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Short-term reconstruction of fragmentation events in Low Earth Orbit using uncertainty propagation

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Abstract

The growth of Earth-orbiting objects represents a major concern for the safety of space operations. Despite the adopted mitigation measures, the increase of the number of space debris poses a threat to space missions in terms of risk of collisions among fragments and active satellites. Since not all these events can be predicted in advance, the early analysis of detected fragments generated by in-orbit fragmentations is crucial to characterise fragmentation events and to increase the safety of space missions. The PUZZLE software prototype was developed at Politecnico di Milano to reconstruct recently occurred breakups, estimate the epoch of fragmentation and identify the involved parent objects. The approach takes as input a set of unclassified Two-Line-Element (TLE) data to detect possible fragmentation events occurred in the recent past. It exploits backward propagation of the objects to search for close encounters among them and it uses filtering and pruning criteria to find a common origin in space. Lastly, it simulates the detected fragmentation using the NASA standard breakup model.

In this work, the short-term version of the PUZZLE approach is improved in terms of automatisation of the process for selecting the settings for an automatic recognition of fragmentations. Appropriate values of the thresholds required for the software are searched and validated with sensitivity analyses. The aim is the selection of automatic settings which can be used for any fragmentation in Low Earth Orbit. Moreover, the uncertainty associated to the TLEs is included to improve the characterisation of fragmentation events, as the inaccuracy of the available data about space debris affects the reliability of the software. Preliminary analyses have shown that improving the quality of input data by including uncertainties through additional TLEs leads to a more accurate identification of the event epoch and of the parent objects, reducing the sensitivity to the input control parameters. A Gaussian Mixture Model is investigated together with other uncertainty propagation methods to properly catch the non-linearity of the problem while keeping a reduced computational cost. The improved approach will be applied to reconstruct known breakups and to carry out sensitivity analyses for optimal automatic settings. The performances of the updated software will be measured and compared with the ones of the original version, and the operational efficiency of the approach for the Space Traffic Management network will be assessed.
Keywords: Space debris, Fragmentations, Low Earth Orbit, Uncertainty

Nomenclature

a [km]	Semi-major axis
e [-]	Eccentricity
i [rad]	Inclination
Ω [rad]	Right Ascension of the ascending node
ω [rad]	Argument of pericentre
θ [rad]	True anomaly

Acronym/Abbreviations

LEO	Low Earth Orbit
TLE	Two-Line Element
ASI	Italian Space Agency
GMM	Gaussian Mixture Model
SST	Space Surveillance and Tracking
GA	Genetic Algorithm
MOID	Minimum Orbital Intersection Distance

1. Introduction

The growing number of debris in space is becoming a concerning threat for space safety and for the sustainability of space activities, causing collisions and explosions to occur with a greater frequency. The issue has increased over the last decades because of significant changes in space traffic, particularly in the Low Earth Orbit (LEO) region. This is the most affected orbital region as it is densely populated by objects, hence it has a higher probability of collision. It is estimated that in the last 2 decades alone, 11.2 non deliberate fragmentations have occurred each year, causing a population in the order of 90000 objects larger than 1 cm [1]. Specifically, two major breakups occurred in LEO have contributed to a significant increase of debris. These are the breakup of the weather satellite Fengyun 1C occurred in 2007 and the catastrophic collision between the satellites Cosmos 2251 and Iridium 33 in 2009 [2]. Since some of these fragmentations are unpredictable, it is crucial to be able to reconstruct a breakup as soon as few fragments are detected and identify the involved objects to ensure the safety of space operations. This is done by characterising the fragmentation and identifying the debris produced in the impact to reduce the risk posed to other satellites and extrapolate useful information for the design of future missions.

Several past works have focused on different techniques for fragmentation detection and reconstruction. Dimare et al. [3] have developed a tool with this aim exploiting orbital similarity functions and the minimum mutual distances among fragments. Andrisan et al. [4] have devised the Simulation of On-Orbit Fragmentation Tool (SOFT) considering the distribution of fragments according to the type of breakup and the average distance of the fragments. Tetrault et al. [5] formulated a technique based on the distance of each fragment from the centre of mass of the fragmentation cloud and the optimised computation of the ballistic coefficient for each fragment. Montaruli et al. [6] implemented the Fragmentation Epoch Detector (FRED) algorithm using a stochastic approach and selecting the fragmentation epoch candidates with respect to the Minimum Orbital Intersection Distance (MOID). With the same aim, the PUZZLE algorithm has been developed at Politecnico di Milano under a contract with the Italian Space Agency (ASI) for the reconstruction of past fragmentation events. The first version of the algorithm focused on short-term analyses, of the order of days [7]. A second version of PUZZLE was developed for long-term investigations [8, 9, 10] spanning from months to years. The two approaches share the same objective, that is the detection of a fragmentation in terms of epoch estimation and objects involved. They exploit pruning and clustering criteria to filter an object catalogue given as input and keep the objects for which a close encounter was possible. A backward propagation is carried out for the remaining objects through different propagators (according to the analysis), to estimate the epoch of the fragmentation and identify the correct parent object(s). However, Two-Line-Elements (TLEs) are intrinsically uncertain [11], therefore the inaccuracy affects the reliability of the algorithm. For this reason, a first version of an improved short-term algorithm including uncertainty propagation was developed in [12]. This version was further investigated in the present work, with a refined introduction of uncertainties, using a Gaussian Mixture Model (GMM) to generate additional TLEs for a more accurate estimation of the fragmentation epoch. An optimisation algorithm is also employed to select the optimal TLE for a single object. This approach guarantees more precise results if the input TLEs are characterised by high uncertainty. The proposed algorithm is being implemented into an operative software. To this aim, automatic settings to study fragmentations in LEO were investigated by means of a sensitivity analysis.

