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Research paper

Reconstruction of in-orbit breakup events over the short term

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ABSTRACT

The massive growth of space activities over the past decades has lead to an increase of the space debris population, which is threatening the safety of space operations and active satellites. Several fragmentation events caused by catastrophic collisions and explosions have single-handedly contributed to the increase in the number of space debris, highlighting the importance of the reconstruction of such events to gain insight on the breakups and decrease the collision risk they cause to active satellites.

This work proposes a fragmentation reconstruction approach for this purpose, which combines pruning and clustering criteria with a backward propagation to identify the epoch of the fragmentation and the involved objects. The model was tested by using publicly available Two-Line-Elements data. The problem of the unreliability of such data is addressed by modifying the original approach to account for uncertainty, multiplying the original Two-Line-Elements data and choosing the optimal set of objects for the reconstruction. The optimal values of the pruning filters thresholds for the method are also investigated to identify the most effective criteria. Moreover, the reciprocal influence of the filters is assessed through a sensitivity analysis. The proposed approach is applied to two real fragmentation events to compare the accuracy of the reconstruction with and without the uncertainty quantification. The method proves to be effective in the evaluation of the epoch of the breakup as well as the identification of the fragments and their parent.

1. Introduction

The exploitation of space as a resource has progressively expanded over the years, delivering an increasing array of essential services for diverse applications, including daily activities, scientific research, and military operations. Consequently, space traffic is growing, particularly in Low Earth Orbit (LEO), driven by the miniaturisation of space systems and the emergence of large constellations [1]. Such a use of space is coupled with an increase in the number of space debris, which pose a threat to the near-Earth environment because of involuntary collisions that can be potentially generated. Indeed, as estimated by NASA [2], debris from satellite breakups now account for 47% of the total catalogued population of Earth-orbiting objects. The main causes for in-orbit breakups are propulsion-related events and deliberate fragmentations [2]. Explosions are mostly related to residual fuel remaining in tanks or fuel lines and failures of attitude control systems [2], while deliberate fragmentations are associated to interceptions by surface-launched missiles. Among the most catastrophic breakups, which shaped the space debris environment, the Fengyun-1C event occurred in January 2007 belongs to the last category. The breakup was the result of the first successful Chinese anti-satellite weapon test, aimed at destroying the 880 kg weather spacecraft Fengyun 1C, and it is considered the most environmentally impactful event to date [2]. It was estimated that this event alone accounted for an increase in the trackable space objects of 20% [3]. The breakup took place at 863 km of altitude, leading to severe consequences between 700 and 1000 km of altitude, where the debris density was already critical. Anselmo and Pardini [3] analysed the fragmentation event and assessed that up to 80% of the fragments would still be in orbit 9 years after the ASAT test. The third category of causes for in-orbit breakups is accidental collisions. The first ever catastrophic collision between two intact satellites occurred on 10th February 2009, between the Russian Cosmos 2251 and the American Iridium 33, at an estimated velocity of 11.6 km/s [4] and at an altitude of approximately 789 km. To this day, this breakup is responsible for the peak of debris density between 770 and 800 km [2]. Despite extensive debris tracking, which is limited by distance and size to objects from 5–10 cm in LEO and 1 m in Geostationary orbit [5], collisions are difficult to predict in advance. For this reason, a swift reconstruction of the breakup after the event can be essential for operations such as risk assessments, collision avoidance manoeuvres and sensor pointing. Further, an accurate depiction of the initial fragment cloud distribution will drive long-term population evolution and orbital capacity models.

Several approaches were adopted to reverse-engineer fragmentations in terms of epoch of the event and involved objects. Andrisan

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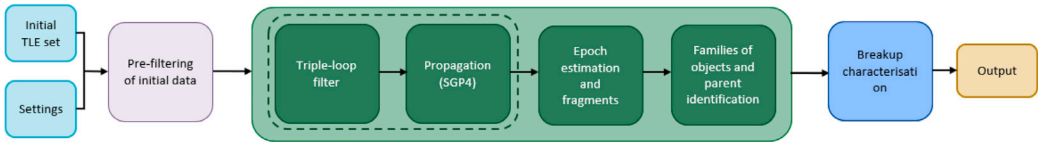


Fig. 1. Block diagram of the PUZZLE model.

