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3 Main Manuscript for

4 Reconfigurable refraction manipulation at synthetic temporal interfaces with
5 scalar and vector gauge potentials

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7 Han Ye^{a,b,1}, Chengzhi Qin^{a,b,1}, Shulin Wang^{a,b,1}, Lange Zhao^{a,b}, Weiwei Liu^{a,b}, Bing Wang^{a,b,*},
8 Stefano Longhi^{c,d,*}, and Peixiang Lu^{a,b,e,*}

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10 ^aWuhan National Laboratory for Optoelectronics and School of Physics, Huazhong University of
11 Science and Technology, Wuhan 430074, China.

12 ^bOptics Valley Laboratory, Hubei 430074, China.

13 ^cDipartimento di Fisica, Politecnico di Milano, Piazza Leonardo da Vinci 32, I-20133 Milano, Italy.

14 ^dIFISC (UIB-CSIC), Instituto de Fisica Interdisciplinar y Sistemas Complejos, E-07122 Palma de
15 Mallorca, Spain.

16 ^eHubei Key Laboratory of Optical Information and Pattern Recognition, Wuhan Institute of
17 Technology, Wuhan 430205, China.

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19 ¹H.Y., C.Q., and S.W. contributed equally to this work

20

21 *Corresponding authors:

22 B. W. (**Email:** wangbing@hust.edu.cn),

23 S. L. (**Email:** stefano.longhi@polimi.it),

24 P. L. (**Email:** lupeixiang@hust.edu.cn).

25

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35 **This PDF file includes:**

36 Main Text
37 Figures 1 to 6
38

39 **Abstract**

40 Photonic gauge potentials, including scalar and vector ones, play fundamental roles in emulating photonic
41 topological effects and for enabling intriguing light transport dynamics. While previous studies mainly
42 focus on manipulating light propagation in uniformly distributed gauge potentials, here we create a series
43 of gauge-potential interfaces with different orientations in a nonuniform discrete-time quantum walk and
44 demonstrate various reconfigurable temporal-refraction effects. We show that for a lattice-site interface
45 with the potential step along the lattice direction, the scalar potentials can yield total internal reflection
46 (TIR) or Klein tunneling while vector potentials manifest direction-invariant refractions. We also reveal the
47 existence of penetration depth for the temporal TIR by demonstrating frustrated TIR with a double lattice-
48 site interface structure. By contrast, for an interface emerging in the time-evolution direction, the scalar
49 potentials have no effect on the packet propagation while the vector potentials can enable birefringence,
50 through which we further create a “temporal superlens” to achieve time-reversal operations. Finally, we
51 experimentally demonstrate electric and magnetic Aharonov-Bohm effects using combined lattice-site and
52 evolution-step interfaces of either scalar or vector potential. Our work initiates the creation of artificial
53 heterointerfaces in synthetic time dimension by employing nonuniformly and reconfigurable distributed
54 gauge potentials. This paradigm may find applications in optical pulse reshaping, fiber-optic
55 communications and quantum simulations.

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58 **Significance Statement**

59 Describing electromagnetic field using scalar and vector gauge-potentials represents one of the
60 fundamental breakthroughs in classical electrodynamics, which is also at the heart of the celebrated
61 Aharonov-Bohm (AB) effect. Realizing the refraction at gauge-potential interfaces can be harnessed to
62 emulate quantum tunneling effects for photons and to mold the flowing of light in a predetermined way.
63 Here, we create a series of gauge-potential interfaces in the temporal lattices and reveal distinct
64 mechanisms for refraction at scalar- and vector-potential interfaces with different orientations. The
65 demonstration of reconfigurable refractions at various gauge-potential interfaces fundamentally expands
66 our capability of manipulating light propagation in synthetic dimensions, which may also find potential
67 applications in the scenarios of optical pulse reshaping, fiber communications, and quantum simulations.

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70 **Main Text**

71 **Introduction**

72 Gauge potentials including scalar and vector ones can endow charged particles with a phase shift even
73 without external fields, which is at the heart of the celebrated Aharonov–Bohm (AB) effect (1). Neutral
74 particles such as photons cannot be directly influenced by the gauge potentials. Nevertheless, artificial

75 gauge potentials introduced by creating photonic analogues of AB phases could manipulate the propagation
76 of photons in a similar manner of charged particles. For instance, bended or index-varying waveguide
77 arrays provide a spatially-distributed scalar potential, which can yield an effective electric field for enabling
78 various control over wave diffraction (2-10). On the other hand, electro-optic modulations imposed into
79 resonators or waveguides can give rise to vector potentials by introducing non-reciprocal phase shifts (11-
80 17). These artificial gauge potentials, either for spatially-inhomogeneous scalar potentials or time-varying
81 vector potentials, provide powerful tools for emulating the coherent transport dynamics of photons, ranging
82 from Bloch oscillations (2-4), dynamic localization (5-7, 15) to Landau-Zener tunneling (8-10). By
83 judiciously designing the spatial distribution of gauge potentials, artificial magnetic field will also emerge,
84 with which one can create topological edge states and emulate the quantum Hall effect of photons (12, 13).
85 In practical applications, the creation of gauge potentials can also lead to a variety of intriguing light
86 control strategies, such as for realizing negative refraction (14), one-way light propagation (16), as well as
87 for developing nonreciprocal devices of optical isolators (11), circulators (14), and routers (17).

88 Recently, vector gauge potentials have been shown to provide a new mechanism of light guiding and
89 localization (18-21), suggesting fresh new ideas in the design of integrated photonic structures with
90 emerging novel functionalities, such as broadband optical switching (22) and dispersionless waveguide
91 coupling (23). Generalized laws of refraction and reflection for discretized light at interfaces between
92 different photonic artificial gauge fields, based on tilted waveguide arrays, have been investigated in (24),
93 whereas demonstration of negative refraction by vector gauge potentials has been reported for sound waves
94 in (25). However, the simultaneous exploitation and the ability to distinguish scalar and vector potentials
95 for strategic manipulation of light refraction have remained elusive in previous studies, where the main
96 limitations arose from the lack of reconfigurability. Furthermore, some more advanced refraction-related
97 effects like tunneling and interference occurring at more complex gauge potential interfaces remain largely
98 unexplored.

99 As a recently emerging field, the photonic lattice in synthetic dimension provides a fertile playground
100 to conveniently tailor the distribution of scalar and vector potentials therein and therefore can serve as a
101 versatile platform to investigate the refraction phenomena with a high degree of reconfigurability. These
102 synthetic lattices can be created by exploiting the internal degrees of freedom of photons, such as frequency
103 (26-33), time (34-39), and orbital angular momentum (40-42). Compared to spatial lattices with fixed
104 potential distributions (43), artificial gauge potentials are more convenient to introduce into synthetic
105 dimensions and, most importantly, they can be reconfigured on demand. For example, artificial vector
106 potentials can be readily introduced into frequency lattices by controlling the phase of dynamic modulation,
107 leading to the advanced control of spectrum evolution, ranging from frequency diffraction, Bloch
108 oscillations (27), refraction (28), time-reversal operation (30) and even for emulating non-Hermitian
109 topological braiding effects (33). These effects can also be implemented in synthetic temporal lattices
110 constructed by two coupled fiber loops. Thanks to the feasibility in controlling the lattice's features such as

111 the on-site energy, coupling phase, and strength, the temporal lattices possess remarkable advantages in
 112 generating scalar and vector potentials, which benefits the demonstration of a variety of classical and
 113 quantum effects ranging from parity-time symmetry (34), non-Hermitian skin effect (37) to the topological
 114 phase transition (38).

115 In this work, by creating a synthetic interface of scalar or vector potential in the synthetic temporal
 116 lattice, we propose and experimentally demonstrate a series of discrete refraction phenomena in a single
 117 and fully reconfigurable setup. It is found that the refraction can be diversely tuned by the orientation of
 118 potential interface. As typical examples, we obtain the temporal total internal reflection (TIR) or Klein
 119 tunneling at a scalar-potential lattice-site interface and refraction-free propagation at the vector-potential
 120 lattice-site interface. This temporal TIR manifests a nonnegligible penetration depth, which is verified by the
 121 frustrated TIR experiment. For an evolution-step interface, we find scalar potential has no effect on the
 122 refraction while vector potential can enable the temporal birefringence effect. Based on this, we further
 123 design a “temporal superlens” and achieve perfect time-reversal operations both for single-site and wave-
 124 packet inputs. Finally, we construct a temporal Mach–Zehnder interferometer by utilizing combined lattice-
 125 evolution scalar- or vector-potential interfaces and demonstrate the prototypes of electric and magnetic AB
 126 effects. The demonstration of reconfigurable refraction at the gauge-potential interfaces fundamentally
 127 expands the capability of manipulating wave packet propagation in synthetic dimensions, with potential
 128 applications in optical pulse shaping, fiber communications, and quantum simulation.

