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EVALUATION OF THE EFFECTS OF THE VOLTAGE HARMONICS ON THE EXTRA IRON LOSSES IN THE INVERTER FED ELECTROMAGNETIC DEVICES

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Abstract - In the present paper a method for the calculation of the extra iron losses occurring in the cores of the inverter fed electromagnetic devices is described: the loss increase, referred to the sinusoidal operation, is evaluated, analysing its dependence on the inverter and on the lamination characteristics.

INTRODUCTION

The main advantages connected with the employment of a variable frequency supply are a significant increase of the voltage frequency supplying the loads (that makes more convenient their sizing) and an increase of the efficiency of the systems; moreover, the amplitude and frequency regulation allows to achieve wider and more favourable control characteristics.

In a lot of cases, the practical implementation of a variable frequency drive requires the use of a voltage source inverter: the most popular kind is that based on the Pulse Width Modulation (PWM), whose employment is mainly preferred for the following reasons:
- the voltage distortion is greatly reduced;
- the significant part of the spectrum is substantially transferred towards high frequencies.

It follows that the absorbed currents have quasi sinusoidal waveforms, and therefore the following occurs:
- the winding losses are just slightly higher than those occurring in the pure sinusoidal operation;
- in case of motors, the torque is almost ripple-free.

On the other hand, the high amount of pulses generated by the PWM inverter causes some negative effects too, among which:
- increase of the dielectric stresses of the insulating materials;
- increase of the losses in the ferromagnetic cores.

In the literature, the effects of the inverter supply on the winding losses and on the torque ripple have been analysed widely [1,2,3,4,5], while more limited efforts have been devoted till now to the theoretical analysis of the effects on the iron losses [6]: in this paper, a method for the calculation of these losses is developed, as a function of the inverter and of the lamination characteristics.

Before the analysis of the phenomenon, the models of the iron losses in periodic operation are studied, and the procedures for the identification of the parameters of the loss contributions are defined.

The following aspects are examined:
- inverter operation with synchronous PWM control (i.e., with integer value of the frequency ratio m_f between the carrier frequency and the modulation frequency), and asynchronous PWM control;
- effects of the non-uniform distribution of the eddy currents in the lamination width at different harmonic frequencies;
- influence of m_f and of m_a (modulation ratio, between the peak values of the modulation signal and of the carrier signal);
- features of the inverter voltage spectrum and definition of a frequency "shifting" law of this spectrum, as a function of m_f;
- dependence of the iron losses increase factor on the characteristics of the employed lamination;
- waveforms and iron losses in case of star or delta connections of the windings, for three-phase loads.

MODEL OF THE IRON LOSSES

A lot of recent studies [7,8,9] lead to the following described model of the p.u. mass iron losses [W/kg] in a magnetic lamination under periodic non-sinusoidal magnetization with period T_1 ($\Rightarrow f_1 = 1/T_1$); indicated with $B = B(t)$ the flux density waveform in a generic position of the core and with $\hat{B}$ its instantaneous peak value, the average specific loss in each cycle equals:

$$p_f = p_{hc} + p_{ec} + p_a = k_{hc}(\hat{B}) \cdot K_m(B) \cdot f_1 + k_{ec} \cdot \frac{1}{T_1} \cdot \int_0^{T_1} \left(\frac{dB}{dt}\right)^2 dt + k_a \cdot \frac{1}{T_1} \cdot \int_0^{T_1} \left|\frac{dB}{dt}\right|^{1.5} dt \quad (1)$$

The three terms of eq.(1) have the following meaning:
- p_{hc} represents the classical hysteresis losses and it is proportional to the fundamental frequency f_1 of the B(t); $k_{hc}(\hat{B})$ is the energy that would be lost in one cycle if the waveform would be sinusoidal, with a flux density peak value $\hat{B}$; $K_m(B)$ is a coefficient, usually higher than unity, that depends on the waveform of B(t): it takes into account the possible presence of minor hysteresis loops during the major loop, and it can be evaluated as follows [10]:

$$K_m(B) = 1 + \alpha_h \cdot \sum_i \Delta B_i / \hat{B} \quad , \quad (2)$$

where $\alpha_h \approx 0.65$, ΔB_i is the amplitude of the i^{th} flux density local reversal of B(t) within one half wave;
- p_{ec} are the eddy current losses in the classical formulation: they occur in the lamination cross section because of the field of e.m.f.s induced by dB/dt; k_{ec} is given by:

$$k_{ec} = d^2 / (12 \cdot \rho_f \cdot \gamma) \quad , \quad (3)$$

with d and γ width and mass density of the lamination respectively and ρ_f lamination resistivity; if the percentage of Silicon Si% included is known, ρ_f can be estimated as follows:

$$\rho_f \approx 0.14 + 0.113 \cdot Si\% \quad [\mu\Omega m] \quad ; \quad (4)$$

- p_a represents the so-called anomalous losses, due to losses phenomena that occur at very small scale, caused by the displacement of the Bloch walls of the magnetic domains [8].

In case of sinusoidal operation, eq.(1) becomes:

$$p_{fs}(\hat{B}_1, f_1) = k_{hc}(\hat{B}_1) \cdot f_1 + K_{ec} \cdot (\hat{B}_1 \cdot f_1)^2 + K_a \cdot (\hat{B}_1 \cdot f_1)^{1.5} \quad , \quad (5)$$

$$\text{with:} \quad K_{ec} = 2 \cdot \pi^2 \cdot k_{ec} \approx 19.74 \cdot k_{ec} \quad , \quad (6)$$

$$\text{and:} \quad K_a = \nu \cdot k_a = \sqrt{2 \cdot \pi} \cdot \int_0^{2 \cdot \pi} |\cos(x)|^{1.5} dx \cdot k_a \approx 8.76 \cdot k_a \quad . \quad (7)$$

In the present analysis, reference will be made to eq.s (1) and (5), but limited to the first two terms only. In order to clarify this assumption, the following remarks must be considered:
- on the one hand, the reduction to just two contributions can be explained with the approximation to 2 of the exponent 1.5 of the anomalous losses, thus incorporating them in the classical eddy current losses: this implies a higher values of these losses, and therefore a conservative over-estimation of the total iron losses;
- on the other hand, some theoretical-experimental analyses [11] suggest a further subdivision of the anomalous losses in two contributions: both these terms are again due to losses phenomena occurring at a small scale, but, about their dependence on $\hat{B}$ and on f_1, they can be reduced to the classical models ($p_a = p_{ah} + p_{ae}$, with p_{ah} = hysteresis anomalous losses, p_{ae} = eddy current anomalous losses).

In conclusion, the following can be written:

$$p_f = p_h + p_e = K_h(\hat{B}) \cdot K_m(B) \cdot f_1 + k_e \cdot \frac{1}{T_1} \cdot \int_0^{T_1} \left(\frac{dB}{dt}\right)^2 dt \quad , \quad (8)$$

$$p_{fs} = p_{hs}(\hat{B}_1, f_1) + p_{es}(\hat{B}_1, f_1) = K_h(\hat{B}_1) \cdot f_1 + K_e \cdot (\hat{B}_1 \cdot f_1)^2 \quad , \quad (9)$$

for the distorted and the sinusoidal periodic operation respectively; the function K_h is similar to the k_{hc}, but it includes the hysteresis anomalous losses, while K_e (linked to k_e by the same coefficient of eq.(6)) includes the eddy current anomalous losses.

