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A comparison between time-invariant and time-periodic stability analyses of wind turbines

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Abstract.
Stability analysis of wind turbines is of critical importance. The current standard methodology is modal analysis based on a linear time-invariant (LTI) dynamical system, derived using the multi-blade coordinate (MBC) transformation. This method does not fully capture the periodic parameter variations in wind turbine dynamics induced by the rotor rotation. Previous research has introduced methodologies based on a linear time-periodic (LTP) system representation. A key open question is under which conditions the LTI approximation becomes insufficient for accurately characterizing the stability of wind turbines. As a first step toward answering this question, this paper aims to establish a baseline comparison between the MBC-LTI and an identification-based LTP methodologies. The identification-based approach generates a reduced-order LTP model and performs Floquet stability analysis on it. The results are directly compared with those obtained using the LTI-based approach. In both methodologies, the aeroelastic response is simulated using the OpenFAST multi-physics solver, ensuring direct comparability. The identification-based LTP methodology successfully extracts the fore-aft and side-side bending modes as well as the in-plane whirling modes of the turbine. Results indicate good agreement with the standard approach and, additionally, reveal more nuanced periodic dynamic behavior. However, the study also identifies practical challenges related to the robustness of the method.

1. Introduction

As the wind energy sector shifts to wind turbine designs with increasing size, structural stability margins during operation decrease. Moreover, offshore wind energy adds complexity to the structural dynamics through hydro-elastic coupling of the submerged structure. It is therefore important for stability analysis tools to be able to accurately predict the stability of the structure.

The conventional stability analysis methodology relies on the Multi Blade Coordinate (MBC) or Coleman transformation, which transforms the degrees of freedom of the dynamical system from the rotating to the fixed reference frame [1]. In doing so it enables a linear time-invariant (LTI) state-space representation of the system. The study of stability of LTI systems is performed using eigen-decomposition of the dynamic matrix [2]. Although this methodology is suitable for most cases, the MBC transformation only results in an LTI system if the rotor is "isotropic". This means that periodicity introduced by effects such as gravity, tower shadow, horizontal or vertical wind shear, mass or aerodynamic imbalances, cannot be fully eliminated by the MBC transformation. The result of the MBC in such cases is a linear time-varying system and more specifically a linear time-periodic (LTP) system [1]. Coleman-based stability analysis methodologies overcome this by averaging the time varying dynamic matrix [1], [3]. The resulting averaged system is then treated as LTI for stability analysis. It can be shown that such an approach misses some harmonics (called superharmonics) that are instead resolved by a more rigorous Floquet analysis of the LTP system [4].

Although the classical LTI analysis is well established and widely used, two important questions remain open:

- What is the level of anisotropy that makes the time-averaging not acceptable anymore in terms of accuracy of the stability limits?
- Is it possible for the missed superharmonics to resonate with other modes or with external excitations?

It is the long-term goal of this research to fill these knowledge gaps. This paper represents a first step toward answering the first question posed above by establishing a baseline direct comparison between LTI and LTP stability analyses. Here we consider a conventional wind turbine design in mild periodic conditions, and we compare the results of the LTI analysis with the Floquet LTP identification-based approach developed in [4]. This initial study shows that the two approaches - as expected for this case of limited periodic effects - deliver very similar results, although the LTI approach is blind to some superharmonics revealed by the LTP analysis. Based on this initial validation, future work will move on to provide answers to the two research questions posed above.

2. Methodology

This study implements the system identification algorithm developed in [4] and compares it with the conventional stability analysis using the linearization capabilities of OpenFAST.

2.1. MBC - LTI stability analysis

An LTI system is modeled as:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}u(t) \end{cases} , \quad \mathbf{x}(0) = \mathbf{x}_0, \tag{1}$$

where, $\mathbf{x}(t)$ is the state vector as a function of time and $\mathbf{A}$ is the dynamic matrix. The stability of this system is determined by the real part of the eigenvalues of the dynamic matrix: if $\Re(\lambda_j) < 0\,\forall j$ then the system is asymptotically stable [2].

