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XI INTERNATIONAL SYMPOSIUM
ON ELECTROMAGNETIC FIELDS
IN ELECTRICAL ENGINEERING
ISEF 2003

Maribor, Slovenia, 18-20 September 2003

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VOLUME I

Editors:
Slawomir Wiak, Andrzej Krawczyk and Mladen Trlep

ANALYTICAL MODEL OF THE AIR-GAP MAGNETIC FIELD IN DOUBLY SLOTTED ELECTRICAL MACHINES WITH DISTRIBUTED WINDINGS

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Abstract **–** The quantities of interest in an electrical machine (e.m.f. and torque) can be evaluated once known the air-gap flux density. The paper shows how to analytically model this flux density distribution, as a function of time and of the rotor position, considering the actual disposition of slots and conductors and the geometrical characteristics of the air-gap periphery. The winding m.m.f. is evaluated by summing suited step functions; as regards the slots, specific field notch functions are obtained, by analytical field solutions and applying suited superposition techniques. The method is applied to constant air-gap machines, neglecting magnetic saturation.

Basic step model of the m.m.f.

The winding m.m.f. is obtained by adding the contributions of the various coils.
At first, sharp step functions will be employed, subsequently showing how to smooth the field near the coil sides.
The model is valid both for coils with integer pitch and shorted pitch, and for single and double layer windings.
In the following, x is the generic linear coordinate, measured along the air-gap periphery.

M.m.f. of integer pitch coil windings

The m.m.f. space distribution of an integer pitch coil of a winding with a generic N° of poles N_p is expressed by:

$$M_{c\sigma}(x) = \sigma\left[\cos\left(\pi \cdot x/\tau\right)\right] - 1/2 \ , \qquad (1)$$

where $\tau = \pi \cdot D/N_p$ is the pole pitch, D the air-gap diameter and $\sigma(x)$ is the step function:

$$\sigma(x) = 1 \text{ for } x \geq 0, \qquad \sigma(x) = 0 \text{ for } x < 0. \qquad (2)$$

Called $I_t(t) = N_t \cdot i(t)/a$ the coil total current (N_t = N° of turns/coil; $i(t)$= phase current; a = N° of parallel paths), the coil m.m.f. becomes:

$$m_c(x,t) = M_{c\sigma}(x) \cdot I_t(t) = M_{c\sigma}(x) \cdot (N_t/a) \cdot i(t) = \left\{\sigma\left[\cos\left(\pi \cdot x/\tau\right)\right] - 1/2\right\} \cdot (N_t/a) \cdot i(t) \, . \qquad (3)$$

In case of uniformly distributed phase coils, called q the number of coils/(pole-phase) and τ_s the slot pitch, the phase m.m.f. space distribution can be obtained by adding q terms like (1); by adopting a displacement equal to $(q-1)\cdot\tau_s/2$ in order to obtain a phase m.m.f. centred along the phase axis, the following expression follows:

$$M_{f\sigma}(x) = \sum_{k=1}^{q} \left\{\sigma\left[\cos\left\{(\pi/\tau)\cdot\left[x - (k-1)\cdot\tau_s + (q-1)\cdot\tau_s/2\right]\right\}\right] - 1/2\right\} . \qquad (4)$$

Called $i_p(t)$ the instantaneous current of the phase p (p = 1, 2, 3), the three-phase m.m.f. equals:

$$m_{3f}(x,t) = M_{f\sigma}(x)\cdot(N_t/a)\cdot i_1(t) + M_{f\sigma}(x - 2\cdot\tau/3)\cdot(N_t/a)\cdot i_2(t) + M_{f\sigma}(x - 4\cdot\tau/3)\cdot(N_t/a)\cdot i_3(t) \, ; \qquad (5)$$

In case of balanced, three-phase, sinusoidal currents, we have: $i_p(t) = \sqrt{2}\cdot I\cdot\cos[\omega\cdot t - (p-1)\cdot 2\cdot\pi/3]$.

