Aeroacustics computation based on harmonic balance solution

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Abstract. This paper presents a new open-source framework to compute the noise emitted by aerodynamic bodies whose motion is dominated by a specific frequency. This flow behavior is typical of propellers, pitching blades and wind turbines. The reduced order model called harmonic balance is used to compute the unsteady flow solution reducing the computational cost. *K* frequencies are solved by obtaining the conservative variables at *N*-time instances inside one period, where N = 2k+1. The time history of the surface flow solution is reconstructed with a Fourier integration. The Kirchhoff Ffowcs Williams Hawkings integral formulation, implemented in SU2, is used to compute the sound pressure level perceived by farfield observers. The integral formulation propagates the acoustic solution with a computational cost independent of observer distance. The noise emittance of a pitching wing is computed with the proposed framework and compared with a fully time accurate solution showing a very good agreement.

Introduction

Noise emittance due to the airframe is gaining importance in many aeronautical areas, in particular concerning wind turbines, pitching blades and propellers for urban air mobility. Both need to interact with residential neighbors and comply with strict regulations and noise certification requirements. It is necessary to develop reliable and verified noise emission models that can be integrated in design processes. Obtaining an accurate prediction of the noise emmittance is challenging and computationally expensive due to its unsteadiness and turbulent nature. An accurate time-resolved flow solution is required. In a modular style, a CFD-CAA aeroacoustic solver had been implemented in the open-source high fidelity software SU2 [1]. Although high-fidelity models are essential, approaches with a modest level of computational complexity are required to include noise into a design environment. The Kirchhoff Ffowcs Williams Hawkings integral formulation (FWH), implemented in SU2 in the Di Francescantonio version [2], is used to compute the sound pressure level (SPL) perceived by farfield observers. The integral formulation propagates the acoustic solution with a computational cost independent of observer distance. In this work, only the solid surface version is used therefore the surface flow unsteady solution is needed as sound source.

Typically, a fully Unsteady Reynolds Averaged Navier Stokes (URANS) is solved with a dual time stepping method and the flow solution at every time step is stored. The novelty of this work is the introduction of a reduced order model to compute the flow solution and use it as noise sources, reducing the computational effort needed. Quasi-periodic flows, like wind turbines or propellers, which are dominated by a set of frequencies can be efficiently solved with a harmonic balance method. It avoids the time-consuming transient calculation that is typical in time accurate CFD. With respect to other frequency methods, the harmonic balance is a time-spectral method where the k frequencies solved are not necessarily integral multiples of one another. Moreover, it is highly parallelized. The harmonic balance method [3] can well capture the time history of the aerodynamic coefficient with a relatively low number of frequencies solved. The HB matrix and the N-instance, where N = 2k + 1 conservative variable vector are combined to generate a matrix-vector product corresponding to the time derivative of the state variables. To sum up, the problem is formulated and solved as N steady-state problems, advancing in parallel in the pseudo-time and linked by a low order representation of the time derivative. Once N time solutions are obtained a more time accurate flow history inside a single period is reconstructed through a Fourier interpolation. Since for the tonal acoustic only the variables on the solid surface are needed, the interpolation is limited to this portion of the CFD domain.

This paper is structured as follows. In Sections 2 and 3, we present the aeroacustics framework and the numerical tools employed. In Section 4, the results obtained on a pitching wing are presented and the difference between the fully time accurate and the solution obtained with the reduced order method are analyzed. In the end, in Section 5 we summarize the findings and comment on future perspectives.

Harmonic Balance

This section introduces the governing flow equations and the reduced order model used to solve them. After temporal discretization and spatial integration across a control volume, the unsteady compressible Navier-Stoker equations are provided using the finite volume approach and are written as:

$$D_t U|\Omega| + R(U) = 0 \tag{1}$$

Where Ω is the control volume and U the conservative variables. The residual R(U) contains the convective and viscous fluxes integrated over the control volume's interfaces. D_t is the derivative operator with respect to the control volume and to time. A dual time-stepping integration methos is used in SU2, at each physical time instance a steady-state problem is solved in the pseudo-time τ:

$$|\Omega| \frac{\partial U_n}{\partial \tau} + D_t|\Omega| U_n + R(U_n) = 0 \tag{2}$$

When dealing with high computational demanding time accurate method, it becomes interesting to introduce reduced-order models and study the range of applicability. Harmonic balance (HB) can be used to solve quasi-periodic flows, dominated by a set of frequencies that are not necessarily integral multiples of one another. The theory about HB, here briefly presented, was implemented in SU2 software by [3]. The harmonic balance time operator $\mathcal{D}_t$ must be introduced.