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130

131 **Results**

132 **Constructions of scalar and vector potentials in the temporal lattices.** We start from the theoretical
 133 model of introducing artificial gauge potentials into synthetic temporal lattices. Consider a coupled fiber-
 134 loop circuit, as shown in Fig. 1A, where an incident pulse traveling in the longer and shorter loops can be
 135 mapped conceptually into a “node-link” model of synthetic temporal lattice, as displayed in Fig. 1B. Details
 136 of experimental setup and related theoretical model are given in SI, sections 1 and 2. The circulating
 137 number of the pulses in the fiber loops corresponds to the time evolution step m in the lattice, and the
 138 relative positions of the pulses within one step is denoted by the lattice site n . A pulse hops from step m and
 139 position n to step $m+1$ and $n-1(n+1)$ in the lattice after finishing a circulation in the short (long) loop. To
 140 introduce a synthetic vector potential, we apply opposite phase modulations $\phi_v = \phi$ and $\phi_u = -\phi$ in long and
 141 short loops (21). The pulses then acquire phase shifts of ϕ and $-\phi$ during rightward and leftward hopping.
 142 Such a direction-dependent phase shift accompanying light hopping is analogous to a Peierls’ phase and
 143 corresponds to a vector potential of $A = (\phi_v - \phi_u)/2 = \phi$. On the contrary, when the modulations applied in the
 144 two loops are in phase, i.e., $\phi_v = \phi_u = \phi$, the pulse will then acquire identical phase shift during leftward and
 145 rightward hopping. Consequently, a synthetic scalar potential $\varphi = (\phi_v + \phi_u)/2 = \phi$ can also be constructed in
 146 the lattice. It is worth noting that the construction of vector potentials here is reminiscent of previous

147 studies on creating vector potentials in synthetic frequency dimension (26-32), both requiring direction-
 148 dependent phase shifts. However, the construction of scalar potentials using direction-independent phase
 149 shift cannot find counterpart in frequency dimension, which is unique to our synthetic temporal lattice.
 150 Accordingly, the refractions relying on scalar-potential interfaces cannot be achieved using synthetic
 151 frequency lattices.

152 The pulse evolution in the lattice under scalar and vector potentials is governed by the following
 153 evolution equation (sec.2 in SI)

$$154 \quad \begin{aligned} u_n^{m+1} &= [\cos(\beta)u_{n+1}^m + i \sin(\beta)v_{n+1}^m]e^{i\phi_u}, \\ v_n^{m+1} &= [\cos(\beta)v_{n-1}^m + i \sin(\beta)u_{n-1}^m]e^{i\phi_v}, \end{aligned} \quad (1)$$

155 where u_n^m and v_n^m denote the pulse amplitudes in the short and long loops at step m and position n , and β is
 156 the coupling angle of the coupler. For uniform potentials ϕ_v and ϕ_u , the lattice displays discretized
 157 translational symmetries both along the evolution time axis (m axis) and the lattice extending direction (n
 158 axis), so that the eigenstates are of Floquet-Bloch form: $(u_n^m, v_n^m)^T = (U, V)^T e^{ikn} e^{im\theta}$, where k and θ are the
 159 transverse Bloch momentum and longitudinal propagation constant, respectively. Note that (k, θ)
 160 constitutes the two axes in the reciprocal momentum space of the temporal lattice denoted by (n, m) .
 161 Substituting the eigen mode into Eq. (1), we can obtain the lattice band structure (SI Appendix, sec. 2)

$$162 \quad \theta_{\pm}(k) = \pm \arccos[\cos(\beta) \cos(k - A)] + \varphi, \quad (2)$$

163 where $\theta_+(k)$ and $\theta_-(k)$ denote the upper and lower branches of the band. According to Eq. (2), one can find
 164 that the physical effect of the scalar potential is to induce a propagation constant shift for an eigen Bloch
 165 mode while the vector potential is to induce a Bloch momentum shift. As a consequence, the scalar and
 166 vector potentials could induce the total band structure shifts along vertical (quasi-energy) and horizontal
 167 (Bloch momentum) directions, as displayed in Figs. 1C and 1D. In the following, we will construct
 168 heterointerfaces in the temporal lattice by applying nonuniformly-distributed scalar and vector potentials.
 169 In terms of interface orientation, we consider two basic types of interfaces: the lattice and evolution ones,
 170 which are constructed by introducing an abrupt change of scalar or vector potential along the lattice
 171 extending direction “ n ” and the time evolution direction “ m ”, respectively. Note that although the lattice-
 172 site index n resembles a spatial coordinate, it is physically a time slot index denoting the relative delay or
 173 advance between the pulses within one step (see sec. 2 in SI). In this sense, both the lattice-site and
 174 evolution-step interfaces still belong to temporal interfaces in terms of the physical time variable. As we
 175 will demonstrate below, these two different interfaces can yield a series of distinct refraction effects as light
 176 propagates through them.

177

178 Refraction at lattice-site interfaces

179 The lattice-site interface can be constructed by introducing a scalar or vector potential step along the lattice
 180 direction “ n ”. We first consider the interface formed by the scalar potentials, which is φ_1 for $n \geq 0$ and φ_2

181 for $n < 0$, as Fig. 2A displays. The potential difference is given by $\Delta\varphi = \varphi_2 - \varphi_1$, for which the band
182 structure undergoes a vertical shift of $\Delta\varphi$ between the two sides of the interface. For a Gaussian-shaped
183 wave packet incident from the right side ($n > 0$), it will generally experience a refraction at the interface,
184 generating both a refraction and a reflection of wave packet, respectively. In direct analogy to the refraction
185 at a spatial interface obeying Snell's law, i.e., the conservation of tangential wave vector along the interface
186 direction, the refraction here along the lattice temporal interface is also governed by the Snell's law, i.e.,
187 the conservation of longitudinal propagation constant θ . Specially, for a relatively small (or large) potential
188 difference $\Delta\varphi$, the upper band of the incident packet at right side will match the upper (or lower) band at
189 left side, giving rise to the intraband (interband) tunneling at the interface and hence the occurrence of
190 refraction. Particularly, such an interband tunneling phenomenon provides the temporal analogue of Klein
191 tunneling, where a particle tunnels through a potential step without quantum decay by turning into its anti-
192 particle (44). Here in our case, the upper and lower bands just play the roles of the particle and its anti-
193 particle. On the contrary, for a moderate potential difference, the band structure at left side falls into the
194 band gap at the right side, such that the refracted packet vanishes and total internal reflection (TIR) will
195 occur. In experiments, we choose $\Delta\varphi = 0.5\pi$ and 0.8π for demonstrating above two cases, where the
196 refraction processes are shown in Figs. 2A and 2B. For $\Delta\varphi = 0.5\pi$, no refracted beam exists, clearly
197 indicating the occurrence of TIR. While for $\Delta\varphi = 0.8\pi$, the Klein tunneling occurs. Also note that the
198 refracted beam manifests a direction derivation with respect to the incident one, suggesting the abrupt
199 change of group velocity during the refraction process. Finally, by continuously varying $\Delta\varphi$ from 0 to 2π ,
200 we also obtain the general formula of the power transmission coefficient (44) (SI Appendix, section 3)

$$201 \quad T_{\pm} = \frac{(1 - e^{2\lambda_0})(1 - e^{\pm 2\lambda_2})}{(\pm 1 - e^{\lambda_0} e^{\pm \lambda_2})^2}, \quad (3)$$

202 where $\lambda_0 = \text{arsinh}[\cot(\beta)\sin(k_i)]$, $\lambda_2 = \text{arsinh}[\cot(\beta)\sin(k_r)]$ with k_i , k_r being the Bloch momenta of the
203 incident and refraction packets. “+(-)” denotes that the refracted packet appears in the upper and lower
204 bands, corresponding to the intraband and interband tunneling cases. The reflection is then given by $R =$
205 $1 - T$. The transmission/reflection coefficients are determined by the incident Bloch momentum and the
206 relative scalar potentials at two sides. The results here are also similar to spatial refractions described by
207 Fresnel's equations, where the transmission/reflection coefficients are determined by the incident angle and
208 the relative refraction indices at both sides of the interface. The measured transmission and refraction
209 coefficients are also shown by the red and blue dots in Fig. 2C, which can coincide well with the theoretical
210 curves. Specially, the TIR occurs in the regions of $0.25\pi < \Delta\varphi < 0.75\pi$ and $1.25\pi < \Delta\varphi < 1.75\pi$, outside of
211 which the refraction can take place (SI Appendix, section 4).