In practice, most tools capable of stability analysis, such as OpenFAST [5], linearize the equations of motion at several rotor azimuth positions over one rotor period T [3]. The result is a set of N matrices, each representing the linearized dynamics at a specific azimuth. Applying the MBC transformation converts this periodic system into the fixed reference frame. Averaging the transformed matrices over all azimuths yields an equivalent LTI system of the form (1), which can then be used for stability and modal analysis.

2.2. PARX - LTP stability analysis

To overcome the limitations of the LTI approach, a system identification methodology for stability analysis was developed in [4].

In this approach, the nonlinear system is perturbed by a force or displacement so as to excite the modes of interest. A reduced-order Periodic ARX (PARX) time-series prediction

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model is then fitted to the response of the system, $\hat{z}(k)$, yielding an equivalent LTP state-space representation. Finally, stability analysis is performed using Floquet theory [6], [7].

The PARX sequence for the predicted output time-series $z(k)$ and an input $u(k)$ can be written as:

$$z(k) = \sum_{i=1}^{N_a} a_i(k)\ z(k-i) + \sum_{j=0}^{N_b}\ b_j(k) u(k-j) + \eta(k), \tag{2}$$

where, $a_i(k)$ and $b_j(k)$ are periodic coefficients, which can be expanded in a Fourier series to ease their estimation:

$$a_i(k) = a_{i0} + \sum_{l=1}^{N_{F_a}} \left[a_{i_l}^c \cdot \cos(l\psi(k)) + a_{i_l}^s \cdot \sin(l\psi(k)) \right], \tag{3a}$$

$$b_j(k) = b_{j0} + \sum_{m=1}^{N_{F_b}} \left[b_{j_m}^c \cdot \cos(m\psi(k)) + b_{j_m}^s \cdot \sin(m\psi(k)) \right]. \tag{3b}$$

The number of terms included in the autoregressive and exogenous part, N_a and N_b respectively, as well as the number of frequencies included in each term, N_{F_a} and N_{F_b}, are referred to as the *complexity* of the PARX model.

From equation (3) it follows that the unknown coefficients or parameters of the time-series prediction model are: a_{i0}, $a_{i_l}^c$, $a_{i_l}^s$, b_{j0}, $b_{j_m}^c$, $b_{j_m}^s$. These coefficients are determined by solving an optimization problem that minimizes the error between the output of the model, $z(k)$, and the measured output, $\hat{z}(k)$. The objective function is defined as:

$$J = \sum_{k=1}^{N} (z(k) - \hat{z}(k))^2. \tag{4}$$

Next a LTP system of the form

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)u(t), & \mathbf{A}(t+T) = \mathbf{A}(t),\, \mathbf{B}(t+T) = \mathbf{B}(t), \\ \mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t) + \mathbf{D}(t)u(t), & \mathbf{C}(t+T) = \mathbf{C}(t),\, \mathbf{D}(t+T) = \mathbf{D}(t) \end{cases} \tag{5}$$

is realized. The matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ are defined in the following way:

$$\left[\begin{array}{c|c} \mathbf{A}(k) & \mathbf{B}(k) \\ \hline \mathbf{C} & \mathbf{D}(k) \end{array}\right] = \left[\begin{array}{ccccc|c} 0 & 0 & \cdots & 0 & \alpha_n(k) & \beta_n(k) \\ 1 & 0 & \cdots & 0 & \alpha_{n-1}(k) & \beta_{n-1}(k) \\ 0 & 1 & \cdots & 0 & \alpha_{n-2}(k) & \beta_{n-2}(k) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & \alpha_1(k) & \beta_1(k) \\ \hline 0 & 0 & \cdots & 0 & 1 & \beta_0(k) \end{array}\right], \tag{6}$$

where,

$$\alpha_j(k) = a_j(k+j), \quad \forall j \in (1, \ldots, N_a), \tag{7a}$$

$$\beta_0(k) = b_0(k), \tag{7b}$$

$$\beta_j(k) = b_j(k+j) + a_j(k+j)\, b_0(k), \quad \forall j \in (1, \ldots, N_b). \tag{7c}$$

This formulation ensures that the state-space system has the same time response as the PARX time-series predictor.