M.m.f. of shorted pitch coils

Considering a shorted pitch coil, called τ_c the coil pitch, U_1 and U_2 the amplitude of the positive and negative coil m.m.f. half-waves respectively, the coil m.m.f. space distribution is described by:

$$M_{c\sigma.sh}(x) = U_1 = 1 - \frac{\tau_c}{2\cdot\tau} \quad \text{for} \quad -\frac{\tau_c}{2} < x < \frac{\tau_c}{2} \ , \quad M_{c\sigma.sh}(x) = -U_2 = -\frac{\tau_c}{2\cdot\tau} \quad \text{for} \quad \frac{\tau_c}{2} < x < 2\cdot\tau - \frac{\tau_c}{2} \ . \qquad (6)$$

The representation of eq. (6) by the step function leads to:

$$M_{c\sigma.sh}(x)=\sigma\left\{\cos\left(\pi\cdot x/\tau\right)-\cos\left[\pi\cdot\tau_c/\left(2\cdot\tau\right)\right]\right\}-1/2+\left(\tau-\tau_c\right)/\left(2\cdot\tau\right). \qquad (7)$$

Sometimes it is useful to express the peripheral extensions in terms of number of slots; indicated with c_p and y_c the number of slots/pole and the coil pitch, eq. (7) can be rewritten by substituting τ_c / τ with y_c / c_p.
Of course, eq. (7) reduces to eq. (1) in case of integer pitch coils ($\tau_c = \tau$, or $y_c = c_p$); moreover, the extensions to time dependence and to phase and three-phase m.m.f. expressions follow expressions similar to eq.s (3) – (5).

M.m.f. of two layer windings, with shorted pitch coils

The m.m.f. of a double layer winding with shorted pitch coils can be easily reduced to the sum of the m.m.f.s $M_{la\sigma1}(x)$ and $M_{la\sigma2}(x)$ of two single layer windings with integer coil pitch (with connected active sides within the same layer); the two layers are simply displaced by a number of slots ε_s equal to the original pitch reduction. This imaginary winding transformation can be justified as follows: the m.m.f. distribution depends only on the distribution of the currents in the slots and the current distribution is the same in the original winding and in the transformed one. Thus, each layer m.m.f. is given by equations like (4); moreover, in order to obtain a resultant phase m.m.f. $M_{f\sigma1\text{-}2}(x)$ centred along the phase axis, an $\varepsilon_s\cdot\tau_s/2$ space displacement must be applied. It follows:

$$M_{f\sigma1-2}\left(x\right)=M_{la\sigma1}(x)+M_{la\sigma2}(x)=M_{f\sigma}(x-\left(\varepsilon_s\cdot\tau_s\right)/2)+M_{f\sigma}(x-\left(\varepsilon_s\cdot\tau_s\right)/2-\varepsilon_s\cdot\tau_s). \qquad (8)$$

Field functions and their superposition

The basic analytical approach employs the method of the conformal transformations. As known, a classical example is the problem of a surface with one indefinitely-deep single slot, separated from a faced smoothed surface throughout a constant air-gap g; among the surfaces, a constant scalar magnetic potential difference (m.p.d.) U is applied. This problem, studied by Carter for unsaturated magnetic cores [1], leads to express the position $p(w)$ within the air-gap and the corresponding flux density $B_1(w)$ as a function of an auxiliary complex parameter w. To our aims, we are interested in analysing B_1 just along the smoothed surface ($p(w) \equiv x(w)$): thus, a more suitable expression can be obtained by substituting the parametric formulation with a function $B_1(x)$, interpolating a suited N° of points along x axis (e.g. by means of a cubic spline); $B_1(x)$ is a real function of a real variable (in fact, the flux density vector, always orthogonal to the smoothed surface, has just one component).
Called $B_i = \mu_0\cdot U/g$ "ideal" flux density (as it exists between two smoothed faced surfaces), we define "lost" flux density $B_L(x)$ the difference among B_i and any actual flux density $B(x)$, and call "field function" $\beta(x)$ the ratio among $B(x)$ and B_i: $\beta(x)$ is so named as it describes the field behaviour by means of a p.u. function. For example, the field functions related to $B_1(x)$, and to the lost flux density $B_{L1}(x)$, and the corresponding relations are:

$$\beta_1\left(x\right)=B_1\left(x\right)/B_i\ , \qquad \beta_{L1}(x)=B_{L1}(x)\,/\,B_i\ , \qquad (9)$$

$$B_{L1}\left(x\right)=B_i-B_1\left(x\right)=B_i\cdot\left(1-\beta_1\left(x\right)\right)=B_i\cdot\beta_{L1}\left(x\right). \qquad (10)$$

The problem is how to combine simple field functions like (9), in order to model the actual geometrical structures, in which several slots are disposed along one surface (for now the other surface is maintained smoothed): to this aim, the Principle of Superposition of the Lost Flux Density (PSLB) must be introduced.
The PSLB declares the possibility to sum the lost flux densities of the single slots: the actual lost flux density of a multi-slot structure equals the sum of the lost flux densities due to each slot, as if it were the only existing slot; thus, the actual flux density equals the difference among the ideal flux density and the global lost flux density:

$$B_L\left(x\right)=\sum\nolimits_j B_{L1}\left(x-j\cdot\tau_s\right),\ \ B\left(x\right)=1-B_L\left(x\right)\ \Rightarrow\ \beta_L\left(x\right)=\sum\nolimits_j \beta_{L1}\left(x-j\cdot\tau_s\right)\ ,\ \ \beta\left(x\right)=1-\beta_L\left(x\right)\ , \qquad (11)$$

with j extended to all the slots. The PSLB is valid also in case of intersecting lost flux density curves, due to very near adjacent slots, and it can be applied also to other geometrical "disturbances", as the interpolar zones.
The PSLB has not been demonstrated yet, but several FEM tests have shown its correctness [3, 4, 5, 6].

Air-gap flux density distribution

The analysis is aimed to obtain the air-gap flux density distribution among two toothed surfaces, with any disposition of the currents in the slots: to this aim, some simple, basic situations must be progressively analysed.
Called x and y the linear peripheral coordinates along the stator and rotor surfaces respectively, and called z the position of the rotor origin with respect to the stator origin, it follows:

$$x = y + z. \qquad (12)$$

Two slotted surfaces, among which a constant m.p.d. U is applied

The following functions should be defined (subscripts $_S$, $_R$, $_L$ indicate: Stator, Rotor, Lost):

– $B_S(x)$ and $B_{LS}(x)$: flux density and lost flux density due to a slotted stator surface faced to a smoothed rotor one, evaluated along the rotor smoothed surface ($\beta_S(x)$ and $\beta_{LS}(x) = 1-\beta_S(x)$ are the related field functions);

– $B_R(y)$ and $B_{LR}(y)$: flux density and lost flux density due to a slotted rotor surface faced to a smoothed stator one, evaluated along the stator smoothed surface ($\beta_R(y)$ and $\beta_{LR}(y) = 1-\beta_R(y)$ are the related field functions);

– $B(x,y)$ and $B_L(x,y)$: resultant flux density and lost flux density, in case of both slotted surfaces, assumed as purely radial, considered at half air-gap width ($\beta(x,y)$ and $\beta_L(x,y) = 1-\beta(x,y)$ are the related field functions).

The field functions $\beta_{LS}(x)$ and $\beta_{LR}(y)$ could be obtained by the previously described analytical procedure, and applying the PSLB: indeed, once operated the superposition (11), it is more suited to employ an interpolating function. Good results have been achieved by using exponential periodic functions.

For the stator field function, called τ_{sS} the slot pitch, the following equation was used:

$$\beta_{LS}(x) = \beta_{LSo} \cdot \exp\left\{-\left[\nu_S \cdot \sin^2(\pi \cdot x/\tau_{sS})\right]^{\lambda_S}\right\} \quad , \tag{13}$$

where β_{LSo}, ν_S, and λ_S can be evaluated by parameter identification, from analytical or FEM solution results.