It is important to point out that the expression of p_{es} in eq.(9) is acceptable if the feeding frequency f_1 is not particularly high; in fact, the dependence of p_{es} on the frequency squared is valid just within a first range of the frequency increase. Above a certain limit, the non-uniform eddy current distribution in the lamination width becomes important, and the eddy currents react against the same inducing field, thus reducing the corresponding eddy losses; from some classical analytical studies [12], the following can be written:

$$p_{es}(\hat{B}_1, f_1) = K_e \cdot \hat{B}_1^2 \cdot f_1^2 \cdot k_{fe}(f_1) \quad , \quad (10)$$

where:

$$k_{fe}(f) = \frac{3}{\xi(f)} \cdot \frac{\sinh(\xi(f)) - \sin(\xi(f))}{\cosh(\xi(f)) - \cos(\xi(f))} \; , \; \text{with} \;\; \xi(f) = \frac{d}{\delta_f(f)} \; , \quad (11)$$
$$\text{and} \quad \delta_f(f) = \sqrt{\rho_f / (\pi \cdot \mu \cdot f)} \quad .$$

δ_f is the iron penetration depth, evaluated with a suited constant average permeability μ (for μ, the value of the apparent permeability deduced from the normal magnetization curve can be adopted, at the operating flux density): eq.(11) shows that the reduction coefficient k_{fe} equals unity at the industrial frequencies, while it decreases when f increases (see Fig.1).

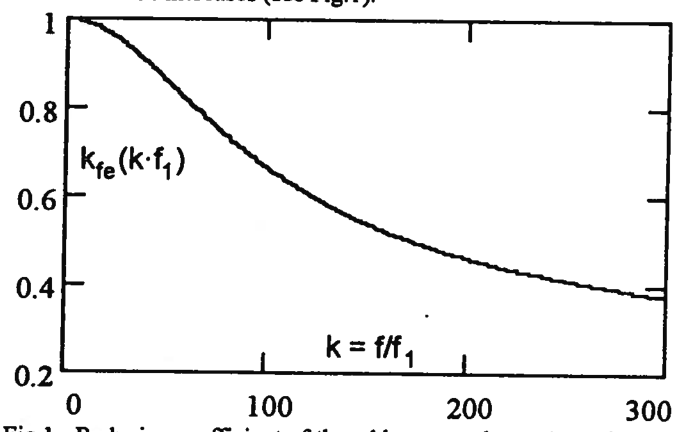

Fig.1 - Reducing coefficient of the eddy current losses $k_{fe}(k \cdot f_1)$, as a function of the harmonic order k, for a lamination magnetized at f_1 = 50 Hz, characterised by: d = 0.5 mm; ρ_f = 0.28 $\mu\Omega$m; μ_r = 1200.

On the basis of eq.(9) and of the results of the Epstein test at the test frequency f_t (for example, f_t = 50 Hz, at which k_{fe} = 1) the coefficient K_e and the function K_h can be determined; in fact:

- usually, the data sheets give the curve of the specific losses p_{fs_t} at the frequency f_t and the value of the hysteresis losses p_{hs_o} for a particular flux density value:

$$p_{fs_t} = p_{fs}(\hat{B}_1, f_t) \quad ; \quad p_{hs_o} = p_{hs}(\hat{B}_{1o}, f_t)$$

- thus, the coefficient K_e can be evaluated:

$$K_e = \left(p_{fs}(\hat{B}_{1o}, f_t) - p_{hso}\right) / \left(\hat{B}_{1o}^2 \cdot f_t^2\right) \quad ;$$

- finally, the function $K_h(\hat{B}_1)$ can be obtained as follows:

$$K_h(\hat{B}_1) = \left(p_{fs}(\hat{B}_1, f_t) - K_e \cdot \hat{B}_1^2 \cdot f_t^2\right) / f_t \quad .$$

Moreover, it must be observed that, to the aim of evaluating the iron losses of actual electromagnetic devices, various inaccuracies exist (for example: losses increase due to the lamination punching, additional losses due to flux pulsations in the teeth of the rotating machines,...). As regards the estimation of the coefficient that takes into account the additional losses due to the punching, the following remarks can be made:

- the iron losses increase, mainly localised along the punching borders of the laminations, is due to various causes, among which:
  - presence of flashes produced during the punching process;
  - grain stresses created along the punching borders;
- the first phenomenon, that causes short circuits between laminations, increases the eddy current losses;
- the second phenomenon causes, at least partially, also an increase of the hysteresis losses;
- it is difficult to evaluate "a priori" theses losses enlargement, strongly dependent on technological parameters: a suited coefficient can be empirically estimated, based on iron losses evaluations and measurements of already manufactured devices;
- moreover, it is also difficult to estimate the sharing of this losses increase between hysteresis and eddy currents;
- again for the sake of simplicity and for conservative reasons, it is preferable to refer this increase just to the eddy current loss contribution: as a matter of fact, when the frequency increases, this corresponds to overestimate the iron losses.

In conclusion, for the specific eddy losses the eq.s (8) and (9) are adopted: for the evaluation of the total eddy current losses, they must be multiplied for a suited increasing factor due to the punching (K_{pun}), considered constant.

Let us suppose that in the considered core point the flux density waveform is periodic and it can be expressed as follows:

$$B(t) = \sum_{k=1}^{\infty} \hat{B}_k \cdot \sin(k \cdot \omega_1 \cdot t + \varphi_k) \quad . \quad (12)$$

The absolute peak value $\hat{B}$ reached by B(t) during the period T_1, depends both on the amplitudes and on the phases of eq.(12).

The first term of eq.(8) can be written in the following manner:

$$p_h = K_h(\hat{B}_1) \cdot f_1 \cdot \left[\left(K_h(\hat{B}) / K_h(\hat{B}_1)\right) \cdot \left(1 + \alpha_h \cdot \sum_i \Delta B_i / \hat{B}\right)\right] \; , \quad (13)$$

or, in other terms:

$$p_h = p_{h1} \cdot \left[\left(K_h(\hat{B}) / K_h(\hat{B}_1)\right) \cdot \left(1 + \alpha_h \cdot \sum_i \Delta B_i / \hat{B}\right)\right] \quad ; \quad (14)$$

p_{h1} represents the hysteresis losses in sinusoidal operation at the fundamental frequency. Eq.(14) suggests that:

- the hysteresis losses in distorted operation equal the sinusoidal operation hysteresis losses (with the fundamental peak value), multiplied by a coefficient that depends on the absolute peak value and on the p.u. amplitude of the minor hysteresis loops;
- there is no dependence on the frequency of the harmonics, but only on their amplitudes and on their phases.