Lastly, Floquet stability analysis is performed on the autonomous identified LTP system,

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t). \tag{8}$$

The main outcomes of this analysis are the characteristic exponents η_j, and the mode shape matrix $\mathbf{Z}_{j_n}$. The characteristic exponents are analogous to the eigenvalues of LTI systems, while the mode shape matrix serves as the periodic counterpart of LTI eigenvectors. The characteristic exponents provide information on modal frequencies and damping ratios. However, in contrast to the LTI case, these quantities are not uniquely defined. This non-uniqueness arises from the multivalued nature of the complex logarithm used in the definition of the characteristic exponents. As a result, each Floquet mode is characterized by an infinite number of frequencies.

To interpret which of these frequencies play a dominant role in the response of the system, an output-specific modal participation factor is introduced. This metric quantifies the relative contribution of each harmonic to a specific system output. As discussed in [8], "the closer the participation of a certain harmonic of a given mode is to one, the more that mode behaves as invariant." Conversely, when the participation is distributed among several harmonics, the corresponding mode exhibits pronounced time-periodic behavior. Importantly, "if a given participation factor is predominant with respect to the others, a single frequency characterization could be accepted." Note that participation factors reflect the contribution of a specific superharmonic to the response of the system but do not influence its stability. The superharmonics of a given mode may be either stable or unstable; however, they always exhibit the same stability characteristics collectively. A detailed mathematical formulation and justification of the participation factor are beyond the scope of this work and can be found in [8].

3. Results

In order to compare the two methods, a well known and realistic model was considered. We simulate the NREL 5 MW Reference wind turbine described in [9], with aerodynamic loading and a variable pitch controller. The simulation was conducted for an 11.4 m/s steady wind, which is the rated wind speed of this turbine and results in an angular velocity of $\Omega = 12.01$ rad/s.

For the MBC-LTI results the linearization capabilities of OpenFAST and the Python scripts provided by NREL were used [10]. After reaching a (periodically) stable operating point, the system was linearized at 36 azimuth positions, equally spaced every 10 degrees of rotation to cover one full period. The MBC transformation was then applied to every matrix, and the resulting matrices were averaged. Finally, eigendecomposition was performed on the averaged matrix as described in section 2.1.

All directions and moments reported in the results are defined with respect to the reference coordinate system shown in figure 1.

3.1. Tower fore-aft response

The implementation of the PARX-LTP approach begins with the identification of the dynamical system. For that purpose a transient simulation was run. After the system had reached a steady operating point at 250 s, an instantaneous impulse force was applied at the tower top in the fore-aft (X) direction to excite side-side modes. The fore-aft tower base moment, M_Y, was recorded from 250 seconds, after the perturbation had ended, until the completion of 30 periods to capture the decay of the oscillatory behavior of the structure. The signal was resampled using 50 values per period to capture the frequency range of interest. This also reduced the amount of high frequency content and the number of data points to be processed. The parameters of the

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model were chosen as $N_a = 4$, $N_{F_a} = 1$, $N_b = 1$, and $N_{F_b} = 15$. Figure 2 shows that the model is able to capture the dynamic response of the system accurately. After the realization of the PARX time-series predictor to a state-space system according to equation 6, Floquet stability analysis was performed. The results of the conventional MBC-LTI and PARX-LTP approaches are summarized in table 1. Note that for the summary of the results of the Floquet analysis, only superharmonics with a participation factor greater than 0.01 were included.

Good agreement between the MBC-LTI and the PARX-LTP stability analyses can be observed both in the eigenfrequencies and their damping ratios. The damping ratios of the PARX-LTP procedure are consistently slightly lower than those of the MBC-LTI approach. This trend is observed throughout this work.

Table 1. Tower fore-aft eigenfrequencies, damping ratios, and participation factors. LTP values shown with corresponding LTI values in parentheses.

Mode	Frequency [Hz] LTP (LTI)	Damping Ratio LTP (LTI)	Participation LTP
1st Tower Fore-Aft	0.337 (0.337)	0.069 (0.072)	0.989
	2.711	0.021	0.027
2nd Tower Fore-Aft	2.911 (2.908)	0.019 (0.021)	0.944
	3.112	0.018	0.028

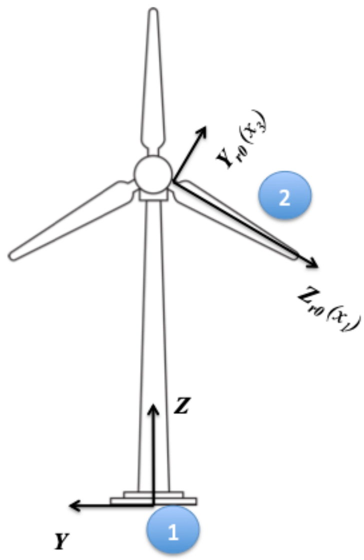

Figure 1. Inertial and blade-attached coordinate systems in OpenFAST. Illustration by Al Hicks, NREL. [5].

