terms; thus:

$$T_m(z,t)=-\Lambda_g\cdot(N_t/a)\cdot N_f\cdot i_f(t)\cdot\sum_{p=1,2,3} i_p(t)\cdot Y_{mp}(z) \quad , \qquad \text{with} \tag{13}$$

$$Y_{mp}(z)=\int_0^{\pi\cdot D}\left\{\begin{array}{l}M_{fS}\left[x-(p-1)\cdot 2\cdot\tau/3\right]\cdot\beta_{eS}^2(x)\cdot\beta_{eR}(x-z)\cdot\\ \cdot\left[2\cdot M_{fR}(x-z)\cdot D_x\beta_{eR}(x-z)+\beta_{eR}(x-z)\cdot D_x M_{fR}(x-z)\right]\end{array}\right\}\cdot dx \quad ; \tag{14}$$

$$T_S(z,t)=-\Lambda_g\cdot(N_t/a)^2\cdot\sum_{p,u=1,2,3} i_p(t)\cdot i_u(t)\cdot Y_{Spu}(z) \quad , \ \text{with} \tag{15}$$

$$Y_{Spu}(z)=\int_0^{\pi\cdot D}\left\{M_{fS}\left[x-(p-1)\cdot\frac{2\tau}{3}\right]\cdot M_{fS}\left[x-(u-1)\cdot\frac{2\tau}{3}\right]\cdot\beta_{eS}^2(x)\cdot\beta_{eR}(x-z)\cdot D_x\beta_{eR}(x-z)\right\}\cdot dx\,; \tag{16}$$

$$T_R(z,t)=-\Lambda_g\cdot N_f^2\cdot i_f^2(t)\cdot Y_R(z) \quad , \qquad \text{with} \tag{17}$$

$$Y_R(z)=\int_0^{\pi\cdot D}\left\{\begin{matrix}M_{fR}(x-z)\cdot\beta_{eS}^2(x)\cdot\beta_{eR}(x-z)\cdot\\ \cdot\left[M_{fR}(x-z)\cdot D_x\beta_{eR}(x-z)+\beta_{eR}(x-z)\cdot D_xM_{fR}(x-z)\right]\end{matrix}\right\}\cdot dx \quad . \tag{18}$$

Equations (13)-(18) suggest the following remarks:

- the space dependent functions $Y_{mp}(z)$, $Y_{Spu}(z)$, $Y_R(z)$ (p, u = 1,2,3) can be evaluated off line just once, for a suited number of rotor positions z, subsequently interpolating the calculated points; thus, when the time dependence is to be taken into account, these space quantities can be considered as known functions;
- $Y_{mp}(z)$, $Y_{Spu}(z)$, $Y_R(z)$ are able to correctly model the local stator and rotor slotting field effects (including the effects of partial slot facings [1]), and the actual field and armature winding structures: this property ensures an accurate modelling of all the torque harmonic contributions, including the well known toothing and cogging torque harmonics, particularly noisy in synchronous machines;
- of course, the adopted approach is rigorously valid just supposing perfectly unsaturated operation, because it implies the application of the superposition principle;
- moreover, it should be noted that, in general, no closed forms can be found for $Y_{mp}(z)$, $Y_{Spu}(z)$, $Y_R(z)$.

Closed form torque formulation for ideally unslotted structures and stepped m.m.f.s

The previous equations can be greatly simplified, leading to closed form expressions of the torque contributions, if the following simplifying hypotheses are adopted:

- the slot openings are considered very small, compared with the air-gap width: this hypothesis implies that:

$$\beta_S(x)=1 \;\;;\;\; d\beta_S(x)/dx=0 \;\;;\;\; \beta_R(x-z)=1 \;\;;\;\; \partial\beta_R(x-z)/\partial x=0 \qquad \text{for every } x \text{ and } z; \tag{19}$$