et al. [6] and Tetreault et al. [7] modelled the epoch of the event and its location by computing the centre of the mass of the debris cloud and the distance of the single objects from it. In Tetreault et al. [7] the effect of a variable computation of the ballistic coefficient of fragments was assessed, indicating how the estimation of this parameter is crucial for accuracy of backward propagation. For fragmentation reconstruction, grouping objects together and evaluating distances is necessary to assess whether the objects originated from a given event. For this purpose, Dimare et al. [8] analysed orbital similarity functions, previously used to find the common origin of asteroids, applying them to study clouds of near-Earth objects and deeming the distance criterion by Southworth and Hawkins [9] and the modification proposed by Jopek [10] as the most suitable to identify similarities between objects. In Montaruli et al. [11] the Minimum Orbital Intersection Distance (MOID) was used to cluster objects in time and the Earth Mover's Distance [12] exploited to compute the statistical distance for each cluster. The tool provides a stochastic approach for fragmentation reconstruction in terms of fragmentation epoch candidates. It also points out a possible flaw of the MOID as a metric, as it is found to be very sensitive to the similarity in orbital planes between the orbits of the observed fragment and the parent. The orbital plane as a parameter for fragmentation reconstruction was also evaluated in Itaya et al. [13], due to the fact that it presents very small errors also in long-term propagation. By combining close approach analysis and the constraint equation derived from the analysis of the orbital planes, a method to estimate the fragmentation epoch was found. The close approach analysis was also exploited in Romano et al. [14,15], who developed the PUZZLE method to detect possible fragmentations and characterise them in terms of objects involved, fragmentation epoch and mass and energy. The conjunction analysis served the purpose of a catalogue screening to assess the possible objects originated in the event, followed by clustering criteria to identify families of fragments and a matchmaking routine for parent identification. This work, which was valid for short-term reconstructions, was later extended by Muciaccia et al. [16,17] for long-term backward propagation, by developing additional pruning filters for catalogued and uncatalogued objects. The drawback of these approaches is the lack of uncertainty quantification, since, as pointed out by Flohrer et al. [18], Two-Line Element (TLE) data are intrinsically uncertain. Detomaso et al. [19] accounted for this aspect by introducing in PUZZLE a preliminary uncertainty evaluation approach, which resulted effective but compromised the computational time of the method.

Within this context, this work aims at extending the PUZZLE model previously developed at Politecnico di Milano for the reconstruction of fragmentations in the short-term, by optimising the uncertainty quantification model and the thresholds required for the detection of the fragmentation.

In this paper, Section 2 introduces the PUZZLE approach originally devised in Romano et al. [14], going into detail for each step of the fragmentation reconstruction routine. Section 3 introduces uncertainty quantification into PUZZLE, explaining the two additional steps required, i.e. the introduction of synthetic TLEs and the choice of the optimal TLE set. Section 4 presents the sensitivity analysis carried out on the pruning thresholds, to optimise the outcome of the reconstruction. Section 5 applies the PUZZLE method to real fragmentation cases, both using the original method without uncertainties and the one with uncertainty quantification. Lastly, Section 6 draws the conclusions of the work and outlines possible future developments.

2. The PUZZLE model

This section introduces the PUZZLE model for the reconstruction of in-orbit fragmentations in the short-term (up to two weeks), which was developed at Politecnico di Milano by Romano et al. [14,15] initially under an Italian Space Agency funded project. The method has two main objectives: the detection of a fragmentation from catalogued TLE data and the reconstruction of the event, when it is detected. It does not require *a priori* knowledge on the fragmentation and it is able to find possible clusters of objects generated in a breakup backward in time. The model was developed to be agnostic to the kind of input data which is provided, however the availability and accessibility of TLEs enabled their use for the validation of the model. The approach is based on a series of pruning criteria to detect outliers in the input data and to analyse the orbital characteristics of the objects, aiming at finding possible close encounters which could indicate a breakup event. In the positive instance, clustering algorithms are employed to detect families of objects and match them to their parent(s). The National Aeronautics and Space Administration (NASA) standard Breakup model [20] is used as the last step to simulate the fragmentation in a forward fashion from the information gained with the reconstruction, in order to obtain the mass and energy involved in it. The structure of PUZZLE can be seen in Fig. 1. The architecture of the PUZZLE method will be described hereafter.