212 On the other hand, the lattice-site interface can also be constructed by substituting φ_1 and φ_2 with A_1
213 and A_2 , such that the potential difference becomes $\Delta A = A_2 - A_1$. The band structure at left side will
214 undergoes a horizontal shift of ΔA with respect to that at right side. Likewise, by applying Snell's law, i.e.,
215 the longitudinal propagation constant conservation, one can find that refraction will always exist due to the

216 complete band structure overlap at two sides, thus preventing the occurrence of TIR. In experiments, we
 217 choose $\Delta A = 0.3\pi$ and π , where beam refraction and reflection are always observable, as shown in Fig. 3D
 218 and 3E. Moreover, in contrast to above lattice-site interface formed by scalar potentials where the refracted
 219 packet experiences an abrupt group velocity change, the packet here can maintain its propagation direction
 220 at the lattice-site interface formed by vector potentials. The general transmission coefficient is given by (SI
 221 Appendix, section 5)

$$222 \quad T = \frac{\cosh(2\lambda_0) - 1}{\cosh(2\lambda_0) - \cos(\Delta A)}. \quad (4)$$

223 The transmission and reflection versus ΔA are shown in Fig. 2F. The transmission reaches minimum at ΔA
 224 $= \pi$ and one can't observe TIR like that in the interface constructed by scalar potentials.

225 The TIR in temporal lattice here is reminiscent of its counterpart in real space. A prominent feature of
 226 spatial TIR is the existence of evanescent wave penetrating the reflection interface at a wavelength scale
 227 depth. Here we design a frustrated total internal reflection (FTIR) scenario as a criterion to verify the
 228 existence of evanescent wave accompanying our temporal TIR. We construct a double lattice-site interface
 229 structure composed of a narrow gap with scalar potential φ_1 and width Δn sandwiched by two semi-infinite
 230 regions with scalar potential φ_2 , as shown in Figs. 3A and 3B. The potential difference is thus $\Delta\varphi = \varphi_2 - \varphi_1$,
 231 which is fixed at $\pi/2$ in our implementation. The incident wave packet is injected from the right side and
 232 experiences TIR at first interface. Similar to the real-space TIR, the packet can partially penetrate to the gap
 233 region. If the gap is narrow enough, the wave can further penetrate to the left side of the gap. The tunneling
 234 process is analogy to the FTIR in real space. As $\Delta n = 2$, the gap is relatively large and there are seldom
 235 waves transmitted. Most of the waves are reflected by the interface. However, if we decrease the gap width
 236 to $\Delta n = 1$, the beam can partially penetrate to the left region. The transmission coefficient versus the gap
 237 width is shown in Fig. 3C, which decreases gradually with the increase of gap width. The experiment
 238 clearly verifies that the penetration depth is at a scale of $\Delta n \sim 1$ for our temporal TIR.

239

240 **Refraction at evolution-step interfaces**

241 As another interface orientation, an evolution-step interface can also be constructed by introducing a
 242 potential step along the direction of time evolution. We firstly consider the interface formed by scalar
 243 potentials, which are assumed to be φ_1 for $m \leq 60$ and φ_2 for $m > 60$, as Fig. 4A displays. The potential
 244 difference is given by $\Delta\varphi = \varphi_2 - \varphi_1$. As the wave packet is incident from the bottom, its propagation is not
 245 influenced by the interface, as shown in Fig. 4A. Figure 4B depicts the band structure of the top region,
 246 which undergoes a vertical shift of $\Delta\varphi = \pi$ with respect to the bottom region. For the evolution-step
 247 interface, the Snell's law is that the transverse Bloch momentum should be conserved. Still considering the
 248 incidence of an upper band mode, it will excite the mode at the same band in the top region. The influence
 249 of $\Delta\varphi$ on the band occupancies in the top region is depicted in Fig. 4C. One sees that the occupancies are
 250 independent on $\Delta\varphi$ and remain the same with that in the bottom region. The output field intensities are

251 measured at $m = 120$ and the profile keeps unchanged as $\Delta\varphi$ varies, experimentally verifying the triviality
 252 of evolution-step interface constituted by scalar potentials, as illustrated in Fig. 4D.

253 The evolution-step interface can also be constructed by introducing vector potentials. As shown in Fig.
 254 4E, the vector potentials are assumed to be A_1 for $m \leq 30$ and A_2 for $m > 30$. The potential difference reads
 255 $\Delta A = A_2 - A_1$. For $\Delta A = \pi/2$, the incident wave packet splits into two branches with opposite propagation
 256 directions. The band structure in the top region undergoes a horizontal shift by $\Delta A = \pi/2$, as depicted in Fig.
 257 4F. Likewise, the Bloch momentums along the lateral position direction should also be conserved. For the
 258 incidence of an upper band mode, the modes of upper and lower bands in the top region are generated
 259 simultaneously, with their occupancies P_+ and P_- given by (SI Appendix, section 6)

$$260 \quad P_{\pm} = \frac{\cosh(\lambda_0 \pm \lambda_1) \pm \cos(\Delta A)}{2 \cosh(\lambda_0) \cosh(\lambda_1)}, \quad (5)$$

261 where $\lambda_0 = \operatorname{arsinh}[\cot(\beta)\sin(k_i)]$, $\lambda_1 = \operatorname{arsinh}[\cot(\beta)\sin(k_i - \Delta A)]$. As ΔA varies from 0 to 2π , the measured data
 262 of P_+ and P_- are depicted in Fig. 4G, which can coincide well with the theoretical analysis. For $\Delta A = \pm\pi/2$,
 263 the upper and lower band occupancies are identical, where the incident wave packet splits equally into two
 264 branches. The output field intensity measured at $m = 120$ undergoes an oscillating transverse motion as ΔA
 265 varies, as illustrated in Fig. 4H. This can be well explained by the opposite periodic variations of the group
 266 velocities of the upper and lower band modes, $v_{g,\pm} = \partial\theta_{\pm}(k - \Delta A)/\partial k$ as ΔA varies. The beam splitting here is
 267 analogous to the time refraction and reflection at a temporal interface, which is induced by an abrupt
 268 change of refraction index at specific time instant in a spatially-homogeneous medium (45, 46). Here the
 269 splitted wave packet occupying the lower band corresponds to reflected beam while the other one
 270 occupying the upper band corresponds to the refracted beam. Unlike the models of Refs. (45, 46), where
 271 the time dimension is continuous, our temporal lattice is discrete both in the lattice (n) and evolution (m)
 272 directions. Accordingly, the time refraction and reflection in our system act to a discretized pulse train,
 273 rather than to continuous light waves as in Refs. (45,46). The evolution-step interface of gauge potentials in
 274 temporal lattice opens a new avenue to emulate refraction and reflection in a synthetic time dimension with
 275 full reconfigurability, which is hard to implement in continuous spatial systems.