Given $\bar{\omega}$ the vector of K frequency to be solved, the N flow solution U will be obtained at the t_n time instance in the period T, where $N = 2K + 1$ and $t_n = (n-1)\,T/N$. Being E the Discrete Fourier Transform (DFT) matrix, defined as:

$$E_{k,n} = \frac{1}{N} e^{-i\omega_k t_n} \tag{3}$$

The spectral operator matrix H is found to be:

$$\mathrm{H} = \mathrm{E}^{-1}\,\mathrm{DE} \text{ where } \mathrm{D} = \mathrm{diag}(\bar{\omega}) \tag{4}$$

Defined $\bar{U}$ as the vector containing the conservative variables for all the N time instance, the harmonica balance operator is given by:

$$\mathcal{D}_t(\bar{U}) = \mathrm{H}\,\bar{U} \tag{5}$$

Replacing the harmonic balance operator in equation (2) and defining q as the pseudo time step it is obtained that each time instance is solved in a steady-state manner, with all the time instances marching with the corresponding local pseudo-time step:

$$|\Omega|\frac{\partial U_n}{\partial \tau} + |\Omega|\text{H}\ \bar{U} + R\left(U_n^{q+1}\right) = 0 \tag{6}$$

In conclusion with harmonic balance, we obtain a discrete flow solution in a flow period with a time step $\Delta t = T/(N-1)$ without computing the typical transitory of the unsteady flows. However, concerning acoustic propagation a more time accurate surface flow solution is needed. To obtain it, a Fourier interpolation is performed for each conservative variable ϕ at each surface node of the grid. An arbitrary time resolution Δt^* related to a larger number of time instances inside N^* the period can be selected and the relative ϕ^* are obtained with:

$$\phi^* = E^{*-1}(E\phi) \tag{7}$$

where E^{*-1} is the bigger IDFT rectangular matrix of dimension $N^* \times N$.

Acoustic Formulation

The Ffowcs Williams-Hawkings (FW-H) equation is solved in this work to transmit pressure changes on the emission surface to observers in the farfield. The so-called "wind tunnel configuration" is taken into consideration, in which the source and observer move simultaneously while maintaining a constant distance between them. In a modular style, the FWH formulation implemented in C++ [4] takes as input the interpolated flow solution obtained with HB. Concerning SU2, the version of the integral formulation proposed by Di Francescantonio is implemented, which is an extension of the Farassat work [5] to general moving surfaces in which combines the positive aspect of the Kirchhoff and FWH equations:

$$4\pi p' = \int_S \left[\frac{\rho_0\left(\dot{U}_i n_i + U_i \dot{n}_i\right)}{r|1-M_r|^2}\right]_{ret} dS + \int_S \left[\frac{\rho_0 U_i n_i K}{r^2|1-M_r|^3}\right]_{ret} dS$$
$$+ \frac{1}{c}\int_S \left[\frac{\dot{F}_i \hat{r}_i}{r|1-M_r|^2}\right]_{ret} dS + \int_S \left[\frac{F_i\hat{r}_i - F_i M_i}{r^2|1-M_r|^2}\right]_{ret} dS + \frac{1}{c}\int_S \left[\frac{F_i\hat{r}_i K}{r^2|1-M_r|^3}\right]_{ret} dS$$

where

$$K = \dot{M}_i\hat{r}_i r + M_r c - M^2 c,$$
$$U_i = u_i + \left[(\rho/\rho_0) - 1\right](u_i - v_i),$$
$$\hat{r}_i = r_i/r.$$

M stands for the Mach number, r for Euclidean observer distance from the source node, u is the flow velocity, v represents the grid velocity and P_{ij} stands for the perturbation stress tensor. The derivatives are calculated in the time that the observer hears the noise signal, which is known as the retarded time. This formulation can handle both permeable and solid surfaces; a solid surface is one that prevents mass transit through it, such as the aerodynamic body itself. The loading noise contribution is connected to the final three components, whereas the first two terms are related to thickness noise contribution.