By inserting eq.(12) in the expression of p_e in (8), one obtains:

$$p_{e\ell} = K_e \cdot f_1^2 \cdot \sum_{k=1}^{\infty} k^2 \cdot \hat{B}_k^2 \quad . \quad (15)$$

Dividing and multiplying the 2nd member of (15) by $\hat{B}_1$ gives:

$$p_{e\ell} = K_e \cdot f_1^2 \cdot \hat{B}_1^2 \cdot \sum_{k=1}^{\infty} \left(k \cdot \frac{\hat{B}_k}{\hat{B}_1}\right)^2 = p_{e1} \cdot \sum_{k=1}^{\infty} \left(k \cdot \frac{\hat{B}_k}{\hat{B}_1}\right)^2 \; , \quad (16)$$

where p_{e1} represents the eddy current losses in sinusoidal operation at the fundamental frequency.

In eq.s (15) and (16) the subscript ℓ (ℓ = limit) has been added to indicate that the non-uniform eddy current distribution is not considered; in order to take it into account, it is sufficient to multiply each term by the corresponding coefficient k_{fe}:

$$p_e = p_{e1} \cdot \left[\sum_{k=1}^{\infty} \left(k \cdot \frac{\hat{B}_k}{\hat{B}_1} \right)^2 \cdot k_{fe}(k \cdot f_1) \right] \quad . \qquad (17)$$

Eq.s (16) and (17) suggest that:

- in distorted operation the eddy current losses can be evaluated by adding the separated contribution of each harmonic;
- the coefficient of losses increase, referred to the sinusoidal operation, depends on the harmonic order, on the spectrum of the p.u. flux density amplitudes and on the coefficient k_{fe};
- the structure of this increasing coefficient shows that:
  - in the limit situation of eq.(16) (k_{fe} = 1 always) no dependence on the harmonic frequency values occurs, but only on their harmonic order and amplitudes;
  - by considering the reducing effects of k_{fe}, the increase of the eddy current losses is attenuated when increasing the frequency band of the harmonic spectrum.

LINKS BETWEEN THE WAVEFORMS OF v(t), e(t), B(t)

The evaluation of the iron losses requires to know the waveform B(t) in each point of the core; on the other hand, considered the inverter characteristics, the waveform v(t) can be determined.

The link between B(t) and v(t) depends on the constructional characteristics of the supplied device and on its operating conditions.

As regards the operation, considering that usually the inverter fed electromagnetic devices are inductive type loads, for them the following, well known, equation can be written:

$$e(t) = v(t) - R \cdot i(t) - L \cdot \frac{di}{dt} \quad , \qquad (18)$$

with e(t) e.m.f. induced in the supplied winding, R internal resistance, L leakage inductance at the winding terminals and i(t) current absorbed; commonly R·i(t)+L·di/dt is called primary voltage drop.

Considered that R and L can be assumed roughly constant, eq.(18) can be decomposed in phasor equations at the harmonic frequencies:

$$\bar{E}_k = \bar{V}_k - R \cdot \bar{I}_k - j \cdot k \cdot \omega_1 \cdot L \cdot \bar{I}_k \quad , \qquad (19)$$

with $\omega_1 = 2 \cdot \pi \cdot f_1$; it is useful to define the following ratio:

$$K_{Vk} = E_k / V_k \quad , \qquad (20)$$

whose value must be determined by eq.(19), for each harmonic.

Once obtained e(t), in order to evaluate B(t) in each core point it is necessary to know the type of load; to this aim, it is useful to classify the devices into two categories, depending on the fact that the supplied windings are concentrated or distributed windings; some examples of the first type are the static devices (for example, transformers and reactors), while the a.c. rotating machines belong to the second category.

For a static device, the B(t)-e(t) link is the following one, for the waveform and for the spectrum amplitudes respectively:

$$B(t) = -\int e(t)dt \Big/ (N_t \cdot A_{fe})\,;\; \hat{B}_k = \left(\sqrt{2} \cdot E_k\right) \Big/ (k \cdot \omega_1 \cdot N_t \cdot A_{fe})\,; \qquad (21)$$

in eq.s (21) N_t is the number of turns of the winding disposed around the core column having cross section A_{fe} and E_k is the RMS value of the k^{th} harmonic e.m.f.; moreover, B(t) has the same values in each point of the considered section (and in all the column sections if all the sections have the same cross-section areas).

An example of device with distributed windings is the induction motor: its "T" equivalent circuit is adopted here, with the magnetizing and iron losses branches derived after the primary impedance. Let be L_r, R_r and L_s, R_s the phase rotor and stator equivalent parameters (the last ones including the feeding cable too).

Considering that in this case these parameters can be again assumed roughly constant with the frequency, eq.(19) allows the evaluation of E_k, once known the spectrum of V_k generated by the inverter, as a matter of fact:

- as regards the fundamental harmonic E_1, it can be determined from V_1 by applying eq.(19) for k = 1:

$$\bar{E}_1 = \bar{V}_1 - (R_s + j \cdot \omega_1 \cdot L_s) \cdot \bar{I}_{s1} \quad ; \qquad (22)$$

thus, the ratio:

$$K_{V1} = E_1/V_1, \qquad (23)$$

must be obtained by applying eq.(22) to the examined condition;

- for the calculation of the harmonics E_k, it must be remarked that, with $m_a \leq 1$, the first harmonics present in the waveform of v(t) are around the carrier frequency (which is quite higher compared with that feeding the motor): therefore, one realises that, on the basis of the equivalent circuit and neglecting the effect of the derived branch, the following approximations are valid as for the slip and the parameters at the frequency $f_k = k \cdot f_1$:

$$s_k \approx 1 \;; \qquad R_s << k \cdot \omega_1 \cdot L_s \;; \qquad R_r << k \cdot \omega_1 \cdot L_r \;; \qquad (24)$$

therefore, it is possible to write:

$$\bar{E}_k \approx \bar{V}_k \cdot K_{Vk} \;, \text{ with } K_{Vk} = L_r/(L_s + L_r) \quad \text{for } k > 1 \,. \qquad (25)$$

It must be observed that, while K_{V1} depends on the operating conditions, K_{Vk} does not depend on them for k > 1 (it can be shown that, even if with different values of K_{Vk}, a similar feature is valid in the case of static electromagnetic devices).

The presence of E_1 and of E_k at the terminals of the derived branch implies the circulation of magnetizing currents that produce m.m.f.s rotating in the air-gap (respectively fundamental and harmonics): here, the harmonic fields with sinusoidal spatial distribution are considered only for each harmonic frequency, neglecting the higher order spatial contributions. These m.m.f. fields, produced by the time harmonics, have the same number of poles of the main field: therefore for them (and for the corresponding e.m.f.s) the same winding factor f_w of the main field must be adopted. These m.m.f. fields produce corresponding flux density fields, whose maximum spatial value at the air-gap equals:

$$\hat{B}_{\delta_k} = E_k \Big/ \left(k \cdot \sqrt{2} \cdot f_1 \cdot f_w \cdot U \cdot \tau \cdot \ell\right) \quad , \qquad (26)$$

where U is the number of series connected active conductors per phase, τ the pole pitch, ℓ the length of the lamination stack.