2.1. TLE reading and prefiltering

PUZZLE takes as input TLE data, which is downloaded from a publicly available and regularly updated catalogue (mainly Space-Track [21]). The first module of the approach reads the data and pre-processes them, to detect possible outliers in the set which would compromise the accuracy of the analysis. Indeed, the quality of TLEs can vary from object to object but also for the same object. To mitigate the issue, the TLEs belonging to the same object, i.e. having the same NORAD ID, are selected and further analysed with the prefiltering method first devised by Lidtke et al. [22] for re-entry predictions of GTO objects. This consists in a series of subsequent filters with the following objectives:

- detecting and removing TLEs which result from a correction of the previous element;
- defining large time gaps between TLEs to search for outliers in these windows;
- removing the TLEs whose mean motion evolution within the analysed temporal window is not coherent;
- removing the TLEs whose eccentricity and inclination are considered outliers with respect to given thresholds;
- removing TLEs whose B* coefficient is negative.

The PUZZLE method allows the possibility of deciding which filtering steps to perform, to increase the flexibility of the method based on the knowledge of the input TLEs. Moreover, only one TLE per object is retained to aid the following identification of the objects involved in a fragmentation event and all TLEs are propagated to the same initial time using the Standard General Perturbations 4 (SGP4) propagator [23].

2.2. Pruning and backward propagation

In the next step of the fragmentation search, the orbital characteristics of the objects are analysed to discern the ones for which a close encounter was possible in a given temporal window. The adopted method couples a triple-loop, which was originally developed in Hoots et al. [24] for conjunction analysis, with a backward propagation carried out with SGP4.

The purpose of the triple-loop is to prune the objects whose orbital geometry is not compatible with a close approach. It consists of three subsequent filtering steps which are applied to each couple of objects under analysis. The first two filtering steps are geometrical, whereas the last one is temporal. The first filter compares the two orbits of the objects by checking the heights of the apogees and the perigees. Defining:

$$q = \max(r_{P,1}, r_{p,2}) \tag{1}$$

$$Q = \min(r_{A,1}, r_{A,2}) \tag{2}$$

and given a threshold Δ, the two objects pass the filter if they satisfy:

$$q - Q \leq \Delta. \tag{3}$$

The second filtering step computes the MOID between the two objects which have passed the previous filter to assess whether it is below a given threshold. In PUZZLE, the MOID is computed following the algebraic formulation of Gronchi [25], which speeds up the computation. This filtering step is crucial because the MOID corresponds to the minimum possible geometric distance between two orbits, hence, as in the previous filter, if it is above a given threshold the close encounter is deemed impossible and the secondary object is discarded from further consideration. This is the most stringent filtering step, which prunes the majority of the objects. The last step is a time filter, accounting for the fact that even if two orbits are geometrically compatible for a close encounter, the two objects must pass within the critical orbital windows simultaneously for a close approach to occur. Two angular windows around the positions of the MOID on the two orbits are computed, as shown in Fig. 2. This step is necessary because a close encounter does not necessarily occur at the MOID, hence an angular window is considered. The angular windows can then be converted into time windows by means of Kepler's equation [26], obtaining the temporal windows in which the two objects pass through the possible close encounter region in a single revolution. By adding multiples of the orbital period of the objects to the time windows, a sequence of intervals of interest are generated. They are checked for possible overlaps in a given temporal window, which indicate a candidate close approach.

If the pair of analysed objects pass all filters, PUZZLE establishes that a close approach was possible, otherwise their TLEs are discarded. In the positive instance, the actual miss distance and time of close approach for the pair of elements are computed, similarly to the method in Hoots et al. [24]. The overlapping time windows indicate a good candidate time for the close encounter considering the midpoint of the overlap, which is set as the starting point of an iterative procedure based on a Newton's method to find the time when the minimum distance between the two orbits is reached. Indeed, considering $r_1(t)$ and $r_2(t)$ as the position vectors of the two objects at time t, $v_1(t)$ and $v_2(t)$ as the velocity vectors and $a_1(t)$ and $a_2(t)$ as the accelerations, the square of the relative distance between the two objects – dropping the dependence with respect to time – is given by:

$$d^2 = (r_1 \cdot r_1) + (r_2 \cdot r_2) - 2(r_1 \cdot r_2) \tag{4}$$

The minimum of the distance is sought, looking for zeros of its derivative:

$$\frac{\mathrm{d}d^2}{\mathrm{d}t} = (r_1 \cdot v_1) + (r_2 \cdot v_2) - (v_1 \cdot r_2) - (r_1 \cdot v_2). \tag{5}$$