276 By taking advantage of the evolution-step interface formed by vector potential steps, we can realize
 277 time reversal of pulse evolution. As shown in Fig. 5A, a vector potential of A_2 is applied in the time interval
 278 Δm beginning from $m = 60$ during the pulse evolution and that in the rest is denoted by A_1 . The potential
 279 difference between adjacent regions is thus $\Delta A = A_2 - A_1$. In order to manipulate the band structures more
 280 flexibly, the coupling ratio of the fibers is also changed with time, which is set as β_2 in the disturbed time
 281 interval Δm and the rest is set as β_1 . Here we choose $\Delta m = 1$, $\Delta A = \pi/2$, $\beta_1 = \pi/4$, and $\beta_2 = \pi/2$. For the
 282 incidence of a Gaussian shaped wave packet with a Bloch momentum $k = \pi/2$ in the upper band, it moves
 283 backward initially and undergoes a mirror reversion after passing through the disturbed time interval,
 284 featuring negative refraction. Note that only a single time step is utilized to vary the vector potential and
 285 coupling ratio while the undisturbed regions on both sides are identical. Therefore, the temporal interface

286 mimicked by a single time step behaves as a superlens with infinitesimal thickness in real space. The
 287 temporal superlens is also applicable to the imaging of a point source that contains all Fourier components,
 288 i.e., Bloch modes in the whole Brillouin zone. As shown in Fig. 5B, for a single pulse incidence, i.e., for
 289 excitation of a single lattice site at $n = 0$, discrete diffraction is observed at first region, with subsequent
 290 refocusing onto a single site after passing through the superlens. Note that there exists perceptible
 291 asymmetry in the imaging pattern along the lattice direction. This is attributed to the unequal excitation of
 292 upper and lower bands from single-loop incidence. Since each Bloch-wave component in upper and lower
 293 bands possesses opposite group velocities, their interference with unequal band occupations gives rise to
 294 this asymmetric evolution pattern. To obtain symmetric evolution patterns, we need to simultaneously
 295 excite from both fiber loops with appropriate relative amplitude and phase. The imaging effect can be
 296 explained by the reversion of Bloch modes. As the wave packet passing through the interface, the Bloch
 297 mode in the upper band converts to the lower one as we change both the gauge potential and the coupling
 298 ratio within a single time step, as illustrated in Fig. 5C. The group velocity undergoes a mirror reversion
 299 after passing through the temporal superlens. The details can be found in the SI Appendix (SI Appendix,
 300 section 7).

301

302 **Electric and magnetic AB effects**

303 In above implementations, we apply a scalar or vector potential alone and have experimentally verified
 304 their unique and distinct functionalities in manipulating temporal refractions at a specific interface. In this
 305 section, we show that the combination of lattice and evolution scalar- or vector-potential interfaces can
 306 benefit to the construction of a Mach–Zehnder interferometer (MZI) in time dimension, and hence for
 307 emulating the celebrated electric and magnetic AB effects. As depicted in Fig. 6A and 6B, the electric and
 308 magnetic AB effects can be accomplished by using MZI applied with scalar and vector gauge potentials,
 309 respectively. The gauge potential difference, and hence the phase difference between the two arms
 310 determines the final output intensity. In our setup, we utilize a lattice-site interface at $n = 0$ formed by
 311 vector potentials to play the role of 50:50 beam splitter and a superlens at $m = 60$ for the beam reversal
 312 operation, as shown in Fig. 6C. The beam interference at the output end is thus determined by the total
 313 phase difference in the two arms, consisting of the propagation phases accumulating in the two paths and
 314 the abrupt phase jumps acquired at the lattice-site interface at $n = 0$ and evolution-step interface at $m = 60$.
 315 Firstly, we consider the MZI without propagation phase. Taking the rightward output beam, along path 1
 316 the packet acquires a phase jump of π for reflection, 0 at the superlens and π for reflection at output end,
 317 giving rise to total phase jump of $\phi_1 = \pi + 0 + \pi = 2\pi$. For path 2, the phase jump is 0 for transmission, π at the
 318 superlens and π for transmission at output end, such that $\phi_2 = 0 + \pi + \pi = 2\pi$. So the phase jump difference
 319 between the two paths is $\phi_1 - \phi_2 = 0$, which gives rise to the constructive interference (SI Appendix, section
 320 8). Similar procedure is also applicable to the analysis of the destructive interference for the leftward output
 321 beam.

322 Based on this, we then introduce the propagation phase to simulate AB effects. Firstly, a scalar
 323 potential difference is imposed on the interferometer arms. The scalar potentials in the two arms are
 324 denoted by φ_1 for $n < 0$ and φ_2 for $n > 0$. The potential difference is denoted by $\Delta\varphi = \varphi_1 - \varphi_2$. In the
 325 experimental implementation, the scalar potentials are applied at $30 \leq m < 90$ with a time interval $\Delta m = 60$,
 326 as depicted in Fig. 6D. Then the accumulated phase should be $\Delta\phi = \Delta m \cdot \Delta\varphi$. As $\Delta\phi = \pi$, the output
 327 intensities of wave packets undergo a constructive interference and the output intensity reaches maximum
 328 at $n < 0$. By varying $\Delta\varphi$, the wave packet will appear at a symmetric position $n > 0$ and the intensity
 329 experiences a periodic oscillation, as illustrated in Fig. 6E. The magnetic AB effect can also be
 330 implemented by employing MZI applied with vector potentials, as depicted in Fig. 6F. A vector potential
 331 difference is imposed on the interferometer arms. The vector potentials are assumed to be A_1 in $30 \leq m < 60$
 332 and A_2 in $60 < m \leq 90$, both with a time length $\Delta m = 30$. The potential difference reads $\Delta A = A_1 - A_2$. From
 333 Fig. 6F, one sees that the accumulated phase difference should be $\Delta\phi = 2\Delta n \cdot \Delta A$, where $\Delta n = v_g \Delta m$ with the
 334 group velocity $v_g = \sqrt{2}/2$. As $\Delta\phi = \pi$, the wave packets also undergo a constructive interference at $n < 0$
 335 while a deconstructive interference at $n > 0$, as illustrated in Fig. 6G. When $\Delta\phi$ varies from 0 to 2π , the
 336 output intensities corresponding to the superposed wave packets with backward and forward evolutions are
 337 measured experimentally. The data are depicted in Fig. 6G, where the output intensities vary as a function
 338 of $\Delta\phi$ sinusoidally, evidently verify the interference affected by the vector potentials.

339

340

341 **Conclusion**

342 In summary, we have experimentally constructed reconfigurable lattice-site and evolution-step interfaces
 343 using inhomogeneous scalar and vector potentials in the temporal lattice, by which the unique features of
 344 refraction processes with respect to these two potentials are demonstrated in a reconfigurable setup. The
 345 TIR and FTIR mediated by scalar potential at the lattice-site interface are achieved, while the transmission
 346 always exists for vector potential. The vector potential redistributes the band occupancies at the evolution-
 347 step interface, resulting in wave-packet splitting upon impinging the interface. By abruptly changing the
 348 vector potentials in a single time step, we also construct an evolution superlens and demonstrate perfect
 349 time-reversal operation of pulse evolution. We further emulate the celebrate electric and magnetic AB
 350 effects by virtue of the combined lattice and evolution-step interfaces. Our results reveal the uniqueness of
 351 scalar and vector potentials in controlling the refraction of wave packets at synthetic interfaces and
 352 demonstrate the ability to reconfigure the refraction scenario, realizing different functionalities with the
 353 same device. Moreover, due to the flexibility in designing the interface orientation and configuration, our
 354 system is expected to serve as an ideal platform to emulate various quantum mechanical phenomena such
 355 as quantum tunneling effects. Finally, the fully reconfigurable temporal refraction could lead to many
 356 applications of pulse reshaping, multiplexing, and manipulation used for optical communication and

357 quantum information processing, where reconfigurability and functional complexity have become
358 increasingly demanding.

359

360

361 **Materials and Methods**

362 **Experimental implementation.** The experimental setup comprises of two fiber loops which are connected
363 by a variable fiber optical coupler (VOC), as Fig. 1A displays. The coupling ratio between two loops can be
364 controlled by applying electrical signal on the VOC via an arbitrary waveform generator (AWG). Each
365 loop comprises a spool of single mode fiber, corresponding to the average roundtrip time of approximately
366 25 μ s for the two loops. The length difference between the two loops is introduced by inserting an
367 additional optical fiber patch cord in the long loop, which give rise to time difference of approximately 0.15
368 μ s between the round-trip times in the two loops. To feed the circuit, a 50 ns long pulse is coupled into the
369 long loop, which is prepared by modulating the output light beam from a 1550 nm distributed-feedback
370 laser. During the circulation in the loops, the optical loss can be compensated by erbium-doped fiber
371 amplifiers (EDFAs) in the two loops. To suppress the transient of the EDFA, the signal pulses is mixed
372 with a high-power 1530 nm pilot light before entering the EDFA. Behind EDFA, the 1530 nm pilot light
373 and spontaneous emission noise in the amplification process are removed by a band-pass filter. Besides, we
374 employ the polarization beam splitter and polarization controllers in the loops to monitor and control the
375 polarization of the light signal. The optical isolators are used to ensure the unidirectional circulation in both
376 loops. The Mach-Zehnder intensity modulators set in fiber loops serve as the optical switches. The phase of
377 pulses is controlled by phase modulators in the loops, which are driven by AWGs. Thanks to the flexible
378 tunability of electrical signal generated by AWGs, the effective gauge potentials deriving from the phase
379 modulations in two loops can be arbitrarily constructed in the synthetic temporal lattice. In order to detect
380 and record the pulse evolutions in the two loops, we couple a small portion of light signals from both loops
381 and detect them with photodetectors. The output voltages of the photodetectors are sampled by an
382 oscilloscope. A more detailed description of the experimental platform is included in SI Appendix (SI
383 Appendix, section 1).