On the basis of the amplitude and of the phase relations expressed by eq.s (22), (25) and (26) and considering the rotation ways of the described rotating fields, it is possible to obtain the waveform of the air-gap flux density field in each angular position along the stator periphery; neglecting the presence of the teeth, we can write:

$$B\delta(t) = \sum_{k=1}^{\infty} \hat{B}_{\delta_k} \sin(k \cdot \omega_1 \cdot t + \varphi_{\delta_k}) \quad . \qquad (27)$$

In order to determine the iron losses due to the inverter supply it is necessary to calculate the flux density B(t) in each point of the stator core, given by eq.(12), starting from $B_\delta(t)$: as a matter of fact, while the peak amplitude spectrum allows to evaluate the eddy current losses (by means of eq.(17)), the knowledge of the B(t) waveform is necessary in order to evaluate the hysteresis parameters $\hat{B}$ and K_m by eq.(14).

While the calculation of the peak flux densities of eq.(12) is easy to perform (they can be evaluated from the corresponding peak flux densities of eq.(27) as for the main field, i.e. taking into account the ratio of the equivalent magnetic cross sections), the determination of the phases φ_k, necessary in order to obtain B(t), is more complicated indeed, because the phases depend on the space and time composition of the fields at the air-gap.

Not considering, for now, the calculation of the hysteresis losses, from the analysis of eq.(21) and (26) one can recognise the validity of the following relation:

$$\frac{E_k}{E_1} = k \cdot \frac{\hat{B}_k}{\hat{B}_1} \quad . \qquad (28)$$

Thanks to eq.(28), eq.s (16) and (17) can be transformed as follows:

$$\frac{p_{e\ell}}{p_{e1}} = \sum_{k=1}^{\infty} \left(\frac{E_k}{E_1} \right)^2 \;, \quad \frac{p_e}{p_{e1}} = \left[\sum_{k=1}^{\infty} \left(\frac{E_k}{E_1} \right)^2 \cdot k_{fe}(k \cdot f_1) \right] . \qquad (29)$$

Eq.s (29) suggest the following, important remarks:

- considering that eq.s (29) include global electrical quantities (E_k) instead of the local peak flux densities of eq.s (16) and (17), they can be used not only for the evaluation of the local specific losses, but also of the total eddy current losses; therefore, one can write:

$$\frac{P_{e\ell}}{P_{e1}}=\frac{p_{e\ell}}{p_{e1}}=\sum_{k=1}^{\infty}\left(\frac{E_k}{E_1}\right)^2\ ;\ \frac{P_e}{P_{e1}}=\frac{p_e}{p_{e1}}=\left[\sum_{k=1}^{\infty}\left(\frac{E_k}{E_1}\right)^2\cdot k_{fe}(k\cdot f_1)\right] \quad ;(30)$$

- the losses ratios of eq.(30) depend on the values of E_k/E_1 : thus, the losses increase, referred to the sinusoidal operation, does not depend on the absolute amplitude, but on the p.u. amplitudes of the inverter voltages V_k (that are close to the e.m.f.s E_k);
- eq.s (30) are valid also in case the losses increasing factor due to the punching (K_{pun}) is to be considered: to this aim, it is sufficient to adopt the same value of this factor for all the harmonics.

FEATURES OF A THREE-PHASE PWM INVERTER

In the present analysis the inverter is supposed to be a voltage source inverter (VSI) of the PWM type, with sinusoidal modulation. As known, the modulation of the pulse duration (necessary to produce an output voltage with a low harmonic content) is obtained by operating the interference between a sinusoidal modulating signal and a triangular carrier signal: their intersections control the valve switchings; the frequency of the output voltage is regulated by varying the modulating signal frequency, equal to the output frequency; the output amplitude can be regulated by modifying the modulating signal amplitude. Let us define the following ratios:
- m_a, peak ratio between modulating and carrier signals;
- m_f, ratio between the frequencies of carrier and modulating signals: $m_f = f_c/f_1$.

Usually the inverters are operated by regulating m_a in the so-called linear range ($0 \le m_a \le 1$), because in this range the following occurs:
- the fundamental harmonic voltage is proportional to m_a :

$$V_1 = V_{1M}\cdot m_a\ ,\quad \text{with } V_{1M}=\frac{V_d}{2\cdot\sqrt{2}}\ ,\quad \text{for } 0\le m_a\le 1 \quad (31)$$

where V_{1M} represents the RMS value of V_1 for $m_a = 1$;
- all the voltage harmonics whose frequencies are between the fundamental frequency and a frequency near the carrier frequency are zero.

In this paper the situation described by eq.(31) is always supposed. As regards m_f, there are two possible control modes:
- control of the synchronous type; m_f is integer: in this case, if the output frequency is regulated, also the carrier frequency must be varied, in such a way to maintain m_f at a constant integer value;
- control of the asynchronous type; $m_f = f_c/f_1$ has a generic value: in this case the carrier frequency f_c is a constant (with advantages and simplification of the control system), while f_1 can be varied in a continuos manner.

The asynchronous PWM technique is frequent in the middle-small rating inverters, that use BJT, MOSFET or IGBT transistors: these valves can be operated with high switching frequencies, allowing the adoption of high carrier frequencies (up to and above the audible range). The consequence of the asynchronous PWM control is that, being f_c non multiple integer of f_1 , some voltage components that are sub-harmonics of the fundamental raise in the output inverter voltage: among the negative effects there are the sub-harmonic currents, the risk of magnetic saturation, the low frequency torque ripple. The weight of these effects depends on f_c : if f_c has a high value these phenomena are more limited, because they are shifted towards higher frequency ranges and then they are reduced, both for electromagnetic effect (higher load inductive reactance) and mechanical effect (higher inertia filtering attenuation).

The synchronous PWM technique is adopted in case of low values of m_f : this is necessary in the high rating inverters, for which, due to the characteristics of the used valves (usually GTO), the carrier frequency (linked to the switching frequency) is low.

However, with every value of m_f (high or low, integer or not), the presence of the sub-harmonic voltages is of no importance as regards the effect on the iron losses, because:
- the amplitudes of these sub-harmonics are usually remarkably lower than the fundamental amplitude;
- their frequency is significantly lower compared with f_1 .

In conclusion, the iron losses due to the possible presence of the sub-harmonics are always negligible compared with the contributions of the fundamental and of the higher harmonics; thus, the conclusions obtainable when m_f integer is supposed are substantially valid also for the asynchronous PWM control; thus, in the following, only the synchronous PWM control will be considered.

Let us consider the inverter scheme shown in Fig.2.

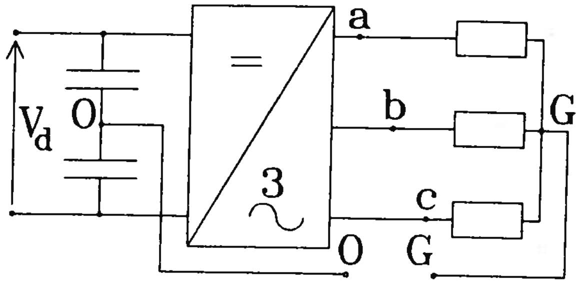

Fig.2 - Three-phase inverter scheme and terminal denominations.

In Fig.2 the point O is at the intermediate voltage of the d.c. bus and G represents the theoretical centre of the output three-phase voltage system (as known, G coincides with the actual centre of a star connected, symmetrical three-phase load).