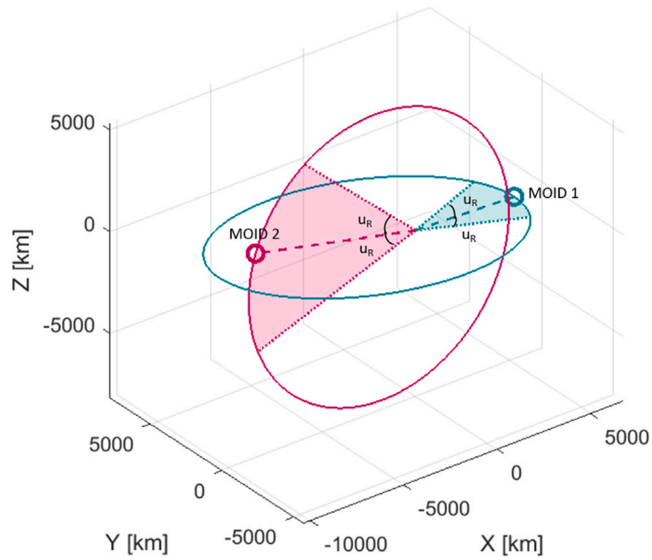


Fig. 2. Representation of third triple-loop filter defining angular windows around the MOIDs on the two orbits.

To account for the effect of orbital perturbations on the Keplerian elements of the objects, the previous procedure is coupled with a backward propagation carried out with the analytical SGP4 model [23], as the TLE averaged orbital data is specific to this model. It considers secular and periodic variations due to Earth oblateness, solar and lunar gravitational effects, gravitational resonance effects, and orbital decay using a drag model. The propagation allows to find clusters of objects backwards in time, identifying the possible convergence of their orbital elements to indicate a close approach. As the SGP4 propagator analytically models perturbations, the accuracy of the propagation is limited to a maximum of two weeks, thus PUZZLE short-term is used for swift reconstructions of fragmentations after at most 14 days from their occurrence. After the triple-loop identifies the overlapping time windows and the time and distance of close approach are computed, if there are multiple overlapping windows, the two objects are propagated backwards to the first time of close approach, to recompute the possible close encounter windows and the miss distance. The procedure is carried out repeatedly until the end of the indicated search window, and the overall minimum distance of close approach is selected as the actual miss distance.

2.3. Epoch identification and clustering

In this module, the PUZZLE model is able to estimate the epoch of the event and identify the families of objects involved in it with the corresponding parent(s).

The breakup epoch is computed by exploiting the results of the close encounter search. The time interval for the fragmentation reconstruction – which is an input for the approach – is divided into bins. The previously computed times of close approach are subdivided into the same bins, allowing to count the number of close approaches into the temporal bins and thus selecting the real epoch of fragmentation as the bin with the highest concentration.

The method discards the objects whose close encounter occurred outside of the event epoch window, leading to a subset of elements belonging to the fragmentation. These are clustered into families of objects according to their orbital elements, looking for a similarity to assess a common origin. The Hierarchical Clustering Method (HCM) by Zappalà et al. [27] (already used for asteroid families identification) is the proposed approach for this purpose. The HCM assigns a distance function δv (with the dimension of a velocity) between any pair of objects to assess their similarity in the orbital elements space, which is the proper elements space for asteroid families and the Keplerian elements space in the PUZZLE approach. In the original HCM method, δv is defined in terms of differences of the proper semi-major axis a, eccentricity e and inclination i, whereas in PUZZLE the definition is modified to take into account osculating orbital elements coming from the TLE data with the addition of the RAAN Ω and the argument of pericentre ω. The addition of the latter orbital elements is needed for Earth-orbiting objects which have a wider range of these values when compared with asteroids [28]. The final δv between each pair of objects is defined as follows:

$$\delta v = na\left[k_1\left(\frac{\delta a}{a}\right) + k_2(\delta e)^2 + k_3(\delta i)^2 + k_4(\delta\Omega)^2 + k_5(\delta\omega)^2\right]^{1/2} \tag{6}$$

where n is the mean motion and the k_j coefficients are of the order of unity, depending on each parameter to which they are associated. Similarly to the metric proposed by Zappalà et al. [27], the δv is related to the velocity increment involved in the separation of the orbits of the objects in the breakup event. Once the similarity function is computed for a pair of objects, four steps follow: (1) the two closest objects, j and k, are identified; (2) they are grouped together in a single family, therefore they are treated as a single object $(j+k)$; (3) update the rest of the distances considering $\delta v(j+k, i)$ the distance between $j+k$ and any other object i defined as the minimum between $\delta v(j, i)$ and $\delta v(k, i)$; (4) if at least two objects do not belong to the family, return to the first step. The process is repeated until all the objects are assigned to a family. In this way, all objects are connected according to their original families.