384

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391 **References**

392 1. Y. Aharonov, D. Bohm, Significance of electromagnetic potentials in the quantum theory.

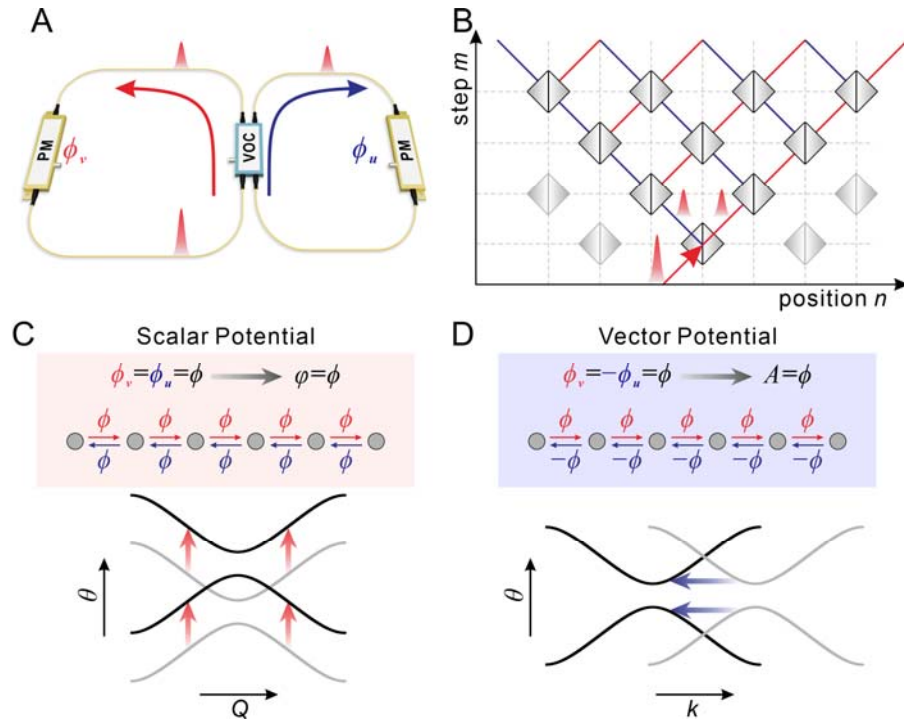
- 393 *Phys. Rev.* **115**, 485-491 (1959).
- 394 2. U. Peschel, T. Pertsch, F. Lederer, Optical Bloch oscillations in waveguide arrays. *Opt. Lett.*
395 **23**, 1701-1703 (1998).
- 396 3. G. Lenz, I. Talanina, C. M. de Sterke, Bloch oscillations in an array of curved optical
397 waveguides. *Phys. Rev. Lett.* **83**, 963-966 (1999).
- 398 4. T. Pertsch, P. Dannberg, W. Elflein, A. Bräuer, F. Lederer, Optical Bloch oscillations in
399 temperature tuned waveguide arrays. *Phys. Rev. Lett.* **83**, 4752-4755 (1999).
- 400 5. S. Longhi *et al.*, Observation of dynamic localization in periodically curved waveguide
401 arrays. *Phys. Rev. Lett.* **96**, 243901 (2006).
- 402 6. A. Szameit *et al.*, Polychromatic dynamic localization in curved photonic lattices. *Nat. Phys.*
403 **5**, 271-275 (2009).
- 404 7. A. Joushaghani *et al.*, Generalized exact dynamic localization in curved coupled optical
405 waveguide arrays. *Phys. Rev. Lett.* **109**, 103901 (2012).
- 406 8. H. Trompeter *et al.*, Visual observation of Zener tunneling. *Phys. Rev. Lett.* **96**, 023901
407 (2006).
- 408 9. S. Longhi, Optical Bloch oscillations and Zener tunneling with nonclassical light. *Phys. Rev.*
409 *Lett.* **101**, 193902 (2008).
- 410 10. F. Dreisow *et al.*, Bloch-Zener oscillations in binary superlattices. *Phys. Rev. Lett.* **102**,
411 076802 (2009).
- 412 11. K. Fang, Z. Yu, S. Fan, Photonic Aharonov-Bohm effect based on dynamic modulation. *Phys.*
413 *Rev. Lett.* **108**, 153901 (2012).
- 414 12. K. Fang, Z. Yu, S. Fan, Realizing effective magnetic field for photons by controlling the phase
415 of dynamic modulation. *Nat. Photonics* **6**, 782-787 (2012).
- 416 13. S. Mittal *et al.*, Topologically robust transport of photons in a synthetic gauge field. *Phys.*
417 *Rev. Lett.* **113**, 087403 (2014).
- 418 14. K. Fang, S. Fan, Controlling the flow of light using the inhomogeneous effective gauge field
419 that emerges from dynamic modulation. *Phys. Rev. Lett.* **111**, 203901 (2013).
- 420 15. L. Yuan, S. Fan, Three-dimensional dynamic localization of light from a time-dependent
421 effective gauge field for photons. *Phys. Rev. Lett.* **114**, 243901 (2015).
- 422 16. K. Fang *et al.*, Generalized non-reciprocity in an optomechanical circuit via synthetic
423 magnetism and reservoir engineering. *Nat. Phys.* **13**, 465-471 (2017).
- 424 17. C. Qin, A. Alù, Z. J. Wong, Pseudospin-orbit coupling for chiral light routings in gauge-flux-
425 biased coupled microring resonators. *ACS Photonics* **9**, 586-596 (2022).
- 426 18. Q. Lin, S. Fan, Light guiding by effective gauge field for photons. *Phys. Rev. X* **4**, 031031

- 427 (2014).
- 428 19. S. Slussarenko *et al.*, Guiding light via geometric phases. *Nat. Photonics* **10**, 571-575 (2016).
- 429 20. Y. Lumer *et al.*, Light guiding by artificial gauge fields. *Nat. Photonics* **13**, 339-345 (2019).
- 430 21. A. V. Pankov, I. D. Vatik, D. V. Churkin, A. A. Sukhorukov, Observation of localized modes
- 431 at effective gauge field interface in synthetic mesh lattice. *Sci. Rep.* **9**, 3464 (2019).
- 432 22. I. A. D. Williamson, S. Fan, Broadband optical switch based on an achromatic photonic gauge
- 433 potential in dynamically modulated waveguides. *Phys. Rev. Appl.* **11**, 054035 (2019).
- 434 23. W. Song *et al.*, Dispersionless coupling among optical waveguides by artificial gauge field.
- 435 *Phys. Rev. Lett.* **129**, 053901 (2022).
- 436 24. M.-I. Cohen *et al.*, Generalized laws of refraction and reflection at interfaces between
- 437 different photonic artificial gauge fields. *Light Sci. Appl.* **9**, 200 (2020).
- 438 25. Y. Yang *et al.*, Demonstration of negative refraction induced by synthetic gauge fields. *Sci.*
- 439 *Adv.* **7**, eabj2062 (2021).
- 440 26. L. Yuan, Y. Shi, S. Fan, Photonic gauge potential in a system with a synthetic frequency
- 441 dimension. *Opt. Lett.* **41**, 741-744 (2016).
- 442 27. L. Yuan, S. Fan, Bloch oscillation and unidirectional translation of frequency in a dynamically
- 443 modulated ring resonator. *Optica* **3**, 1014-1018 (2016).
- 444 28. C. Qin *et al.*, Spectrum control through discrete frequency diffraction in the presence of
- 445 photonic gauge potentials. *Phys. Rev. Lett.* **120**, 133901 (2018).
- 446 29. A. Dutt *et al.*, A single photonic cavity with two independent physical synthetic dimensions.
- 447 *Science* **367**, 59-64 (2020).
- 448 30. C. Qin, B. Wang, Z. J. Wong, S. Longhi, P. Lu, Discrete diffraction and Bloch oscillations in
- 449 non-Hermitian frequency lattices induced by complex photonic gauge fields. *Phys. Rev. B*
- 450 **101**, 064303 (2020).
- 451 31. H. Chen *et al.*, Real-time observation of frequency Bloch oscillations with fibre loop
- 452 modulation. *Light Sci. Appl.* **10**, 48 (2021).
- 453 32. J. Yang *et al.*, Phonon-induced anomalous gauge potential for photonic isolation in frequency
- 454 space. *Optica* **8**, 1448-1457 (2021).
- 455 33. K. Wang, A. Dutt, C. C. Wojcik, S. Fan, Topological complex-energy braiding of non-
- 456 Hermitian bands. *Nature* **598**, 59-64 (2021).
- 457 34. A. Regensburger *et al.*, Parity-time synthetic photonic lattices. *Nature* **488**, 167-171 (2012).
- 458 35. M. Wimmer, H. M. Price, I. Carusotto, U. Peschel, Experimental measurement of the Berry
- 459 curvature from anomalous transport. *Nat. Phys.* **13**, 545-550 (2017).
- 460 36. M. Wimmer, U. Peschel, Observation of time reversed light propagation by an exchange of