The paper is organised as follows. Section 2 describes the general architecture and methodologies of the PUZZLE algorithm. Section 3 presents the update of the short-term PUZZLE with the introduction of uncertainties with respect to the previous work in [12], going into details about the operations in each additional module. The results of the updated routine are compared to the results of the original one to highlight advantages of including uncertainty in the model. Section 4 proposes a sensitivity analysis on the thresholds of the most important parameters to make the algorithm an operative software ready to be used by an operator. The results are analysed and the optimal values of the thresholds are defined.

2. PUZZLE algorithm

The PUZZLE algorithm was developed at Politecnico di Milano with the aim of detecting occurred breakup events (in terms of fragmentation epoch, fragments and parent identification) and of characterising the events in terms of energy, mass and orbital elements [8]. The tool was designed to analyse a set of input TLEs of unclassified objects to detect a possible fragmentation, without knowing *a priori* that the breakup actually occurred. Two versions of the algorithm were developed in the past: the short-term version [7] and the long-term version [8], which are able to reconstruct fragmentations after, respectively, a few days up to two weeks and months up to years. The two algorithms employ different techniques, however the general idea is the same. They were tested and validated in LEO and, particularly for the long-term version, the natural features of this region were exploited for the design.

The algorithm takes as input a set of TLEs and preprocesses it by removing statistical outliers with filters, following the approach of Lidtke et al. [13]. Then, pruning criteria are used to discard the TLEs of objects which could not have been involved in a fragmentation because a close approach was not possible for them. The pruning criteria common to both routines are applied through the triple-loop filter, similarly to Hoots et al. [14]. The triple-loop filter is composed of three pruning steps applied sequentially: two geometric ones and a time filter. The first is an apogee-perigee pruning criterion, which checks if the relative geometry of the objects (analysed in pairs) is compatible for a close encounter. To pass it, the maximum of the two perigees of the analysed couple of objects q and the minimum of the two correspondent apogees Q have to fulfill the following relation:

$$q - Q \leq \Delta \quad (1)$$

where Δ is a given threshold. The second pruning criterion involves the computation of the MOID with the analytical formulation proposed by Gronchi et al. [15] to check if it is below a specific threshold. Indeed, the MOID corresponds to the minimum possible geometric distance between two orbits, hence if it is above a given threshold the close encounter is deemed impossible. The last pruning criterion checks whether two objects can be in the same angular window around the position of the MOID on the two orbits at the same instants. This step is necessary because the MOID represents the minimum theoretical distance between the two orbits, however the close encounter does not necessarily occur at the MOID, hence an angular window is considered. The angular windows are defined through an aperture angle, as shown in Fig. 1. The angular windows are then converted into time windows and PUZZLE verifies the possibility of close encounters. Both versions

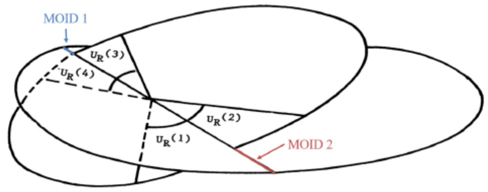

Fig. 1. Definition of angular windows around the MOID. Image from [7].

of the algorithm perform a backward propagation of the objects. The short-term algorithm uses osculating orbital elements with the Standard General Perturbations 4 (SGP4) propagator [16].
The remaining objects are clustered into families with a Hierarchical Clustering Method similar to the one of Zappala et al. [17] and a matchmaking routine is performed to assign the correct parent object from the catalogue to each family comparing the positions of the objects in the catalogue with the ones of the objects in the families.
The last step of the algorithm is the modelling the fragmentation with the NASA standard Breakup Model [18] to estimate the number and the distribution of the physical characteristics of generated fragments and the total mass involved in the event.

3. Update of short-term PUZZLE algorithm with uncertainty introduction

This section is devoted to the description of the improved short-term PUZZLE algorithm by means of the introduction of uncertainties. The idea, in a first simplified version, was proposed and implemented in [12], where three plug-in blocks were added to the original PUZZLE algorithm to account for uncertainty propagation. This updated routine was used to reconstruct Cosmos 1408 breakup from uncertain observation data [19], proving its advantage in terms of ability of characterising the breakup with uncertain TLEs. The approach presented in this work decreases the computational effort required by the algorithm in [12] by introducing only two additional modules with respect to the original algorithm, as seen in Fig. 2 (in blue).