To verify the presence of a fragmentation, the physical distance between the objects belonging to a family is computed for each family at the estimated fragmentation time. The minimum distance within a family is compared against a threshold to assess whether a fragmentation was likely. Indeed, a low distance between objects at the possible fragmentation time is a necessary condition for the identification of a breakup event. The families of objects are then matched with the corresponding parent by computing the distance between the objects in the cluster and all the objects in the input catalogue. The catalogued objects closest to the family of fragments are selected as possible parent candidates, taking the final parent as the object with the lowest NORAD. However, the PUZZLE method also allows skipping the parent-matching process when the parent is known *a priori* and provided as input.

2.4. Fragmentation modelling

The last phase of the PUZZLE approach aims at modelling the fragmentation event forward, as opposed to the rest of the method, to characterise the distribution of the generated fragments and estimate their number starting from the masses involved and the position of the retrieved parent(s). For this reason, the NASA Standard Breakup Model [20] is used. In this way, the number and distribution of physical attributes of the fragments is computed according to the type of fragmentation (collision or explosion). In the applications of this work this module was not included, however it is part of the original PUZZLE formulation [28].

3. Uncertainty quantification in PUZZLE

The PUZZLE model described thus far is effective for the reconstruction of fragmentation events, however it lacks the quantification of uncertainty which is intrinsic to TLE data and amplified in the propagation phase. A preliminary introduction of the concept of uncertainty in the PUZZLE approach was developed in Detomaso et al. [19], where three add-on blocks where included in the algorithm. The drawback of the method consists in the redundancy of some steps, which burden the computational effort of the updated method. For this reason, an alternative version of the uncertainty quantification algorithm is proposed, introducing two steps into PUZZLE: the introduction of uncertainties and the optimal TLE selection.

3.1. Introduction of uncertainties

The main concern when dealing with uncertain TLE data for fragmentation reconstruction is an erroneous estimation of the epoch of the event, as the close encounters epoch estimation may be wrong. To overcome the issue, the first additional step in the algorithm introduces synthetic TLEs in addition to the ones in the input catalogue. Following the approach in [19], the synthetic TLEs are generated starting from the input ones and they are added to the analysis after the pre-filtering module and before the triple-loop and backward propagation. This ensures that any outlier in the original set was already discarded and that the pruning phase of the method is able to preserve in the analysis the TLEs of the objects actually involved in the fragmentation event, with more stringent thresholds with respect to the original version of PUZZLE. As a result, more TLEs of objects unrelated to the event are discarded and the fragmentation epoch estimation is more accurate due to the concentration of close encounters in the correct temporal bin, as the TLEs were multiplied. This process is particularly beneficial for the fragmentation cases for which the TLEs are very uncertain or when only few TLEs are available.

The generation of new TLEs starting from the input ones is performed by selecting each object and producing a random normal distribution either on all components of its Cartesian state or on each of its Keplerian elements with a given level of uncertainty α. The distribution is centred in the nominal state of the object and the standard deviation is given by:

$$\sigma_j = \alpha p_j \tag{7}$$

where p_j is the nominal value of each orbital parameter considered. The synthetic TLEs are generated by modifying the original ones in terms of Keplerian parameters, while keeping the rest of the properties such as the epoch of the TLE. In this way, the uncertainties related to the state of the object in the TLE are averaged out by the multiplication of the TLEs themselves, modifying the state within a range with respect to the original one. To this aim, a Gaussian Mixture Model (GMM) is used to fit the distributions. The GMM is particularly suited for this problem because it allows to properly catch the effects of nonlinearity while maintaining a relatively small computational effort. This approach approximates a given Probability Distance Function (PDF) - in this instance, the PDFs of the distributions in orbital elements — with a finite sum of weighted Gaussian density functions [29]. Each of the components is characterised by a mean and a covariance matrix; thus the mean of each Gaussian kernel, in terms of orbital elements, can be used to modify the state of the original TLE, generating the synthetic one. For each input object, a new TLE is generated for each kernel and a further TLE is generated by using the mean of all the kernels. Ultimately, the number of TLEs in the analysis is equal to $(N+2)k$, where N is the number of kernels and k is the initial number of TLEs. Artificial TLEs are assigned fictitious NORAD IDs starting from 100000, in order to recognise them later on in the process. The GMM becomes more accurate the higher the number of kernels [29], i.e. Gaussian components. However, no more than 10 are considered feasible due to the computational effort they imply.