- 461 eigenstates. *Sci. Rep.* **8**, 2125 (2018).
- 462 37. S. Weidemann *et al.*, Topological funneling of light. *Science* **368**, 311-314 (2020).
- 463 38. S. Weidemann, M. Kremer, S. Longhi, A. Szameit, Topological triple phase transition in non-
464 Hermitian Floquet quasicrystals. *Nature* **601**, 354-359 (2022).
- 465 39. S. Wang *et al.*, High-order dynamic localization and tunable temporal cloaking in ac-electric-
466 field driven synthetic lattices. *Nat. Commun.* **13**, 7653 (2022).
- 467 40. F. Cardano *et al.*, Detection of Zak phases and topological invariants in a chiral quantum walk
468 of twisted photons. *Nat. Commun.* **8**, 15516 (2017).
- 469 41. X. W. Luo *et al.*, Synthetic-lattice enabled all-optical devices based on orbital angular
470 momentum of light. *Nat. Commun.* **8**, 16097 (2017).
- 471 42. L. Yuan *et al.*, Photonic gauge potential in one cavity with synthetic frequency and orbital
472 angular momentum dimensions. *Phys. Rev. Lett.* **122**, 083903 (2019).
- 473 43. A. Szameit *et al.*, Fresnel's laws in discrete optical media. *New J. Phys.* **10**, 103020 (2008).
- 474 44. S. Longhi, Klein tunneling in binary photonic superlattices. *Phys. Rev. B* **81**, 075102 (2010).
- 475 45. E. Lustig *et al.*, "Towards photonic time-crystals: Observation of a femtosecond timeboundary in
476 the refractive index," in *2021 Conference on Lasers and Electro-Optics (CLEO)* (IEEE,
477 Piscataway, NJ, 2021), pp. 1-2
- 478 46. H. Moussa *et al.*, Observation of temporal reflections and broadband frequency translations at
479 photonic time-interfaces. arXiv [Preprint] (2022). <http://dx.doi.org/10.48550/arXiv.2208.07236>
480 (accessed 1 August 2022).
- 481

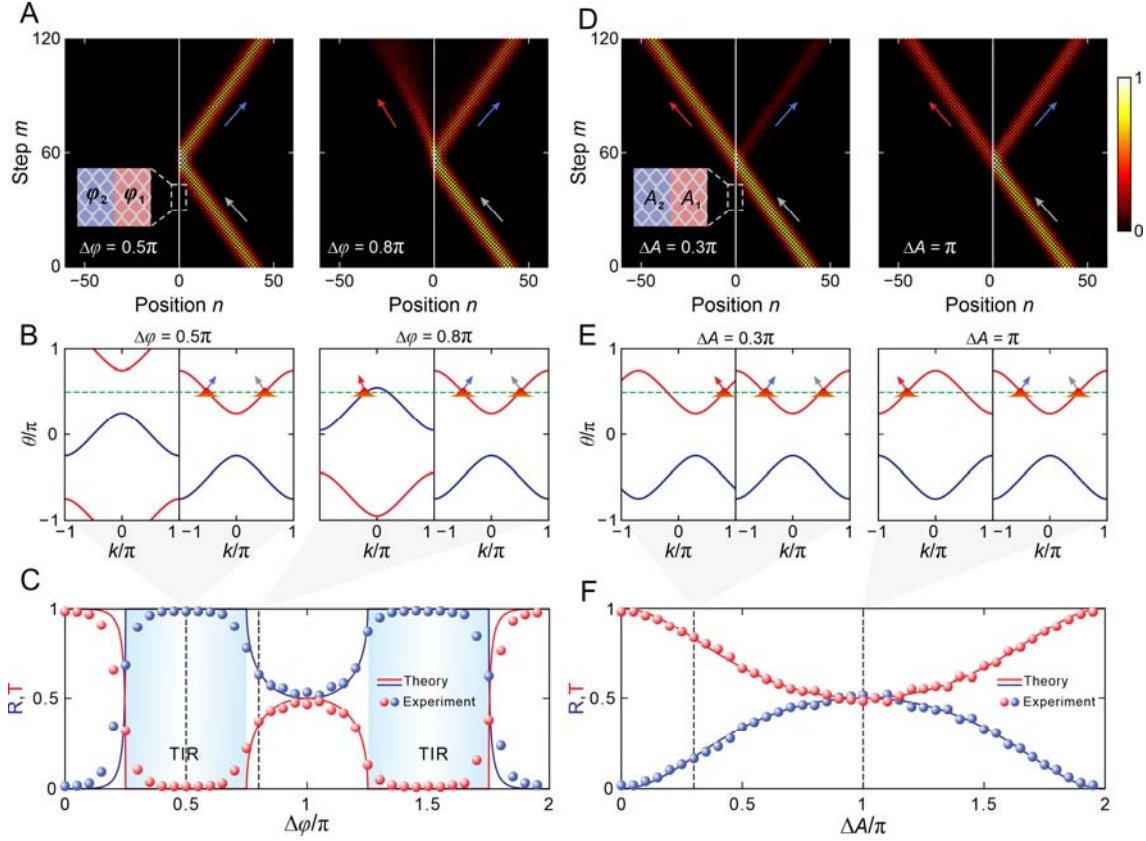
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Figures and Tables



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Fig. 1. Experimental implementation of effective scalar and vector potentials in synthetic temporal lattice. (A) Schematic sketch of the experimental setup used for implementing synthetic temporal lattice. Two fiber loops with slightly different lengths are connected by a variable optical coupler (VOC). Phase modulators (PMs) in long and short loops control the phase of pulses via ϕ_v and ϕ_u , respectively. The arrows represent the propagation direction of pulses. (B) Synthetic temporal lattice mapping from the pulse evolutions in a. Red (Blue) arrow indicates rightward (leftward) hopping in the lattice corresponding to a circulation in the long (short) loop. (C) Identical phase modulations $\phi_v = \phi_u$ in two loops bring about effective scalar potential φ in synthetical temporal lattice, by which the band structure is shifted vertically. (D) Effective vector potential A is mediated by opposite phase modulations $\phi_v = -\phi_u$ in the loops. The band structure undergoes a horizontal displacement in presence of effective vector potential.