Thanks to the G properties (in particular: $v_{aG}+v_{bG}+v_{cG} = 0$), for the instantaneous voltages of the circuit of Fig.2 we can write:

$$\begin{aligned} v_a = v_{aG} &= (2\cdot v_{aO} - v_{bO} - v_{cO})/3 = (v_{ab}+v_{ac})/3 \\ v_b = v_{bG} &= (2\cdot v_{bO} - v_{cO} - v_{aO})/3 = (v_{bc}+v_{ba})/3 \\ v_c = v_{cG} &= (2\cdot v_{cO} - v_{aO} - v_{bO})/3 = (v_{ca}+v_{cb})/3 \end{aligned} \quad . \quad (32)$$

As an example, Fig.3 shows the line-to-line and the line-to-neutral voltages of an ideal inverter (i.e. equipped with perfect switches, having instantaneous switching), with m_f= 15 and m_a = 0.8.

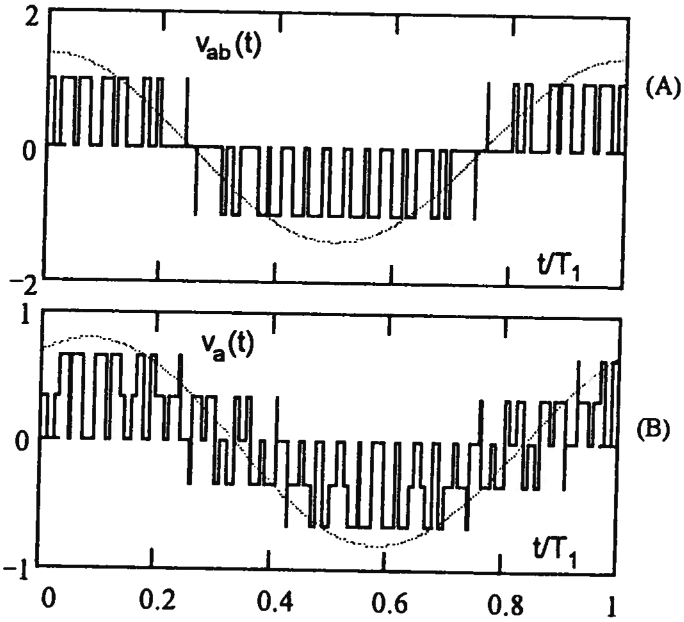

Fig.3 - Line-to-line (A) and line-to-neutral (B) voltages of an ideal PWM inverter with sinusoidal modulation; m_f= 15; m_a = 0.8.

Fig.4 shows the corresponding ideal flux linkages, expressed as follows:

$$\psi_{ij}(t) = \int v_{ij}(t)dt \quad ;$$

this equation corresponds to neglect the voltage drop across the load primary impedance, given by eq.(18).

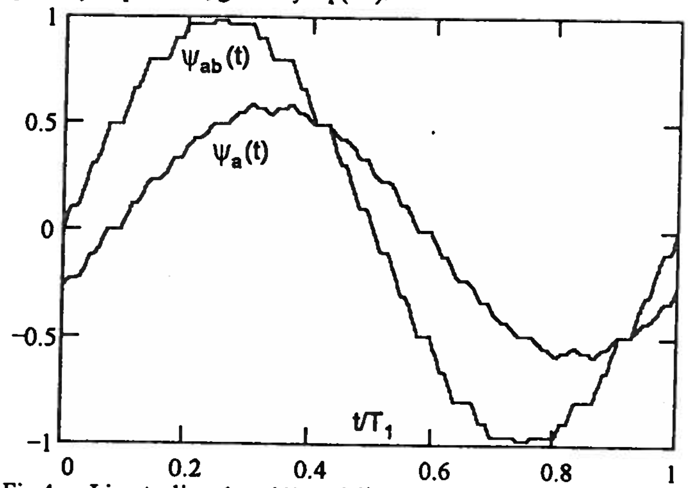

Fig.4 - Line-to line ($\psi_{ab}(t)$) and line-to-neutral ($\psi_a(t)$) ideal flux linkages produced by an ideal PWM inverter with sinusoidal modulation: $m_f = 15$; $m_a = 0.8$ (values referred to ψ_{ab_M}).

Fig.5 shows the voltage amplitude spectrum of an ideal inverter, with $m_f = 51$, $m_a = 0.8$.

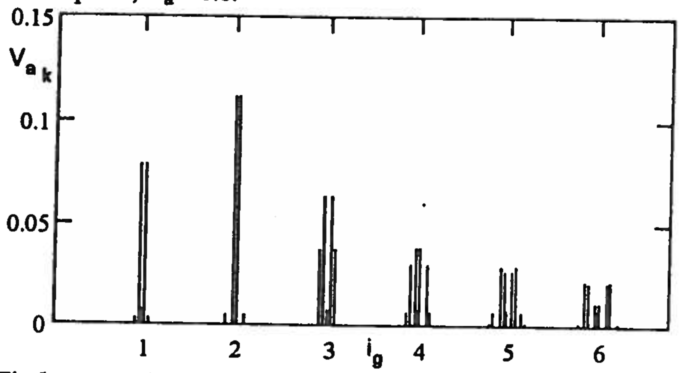

Fig.5 - Amplitude spectrum of the line-to-neutral voltage of an ideal PWM inverter with sinusoidal modulation, as a function of the group index i_g ($i_g = k/m_f$); the amplitudes are RMS values, expressed in p.u., referred to the voltage V_d : $m_f = 51$; $m_a = 0.8$.

About the hypothesis of ideal inverter, the following remarks occur:
- the actual voltages are different from the ideal ones of Fig.3, substantially because of the lower front rising gradients (thanks to the presence of the snubber circuits), and for the presence of the spikes in the instants of the voltage level changing;
- correspondingly, the actual amplitude spectrum differs from the ideal one, shown in the example of Fig.5.

However, the influence of the described phenomena becomes important mainly on the higher order harmonics, whose effects are already damped by the coefficient $k_{fe}(f)$.

The systematic analysis of the waveforms and of the amplitude spectra of an ideal inverter has given the following results:
- the harmonic orders present in the line-to-neutral voltages are the same of the line-to-line voltages, as shown by eq.s (32);
- the value of m_f has the following influence on the voltage waveforms (and on the waveforms of the ideal flux linkage too):
  - m_f even, multiple of 3: the voltage system is symmetrical, but there is a difference among the positive and the negative half waves, index of the presence of even harmonics;
  - m_f odd, non multiple of 3: the even order harmonics are absent, but the voltage system is not symmetrical:
  $$(v_a(t) \neq v_b(t+T_1/3) \neq v_c(t+2 \cdot T_1/3)):$$
  in fact, due to the fact that m_f does not include 3, a non repetitive sequence occurs in the intersections of the triangular carrier signal with the 3 phase modulating signals;
  - m_f even, non multiple of 3: even harmonics are present and the line-to-neutral voltages are non-symmetrical;
  - m_f odd, multiple of 3: the even harmonics are not present and the line-to-neutral voltages are symmetrical;

  thus, in case of synchronous PWM control, the best choice corresponds to m_f odd and multiple of 3;
- for every value of m_f, the amplitudes of the voltage harmonics have always the same RMS values in the three phases (also in case of non-symmetrical three-phase system): considering that the eddy current iron losses depend on the RMS values, the possible non symmetry does not affect these losses;
- the harmonics are disposed around the carrier frequency ($f_c = m_f f_1$) and around its multiple integer: thus, the harmonics are grouped around each integer value of the group index i_g;
- the harmonic orders corresponding to the non-zero amplitudes of these harmonics depend on the value of m_f, but their disposition around the group index and their amplitude does not depend on the value of m_f, but only on the value of m_a;
- for every value of m_f, there is a marked difference of waveforms between the line-to-neutral flux linkage and the line-to-line one, as one can realise by observing the diagrams in Fig.4: while the first one shows a non-monotone behaviour within each half wave (i.e., there are instants in which the flux, even if maintaining its own sign, has some local oscillations), the line-to-line flux linkage presents a monotone behaviour; considering that the flux linkage waveform is proportional to the flux density waveform, the described waveform oscillations correspond to some hysteresis minor loops.

