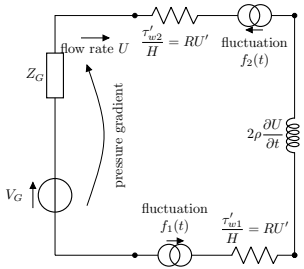
Turbulent channel flow DNS: Looking beyond the periodic box



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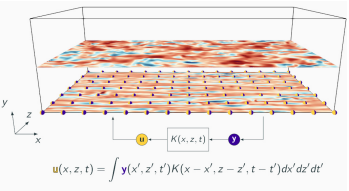
RV60, GSSI, Oct 16 2026

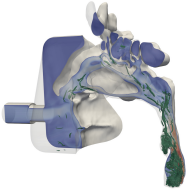
What is a suitable topic for RV60?

Turbulence, flow control?



- DNS, statistics
- Active/Passive Control

The nose flow?



- Geometrical complexity
- Clinical implications

IBM DNS/LES solver?

- www.amphibious.cfd
- Active development

Back to the "good old times": channel flow DNS

- DNS of doubly periodic channels is now established
- KMM87 set the standard
- Is there anything new left?

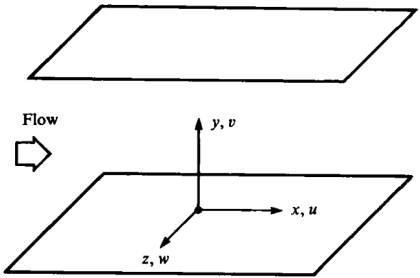


FIGURE 1. Coordinate system in channel.

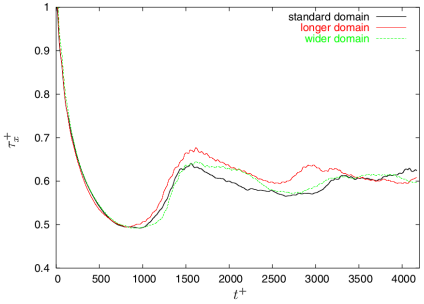
Understanding the forcing term

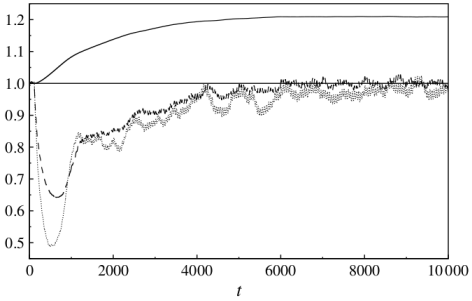
NS equations alone cannot push fluid through a periodic duct: a forcing term is needed

- Several options: CPG, CFR, (CPI)
- CFR/CPG differences known in practice
- Minor/no effect on flow statistics

Transients in drag-reduced flows

Differences between CFR and CPG





Left: Quadrio & Ricco JoT 2003 (CFR). Right: Ricco et al JFM 2012 (CPG)

Can we understand/predict these transients?

Can we obtain information about the transients of a system by just looking at the frequency spectrum of its natural fluctuations?

- Yes, for a linear system driven by white noise
- Yes, in nonequilibrium thermodynamics (fluctuation-dissipation theorem)

Do we have scale separation here?

- The geometric scale h is separated from the infinite length
- Phenomena at scales $> h$ and times $> h/u_\tau$ are separated
- Turbulence is white noise (just as in statistical physics) for the slowest time scales

An equation for the spatial mean flow rate

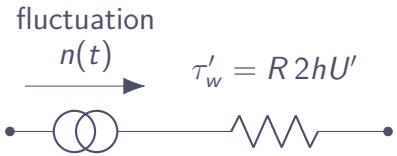
$$\rho \frac{\partial U}{\partial t} + F_x + \frac{\tau_{w2} - \tau_{w1}}{2h} = 0$$

A general forcing can be written as a linear combination of pressure gradient and bulk velocity:

$$F_x = \frac{\partial p}{\partial x} - Z_G U$$

Z_G is a (possibly complex) generator impedance
CPG: $Z_G = 0$; CFR: $Z_G = \infty$; CPI: $Z_G = 2\tau_w / U$