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Numerical Simulation

To validate the capability of the proposed framework to predict tonal noise emitted by quasi period flow a pitching straight wing has been simulated. The wing has a chord of 1m and a span of 3m, the section is a NACA6410 airfoil. In standard conditions of pressure and temperature, the freestream Mach number is M = 0.796, resulting in a Reynold number of 12.56 E6. Jameson-Schmidt-Turkel convective scheme is used with the $2th$ order artificial dissipation term set at 0.02 and the $4th$ = 0.02. SA one equation turbulence model is applied. A hybrid grid of around 1 million elements is generated with Pointwise, with a first cell high low enough to obtain y+<1 on the entire surface. The rigid motion is imposed by a prescribed time-varying angle of attack, calculated with:

$$\alpha(t) = 1.06 \sin(\omega t) + \alpha_0$$

where $\alpha_0 = 0°$ and $\omega = 109.339$.

The test case is simulated both with a standard fully time accurate URANS with 100 time steps per period and with the Harmonic Balance method. Concerning HB, the solution is obtained both for 3- and 5-time instances corresponding to a frequency vector of $\overline{\omega} = [0; \pm\omega_1]$ in the first case and $\overline{\omega} = [0; \pm\omega_1; \pm\omega_2]$ in the second one, with $\omega_1 = \omega = 109.339$ *and* $\omega_2 = 2\omega_1$. After, with the Fourier interpolation the surface flow time solution with the same Δt of the URANS is obtained. For every time instance in the HB simulation, the residual density must be reduced by eight orders of magnitude.

The next figures show the comparison of the lift and drag coefficient. Concerning the C_l a very good matching is obtained already with only three frequencies, instead for the C_d it is necessary to increase the length of $\overline{\omega}$ to reduce the relative error.

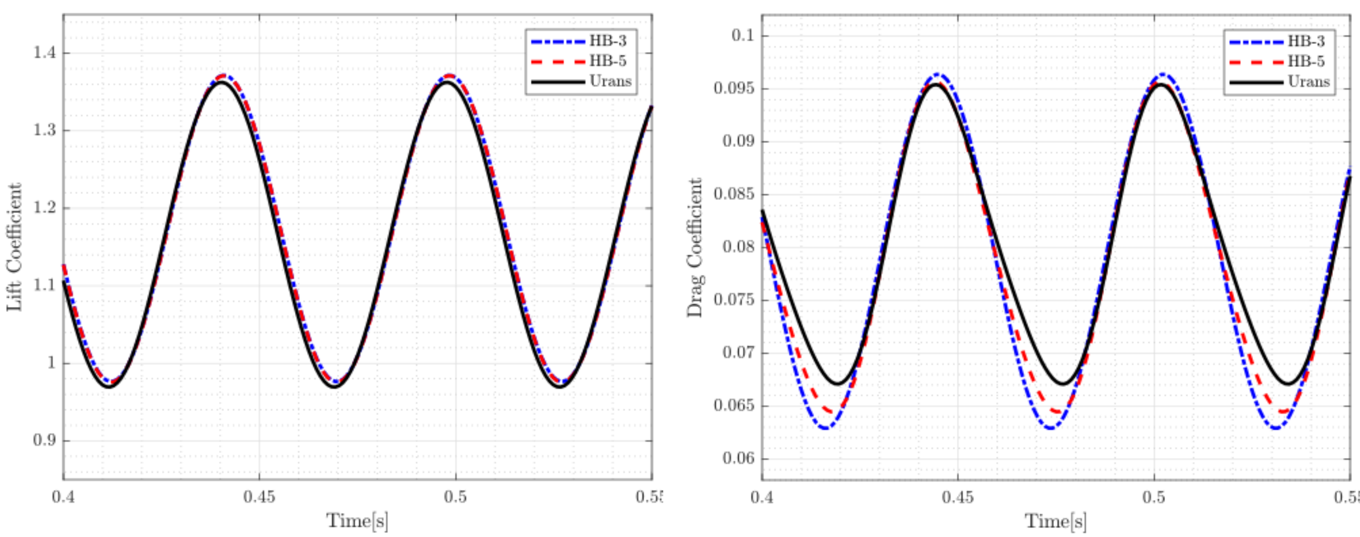

Figure 2 Lift (left) and drag (right) coefficient comparison between HB and Urans for one period of oscillation.

The sound pressure level (SPL) in dB perceived by 17 microphones is computed with the FWH module. The observers are equally distributed on an arch with radius 10m, centered in the middle of the span and the chord, on a plane perpendicular to the wing.