EVALUATION OF THE IRON LOSSES INCREASE

From the characteristics of the inverter output waveforms and spectra some immediate ideas about the increase of the hysteresis losses can be obtained; there is a correlation between the nature and level of these losses and the winding connections:
- with star connection, the growth of the hysteresis losses, compared with the sinusoidal operation (growth calculable by eq.(14)), depends both on the difference between $\hat{B}$ and the fundamental peak value (that affects the ratio of the K_h values), and on the fluctuations ΔB_i (due to the hysteresis minor loops);
- with delta connection, on the contrary, there are no fluctuations ΔB_i indeed, because dv(t)/dt does not show sign changes during each half wave (see fig.3 A and B): therefore, in this case the hysteresis losses increment is always negligible;
- on the other hand, even in case of star connection the hysteresis losses increment compared with the sinusoidal operation can be quite limited, under the condition that m_f not be too much low; as a matter of fact, in this case we can consider that:

$$\hat{B} \approx \hat{B}_1 \quad \text{and} \quad \Delta B_i \approx 0.$$

As regards the eddy current losses increase, usually more weighty, the application of eq.s (30) requires further evaluations, in order to develop the dependence on m_f and on m_a.

To this aim, it is useful to define the following quantity:

$$q = \left(\frac{V}{V_1}\right)^2 \cdot m_a \quad , \qquad (33)$$

with V_1 RMS value of the 1[st] harmonic of the line-to-neutral voltage, while V is the global RMS value of the waveform v(t):

$$V = \sqrt{\frac{1}{T_1} \cdot \int_0^{T_1} v^2(t) \cdot dt} = \sqrt{\sum_{k=1}^{\infty} V_k^2} \quad . \qquad (34)$$

Table I shows, as an example, the numerical values of V/V_1 and of q, for some values of m_f and of m_a.

Table I - V/V_1 and $q = (V/V_1)^2 \cdot m_a$, for some values of m_f and m_a.

case	m_f	m_a	V/V_1	$q = (V/V_1)^2 \cdot m_a$
1	12	0.8	1.3549	1.4686
2	21	0.8	1.3555	1.4698
3	51	0.2	2.7085	1.4672
4	51	0.4	1.9169	1.4698
5	51	0.6	1.5652	1.4700
6	51	0.8	1.3555	1.4698
7	51	1.0	1.2125	1.4701

The cases 1, 2 and 6, corresponding to $m_a = 0.8$, confirm that V/V_1 does not depend on m_f, but only on m_a; the cases 3 and 7 show the dependence of V/V_1 on m_a, but reveal that q is not affected by m_a. These remarks can be generalised; therefore q can be considered as an absolute constant and, considering the numerical approximations, one can assume:

$$q = \left(\frac{V}{V_1}\right)^2 \cdot m_a \approx 1.47 \ .$$

Once defined the following ratio:

$$\rho_V = \left(\frac{V}{V_{1M}}\right)^2 \ , \quad (35)$$

from eq.(31) one obtains:

$$\rho_V = (V/V_1)^2 \cdot m_a^2 = \rho_V(m_a) \ , \quad (36)$$

and thanks to eq.(33):

$$\rho_V(m_a) = q \cdot m_a \ . \quad (37)$$

The eddy current losses due to the fundamental harmonic voltage only (that coincide with the eddy current losses in pure sinusoidal operation) can be expressed as follows:

$$P_{el} = P_{el}(m_a) = P_{elM} \cdot \frac{E_1^2}{E_{1M}^2} \approx P_{elM} \cdot \frac{V_1^2}{V_{1M}^2} = P_{elM} \cdot m_a^2 \ , \quad (38)$$

where P_{elM} is the value of this loss for $m_a = 1$: in eq.(38) the ratio K_{V1}, given by eq.(23), has been considered roughly constant.

It is now possible to modify the expression of the eddy current limit losses, given by the first of eq.s (30), that neglects the reducing effect of the eddy currents themselves, by remembering the link between voltages and harmonic e.m.f.s:

$$E_1 = K_{V1} \cdot V_1$$

$$\text{and} \quad E_k = K_{Vk} \cdot V_k \quad (k>1),$$

with K_{Vk} = constant for each value of $k > 1$.
Indicated with K_v the following ratio:

$$K_v = K_{Vk}/K_{V1} \ , \quad \text{for } k > 1 \quad (39)$$

by eq.s (34) and (36), the 1st of eq.s (30) changes as follows:

$$\frac{P_{e\ell}}{P_{el}} = \frac{P_{e\ell}(m_a)}{P_{el}(m_a)} = 1 + \sum_{k=2}^{\infty}\left(\frac{E_k}{E_1}\right)^2 = 1 + \sum_{k=2}^{\infty}\left(\frac{K_{Vk} \cdot V_k}{K_{V1} \cdot V_1}\right)^2 =$$

$$= 1 + K_v^2 \cdot \frac{\sum_{k=2}^{\infty} V_k^2}{V_1^2} = 1 + K_v^2 \cdot \frac{V^2 - V_1^2}{V_1^2} = 1 + K_v^2 \cdot \left[\left(\frac{V}{V_1}\right)^2 - 1\right] ,$$

$$\Rightarrow \quad \frac{P_{e\ell}(m_a)}{P_{el}(m_a)} = 1 + K_v^2 \cdot \left(\frac{\rho_V(m_a)}{m_a^2} - 1\right) \ . \quad (40)$$

By substituting eq.(37) in eq.(40), the ratio $P_{e\ell}/P_{el}$ becomes:

$$\frac{P_{e\ell}(m_a)}{P_{el}(m_a)} = 1 + K_v^2 \cdot \left(\frac{q}{m_a} - 1\right) \ . \quad (41)$$

By inserting eq.(38) in eq.(40), one obtains:

$$\frac{P_{e\ell}(m_a)}{P_{elM}} = m_a^2 + K_v^2 \cdot \left(\rho_V(m_a) - m_a^2\right) \ , \quad (42)$$

and, again thanks to eq.(37):

$$\frac{P_{e\ell}(m_a)}{P_{elM}} = m_a^2 + K_v^2 \cdot \left(q \cdot m_a - m_a^2\right) \ . \quad (43)$$

Eq.s (41) and (43), as the corresponding (40) and (42), express in a different way the increase of the eddy current limit losses:

- $P_{e\ell}/P_{el}$ indicates the actual increase of the limit losses referred to the first harmonic contribution, for the same value of m_a: as can be observed by (41), $P_{e\ell}/P_{el}$ tends to infinity when m_a tends to zero;
- $P_{e\ell}/P_{elM}$ indicates the value of the limit losses, referred to the maximum value of the losses due to the fundamental: considering that P_{elM} is a constant, eq.(43) represents the p.u. value of $P_{e\ell}$, with constant reference loss; by observing eq.(43), one realises that also $P_{e\ell}$ tends to zero when m_a tends to zero: thus, the fact that $P_{e\ell}/P_{el}$ tends to infinity when m_a tends to zero means that P_{el} tends to zero more quickly than $P_{e\ell}$.