3.1 Introduction of uncertainties

The uncertainties are first accounted for in the algorithm by generating additional TLEs for all the objects given as input. Following the same approach of [12], this module is introduced after the pre-filtering of the TLEs, such that the outliers were already discarded. The introduction of additional TLEs for each object before the triple-loop guarantees that during the pruning phase the TLEs of the objects involved in the fragmentation have a higher probability of being preserved, allowing also to set stricter thresholds with respect to the original PUZZLE algorithm. The advantage of stricter thresholds is that they can discard more objects unrelated to the fragmentation. This also ensures a better estimation of the event epoch as multiplying the TLEs leads to a higher concentration of close encounters among the objects in the correct time interval. This is especially true in cases for which the input TLEs are affected by high uncertainty or they are very few.
The uncertainties can be introduced both on the Cartesian state or on the Keplerian elements of each object. The uncertainty on the state of the object, α, is used to generate a random normal distribution of each element of the state, either $(r_x, r_y, r_z, v_x, v_y, v_z)$ or $(a, e, i, \Omega, \omega, \theta)$. The distribution is centered in the nominal state of the object and the standard deviation is given by:

$$\sigma_j = \alpha p_j \quad (2)$$

where p_j is the nominal value of each orbital parameter considered. Due to the lack of reference values on the uncertainty α, the same value of uncertainty is considered for all parameters. This is a simplified approach, since a more in-depth analysis should consider different values of uncertainties tuned for each parameter, particularly when the Keplerian elements are considered. This step is carried out for each object under analysis, belonging to the initial TLE set.
Once the distributions are generated, synthetic TLEs must be obtained and added to the analysis. A Gaussian

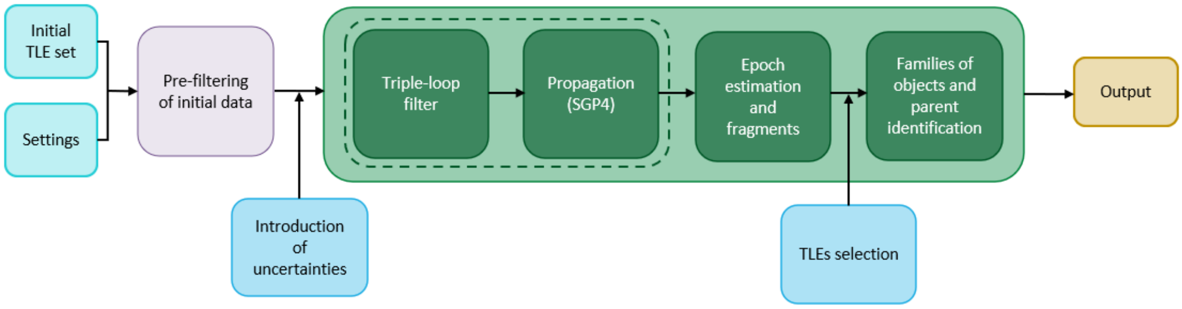

Fig. 2. Block diagram of the updated short-term PUZZLE routine.

Mixture Model is used to fit the distributions in orbital parameters. The GMM allows to approximate a PDF by a finite sum of weighted Gaussian density functions (kernels) [20]. The mean of each Gaussian component is retrieved and used for the generation of the additional TLEs. A new TLE is generated for each kernel, modifying the original TLE using the mean values of the kernel as orbital parameters, either in terms of the Cartesian state or of the Keplerian elements according to the adopted approach. The mean of the whole GMM, i.e. the mean of all kernels, is also used to add one more TLE. The synthetic TLEs are added to the original set, therefore at the end of the process PUZZLE will have a set of $(N+2)k$ TLEs to analyse, where N is the number of kernels and k is the number of initial TLEs. To recognise synthetic TLEs, IDs higher than 100000 are assigned to each of them. The entire process is shown in the block diagram in Fig. 3.

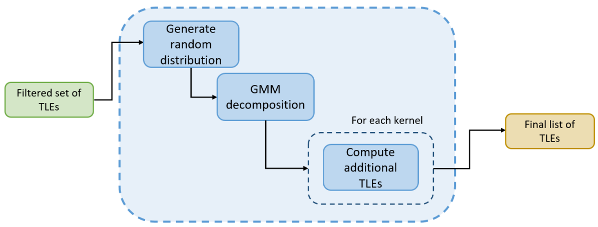

Fig. 3. Block diagram of the introduction of uncertainties.