As regards $\beta_{LR}(y)$, an expression like (13) can be similarly obtained only if the rotor is uniformly slotted (as in the induction motors). In case of an isotropic synchronous machine, this condition is not verified, and a different global slotting model must be developed: after performing the superposition (11), the function $\beta_{LR1}(y)$ should be considered, interpolating the field just within one rotor slot pitch; then, the slotting repetition is obtained by summations, at first extended within one pole ($\beta_{LRp}(y)$), subsequently all along the overall periphery ($\beta_{LR}(y)$):

$$\beta_{LR1}(y) = \beta_{LRo} \cdot \exp\left\{-\left[\nu_R \cdot \left(\left(y-\frac{\tau}{2}\right)\Big/\tau_{sR}\right)^2\right]^{\lambda_R}\right\}, \quad \beta_{LRp}(y) = \sum_{j_r=0}^{c_{pR}-1} \beta_{LR1}\left(y - \left(\frac{c_{pR}-1}{2} - j_R\right)\cdot \tau_{sR}\right), \tag{14}$$

$$\beta_{LR}(y) = \sum_{j_p} \beta_{LRp}(y - j_p \cdot \tau) \quad , \quad j_p = 0,\dots,N_p - 1\,, \qquad \text{where} \tag{15}$$

c_{pR} = N° of rotor slots/pole, τ_{sR} = slot pitch, N_p= N° of poles, β_{LRo}, ν_R, and λ_R to be evaluated as for the stator.

As regards the functions $B(x, y)$ and $\beta(x, y)$, y must be explicitly expressed by eq. (12): in fact, while the rotor position z among the faced structures is indifferent for the evaluation of $\beta_{LS}(x)$ and $\beta_{LR}(y)$ (because one structure is smoothed), in a doubly slotted machine the mutual position z among the structures must be defined; thus:

$$B(x,y) = B(x, x-z) \quad , \quad \beta(x,y) = \beta(x, x-z), \tag{16}$$

so that, for a given rotor position z, B and β are expressed only as a function of coordinate x along the stator.

In order to evaluate the resultant functions $\beta(x, x\text{-}z)$, it could seem reasonable to extend the PSLB, by summing the lost flux density distributions of each structure; on the contrary, various FEM simulations have shown that:

$$\begin{aligned} &\beta_L(x, x-z) \neq \beta_{LS}(x) + \beta_{LR}(x-z) \\ &\beta(x, x-z) = \beta_S(x)\cdot\beta_R(x-z) = \left[1-\beta_{LS}(x)\right]\cdot\left[1-\beta_{LR}(x-z)\right] \end{aligned} \quad . \tag{17}$$

As for the PSLB, also eq. (17) has not been demonstrated yet; just some remarks can explain its soundness:

– from eq. (17), the actual flux density can be obtained as follows:

$$B(x, x-z) = B_i \cdot \beta(x, x-z) = B_i \cdot \beta_S(x)\cdot\beta_R(x-z) = B_S(x)\cdot\beta_R(x-z)\,; \tag{18}$$

thus, the $\beta_R(x\text{-}z)$ function can be considered as the correction field function to be applied to $B_S(x)$;

– the Carter's factor of a doubly slotted structure is similarly evaluated, as the product of the single factors.

Finally, the origin of eq. (18) implies that the resultant flux density is assumed as consisting of a radial component only; in fact, the actual flux density on the cylinder settled at half air-gap width has also tangential components: a priori, this discrepancy could imply certain errors in evaluating some air-gap quantities (such as flux linkages, magnetic energy): various numerical tests have shown that these errors remain acceptably small.

Single coil fed by current in a slotted structure, faced to a smoothed surface

Coming back to the case of a smoothed surface faced to a slotted one (e.g. the stator), consider that just a single coil is fed by current (fig.1): in case of an integer pitch τ, the m.m.f. has the square wave distribution of eq. (1).