- the m.m.f. space distribution is assumed step-wise, with a sharp variation occurring in each slot axis position, whose amplitude depends on the instantaneous total current included in the considered slot; if $\sigma(x)$ indicates the step function:

$$\sigma(x) = 1 \text{ for } x \geq 0, \qquad \sigma(x) = 0 \text{ for } x < 0 \quad , \tag{20}$$

  the stepped m.m.f.s expressions consist of superposed $\sigma(x)$ and $\sigma(y)$ terms, suitably amplified and displaced; for example, in a stator, single-layer, winding, we would have [1]:

$$M_{fS\sigma}(x)=\sum_{j_S=1}^{q}M_{cS\sigma}\left[x-\left(\frac{q-1}{2}-(j_S-1)\right)\cdot\tau_{sS}\right] \;, \qquad M_{cS\sigma}(x)=\sigma\left[\cos(\pi\cdot x/\tau)\right]-1/2 \;, \tag{21}$$

  while the rotor m.m.f. is modelled as follows:

$$M_{fR\sigma}(y)=\sum_{j_R=1}^{c_p}M_{cR\sigma}\left[y-\left(\frac{c_p-1}{2}-(j_R-1)\right)\cdot\tau_{sR}\right] \;, \qquad M_{cR\sigma}(y)=\sigma\left[\cos(\pi\cdot y/\tau)\right]-1/2 \;, \tag{22}$$

  where c_p is the number of rotor slots/pole, τ_{sS} and τ_{sR} the stator and rotor slot pitches respectively.

By adding the subscript σ to all the quantities evaluated according to the previous hypotheses (and indicating with $M_{fS\sigma}(x)$ the m.m.f. of a generic stator winding - one or two layers, integer or shorted pitch -), it follows:

$$Y_{m\sigma p}(z)=\int_0^{\pi\cdot D}\left\{M_{fS\sigma}\left[x-(p-1)\cdot 2\cdot\tau/3\right]\cdot D_xM_{fR\sigma}(x-z)\right\}\cdot dx \;\;; \tag{23}$$

$$Y_{S\sigma pu}(z)\equiv 0, \quad \text{for all the values} \quad p = 1,2,3; \;\; u = 1,2,3\;; \tag{24}$$

$$Y_{R\sigma}(z)=\int_0^{\pi\cdot D}\left\{M_{fR\sigma}(x-z)\cdot D_xM_{fR\sigma}(x-z)\right\}\cdot dx=\int_{M_{fR\sigma}(0-z)}^{M_{fR\sigma}(\pi\cdot D-z)}M_{fR\sigma}(x-z)\cdot d\left(M_{fR\sigma}(x-z)\right)\equiv 0 \;. \tag{25}$$

Thus, the torque expression reduces to the following one:

$$T_\sigma(z,t)=T_{m\sigma}(z,t)=-\Lambda_g\cdot(N_t/a)\cdot N_f\cdot i_f(t)\sum_{p=1,2,3}i_p(t)\cdot Y_{m\sigma p}(z) \qquad , \tag{26}$$

in which $Y_{m\sigma p}(z)$ can be calculated in closed form; in fact, called $\delta(x) = d\sigma(x)/dx$ the Delta of Dirac distribution, the quantity $D_xM_{fR\sigma}(x–z)$ becomes:

$$D_wM_{fR\sigma}(w)=-\sum_{j_R=0}^{c_p-1}\delta\left\{\cos\left[\frac{\pi}{\tau}\cdot\left(w-\left(\frac{c_p-1}{2}-j_R\right)\cdot\tau_{sR}\right)\right]\right\}\cdot\sin\left[\frac{\pi}{\tau}\cdot\left(w-\left(\frac{c_p-1}{2}-j_R\right)\cdot\tau_{sR}\right)\right]\cdot\frac{\pi}{\tau}\;; \tag{27}$$

thus, inserting eq. (27) in (23) and integrating (23) along all the periphery, eq. (26) gives (with N_p = N° of poles):