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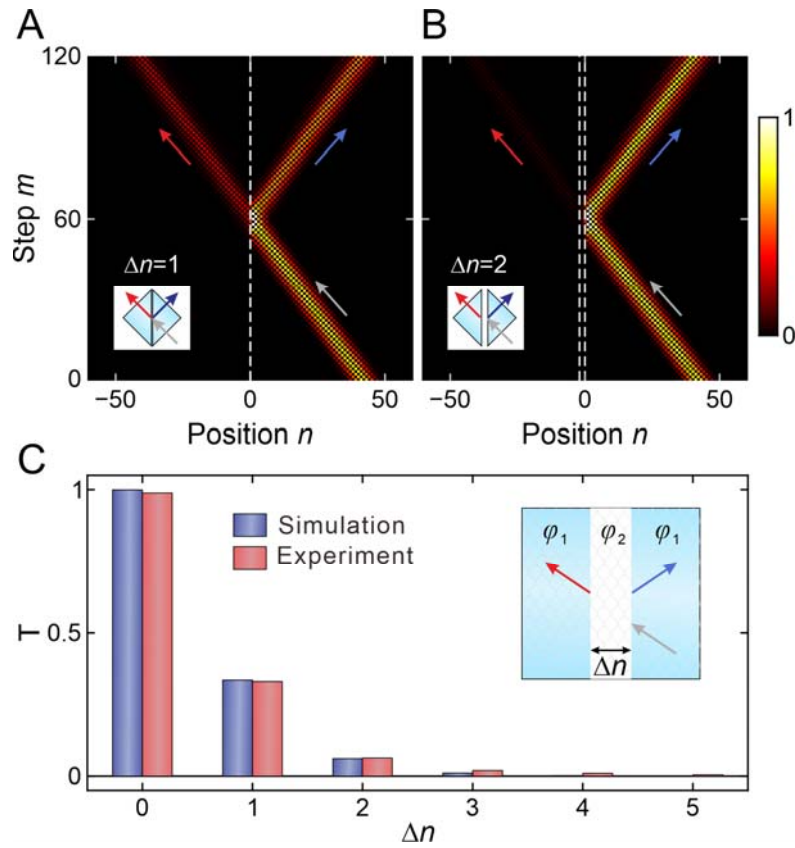
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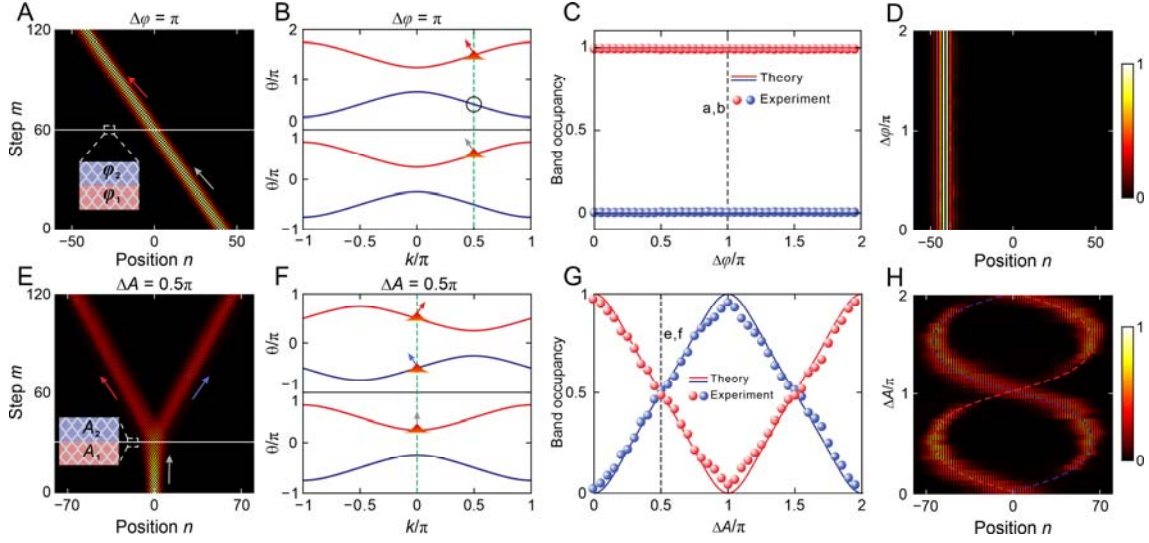
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Fig. 2. Refractions at the lattice-site interfaces. (A) Measured pulse intensity evolution as a wave packet impinges the lattice-site interface constructed by scalar potentials. The inset shows the distribution of scalar potentials. The potential difference is given by $\Delta\phi = \phi_2 - \phi_1$, which is fixed at 0.5π for left panel and 0.8π for right panel. (B) Band structures in two sides of the interface at $\Delta\phi = 0.5\pi$ and 0.8π . Green dashed line represents the propagation constant conservation. The gray, blue and red arrows indicate propagation directions of the incident, reflected and transmitted wave packet, respectively. (C) Theoretical (solid line) and measured (circle) transmission T (red) and reflection R (blue) coefficients as a function of $\Delta\phi$. (D) Measured pulse intensity evolution at lattice-site interface constructed by vector potentials. The potential difference in left and right panels are $\Delta A = 0.3\pi$ and π . The inset illustrates the distribution of vector potential. (E) The band structures at $\Delta A = 0.3\pi$ and π . (F) Theoretical (Solid line) and measured (circle) transmission T (red) and reflection R (blue) coefficients versus ΔA .



510

511 **Fig. 3.** Temporal FTIR. (A and B) The measured pulse intensity evolutions as $\Delta n = 1$ (A) and 2 (B). The
 512 insets show the schematic diagrams of the FTIR. (C) Measured (red bars) and simulated (blue bar)
 513 transmission coefficient as a function of the gap width Δn . The inset shows the distribution of scalar
 514 potentials.

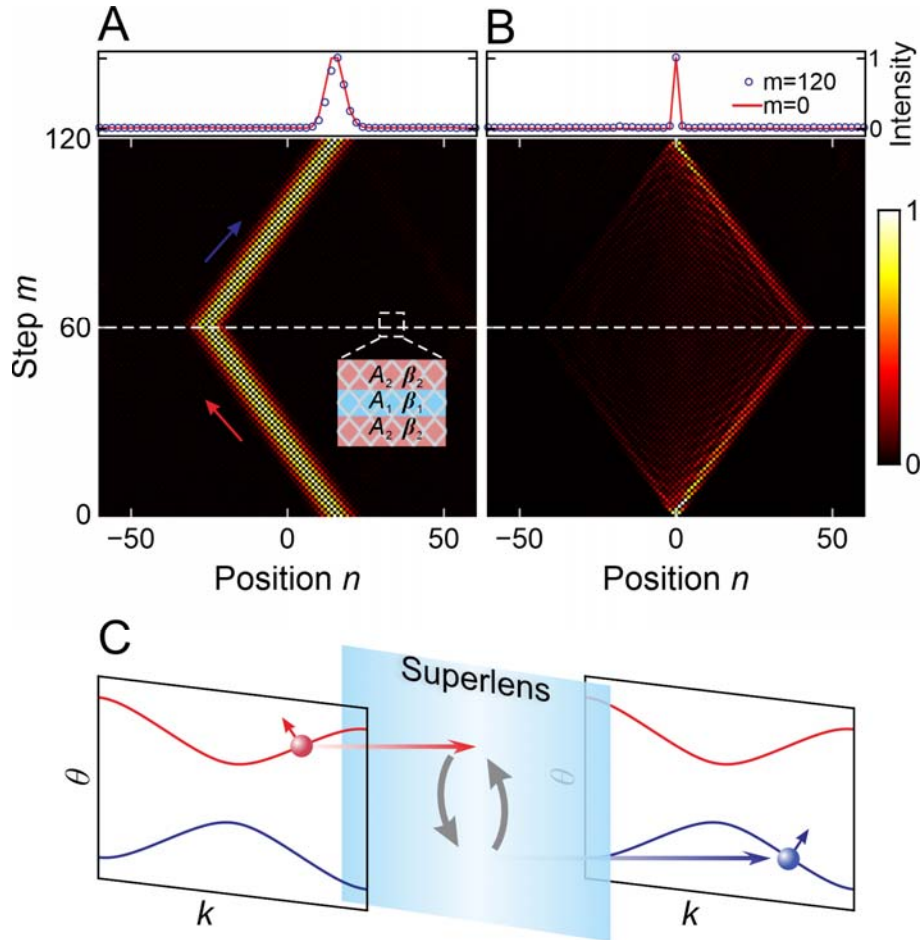


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Fig. 4. Refractions at evolution-step interfaces. (A) Measured pulse intensity evolution as $\Delta\phi = 0.5\pi$. The
 517 inset shows the distribution of scalar potentials at the evolution-step interface. (B) The band structures of
 518 the two sides at $\Delta\phi = 0.5\pi$. The Bloch momentum conservation is indicated by green dashed line. (C)
 519 Measured (circle) and theoretical (solid line) upper band (red) and lower band (blue) occupancies versus $\Delta\phi$
 520 in the top region. (D) Measured output field intensity at $m = 120$ as a function of $\Delta\phi$. (E) Measured pulse
 521 intensity evolution for $\Delta A = 0.5\pi$. (F) The band structures in the top and bottom regions at $\Delta A = 0.5\pi$. (G)
 522 Measured (circle) and theoretical (solid line) upper band (red) and lower band (blue) occupancies versus
 523 ΔA as the wave packet crosses the interface. (H) Measured output field intensity distribution at $m = 120$ as a
 524 function of ΔA .