3.2. Optimal TLEs selection

After the fragmentation is assessed and the time of the event is estimated, the PUZZLE approach identifies the objects involved in the breakup, matching the parent(s) to the fragments. To achieve this, it is ideal to have one TLE per each object, avoiding any possible mistakes due to the repetition of the objects. Conversely, at this point in the reconstruction method, because of the addition of synthetic TLEs in the analysis, a single object could be associated to multiple TLEs that have passed the pruning and clustering steps. To overcome the issue, a selection of the optimal TLEs to retain in the analysis is carried out for each object by solving an optimisation problem. Following the approach in [19], three main factors are accounted for in the optimisation:

- the estimation of the event epoch can be carried out both with the full set of TLEs including the repeated objects – which makes the estimation more precise – and with the restricted set with one TLE per object. The two computed epochs must be as close as possible to give an accurate estimate;
- for each pair of objects which survived the analysis, both the miss distance and the time of close approach are available. It is expected that the objects whose close encounter occurred closest to the estimated fragmentation epoch also have the minimum miss distance;
- it is of interest to the analysis that the highest number of fragments belonging to the fragmentation is recognised by the method. Therefore, for the optimisation problem, the selected TLEs must maximise the retention of objects whose close approach times fall within a specified range from the computed fragmentation epoch.

With these requirements, the optimisation problem is formulated as follows:

$$\min_{x_{TLE}} \quad |t_{full} - t_{restricted}| + \overline{d}_{CA} + \frac{1}{n_{obj}} \tag{8}$$

with t_{full} the fragmentation epoch estimated with the full set of TLEs, $t_{restricted}$ the fragmentation epoch estimated with the restricted set of TLEs without repetitions, $\overline{d}_{CA}$ the mean close approach distance for those objects whose close encounter occurs within an hour with respect to the fragmentation epoch, and n_{obj} is the number of objects retained in the analysis whose close approach is at an epoch comparable to the fragmentation time. The optimisation variable x_{TLE} is the optimal combination of one TLE per object. The optimisation problem is solved through a Genetic Algorithm (GA), considering integer variables and the appropriate bounds. The GA allows to generate at each iteration a new population, which in this instance is a new combination of TLEs considering one per object, randomly selecting individuals from the current population to use as parents for the next iteration.

Once the optimal set of TLEs of objects involved in the fragmentation is selected, PUZZLE can match the parents to the fragments. However, the optimal TLE set is composed of TLEs coming from the initial input set and synthetic TLEs. This may cause an issue for the matchmaking routine as the synthetic TLEs are generated through an averaging process of the GMM which could lead to errors in the state. In [19], the problem was tackled by adding a further optimisation problem to find the parent(s), however including an additional optimisation problem to solve in the fragmentation reconstruction method results in a cumbersome procedure. For this reason, the method proposed here checks if the optimal TLE set is composed by any synthetic TLE and, in the positive instance, it is able go back to the original TLE from which the synthetic one stemmed. This guarantees that the objects most likely involved in the fragmentation event are retained in the analysis while overcoming the issue of the averaging process, ensuring a more reliable parent-to-fragment match at a reduced computational cost.

4. Sensitivity analysis

When using the PUZZLE approach, several thresholds for the pruning filters must be set. Typically, this requires a certain degree of *a priori* knowledge on how the method works, as the settings need to be tuned according to the fragmentation event under analysis and the available input data. However, an optimisation of the input settings can be searched such that most breakup events can be reconstructed with optimal thresholds. These represent a trade-off between the computational time required by the approach (the higher the thresholds, the larger the number that pass the filters and thus the longer the required computational effort) and accuracy of the reconstruction. To this aim, a sensitivity analysis was carried out on the values of the thresholds of the main pruning settings.

4.1. Sensitivity analysis framework

To test possible values of the thresholds, three well-known fragmentation events were chosen as test cases. These are:

- the catastrophic collision of Cosmos 2251 and Iridium 33, occurred on 10th February 2009 [2];
- the explosion of the weather satellite NOAA 16, occurred on 25th November 2015 [2];
- the intentional breakup of Fengyun 1C resulted from the first successful anti-satellite test, occurred on 10th January 2007 [2].