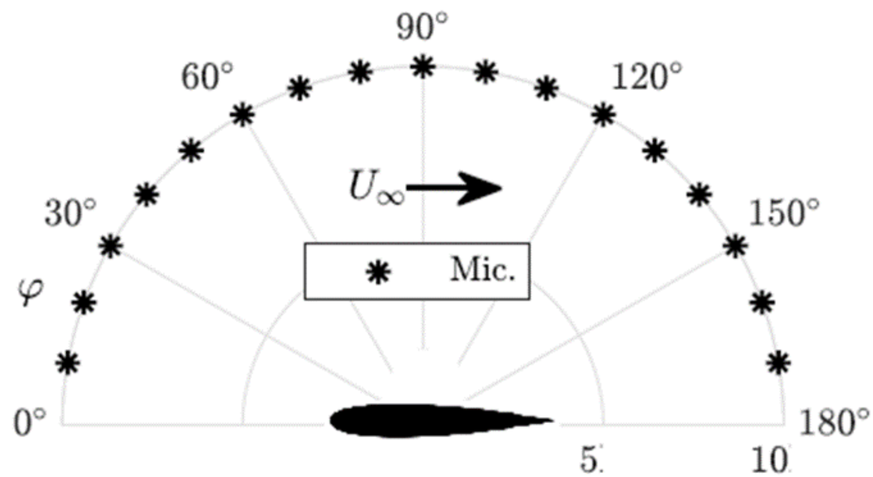

Figure 3 Farfield observer position and reference of system.

First it is reported the pressure perturbation perceived by two observers. The harmonic balance reported is the one obtained with 5 frequencies. It can be observed that there is perfect matching for the microphone positioned at $\varphi = 90°$ instead a higher relative error is found in the downstream direction $\varphi = 170°$, Fig(4). The same trend is reflected for the SPL computed for the entire arch, Fig(5). The relative error is everywhere lower than 1%, which is a remarkable result, Fig(5). There is not only a good matching in the integral value, but also the spectrum directivity plots obtained with the two solutions show are almost identical, see Fig(6).

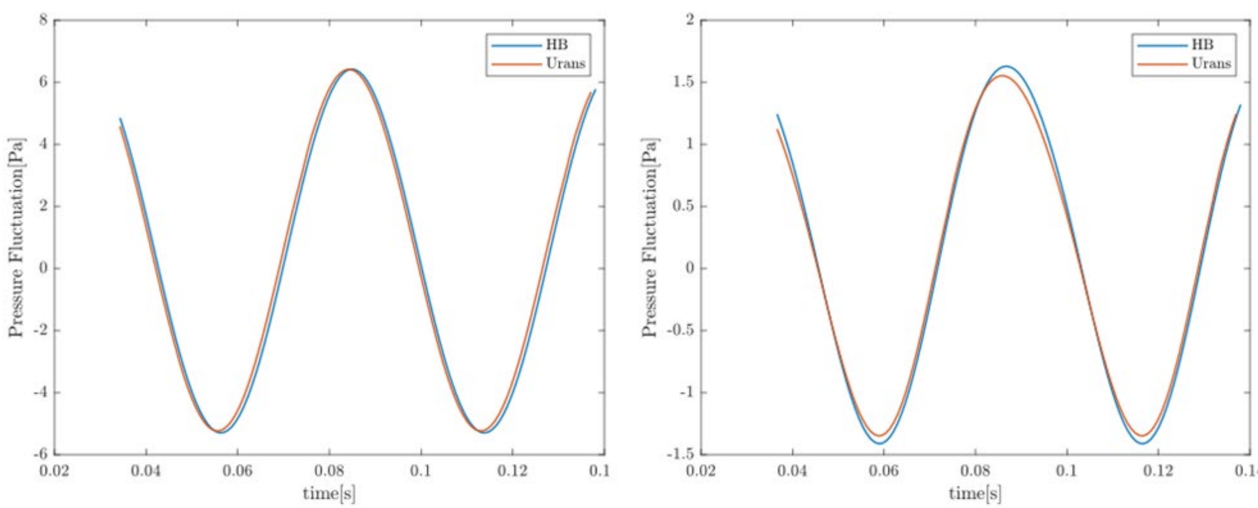

Figure 4 Pressure perturbation perceived by the microphone at $\boldsymbol{\varphi = 90°}$*(left) and* $\boldsymbol{\varphi = 170°}$ *(right).*