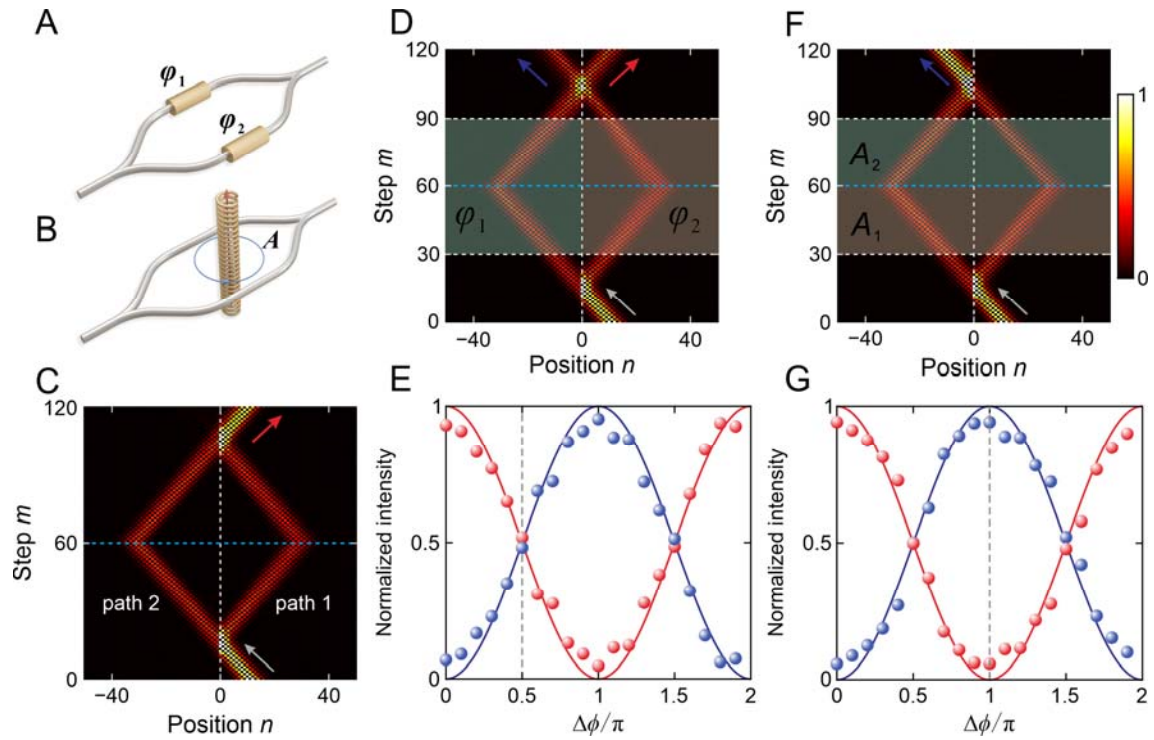
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527 **Fig. 5.** Time reversal induced by band occupancies inversion. (A) Measured pulse intensity evolution for a
 528 wave packet incidence. The inset figure shows the distribution of vector potentials and coupling
 529 parameters. Measured intensity profiles at time step $m = 0$ and $m = 120$ are displayed on the upper panel.
 530 (B) Measured pulse intensity evolution for a single-site excitation. (C) The abruptly change of vector
 531 potential and coupling parameter at time step $m = 60$ can be viewed as a superlens for exchange of
 532 eigenstates.

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534

535 **Fig. 6.** Electric and magnetic Aharonov–Bohm (AB) effects. (*A* and *B*) Schematic diagram of the electric

536 (*A*) and magnetic (*B*) AB effects. (*C*) Temporal Mach-Zehnder interferometer constructed by combined

537 lattice-evolution-step interfaces. The gray and cyan dashed lines represent vector potentials interface and

538 superlens, respectively. The gray and red arrows indicate the incident and output wave packets. (*D*)

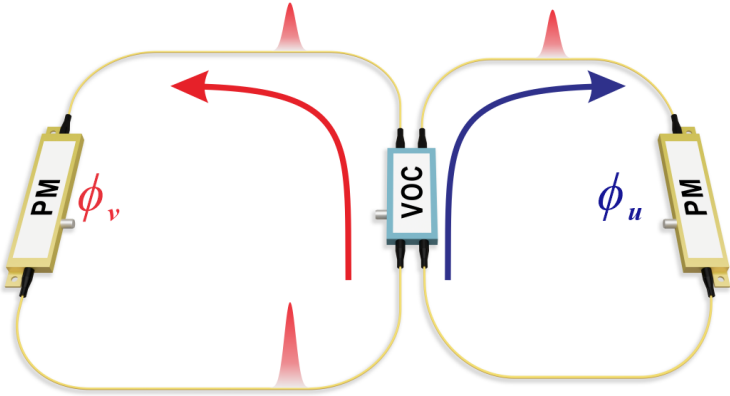
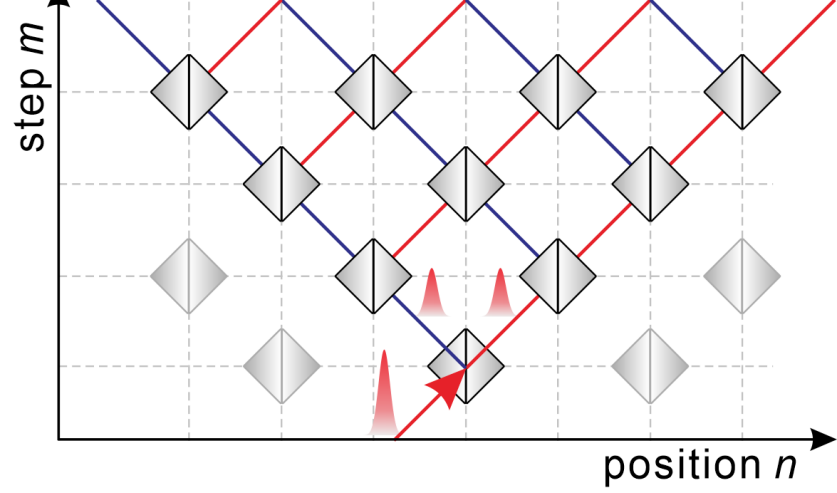
539 Measured pulse intensity evolution for electric AB effect as accumulated phase difference between two

540 arms is $\Delta\phi = \pi$. (*E*) Measured (circles) and theoretical (solid curves) normalized intensity of outputs on the

541 left (blue) and right (right) side of the lattice-site interface as a function of $\Delta\phi_\varphi$. (*F* and *G*), Same as (*D* and

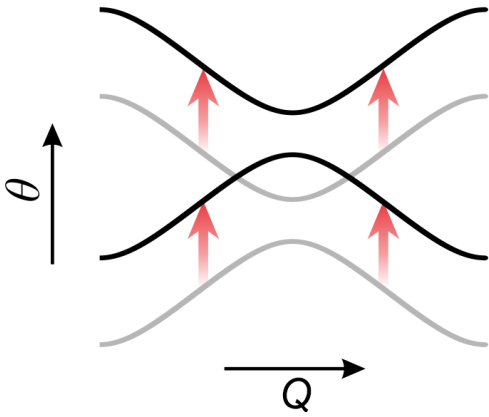
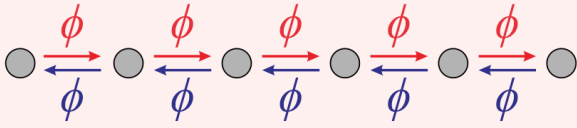
542 *E*) but for magnetic AB effect.

543

A**B****C**

Scalar Potential

$$\phi_v = \phi_u = \phi \longrightarrow \varphi = \phi$$

**D**

Vector Potential

$$\phi_v = -\phi_u = \phi \longrightarrow A = \phi$$