3.2 TLEs selection

After the estimation of the fragmentation epoch, the PUZZLE algorithm selects the fragments belonging to the event and the associated parent(s). For this step, it is necessary to have one TLE per object, hence a selection of the optimal TLE for each object has to be carried out to avoid problems of objects repetition. An optimisation problem is formulated to this aim, taking into account three factors and following the approach of [12]:

- the fragmentation epoch estimated with the full set of TLEs and the one estimated with the restricted set have to be as close as possible;
- the objects whose close encounters occur closest to the fragmentation epoch should have the minimum close approach distance;
- the algorithm should keep the highest number of objects whose time of close approach is comparable with the fragmentation epoch.

Therefore, the optimisation problem for the selection of the optimal TLEs is:

$$\min_{x_{TLE}} \quad |t_{full} - t_{restricted}| + \overline{d}_{CA} + \frac{1}{n_{obj}} \qquad (3)$$

where t_{full} is the estimated epoch with the full set of TLEs, $t_{restricted}$ is the epoch estimated with the selected TLEs, $\overline{d}_{CA}$ is the average of the close approach distances of the objects whose close encounters occur within an hour with respect to the fragmentation epoch and n_{obj} is the number of objects kept by the algorithm because the close approaches are comparable with the fragmentation epoch. The optimisation variable x_{TLE} is the optimal combination of one TLE per object. The problem is solved with Matlab built-in Genetic Algorithm, considering integer variables and the appropriate bounds. The GA generates at each iteration a population of points - in this case, a population of selected TLEs - and randomly selects individuals from the current population as parents for the next generation. The tolerance was here set to 10^{-6} as a trade-off between accuracy and computational effort.
Once the step is completed, the algorithm uses the optimal set of TLEs, which is composed of both original and modified TLEs, to retrieve the parent(s). However, when the original TLEs are modified, the state of each object is affected by errors due to the averaging of the GMM results, which can lead to errors in the matchmaking routine to find the correct parent(s). In the algorithm proposed in [12], the issue was solved by using a further optimisation problem to find the parent(s). The drawback is that this additional step increases the computational effort. For this reason, the algorithm proposed here instead considers the optimal TLEs remaining under analysis at this point of the reconstruction and, if among them there are modified TLEs, it goes back to the original TLE. This ensures that the state is not affected

by the errors of the averaging process, while also keeping in the analysis the objects that most likely were involved in the breakup, due to the previous pruning process done with the additional TLEs. Consequently, the computational cost is reduced as no optimisation problem has to be solved, while giving satisfactory results. The matchmaking routine is carried out as in the original algorithm.

3.3 Results of the updated algorithm

The new version of PUZZLE with uncertainties was tested and validated. The presented test case is relative to the Cosmos 2251 fragmentation, occurred due to a collision with Iridium 33 on 10th February 2009; The TLEs of the fragments were taken from SpaceTrack [21], dating ten days after the events. The test case shows the improvement of the updated routine with respect to the original one. In particular, it proves that the advantage of including the uncertainties is the possibility of carrying out analyses even with few fragments.

The input TLE set contains only 14 TLEs, out of which 9 belong to Cosmos 2251, dated 20th of February 2009, i.e. 10 days after the breakup. The fragmentation is searched in the previous 14 days. Due to the low number of TLEs, a correct reconstruction of the fragmentation is hard to obtain. This is one of the typical cases in which the approach with uncertainties can be useful. To carry out the analysis the uncertainties were added onto the Cartesian state, with a value $\alpha = 0.01$ and 7 kernels for the GMM were used, therefore the total number of TLEs under analysis including the synthetic ones is 126.

Fig. 4 shows the close encounters of the fragments in terms of distance and time. The concentration of close encounters ten days before the TLEs epoch coincides with the fragmentation epoch. The gray dots represent the close encounters of the synthetic TLEs, therefore it is clear that without their addition, the event epoch could not be reconstructed correctly.

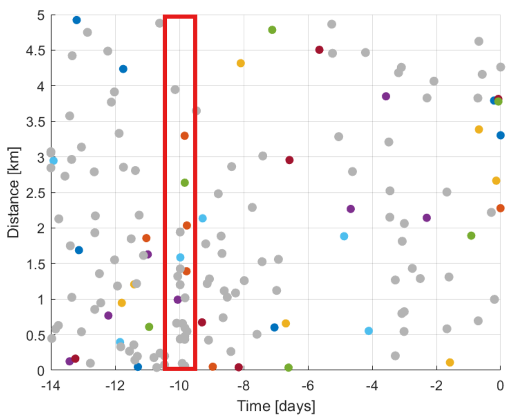

Fig. 4. Close encounters between objects for Cosmos 2251 breakup. The gray dots represent the close encounters of additional TLEs.

The main results of the fragmentation reconstruction are summarised and compared with the results of original PUZZLE in Table 1. The original algorithm is not able to successfully reconstruct the fragmentation, including identifying the parent. In contrast, the updated routine can correctly estimate the event epoch, complete with the time of the breakup, and the parent object. The drawback of the updated algorithm is an increased computational time, due to the higher number of TLEs to analyse. In this test case the computational time is almost six times the one of the original algorithm, although the initial TLE set is very small therefore the computational time is still negligible. This factor has to be taken into account in other cases in which the initial set is larger as the computational effort derived from including additional TLEs could be prohibitive.