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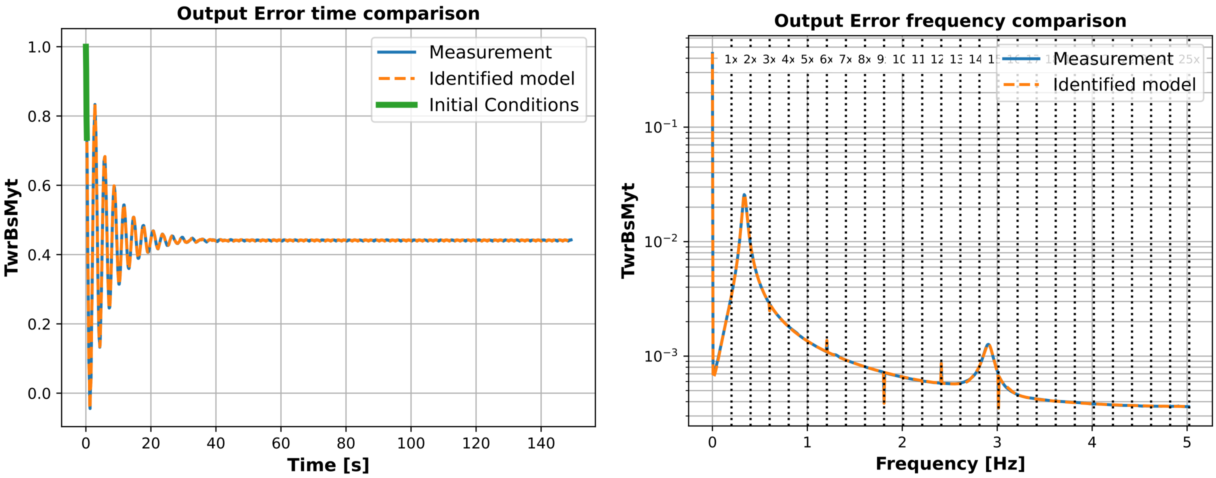


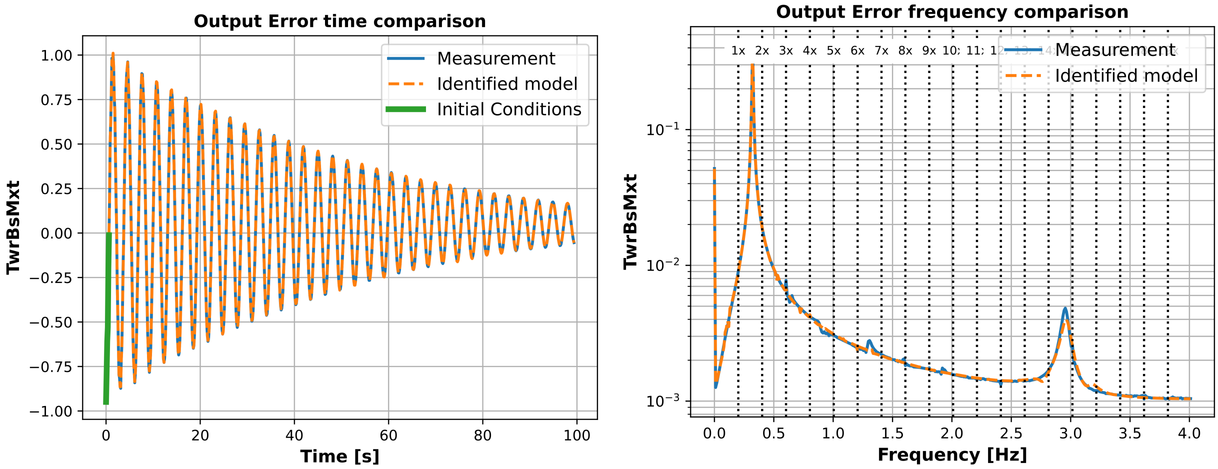
(a) Time domain. **(b)** Frequency domain.