$$T_\sigma(z,t) = N_p \cdot \Lambda_g \cdot \frac{N_t}{a} \cdot N_f \cdot i_f(t) \sum_{p=1,2,3} i_p(t) \cdot \sum_{j_R=0}^{c_p-1} M_{fS\sigma}\left[z + \left(\frac{c_p-1}{2} - j_R \right) \cdot \tau_{sR} + \frac{\tau}{2} - (p-1) \cdot \frac{2 \cdot \tau}{3} \right], \quad (28)$$

in which the stator winding structure (one or two layers, integer or shorted pitch) affects the $M_{fs\sigma}(x)$ function.
For example, in case of a double layer stator winding, with coils having a pitch shorted by ε (expressed in number of slot pitches), the stator phase m.m.f. space distribution $M_{fS\sigma}(x)$ can be expressed as follows [1]:

$$M_{fS\sigma}(x) = M_{f\sigma}(x - (\varepsilon \cdot \tau_{sS})/2) + M_{f\sigma}(x - (\varepsilon \cdot \tau_{sS})/2 - \varepsilon \cdot \tau_{sS}), \quad \text{with} \quad (29)$$

$$M_{f\sigma}(x) = \sum_{k=1}^{q} \left\{ \sigma\left[\cos\left\{ (\pi/\tau) \cdot \left[x - (k-1) \cdot \tau_{sS} + (q-1) \cdot \tau_{sS}/2 \right] \right\} \right] - 1/2 \right\}. \quad (30)$$

Fig.1 shows some results regarding stepped torque waveforms of an isotropic synchronous machine operating with constant field current I_f and rotating at constant speed Ω, for various winding data of a two-layer stator winding structure; curves concerning two different conditions are shown:

- T_h = holding torque, with constant stator currents I_1, I_2, I_3 (in such a way that: $I_1 + I_2 + I_3 = 0$);
- T_n, T_m, T_M = torque waveforms with balanced, three-phase sinusoidal currents, always with the same r.m.s. amplitude, but whose corresponding stator m.m.f. axes form different angles with respect to the rotor m.m.f. axis: this angle is chosen in such a way to give null, medium and maximum average torque respectively.

The curves are in p.u., referred to the average value of the maximum torque. The following remarks can be done:

- even without slot effects, the torque shows some ripple, because the m.m.f. space distribution is not sinusoidal;
- as expected, the higher are q and c_p, the more the torque ripple has lower amplitude and higher frequency: thus, the holding torque approaches a sinusoid and the torque with sinusoidal currents tends to a constant.