Table 1. Cosmos 2251 test case results

	Updated PUZZLE	**Original PUZZLE**
Estimated epoch	10/02/2009 16:54:50	07/02/2009 07:11:07
Detected parent	ID: 22675 (Cosmos 2251)	None
Computational time (s)	89.40	15.50

4. Sensitivity analysis

The project in which this work was completed aimed at developing an infrastructure to be used for Space Traffic Management. The main challenge arising from this for what concerns fragmentation reconstruction is turning the new PUZZLE algorithm into operational software. This requires standardised values of thresholds which can be used by an operator, without the need of knowing the theoretical aspects behind the algorithm. A sensitivity analysis was carried out on the most significant thresholds of parameters affecting the results of PUZZLE considering three different test cases.

4.1 Sensitivity analysis setup

Three major fragmentation events were used as test cases for the sensitivity analysis. The input TLE files are composed of both TLEs belonging to the fragmentation event and TLEs of other objects, in variable proportions so as to have the most general test cases possible. The TLEs were taken from SpaceTrack, ten days after each event. The three cases are the following:

- Cosmos 2251 and Iridium 33 breakup, which has already been used in Section 3. The initial set contains 1842 TLEs, among which there are the TLEs of 276 fragments generated in the collision;
- NOAA 16 explosion, occurred on 25th November 2015. The TLE set contains 5792 TLEs. The

fragments belonging to the fragmentation in the set are 54.

- Fengyun 1C fragmentation, occurred on 10th January 2007. The set contains 1410 TLEs. The Fengyun 1C fragments in the set are 32.

The five thresholds which the sensitivity analysis has focused on concern the pruning process of the software. They are the apogee-perigee filter threshold, the MOID filter threshold, the time filter threshold, the distance margin and the time margin for close approaches. The results will be detailed in the following sections.

4.2 Apogee-perigee distance

The threshold of the apogee-perigee filter was tested with values ranging from 1 km to 50 km. The analysed figures of merit are the results right after the triple-loop filter, i.e. how many objects were discarded with respect to the original number of objects, and the final result of the fragmentation reconstruction, that is the number of fragments correctly identified by the software with respect to the total number present in the initial set and whether or not the parent was correctly recognised.
The results are shown in Fig. 5 and 6. The circles in Fig. 6 indicate that the parent was correctly found, hence the reconstruction was successfully completed.

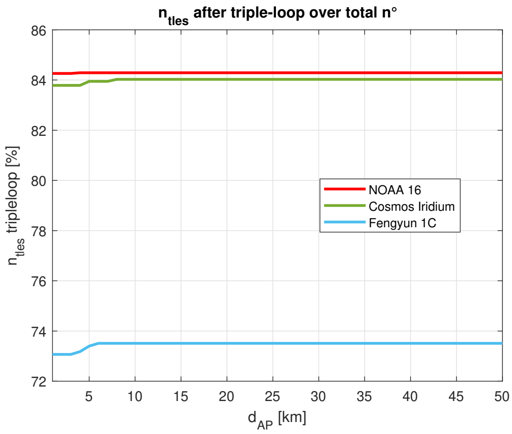

Fig. 5. Percentage of remaining objects after triple-loop with respect to apogee-perigee threshold.

As expected, the outcome of the triple-loop is mostly invariant to the value of the apogee-perigee filter threshold (Fig. 5). This result is due to the fact that in the triple-loop filter the majority of the objects are discarded by the MOID filter, therefore even if large values of this threshold are imposed, keeping many objects, those are then discarded by the MOID filter. This result is reflected also on the final outcome of the fragmentation reconstruction (Fig. 6), which is independent with respect to the apogee-perigee filter threshold. The fragmentation is correctly reconstructed with any value of

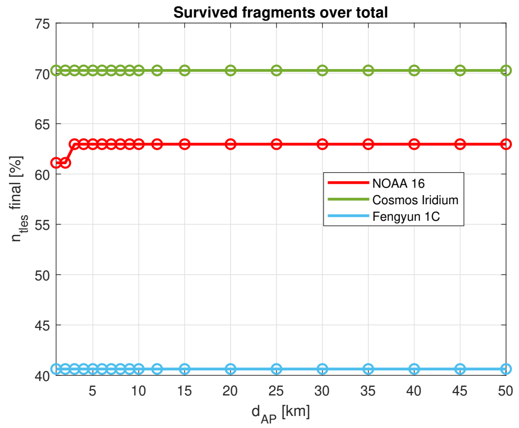

Fig. 6. Percentage of correctly identified fragments at the end of the reconstruction with respect to apogee-perigee threshold.

this threshold, as the circles in the figure indicate that the parent was identified successfully.

4.3 MOID threshold

The MOID pruning criterion is expected to have a significant influence on the final results of PUZZLE, as it discards objects based on the closest theoretical point between two orbits. As for the previous filter, the threshold was tested with values ranging from 1 to 50 km. The same figures of merit were analysed and the results are reported in Fig. 7 and 8.