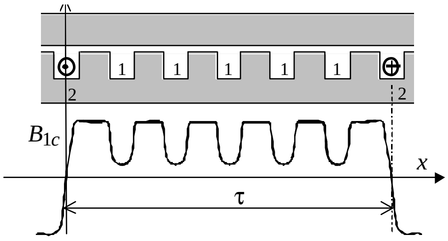

Fig.1 – Flux density distribution produced by a single coil fed by current, disposed in a slotted structure faced to a smoothed one.

Called $B_{1c}(x)$ and $\beta_{1c}(x)$ the flux density and the field function distributions respectively, the $\beta_{1c}(x)$ waveform appears as follows:

- between the coil sides, the flux density is that due to a constant m.p.d., applied among a smoothed and a slotted structure;
- near the current fed slots (N°2 in fig.1), the flux density gradually reverses, with a zero crossing in the position of the slot axes.

Called $B_{in}(x)$ and $\beta_{in}(x)$ the flux density and the corresponding field function evaluated along the smoothed surface in front of each fed slot, it should be observed that this field behaviour is the same of that in the interpolar zone of a salient pole synchronous machine.

Various FEM analyses confirmed this hypothesis: thanks to this property, the field distribution $\beta_{in}(x)$ can be obtained from the conformal transformation solution of the interpolar field of a synchronous machine [2, 3, 4].

The problem is how to combine the notch field function $\beta_s(x)$, which exists in front of the intrapolar slots (slots N° 1 in fig.1), and the field function $\beta_{in}(x)$ in front of the current fed slots (that can be called interpolar slots).

A first way to solve this difficulty could consist in defining the following function, called interpolar function:

$$\alpha(x) = \beta_{in}(x)/\beta_s(x) \quad . \tag{19}$$

In fact, this allows to apply the notch function $\beta_s(x)$, due to slots, to all the slots; as regards the field in front of the interpolar slots, it can be correctly modelled by applying the function $\alpha(x)$ as follows: $B_{in}(x) = B_i \cdot \beta_s(x) \cdot \alpha(x)$.

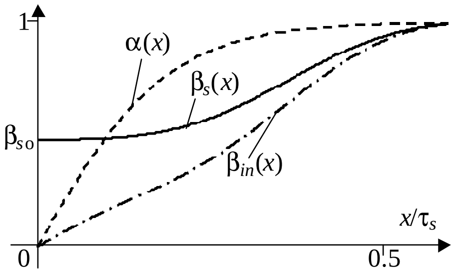

Fig.2 – Field functions in front of an interpolar slot (origin in the slot axis).

The typical shapes of the functions $\beta_{in}(x)$, $\beta_s(x)$ and $\alpha(x)$ are shown in fig.2, within the half slot pitch at the right of the axis of the current fed slot: while $\beta_s(x)$ is an even function, both $\beta_{in}(x)$ and $\alpha(x)$ are odd functions. As suggested by fig.2 and verified by FEM simulations, $\alpha(x)$ can be fairly approximated by:

$$\alpha(x) = \tanh(x/\tau_i) \quad , \tag{20}$$

where τ_i is a space constant, that can be easily estimated as follows:

$$\tau_i \approx \beta_{so} / \left(d\beta_{in}(x)/dx \right)_{x=0} \quad , \tag{21}$$

thanks to the linear behaviour of the functions near the slot axis.

<u>Whole phase winding fed by current in a slotted structure, faced to a smoothed surface</u>

A difficulty that arises in using the interpolar function $\alpha(x)$ is due to the fact that, when all the phase winding coils are fed by current, each fed slot simultaneously appears as intrapolar (when considering the field produced by currents in slots external to it) and interpolar (as regards the field contribution caused by its own current): in the general case, this makes quite complicated the application of $\alpha(x)$ to the flux density distribution.