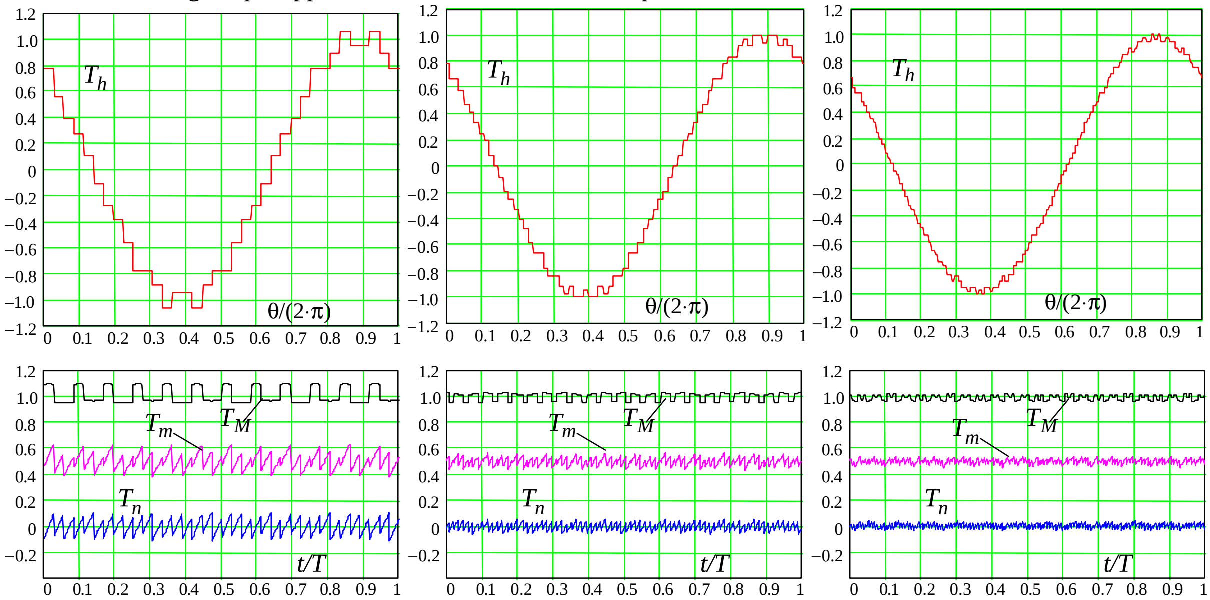

Fig.1 – Stepped torque waveforms of an isotropic synchronous machine with a double layer, shorted pitch stator winding (all the torque values are expressed in p.u., referred to the average value of the maximum torque).
Operating conditions: constant field current I_f; model hypotheses: no slotting effects; stepped m.m.f. distributions.
Top row curves: T_h = holding torque (constant stator currents I_1, I_2, I_3, with: $I_1 + I_2 + I_3 = 0$); bottom row curves: T_n, T_m, T_M = instantaneous torque with null, medium and maximum average value respectively (sinusoidal, balanced stator currents).
Left column: $q = 2$, $\varepsilon = 1$, $c_p = 3$; middle column: $q = 4$, $\varepsilon = 2$, $c_p = 3$; right column: $q = 4$, $\varepsilon = 2$, $c_p = 5$.

Comparison among analytical and FEM evaluation of the torque waveforms

Case of ideally unslotted machine, with stepped m.m.f.s

This first, idealised machine comparison is aimed to show the correspondence among analytical results, connected with the use of eq. (28), and numerical results, obtained by suited simulations, performed by means of

FEM transient tools [3]: fig.2 shows the holding torque waveforms, with constant stator currents (with zero sum), as a function of the angular rotor position θ, for a machine equipped with a single-layer stator winding of q = 3 slots/(pole·phase), with a field winding with c_p = 4 slots/pole (see data in Table I; the FEM holding torque curve have been vertically displaced by T_{offset} = 200 Nm, for visibility reasons).
The numerical FEM curve is fairly similar to the analytical one, except for the following aspects:
- a high frequency ripple arises in the FEM waveform, that can be totally ascribed to numerical noise;
- some notches can be observed in the FEM waveform, occurring because of the small, but non-zero, slot openings adopted in the FEM model (thus, $\beta_{eS}(x)$ and $\beta_{eR}(y)$ are not exactly equal to 1 everywhere).