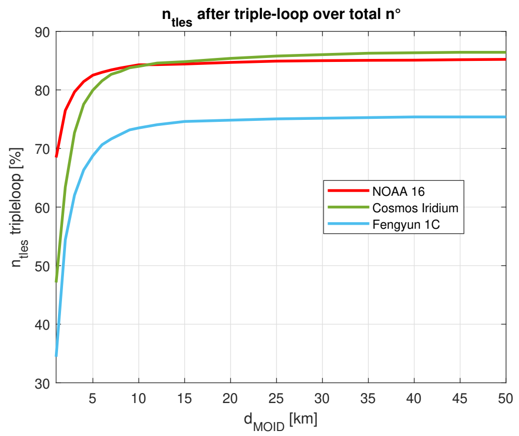

Fig. 7. Percentage of remaining objects after triple-loop with respect to MOID threshold.

The results in Fig. 7 clearly show a larger influence of the MOID filter on the final result of the triple-loop. For all three analysed cases the number of TLEs which survive the filter has a plateau after a threshold of 10 km, indicating that such a value of the threshold is enough to discard all the TLEs that could not have had a close encounter. The final results of the reconstruction (Fig. 8) demonstrate that the value of the MOID threshold affects the outcome of the simulation. The crosses on the

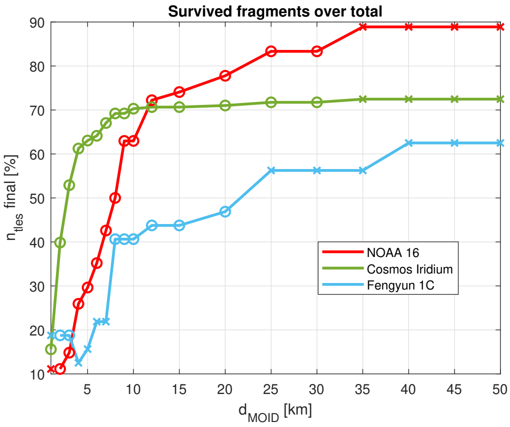

Fig. 8. Percentage of correctly identified fragments at the end of the reconstruction with respect to MOID threshold.

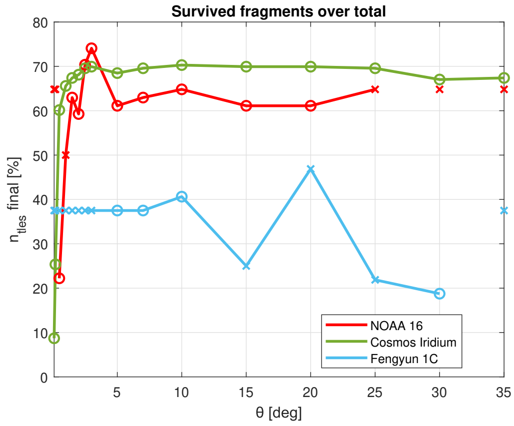

Fig. 10. Percentage of correctly identified fragments at the end of the reconstruction with respect to time filter threshold.

plot indicate that the parent was not recognised, hence for large values of the threshold PUZZLE is not able to pick the correct parent. This can be explained in the fact that when the threshold has large values, a lot of objects survive the triple-loop filter and that makes it difficult for the software to recognise the parent as the closest object to the cluster of families.

4.4 Time filter threshold

The time filter implemented in PUZZLE is based on an angular aperture around the MOID that is then turned into time window, therefore the threshold for this filter is expressed in degrees. The values considered for this sensitivity analysis cover a range from 0.1 to 35 deg, which is a very large angular window as one has to consider $[-\theta, +\theta]$. The results of the metrics are reported in Fig. 9 and 10.

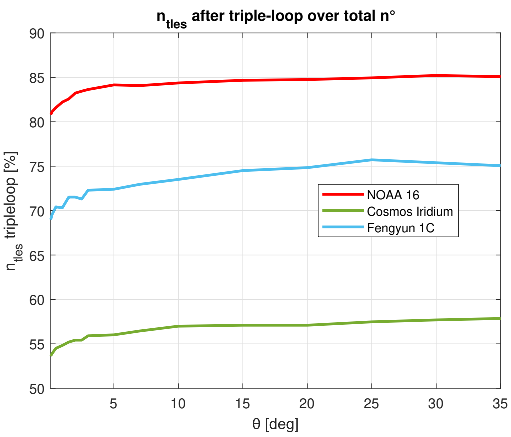

Fig. 9. Percentage of remaining objects after triple-loop with respect to time filter threshold.