The use of $\alpha(x)$ becomes easier if it is associated to the distribution of the m.m.f. rather than to the flux density distribution: in fact, this choice allows to apply $\alpha(x)$ to each contribution of the coil m.m.f. (1), thus transforming the step variations of m.m.f. (implemented in eq.s (1) – (8)) in smoothed variations, according to the interpolar slot field behaviour: in this way, the use of $\alpha(x)$ is very simple, because, when performing the superposition of the coil m.m.f. contributions, $\alpha(x)$ is selectively and automatically implemented.

Of course, even if applied to the m.m.f., in principle $\alpha(x)$ is a flux density correction function.

The application of $\alpha(x)$ to the basic situation of a coil with an integer pitch shows that the product among $\alpha(x)$ and the step square-wave waveform (1) can be conveniently substituted by the following expression:

$$M_m(x) = (1/2) \cdot \tanh\left[k_{m\tau} \cdot \cos(\pi \cdot x/\tau)\right] \quad ; \tag{22}$$

$k_{m\tau}$ is a coefficient set to the value that correctly reproduces the slope of $\alpha(x)$ at the zero crossing; it is always sufficiently high (usually higher than 10) to saturate the tanh value to ± 1 when the cosine function tends to unity.

In case of a shorted pitch coil, the smoothed slope m.m.f. expression corresponding to the stepwise one of (7) is:

$$M_{m.sh}(x) = (1/2) \cdot \left\{ \tanh\left[k_{m\tau} \cdot \left(\cos(\pi \cdot x/\tau) - \cos\left[\pi \cdot \tau_c/(2 \cdot \tau)\right]\right)\right]\right\} + (\tau - \tau_c)/(2 \cdot \tau) . \tag{23}$$

As concerns eq.s (3), (4), (5), (8), they remain unvaried, but in them (22) and (23) should substitute (1) and (7).

<u>Doubly slotted synchronous machine with three-phase stator winding and distributed field rotor winding</u>

Consider an isotropic synchronous machine having a generic three-phase stator winding (one or two layers; integer or shorted pitch) with q slots/(pole-phase), a rotor winding with c_p slots/pole and N_f turns/(field coil), and stator and rotor slot pitches equal to τ_{sS} and τ_{sR}.

Called $M_{fS}(x)$ and $M_{fR}(y)$ the m.m.f. space distributions of one phase stator winding and of the field winding respectively, and indicated with $i_f(t)$ the instantaneous current in the field winding, the following stator, rotor and resultant instantaneous m.m.f.s. can be written:

$$m_S(x,t) = (N_t/a)\cdot \sum_{p=1,2,3} M_{fS}\left[x-(p-1)\cdot 2\tau/3\right]\cdot i_p(t) \quad ; \qquad m_R(y,t) = M_{fR}(y)\cdot N_f \cdot i_f(t) \quad , \qquad (24)$$

$$m(x,y,t) = m_S(x,t) + m_R(y,t) \quad \Rightarrow \quad m(x,x-z,t) = m_S(x,t) + m_R(x-z,t) \quad . \qquad (25)$$

The second formulation of the instantaneous distribution of the total m.m.f. points out that the m.m.f. acting in each point x measured along the stator periphery depends on time (as concerns the current time waveforms) and on rotor position z (that can vary as a time dependent variable too).

Finally, the instantaneous distribution of the air-gap flux density is described by this compact expression:

$$b(x,y,t) = (\mu_0/g)\cdot m(x,y,t)\cdot \beta_S(x)\cdot \beta_R(y) \Rightarrow \; b(x,x-z,t) = (\mu_0/g)\cdot m(x,x-z,t)\cdot \beta_S(x)\cdot \beta_R(x-z)\, , \qquad (26)$$

where $\beta_S(x)$ and $\beta_R(x\text{–}z)$ are the “notch” field functions taking into account the stator and rotor slotting effects, while the interpolar field effects are modelled directly by the tanh zero crossing shape of the m.m.f. terms.