Figure 2. Comparison of simulation-derived virtual measurement and PARX model prediction of the normalized tower base fore-aft moment M_Y.

3.2. Tower side-side response

For the identification of these modes using the *PARX-LTP* approach, a similar procedure was followed as before, the only difference is that the impulse force was applied in the side-side, (Y), direction to excite the relevant modes. The tower base side-side moment, M_X, was measured after the excitation for 20 periods and resampled at a rate of 40 samples per period. The complexity of the PARX model was chosen as follows: $N_a = 6$, $N_{F_a} = 1$, $N_b = 1$, $N_{F_b} = 3$. Figure 3 shows that the model is able to capture the dynamics well, although not perfectly as it can be seen from the small mismatches in the frequency domain in figure 3b. Table 2 summarizes the results of the Floquet analysis of the identified state-space system along with the MBC-LTI results. Only the harmonics with participation factors greater than 0.01 are presented. Notice that the damping of these modes is significantly lower than that of the fore-aft modes presented in table 1, which is expected because the aerodynamics of the wind turbine are taken into account.

The main tower frequency of 0.323 Hz exhibits a participation factor of only 0.7. This indicates that significant contributions come from other harmonics, the most significant being a superharmonic at 0.524 Hz. These contributions are completely ignored by the MBC-LTI methodology. The side-side tower response includes contributions beyond the tower modes. This is likely due to the lower damping of the edgewise blade modes which partake in the response as well. This makes identification of the side-side modes more difficult.



(a) Time domain. **(b)** Frequency domain.

Figure 3. Comparison of simulation-derived virtual measurement and PARX model prediction of the normalized tower base side-side moment M_X.

Table 2. Tower side-side eigenfrequencies, damping ratios, and participation factors. LTP values shown with corresponding LTI values in parentheses.

Mode	Frequency [Hz] LTP (LTI)	Damping Ratio LTP (LTI)	Participation LTP
1st Tower Side-Side	0.122	0.024	0.123
	0.280	0.011	0.020
	0.323 (0.321)	0.009 (0.011)	0.705
	0.524	0.006	0.129
2nd Tower Side-Side	2.763	0.016	0.057
	2.964 (2.955)	0.015 (0.011)	0.876
	3.165	0.014	0.061