It must be observed that eq.(35) can be rewritten by expressing $(V(m_a))^2$ as summation of the RMS squared amplitudes of all the voltage harmonics included in the spectrum of v(t):

$$\rho_V(m_a) = \left(\frac{V(m_a)}{V_{1M}}\right)^2 = \frac{\sum_{k\geq 1}^{\infty}(V_k(m_a))^2}{V_{1M}^2} \ . \quad (44)$$

Eq.(44) allows to take into account the effect of the coefficient $k_{fe}(f)$, by applying eq.(11) for each harmonic frequency present in the spectrum; to this aim the following quantity is defined:

$$\rho_{Vf}(m_a) = \frac{\sum_{k=1}^{\infty}(V_k(m_a))^2 \cdot k_{fe}(k \cdot f_1)}{V_{1M}^2} \ . \quad (45)$$

It can be noted that eq.(44) represents the upper limit of (45), when all the coefficients k_{fe} are considered equal to unity; on the other hand, these coefficients are as lower than unity as higher is the considered harmonic order; thus, the following is always verified:

$$\rho_{Vf}(m_a) < \rho_V(m_a) \ , \quad (46)$$

for every value of m_a.
Just the presence of these coefficients and the fact that they are applied, with different values, to each term of the summation, does not allow to obtain a "closed form" expression from eq.(45), as already obtained with eq.(37) from eq.(36).
On the other hand, thanks to eq.(45), the ratios corresponding to those of eq.s (40) and (42) are transformed as follows:

$$\frac{P_e(m_a)}{P_{el}(m_a)} = 1 + K_v^2 \cdot \left(\frac{\rho_{Vf}(m_a)}{m_a^2} - 1\right) \ , \quad (47)$$

and

$$\frac{P_e(m_a)}{P_{elM}} = m_a^2 + K_v^2 \cdot \left(\rho_{Vf}(m_a) - m_a^2\right) \ . \quad (48)$$

A complication implicit in the application of eq.s (47) and (48) is that $\rho_{Vf}(m_a)$ depends on the value of m_f: as a matter of fact, m_f affects both the spectrum distribution of the amplitudes V_k around the group indexes $i_g = k/m_f$, and the values of the reducing coefficients $k_{fe}(k \cdot f_1)$. It could seem necessary to pre-calculate the spectrum of V_k for each m_f, then applying eq.(45) to the spectrum itself. On the contrary, on the basis of the previous remarks, it is possible to perform the calculation of the spectrum of V_k for one value of m_f only, called "basic frequency ratio" and indicated with m_{fb}: subsequently, a suited law of frequency "shifting" can be applied to this spectrum, in such a way to multiply the quantities $(V_k)^2$ for the correct values of k_{fe}.

This procedure is based on the observation that, for each fixed value of m_a, the amplitude spectrum of V_k is the same when varying m_f, except for the frequency displacement of each harmonic group, whose central axis is displaced proportionally to m_f itself. Thus, it is sufficient to evaluate the values of the harmonic orders k to insert in the factors $k_{fe}(k \cdot f_1)$ included in eq.(45).

Called k_b the harmonic order that in the basic spectrum evaluated with a chosen value m_{fb} corresponds to the V_k harmonic (belonging to the i_g-th group), one can verify that the new harmonic order k that must be assigned to the same harmonic with amplitude V_k of a spectrum corresponding to a new value m_f must be evaluated by means of the following relation:

$$k = k_b + i_g \cdot (m_f - m_{fb}) \quad . \qquad (49)$$

In order to perform correctly the shifting of each harmonic group, the group index of eq.(49) must be calculated as follows:

$$i_g(k_b, m_{fb}) = 1 + \text{floor}\left(\frac{k_b}{m_{fb}} - 0.5\right) \quad , \qquad (50)$$

where the function floor(x) represents the operator "integer part of" x (for example: floor(12.99) = 12; floor(13.01) = 13).

Table II shows some calculation results for a few values of m_f, with m_a = 0.8: reference is made to the lamination Terni 3050 (corresponding to the IEC 700-50 A5 type and to the A.I.S.I. M 47 type), that presents the following values of hysteresis and eddy current losses in sinusoidal operation at f_t = 50 Hz:

$$\hat{B}_{1o} = 1.5 \text{ T} \; : \; p_{h1_o} = 4.58 \text{ W/kg}; \;\; p_{e1_o} = 1.52 \text{ W/kg}.$$

The values of two ratios are considered, regarding the PWM inverter supply compared with the sinusoidal operation, with equal RMS value of the fundamental (K_v = 0.75 has been supposed):

- P_e/P_{e1} = ratio of the eddy current losses only;
- $P_f/P_{f1} = (P_h+P_e)/(P_h+P_{e1})$ = ratio of the total iron losses.

Table II - Eddy current iron losses (P_e) and total iron losses (P_f), referred to the corresponding loss values in sinusoidal operation, with the same RMS value of the first harmonic, for m_a = 0.8, when varying m_f, supposed K_v = 0.75 (lamination Terni 3050).

m_f	P_e/P_{e1}	P_f/P_{f1}
12	1.4172	1.1040
13	1.4154	1.1035
14	1.4081	1.1017
15	1.4005	1.0998
51	1.2781	1.0693
300	1.1159	1.0289

In the first 4 cases the eddy current losses increase is about 40%, with a slight decrease when increasing m_f, thanks to the weak attenuation due to the corresponding decrease of k_{fe} (this fact confirms, even if indirectly, that the amplitude spectrum is displaced with m_f, but the amplitudes do not vary); for the same cases, the global loss increase is 10% roughly.

As can be observed, high values of m_f imply limited increases of the eddy current losses, and then produce weak increases of the total iron losses.

The values of Table II give an interesting idea of the level of iron losses increase, but they refer to a particular value of m_a; thus it is worth to analyse the eddy current iron losses increase as a function of m_a.

Table III shows the values of the eddy current losses with PWM inverter supply with f_1 = 50 Hz, referred to the loss P_{e1M}: according to eq.(38), this loss corresponds to the contribution of the fundamental only, for m_a = 1 (P_{e1M} coincides also with the loss in sinusoidal operation at rated voltage).