As an example of m.m.f. space distributions, in case of single-layer, integer pitch stator windings, the stator and rotor coil m.m.f. expressions can be written as follows:

$$M_{cS}(x) = (1/2)\cdot \tanh\left[k_{mS}\cdot \cos(\pi\cdot x/\tau)\right] \quad ; \quad M_{cR}(y) = (1/2)\cdot \tanh\left[k_{mR}\cdot \cos(\pi\cdot y/\tau)\right], \qquad (27)$$

while the corresponding phase and field winding expressions are given by:

$$M_{fS}(x) = \sum_{j_S=1}^{q} M_{cS}\left[x-\left(\frac{q-1}{2}-(j_S-1)\right)\cdot \tau_{sS}\right]; M_{fR}(y) = \sum_{j_R=1}^{c_p} M_{cR}\left[y-\left(\frac{c_p-1}{2}-(j_R-1)\right)\cdot \tau_{sR}\right]. \qquad (28)$$

Validation of the analytical method by FEM comparison

A lot of global and local tests have been done, performing FEM simulations [6] and analytical evaluations of different geometrical and operating conditions: in the following, just some results are shown, aimed to show the soundness of the adopted approach. Reference is made to a constant air-gap synchronous machine (data in Table 1): even if some quantities are not completely realistic, it is a significant validation test for the described method.

Fig.3 shows the flux density distribution as a function of the electrical angle θ along the stator bore.

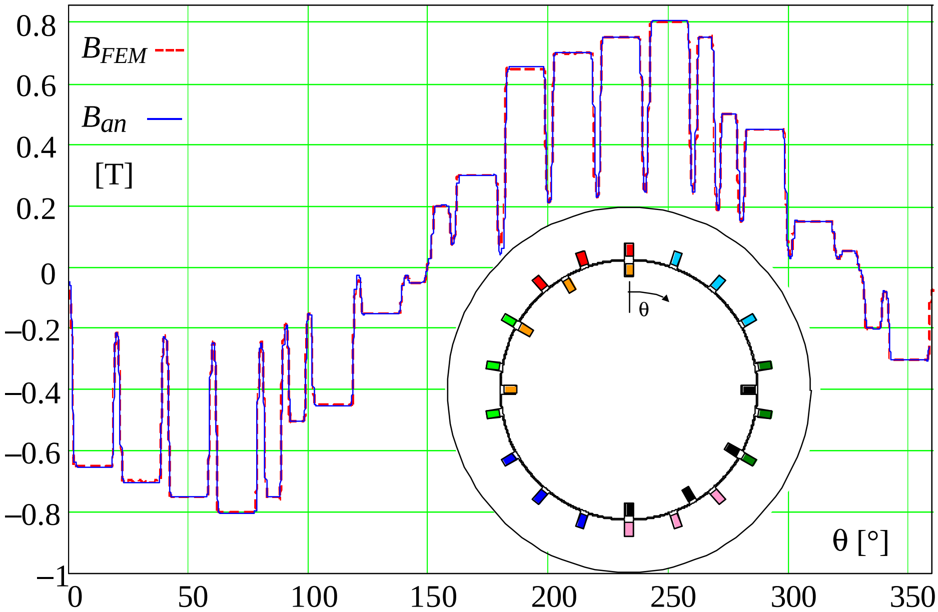

Table 1- Data of the analysed constant air-gap synchronous machine	
N° of poles	2
N° of parallel paths	1
N° of stator slots	18
N° of stator phases	3
N° of winding layers	1
N° of rotor slots/pole	4
mech. angle among rotor slots	30°
stator internal diameter [m]	1
air-gap [mm]	5
stator and rotor slot opening [mm]	30

Fig.3 - Flux density distribution as a function of the angle θ along the stator bore, for the shown rotor position; B_{an} = analytical method; B_{FEM} = FEM calculations (radial component, evaluated at half air-gap width); machine data of Table 1; operating conditions: field slot tot. current = I_{tf} = 1000 A; sinus. stator currents: peak phase slot tot. current = I_{tphM} = 400 A.