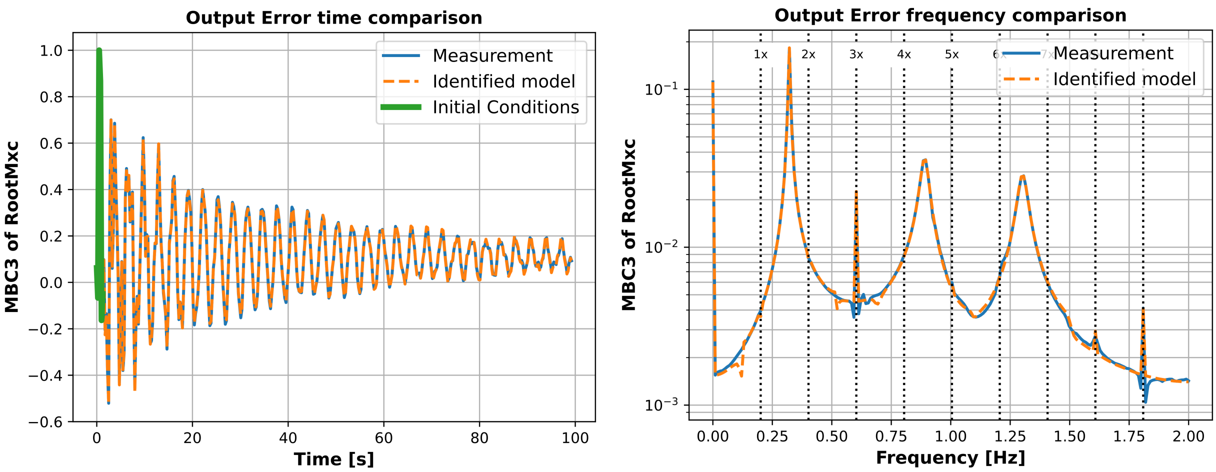
3.3. In-plane whirling response

The in-plane whirling modes were the most difficult to identify using the *PARX-LTP* approach because it is nearly impossible to excite them in a way that does not also excite the tower modes. The first attempt to identify these modes relied on the measurement of the tower base side-side moment M_X, but as it can be seen from figure 3b, the whirling modes are completely overshadowed by the 1st tower side-side mode. The next attempt focused on identifying the modes through the measurement of fore-aft tower base moment, M_Y. Though counterintuitive, the in-plane whirling modes were much more prominent in this signal, because they were not overshadowed by the side-side response of the tower. This approach was effective, but not as effective as the final solution that we present here, because of the complexity of the signal and the number modes that had to be concurrently identified to match the response of the fore-aft tower base moment. Finally, the most effective strategy to identify those modes was a side-side excitation of the tower top with an impulse force, while recording the edgewise moment of the root of every blade. These quantities, noted M_{X_B1}, M_{X_B2}, M_{X_B3}, were then post-processed, to express the measurements in the fixed frame, through the MBC transformation, as given by:

$$\begin{bmatrix} q_0 \\ q_c \\ q_s \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 2\cos\psi_1 & 2\cos\psi_2 & 2\cos\psi_3 \\ 2\sin\psi_1 & 2\sin\psi_2 & 2\sin\psi_3 \end{bmatrix} \begin{bmatrix} M_{XB1} \\ M_{XB2} \\ M_{XB3} \end{bmatrix}. \tag{9}$$

From equation 9 we can see that three signals arise in the fixed reference frame, q_0, q_c and q_s. For the identification of the in-plane forward and backward whirling modes of the rotor either one of the q_c or q_s signals could be used. In this instance, we chose to work with q_c. This procedure does not influence the frequencies or damping factors of the modes, it only affects the participation factors, as demonstrated in [4]. Note that the MBC transformation is used here only as a post-processing tool to transform the signals to the fixed frame in order to reveal the whirling modes, which are a property of the whole rotor and are only observable in the fixed reference frame. This is a much less involved process than transforming the equations of motion in the fixed frame, as done in codes like OpenFAST for implementing the LTI-based approach. From this point onward, the identification procedure is similar. The signal was sampled at a rate of 20 samples per period and recorded for 20 periods. The complexity of the PARX model was chosen as follows: $N_a = 6$, $N_{F_a} = 1$, $N_b = 1$, $N_{F_b} = 9$.

Figure 4 shows that the response of the system includes a strong presence of the tower side-side mode, but the backward and forward whirling in-plane modes are also strongly present. Moreover, the predictions of the model closely match the measured response, which gives confidence in the accuracy of the results. Table 3 summarizes the stability analysis results for the in-plane forward and backward whirling modes. Only the first in-plane whirling modes are available for the *MBC-LTI* approach, since the Elastodyn structural model used by OpenFAST is based on a modal representation of the blades and does not include higher-order edgewise bending modes [5]. Once again for the PARX-LTP approach, only harmonics with participation factors greater than 0.01 are reported. Here, we also observe that the modes do have super-harmonics though not with a large participation factor. Furthermore, the frequencies and the damping values obtained with the MBC-LTI and PARX-LTP approaches are in close agreement. The drivetrain torsional mode, or the collective in-plane mode, could not be successfully identified, probably due to the high amount of damping in the drivetrain, as shown in table 3.



(a) Time domain. **(b)** Frequency domain.

Figure 4. Comparison of simulation-derived virtual measurement and PARX model prediction normalized cosine component of the MBC transformation of the blade edgewise root moment M_X.

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Table 3. In-plane whirling mode eigenfrequencies, damping ratios, and participation factors. LTP values shown with corresponding LTI values in parentheses.