Wall-shear as a noisy resistor



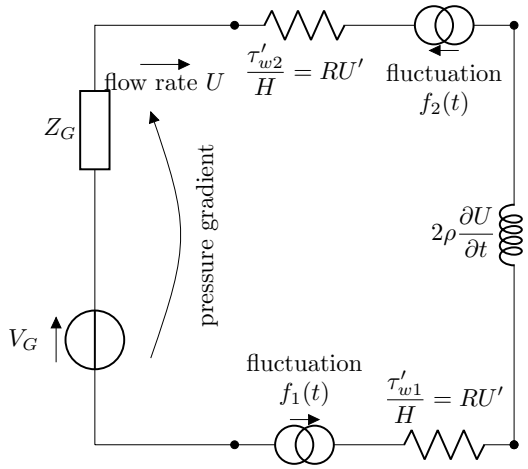
- White-noise assumption follows from scale separation:

$$\tau_w = C_f \rho \frac{U^2}{2} + n(t)$$

- "Macroscopic" (slow) fluctuations can be linearized via the friction law:

$$\tau'_w = R2hU'$$

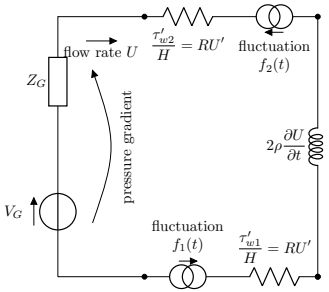
The Turbulent Channel Flow



A first-order LR low-pass filter with noisy resistors

Fluid mechanics

- Pressure gradient
- Flow rate U
- Hydraulic resistance
- Fluid inertia
- Wall-shear fluctuations
- Generator impedance Z_G



Circuit theory

- Driving voltage
- Electric current
- Electrical resistance R
- Inductance L
- Fluctuating voltage sources
- External forcing impedance

All components are independent of the computational-box size!

Orders of Magnitude

For a classical first-order low-pass filter:

- Typical time constant of the τ_w fluctuation: $\sim h/u_\tau$
- Differential resistance estimated as the derivative of Prandtl's law:

$$R = \frac{1}{2h}\frac{d\tau_w}{dU} = \frac{2\overline{\tau}_w}{2h\overline{U}}\left(1+\kappa^{-1}\sqrt{C_f/2}\right)^{-1} \sim \frac{\overline{\tau}_w}{h\overline{U}}$$

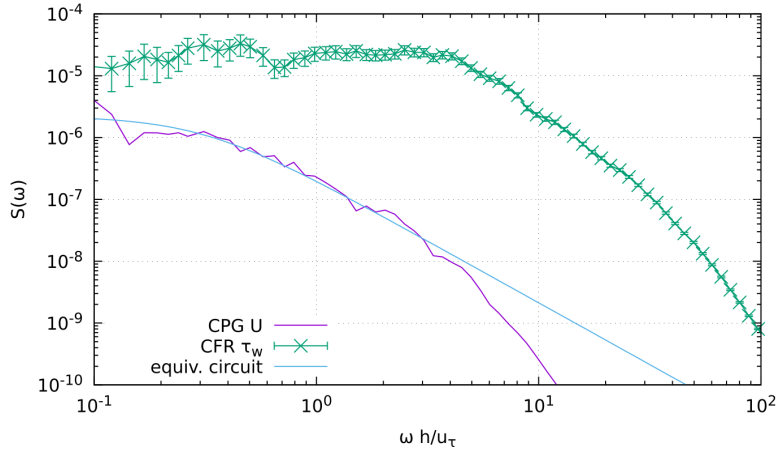
- Time constant of the filter:

$$\frac{L}{2R} = \frac{\rho U h}{2\tau_w} = \frac{1}{2}\frac{U}{u_\tau}\frac{h}{u_\tau}$$

Scale separation! Characteristic time for U is U/u_τ times longer than the characteristic time of τ_w

Numerical test (with a 2016 dataset)

Channel flow at $Re_\tau = 200$; $\Delta t^+ = 0.2$, $T^+ = 150,000$.



(Equivalently, transfer function between τ_w' at CFR and U' at CPG)

What about the periodic boundary conditions?

- In circuit theory, one can consider an infinite lattice of identical impedances
- Its generator impedance contains both a resistive term and a frequency-dependent inertial term. In time domain:

$$Z_G = \frac{2\tau_w}{U} + \rho h \frac{d}{dt}$$

- For the pipe, a 1-d lattice gives $Z_G = \infty$, i.e. CFR!

Conclusions

- The time constant is well predicted by this simple circuit
- The white-noise assumption works well: the scale separation is (unexpectedly) quite large
- Useful info to design simulations (averaging time)
- An infinite-lattice forcing condition goes beyond the periodic box

Spectra and box size

- The ensemble mean of N independent samples has the same statistical mean but $N^{1/2}$ times the fluctuation as an individual sample. However,
- the spectrum of fluctuations remains proportional to itself, and its characteristic frequency and time scale are the same.
- The spatial mean over an interval (or a box in multiple dimensions) of size L has the same statistical mean but $L^{1/2}$ times the fluctuation as an instantaneous and localized value. However,
- the spectrum of fluctuations remains proportional to itself, and its characteristic frequency and time scale are the same.