The data of Table III suggest the following remarks:

- by observing the couples of columns with the same value of m_f, one can conclude that the frequency shifting law, given by eq.s (49) and (50), has an excellent validity: in fact, the difference between corresponding values is always lower than 0.1%;
- the increase of f_c reduces the eddy current losses, thanks to the effect of the reducing coefficient $k_{fe}(k \cdot f_1)$;
- the values of the Table III are p.u.losses, referred to P_{e1M}: if we want to refer the same values to the losses occurring at the actual m_a, as shown in the 2nd column of Table II, it is necessary divide the mentioned ratios by m_a^2.

Fig.5 shows the eddy current losses as a function of m_a, for f_1 = 50 Hz, again referred to P_{e1M}, with m_f as a parameter: the lower limit curve illustrates the values of P_{e1}/P_{e1M} (losses in sinusoidal operation), while the higher limit curve represents the ratio $P_{e\ell}/P_{e1M}$ (limit losses, that is with k_{fe} = 1 for every k); the curves identified with α, β, γ represent the ratio P_e/P_{e1M} for m_f= 15, 51, 300 respectively.

Table III - Eddy current losses with PWM inverter supply, with f_1 = 50 Hz, referred to the loss P_{e1M}: values for m_f= 51 (f_c = 2.55 kHz) and for m_f = 300 (f_c = 15 kHz); comparison among the values evaluated with different values of m_{fb}, by applying the frequency shifting law given by eq.s (49) and (50).

m_a	P_e/P_{e1M} m_f= 51 (m_{fb} = 12)	P_e/P_{e1M} m_f= 51 (m_{fb} = 51)	P_e/P_{e1M} m_f= 300 (m_{fb} = 12)	P_e/P_{e1M} m_f= 300 (m_{fb} = 51)
0.1	0.0362	0.0362	0.0207	0.0207
0.2	0.1045	0.1045	0.0664	0.0664
0.3	0.1917	0.1916	0.1315	0.1315
0.4	0.2937	0.2937	0.2146	0.2146
0.5	0.4084	0.4086	0.3147	0.3148
0.6	0.5354	0.5356	0.4319	0.4320
0.7	0.6741	0.6742	0.5659	0.5660
0.8	0.8253	0.8255	0.7171	0.7172
0.9	0.9914	0.9912	0.8864	0.8863
1.0	1.1775	1.1770	1.0758	1.0756

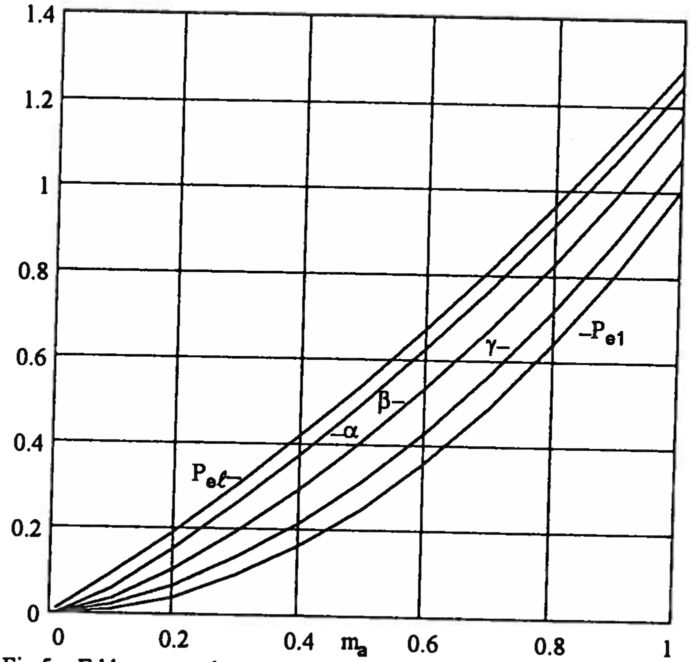

Fig.5 - Eddy current losses as a function of m_a, for f_1 = 50 Hz, referred to P_{e1M}, with parameter m_f: P_{e1} = losses in sinusoidal operation; $P_{e\ell}$ = limit losses (that is with k_{fe} = 1 for every k); α, β, γ = curves of P_e for m_f = 15, 51, 300 respectively.

For the curves of fig.5, the following remarks can be made:

- all the losses decrease when decreasing m_a, but the losses in sinusoidal operation decrease more quickly;
- the curve α ($m_f = 15 \Rightarrow f_c = 750$ Hz) shows that the eddy current losses with $m_a = 1$ are higher of about 25% than the corresponding losses of the first harmonic, while for $m_a = 0.5$ the ratio between these losses is nearly 2; thus, with a value of m_f so low (typical of a high rating GTO inverter), the effect of the inverter is not negligible; moreover, the real losses are not too far from the limit losses;
- the curve β ($m_f = 51 \Rightarrow f_c = 2.55$ kHz) shows that, for the same values of m_a previously considered, the corresponding losses ratios become 1.18 and 1.63 respectively; it is evident the stronger reducing effect of the coefficients k_{fe}, due to the higher value of m_f (typical of medium rating inverters, for example realised with BJT transistors); this implies a significant approaching of the losses curve to those of the sinusoidal operation;
- the curve γ ($m_f = 300 \Rightarrow f_c = 15$ kHz) shows that the losses ratios corresponding to the previous ones are some more reduced, to 1.08 and 1.26 respectively; thus, the high value of m_f (typical of a small-medium rating inverter, realised with IGBT transistors) leads to more reduce the eddy current losses, thanks to the great attenuation applied by the coefficients k_{fe}, as confirmed also by the more pronounced approaching of the curve of the actual losses to the curve of the sinusoidal operation.

CONCLUSIONS

In the present paper the analysis of the extra iron losses produced in the ferromagnetic core of the electromagnetic devices by the inverter supply has been performed.

After a critical analysis of the models of the iron losses, the expressions of these losses have been developed, as a function of the PWM inverter voltage waveform and of its corresponding amplitude spectrum, taking into account the voltage drops, the winding characteristics and connections and the features of the used laminations.

From the analysis, the following conclusions can be obtained:

- from the point of view of the method for the calculation of the extra eddy current iron losses, it is possible to determine one voltage spectrum only, just for one value of the frequency ratio m_f and for a suited number of m_a values: subsequently, in order to evaluate these losses for every value of m_f and m_a, these data can be extrapolated by means of a frequency shifting law;
- when increasing m_f, the extra iron losses due to the PWM inverter supply decrease, because:
  - the hysteresis losses are not too different from those in sinusoidal operation, and they practically coincide with the last ones when there are no minor hysteresis loops (with winding delta connection);
  - the extra eddy current losses, that represent just a portion of the iron losses, decrease because of the reducing effect of the coefficient k_{fe};
- the behaviour of the eddy current losses when the modulation ratio m_a increases is similar to that of the first harmonic contribution (that is similar to the contribution in sinusoidal operation), but it is as nearer to that of the limit losses as lower is m_f.

The research upon these subjects will continue, in particular as regards the following aspects:

- improvement of the model, both for the eddy current losses and for the hysteresis losses;
- analysis of the effect of different strategies of the inverter control (PWM with non sinusoidal modulation, PWM with selective harmonic elimination technique,...);
- comparative analysis of the iron losses due to the "six steps" inverter supply and to the PWM inverter supply;
- experimental validation activities.

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