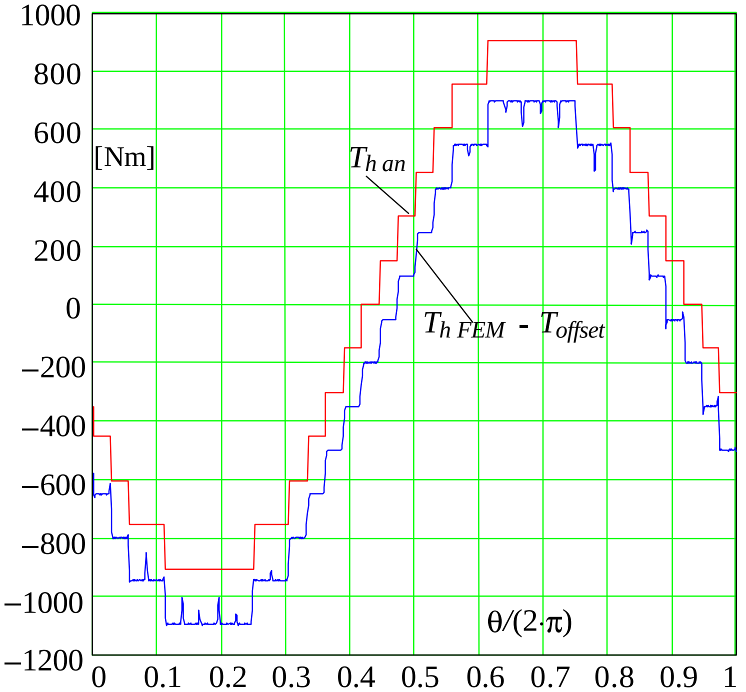

Table 1- Data of the analysed constant air-gap synchronous machine with small slot openings	
N° of poles	2
N° of parallel paths	1
N° of stator slots	18
N° of stator phases	3
N° of winding layers	1
N° of rotor slots / pole	4
mech. angle among rotor slots	30°
stator internal diameter D [m]	1
air-gap [mm]	5
stator, rotor slot openings [mm]	5

Fig.2 – Holding torque simulated waveforms, as a function of the rotor position θ, with constant stator currents (with zero sum); machine data in Table I: almost closed slot openings have been considered for the FEM model (FEM curve vertically displaced by T_{offset} = 200 Nm). $T_{h\,an}(\theta)$ = analytical evaluated waveform (see eq. (28)); $T_{hFEM}(\theta)$ = FEM simulation result.

Case of a doubly slotted machine, with non-stepped m.m.f.s

In this case, equations (13), (15) and (17) should be used, in which the notch field functions and their derivatives are included, and the stator and rotor m.m.f. space distributions $M_{fS}(x)$ and $M_{fR}(y)$ take into account the tanh-wise level variations, modelling the interpolar field behaviour [1].
Consider the holding torque, developed at constant stator currents I_1, I_2, I_3 and at constant field current I_f : to this aim, the stator currents are assumed as evaluated in a chosen instant t_0 ($I_1 = i_1(t_0)$, $I_2 = i_2(t_0)$, $I_3 = i_3(t_0)$). In practice, the holding torque can be calculated from the expression of the total torque (sum of eq.s (13), (15), (17) contributions), putting $t = t_0$, and varying the rotor position z all along the machine periphery:

$$T_h(z) = T_m(z,t_0) + T_S(z,t_0) + T_R(z,t_0) = -\Lambda_g \cdot \left[\begin{array}{l} \dfrac{N_t}{a} N_f I_f \cdot \sum\limits_{p=1,2,3} i_p(t_0) \cdot Y_{mp}(z) + N_f^2 \cdot I_f^2 \cdot Y_R(z) + \\ +(N_t/a)^2 \cdot \sum\limits_{p,u=1,2,3} i_p(t_0) \cdot i_u(t_0) \cdot Y_{Spu}(z) \end{array} \right] . \quad (31)$$

Fig.3 shows the comparison among the holding torque evaluated analytically by means of (31) (solid line), and by a transient FEM simulation (dotted line), as a function of the rotor angular position θ, performed for the same machine geometry described in Table I, and in the same operating conditions, already considered for fig.2, except for the stator and rotor slot openings, here assumed equal to 30 mm (→field constants: $\eta_{\beta S} = \eta_{\beta R}$ =0.66).
The following remarks can be made:
- the analytical and FEM results are very close, confirming the correctness of the developed model;

- the waveform disturbances due to the stator and rotor slotting effects are very important: they correspond to the classical, well known torque tooth harmonics, whose amplitude can be reduced by using skewing only (not considered here, just with the aim to show and estimate the accuracy evaluation of this effect);
- apart from the slotting effects, the envelope holding torque waveform presents torque levels near to those evaluated in the ideal case of fig.2.