Clearly, the results of the triple-loop filter do not strongly depend on the value of the time filter threshold because, as mentioned before, the majority of the objects are filtered by the MOID filter. As for the final results of the fragmentation reconstruction, Fig. 10 shows for Cosmos 2251-Iridium 33 and NOAA 16 an expected behaviour, that is that with very narrow angular windows only a few fragments are detected. The Fengyun 1C case instead shows an irregular behaviour, probably due to the quality of the initial TLEs. With high values of tolerance, PUZZLE is not able to correctly reconstruct and verify the fragmentation (as shown by the crosses without the continuous line). This is true for the cases where very few fragments were available, hence with these thresholds the software preserves too many objects and it is not able to recognise the breakup.

4.5 Distance margin

The distance margin is expected to have a lot of influence on the final result, as it prunes objects according to the distance of their close encounters, therefore it could discard a lot of objects if tight. The margin was tested from 1 km to 50 km and the same metrics as before were evaluated. The results are shown in Fig. 11 and 12.

The results are very similar to those of the MOID threshold. Also in the case of the distance margin the curves show a plateau-like behaviour starting at about 10-15 km (Fig. 11). This suggests that higher thresholds do not allow to discard more objects, as the ones that could be discarded are already rejected at those values. The final results of the simulation (Fig. 12) demonstrate that the outcome of the reconstruction significantly depends on the value of the distance margin, not only in terms of recognised fragments but also in terms of parent identification. When the value of the distance margin threshold is strict (i.e. from 1 to 5 km) the software cannot identify the correct parent as too few objects remain. Similarly, for Cosmos 2215-Iridium 33 and Fengyun 1C the parents are not correctly identified when the tolerance is too large.

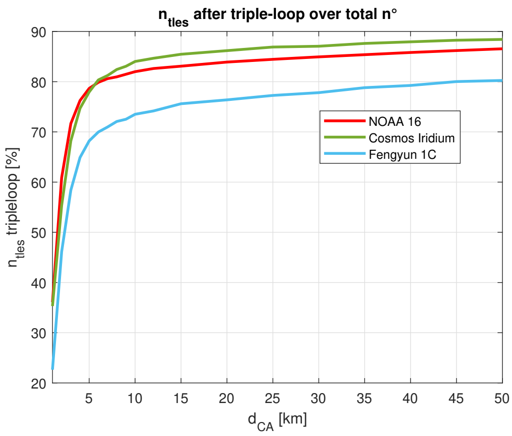

Fig. 11. Percentage of remaining objects after triple-loop with respect to distance margin.

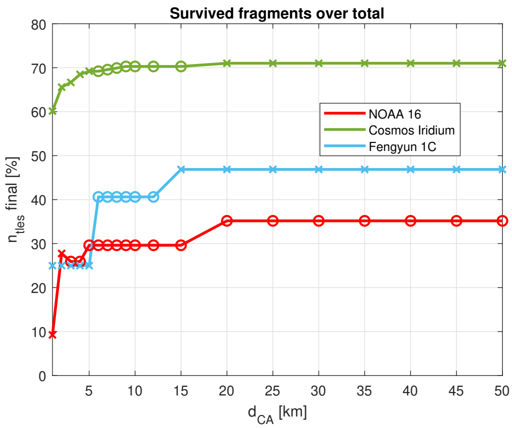

Fig. 12. Percentage of correctly identified fragments at the end of the reconstruction with respect to distance margin.

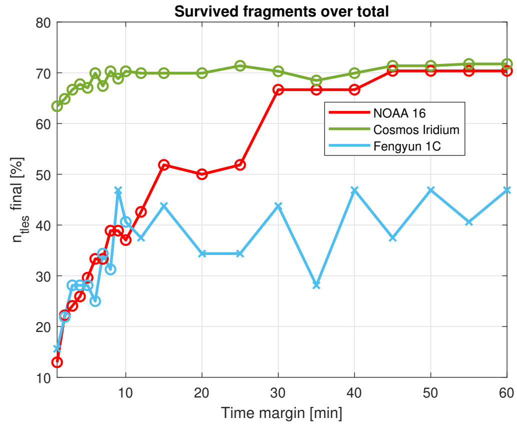

Fig. 13. Percentage of correctly identified fragments at the end of the reconstruction with respect to time margin.

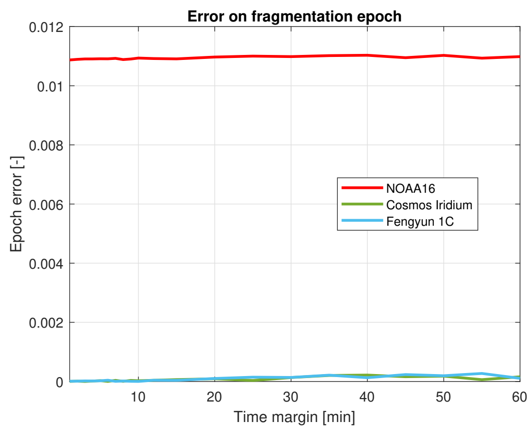

Fig. 14. Weighted error on fragmentation epoch with respect to time margin.