Since the analytical model assumes a purely radial flux density, in the FEM simulations only radial components at half air-gap width have been considered.
As can be noted, the field distributions are very close, giving an effective validation of the described analytical approach; a quantitative estimation of the reciprocal discrepancies can be obtained by evaluating the following accuracy index integrals, indicating p.u. values, performed within one half-wave of the flux density diagrams:

$$I_1 = \frac{1}{\tau} \cdot \int_0^{\tau} \frac{B_{an}(x) - B_{FEM}(x)}{B_{MAX}} dx \quad ; \qquad I_2 = \frac{1}{\tau} \cdot \int_0^{\tau} \frac{\left|B_{an}(x) - B_{FEM}(x)\right|}{B_{MAX}} dx \qquad (29)$$

I_1 gives the average error value of the actual local difference, while I_2 represents the average error value of the absolute value of this difference: the percent values referred to fig.3 are: I_1 = 0.33 %; I_2 = 1.20 %; the first index is lower than the second one, thanks to a partial sign compensation in the local differences, rectified in the I_2.
The level of discrepancy is sufficiently low to suggest that it is caused not only by possible inaccuracies of the analytical method; in fact, it could be partially imputed to the FEM results.
Finally, in order to show the accuracy of the analytical method also in case of partial overlapping of faced slots, fig.4 shows some p.u. flux density distributions (i.e., the function $\beta(x, x-z) = \beta_S(x) \cdot \beta_R(x, x-z)$) evaluated analytically and by FEM simulations (as before, radial component at half air-gap width), under constant p.m.d. U: the size of slot openings and air-gap are the same of Table 1, and five reciprocal positions Δz are shown with steps of 10 mm, starting from the slot axes alignment: as can be seen, the waveforms are quite similar, both as shape and values, confirming the fair soundness of the method.

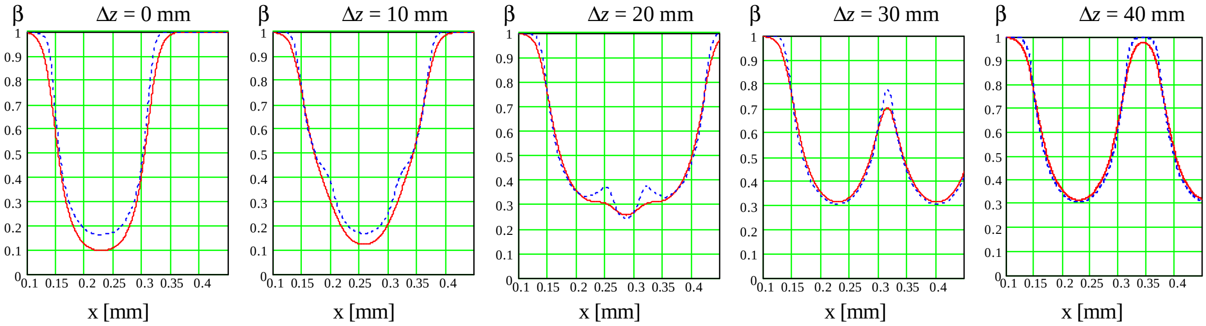

Fig.4 - p.u. flux density distributions (i.e., function $\beta(x, x-z) = \beta_S(x) \cdot \beta_R(x, x-z)$) evaluated analytically (—) and numerically (----: FEM simulations - radial component at half air-gap width -), under constant p.m.d. U; slot openings and air-gap sizes: see Table 1; Δz is the mutual displacement between stator and rotor slots, starting from the slot axes alignment; parameters of the equations (17), (18): stator: β_{LSo}= 0.684; ν_S = 17.2; λ_S =1.77; rotor: β_{LRo}= 0.684; ν_R = 371; λ_R =1.79.

Conclusion

An analytical method for the evaluation of the flux density distribution at the air-gap, taking into account the stator and rotor slotting effects and the actual distribution of the instantaneous currents, has been described: its good level of accuracy has been verified considering a test case, concerning an isotropic synchronous machine.
The described method is well suited to the analytical, accurate, evaluation of e.m.f. and torque waveforms, of great interest in many modelling and design problems concerning slotted electrical machines.

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