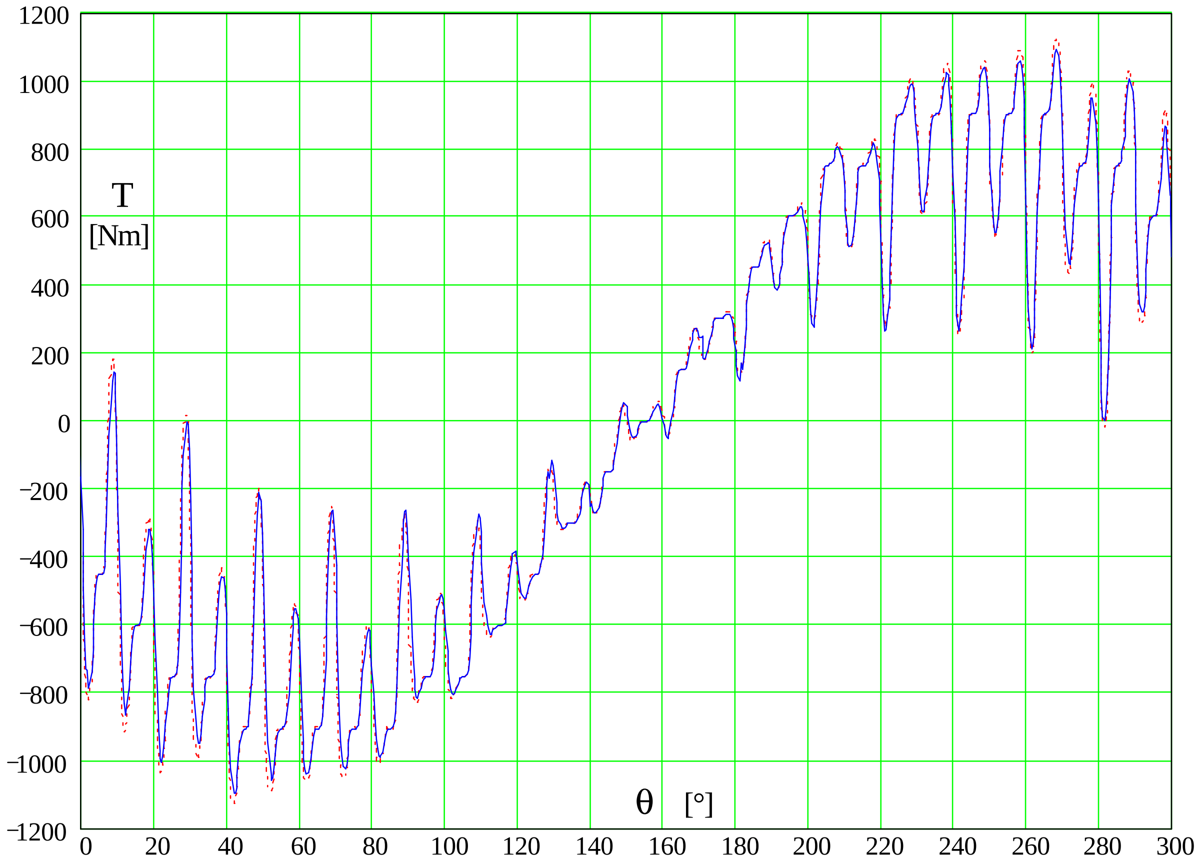

Fig.3 – Holding torque simulated waveforms, as a function of the angular rotor position θ; machine data of Table I, except for the stator and rotor slot openings, here equal to 30 mm: operating conditions corresponding to the same considered in fig.2 simulation; solid line = analytical evaluated waveform (see eq.(31)); dotted line = FEM transient simulation waveform.

Conclusion

An analytical method for the evaluation of the electromagnetic torque waveform of an isotropic synchronous machine has been developed, taking into account the stator and rotor slotting effects and the actual time dependent current waveforms and winding turns space distribution: its good level of accuracy has been verified by means of comparisons with transient FEM simulations.

The described method is well suited to the analytical, accurate, evaluation of torque waveforms in many modelling and design problems of slotted electrical machines: the method is valid not only in steady-state operation, but also in transient conditions, thanks to the separated dependence on space and time variables.

References

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[2] A. Di Gerlando, Gian Maria Foglia, Roberto Perini, " Analytical Evaluation of the Electromotive Forces in Doubly Slotted Synchronous Machines with Distributed Windings", ISEF '03 Conference Proceedings, Maribor, Slovenia, September 18-20, 2003.

[3] Maxwell 2D FEM code, 8.0.27 Version, Ansoft Corporation, Pittsburgh, PA.