4.6 Time margin

The time margin sets the margin (in minutes) on the estimation on the epoch of the event. The threshold was tested starting from 1 min up to 60 min. In the case of the time margin the final result of the fragmentation reconstruction is checked as in the previous cases. Moreover, the error on the estimation of the fragmentation was analysed, considering the following metric:

$$err = \frac{T_{computed} - T_{real}}{T_{TLE} - T_{real}} \tag{4}$$

where $T_{computed}$ is the estimated epoch of fragmentation, T_{real} is the real, known fragmentation epoch and T_{TLE} is the epoch of the TLEs. This formulation allows to take into account the influence of the epoch at which the TLEs are retrieved with respect to the fragmentation epoch. The error on the estimated fragmentation epoch was not considered for the other parameters as no clear relation exists between them and the epoch. The results are displayed in Fig. 13 and 14.

The final results of the simulation (Fig. 13) do not exhibit a uniform behaviour with respect to the time margin, particularly for the Fengyun 1C case, probably because of the quality of the input TLEs, as mentioned earlier. For the other two cases under analysis, the influence of this parameter is limited as the reconstruction is successful for all values of the threshold (circles).
As for the error on the estimated fragmentation epoch (Fig. 14), it is clear that the results are independent from the value of the time margin. For Fengyun 1C and Cosmos 2251-Iridium 33 the error is very close to zero, instead the error for NOAA 16 is higher because PUZZLE reconstructs the fragmentation on the correct date, but two hours before the actual breakup.

4.7 Considerations on sensitivity analysis

The sensitivity analysis allows to define the optimal values of the thresholds for the analysed parameters, such that the fragmentation can be reconstructed correctly in most cases in LEO. Indeed, these values will not be universal since fragmentations have to be analysed on a case-by-case basis, depending on the available data.

For this reason, the optimal values are defined together with possible ranges for the parameters to be tested if a fragmentation is not reconstructed with the optimal parameters. Looking at the final results of PUZZLE reconstruction for all the parameters, the following values of the thresholds are selected:

- for the apogee-perigee filter 5 km, with a range from 5 to 10 km;
- for the MOID filter 10 km, with a range from 5 to 20 km;
- for the time filter 10 deg, with a range from 5 to 15 deg;
- for the distance margin 10 km, with a range from 5 to 15 km;
- for the time margin 10 minutes with a range from 5 to 20 minutes.

These values were selected by taking into account the thresholds that allow to successfully reconstruct all three fragmentations, i.e. identify the right parent and the highest number of fragments. Moreover, when multiple values of thresholds would give very similar results, the lowest value was selected, as higher thresholds imply higher computational effort for PUZZLE. The selected settings allow the operator of the infrastructure to analyse an object catalogue automatically, without having to tune the parameters.
The limitation of this sensitivity analysis is that each parameter was considered separately, varying in a given range while the other thresholds were fixed. For this reason, the reciprocal influence of the parameters was not captured. An extended sensitivity analysis will be carried out in future work considering the most influential and related parameters varying simultaneously, e.g. the MOID threshold and the distance margin, and studying their correlation.

5. Conclusions

The increase in debris population and space traffic is becoming a growing concern for the safety and sustainability of space operations. This is causing fragmentation events (collisions and explosions) to become more frequent, particularly in the LEO region, hence the early detection and reconstruction of such breakups is crucial to handle these situations and reduce the risk posed to other satellites.
The PUZZLE algorithm was developed at Politecnico di Milano with the purpose of detecting and characterising occurred fragmentations in terms of epoch, objects involved and mass. Two versions of the algorithm currently exist, to perform fragmentation reconstruction both in the short and long-term exploiting backward propagation and pruning and clustering criteria to analyse an initial TLE set.
This paper proposed to extend the existing short-term routine to include uncertainties in the model, addressing the inaccuracies associated with TLEs. This approach was based on a pre-existing work, which was modified to reduce the computational effort. The first block generates synthetic TLEs from the initial set using a Gaussian Mixture Model, which builds new TLEs based on the original data by introducing uncertainties either in the Keplerian elements or the Cartesian state of the objects. The second block solves an optimisation problem to select the optimal TLE for each remaining object after the fragmentation epoch estimation. The updated routine was applied to a test case and the results were compared with the original algorithm. This proved that the updated one is able to reconstruct fragmentations also when very few fragments are available, when the original routine fails. The drawback is in the computational effort, as incorporating additional TLEs makes the reconstruction more demanding. Future work will aim at improving the computational burden of the updated algorithm by exploring parallelisation techniques.
The second aim of this work was to make the PUZZLE algorithm an operative software which could be used automatically by operators. This requires standard values of the parameters thresholds, therefore a sensitivity analysis was carried out on the thresholds of the apogee-perigee filter, the MOID filter, the time filter, the distance margin and the time margin, considering three test cases. The results showed that the MOID filter and the distance margin are the most influential parameters. Moreover, standard values and ranges of the thresholds were defined considering the final results of the simulations in terms of identified fragments and parent(s). To improve the sensitivity analysis, more work should be focused on analysing the reciprocal influence of the related parameters and find the optimal combination of thresholds.

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