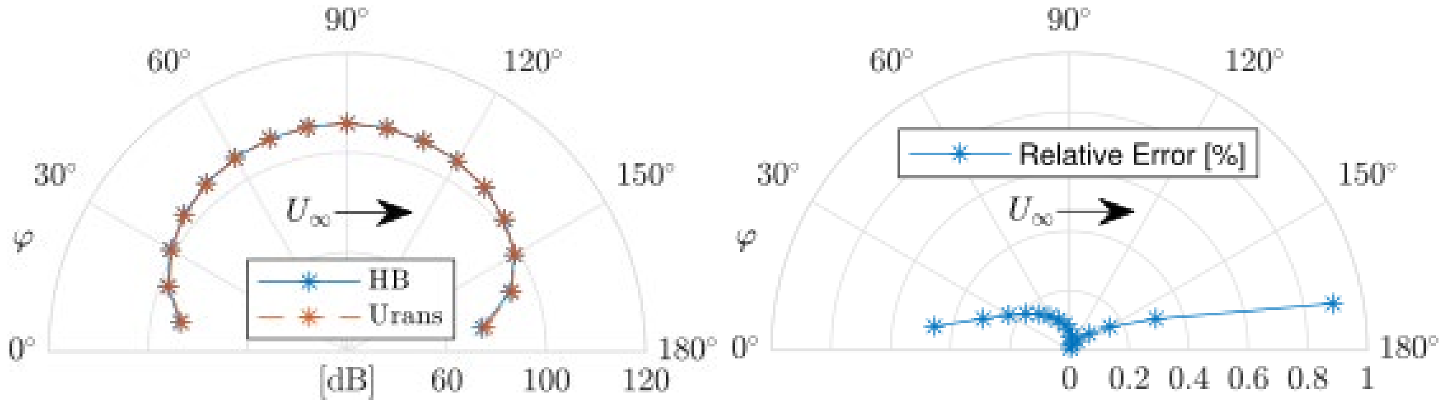

Figure 5 SPL perceived by 17 microphones (left) and relative error between the HB and Urans solution (right)

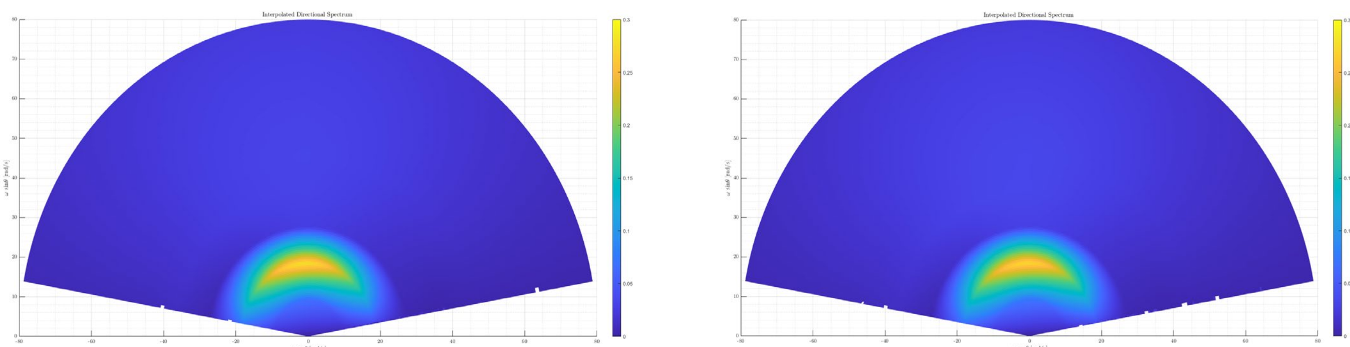

Figure 6 Directivity spectrum obtained with HB solution(left) and Urans solution (right).

Conclusions

In this work, a new open-source framework for computing the sound pressure level perceived by farfield observers is presented. The framework is specific for quasi-periodic flows. The reduced order model Harmonic Balance method is used to compute the flow solution. With this time-spectral method it is not necessary to simulate the transient period typical of an unsteady CFD but the converged solution is directly obtained advancing in the pseudo time. Following, a more time accurate solution is obtained with a Fourier interpolation only for the surface nodes. The acoustic module, where the integral FWH formulation is implemented, takes the interpolated solution as noise sources, and propagates to farfield observers. The proposed framework has been tested on a pitching wing and the results compared with fully time accurate solution obtained with a standard Urans. Five frequencies are sufficient to obtain a good approximation of the integral aerodynamic coefficients. Concerning the acoustics, a good matching is found between the two solutions not only in terms of SPL but also comparing the directivity spectrum. The promising results obtained suggest the application of the proposed framework to more complex cases like propellers or helicopter blades.

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