Mode	Frequency [Hz] LTP (LTI)	Damping Ratio LTP (LTI)	Participation LTP
1st In-Plane Backward Whirl	0.690	0.026	0.016
	0.891 (0.892)	0.020 (0.021)	0.957
	1.092	0.017	0.020
1st In-Plane Forward Whirl	1.302 (1.298)	0.016 (0.014)	0.972
	1.503	0.014	0.017
Drivetrain Torsion	(1.682)	(0.280)	–

4. Conclusions

The PARX-LTP methodology was successfully employed to identify the tower fore-aft, side-side, and in-plane backward and forward whirling modes of this wind turbine model, with results that in general closely match the conventional MBC-LTI methodology. To the best knowledge of the authors, such a direct comparison of these methodologies has not been previously reported. The lack of strong periodic effects and therefore the close agreement of results is likely due to the conservative design of the wind turbine that was chosen, which is not highly flexible, thus limiting the periodic effects in the dynamics. It has therefore been shown that in such cases with simplified inflow conditions and a relatively stiff wind turbine the assumptions of the MBC-LTI methodology remain valid, providing a baseline against which cases with stronger periodic effects can be evaluated.

Notwithstanding the general close match between the two approaches, even in this mildly periodic example we were able to confirm that the MBC-LTI method is blind to some modes. Specifically, the superharmonics of the first tower side-side mode were absent from the results of the conventional analysis, including a mode at 0.524 Hz with a small damping ratio of only 0.006. The possible effects caused by the resonance of such a mode with other structural modes or external excitations deserve further investigation.

The PARX-LTP approach allows for the identification of system dynamics directly from time-series data. It is therefore completely independent of the simulation tool used and very scalable as the computational cost of applying the methodology itself should be the same irrespective of the complexity of the simulation. Furthermore, it does not require linearization of the dynamics or the application of the MBC transformation, which normally needs to be adapted every time the equations of motion are modified. Certain modes could not be identified using this methodology. Specifically, the collective in-plane mode could not be identified due to the large damping provided by the drivetrain, and the blade flapping modes because of the aerodynamic damping associated with their shape.

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5. Discussion & future work

As far as the methodology itself is concerned, several practical challenges were identified. Tuning the models remains challenging, and care must be taken when deciding which excitation is used, how long after the excitation a signal should be recorded, which signal is recorded and most importantly which parameters should be used for the complexity of the PARX model. These variables significantly affect the result of the identified modes, and the theory behind the method provides little practical guidance for selecting them. Often there is no single correct choice, and the quality of the results must be carefully assessed a posteriori. Good agreement between measured and predicted signals does not guarantee the quality of the identified state-space model. Overfitting may still lead to nonphysical modes in the subsequent Floquet analysis. If an overly complex PARX model is used, it may capture the dynamics well, but it does so by introducing more modes than the actual system possesses. This results in spurious, nonphysical eigenmodes. Spurious modes can be detected in the results because they are characterized by a main frequency which has no visible peak in the Fourier transformed response of the system. Some general guidelines for making reasonable choices for these parameters are summarized below:

- The autoregressive terms:
  - N_a: determines the number of modes of the system.
  - N_{F_a}: is still a matter of trial and error though a value between 1 and 5 is recommended.
- The exogenous terms:
  - N_b: is either 0 if there is no wind, or 1 if there is steady wind.
  - N_{F_b}: In case N_b is 1, the number of frequencies included in the N_b terms should be equal to the highest per-rev. resonance that is detectable in the measured signal.
- Sampling rate: it should be chosen according to the highest frequency that needs to be identified and according to the Nyquist rate.
- Signal duration: it should be able to capture well the decay of the modes. Highly damped modes require only little time to decay. On the contrary, modes with low damping as well as modes that are close in frequency require more time in order to be successfully identified.

In practice, it is not always possible to fit a good model using these guidelines for several reasons. These include the non-linear nature of the optimization problem that can make convergence difficult; measured signals that often contain unwanted modes or very weakly participating modes, which complicate convergence; and coalescence of mode peaks when a per-rev. peak is close to a mode peak or when two modes are very close in frequency to each other. A good demonstration of this fact is the side-side tower response presented in section 3.2, where the N_a term was set to 6 instead of 4 as the guidelines would suggest.

Future research will focus on applying the methodology to more modern wind turbine designs that are much more flexible and to operating conditions that include stronger periodic effects, (for example, strong wind shear, or high misalignment angles) where it is expected that the MBC-LTI results will differ more significantly from the PARX-LTP results.

Furthermore, efforts will focus on improving the robustness of the method especially with respect to choosing the appropriate complexity of the PARX time series for identification and

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the optimization of the objective function, in order to make its application easier and more reliable.

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