These events were selected because of the availability of public TLEs within two weeks from their breakups, to comply with the temporal validity of PUZZLE. The input TLE files consist of both TLEs of fragments generated in the event and other unrelated objects, with different ratios. This step was taken to ensure the robustness of the analysis across different scenarios and enhance its validity. For this reason, the first case contains 1842 TLEs, out of which 276 belong to fragments, the second case has 5792 TLEs of which 54 belonging to the fragments and the last case has 1410 TLEs with 32 fragments. The TLEs were downloaded from SpaceTrack [21], ten days after each event. The number of TLEs used for these test cases is lower than the number of fragments produced by the events; however, the analysis is limited to publicly available data within two weeks from the breakups. The analysed thresholds are the ones of the apogee-perigee filter, the MOID filter, the time filter, the distance margin, and the time margin for close approaches. The sensitivity analysis was carried out by testing several values of the threshold, one filter at a time, therefore making each filter vary independently from the others. For the most significant thresholds, the reciprocal influence of the thresholds of the filters was also assessed.

4.2. Apogee-perigee filter

The distance threshold for the apogee-perigee pruning step was tested with values ranging from 1 km to 50 km. Two main results of the analysis are considered as figures of merit, i.e. the number of objects discarded by the triple-loop with respect to the original number of objects in the analysis to see how well the pruning process works and the final results at the end of the analysis in terms of number of fragments correctly identified with respect to the total and whether the correct parent was associated or not. The results are shown in Figs. 3 and 4. The circles in Fig. 4 indicate that the parent was correctly found, hence the reconstruction was successfully completed.

The results show, as expected, that the threshold of the apogee-perigee filter does not greatly impact the outcome of the triple-loop and of the final reconstruction. Indeed, the number of discarded objects and of final recognised fragments is mostly invariant with respect to the threshold. This result was to be expected as the majority of the discarded objects in the triple-loop are due to the MOID filter, which is more stringent, hence even if a high number of objects is kept in the triple-loop by a large value of the threshold for the apogee-perigee filter, a vast portion of them is then rejected by the MOID step. The final result of the reconstruction also shows that the fragmentation is successfully reconstructed with any value of the threshold.

4.3. MOID filter

As in the previous case, the MOID filter threshold was tested with values from 1 km to 50 km. The same results were analysed, to assess the effect of this threshold on the reconstruction method. The MOID step is expected to have a higher impact, as it prunes objects based on the theoretical minimum distance between two orbits, imposing a more strict requirement on the objects. The same two figures of merit as for the apogee-perigee filter were investigated. The results are shown in Figs. 5 and 6.

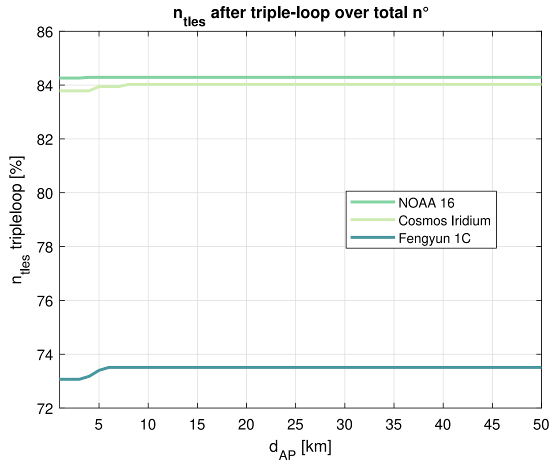


Fig. 3. Percentage of remaining objects after triple-loop with respect to apogee-perigee threshold.

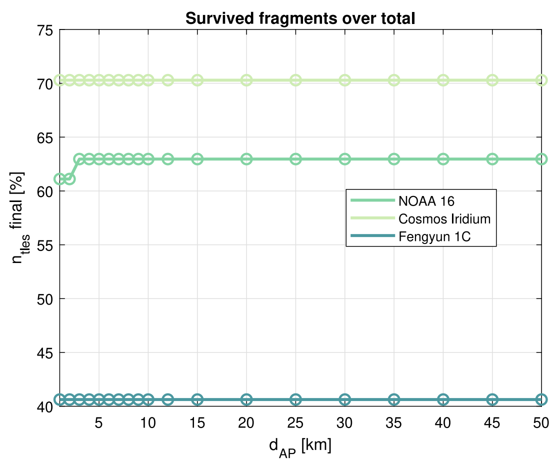


Fig. 4. Percentage of correctly identified fragments at the end of the reconstruction with respect to apogee-perigee threshold. The circles indicate that the parent was correctly found.

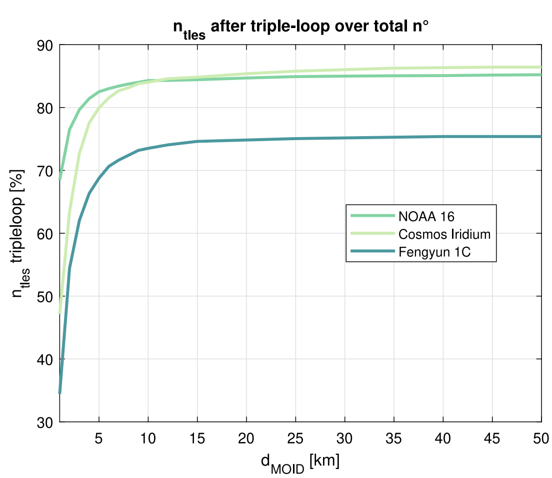


Fig. 5. Percentage of remaining objects after triple-loop with respect to MOID threshold.

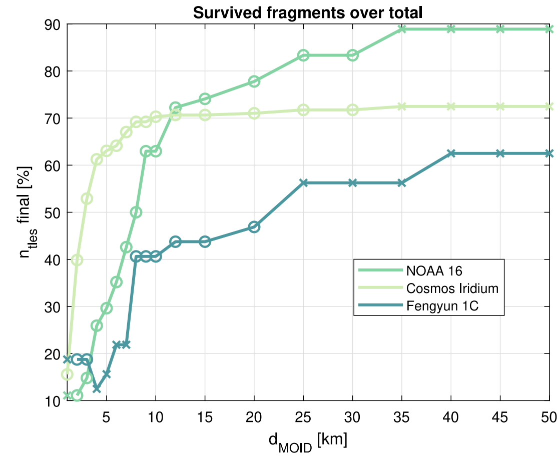


Fig. 6. Percentage of correctly identified fragments at the end of the reconstruction with respect to MOID threshold. The circles indicate that the parent was correctly found, while the crosses on the plot indicate that the parent was not correctly matched to the fragments.

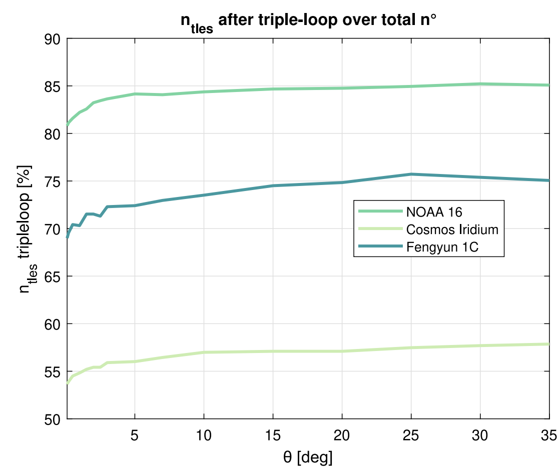


Fig. 7. Percentage of remaining objects after triple-loop with respect to time filter threshold.

Fig. 5 clearly shows that the MOID filter has a higher impact on the triple-loop results, being responsible for the rejection of the majority of discarded objects. In all three test cases, the curves reach a plateau at about 10 km of MOID threshold, suggesting that this value is enough to discard the objects for which a close approach was not possible. As for the final outcome of the fragmentation reconstruction (Fig. 6), it is strongly dependent on this parameter. The crosses on the plot indicate that the parent was not correctly matched to the fragments, which occurs for high values of the threshold. The reason for this is that when the threshold is high, many objects survive the triple-loop filter, hence making it hard for PUZZLE to recognise the parent as the closest object to the fragments families.

4.4. Time filter

The time filter relies on defining an angular aperture around the position of the MOID, then turned into time windows. For this reason, the tested values of the threshold range between 0.1 to 35 deg. The actual angular window is then double the value of the threshold since one has to consider $[-\theta, +\theta]$. The results are reported in Figs. 7 and 8.

The plots indicate that the pruning effect of the triple-loop is not heavily dependent on the time filter threshold value, as the curves in Fig. 7 demonstrate a relatively stable behaviour, with only minor fluctuations, suggesting that their values remain nearly constant within the given angular interval. As for the final outcome of the fragmentation reconstruction, the two test cases of NOAA 16 and Cosmos

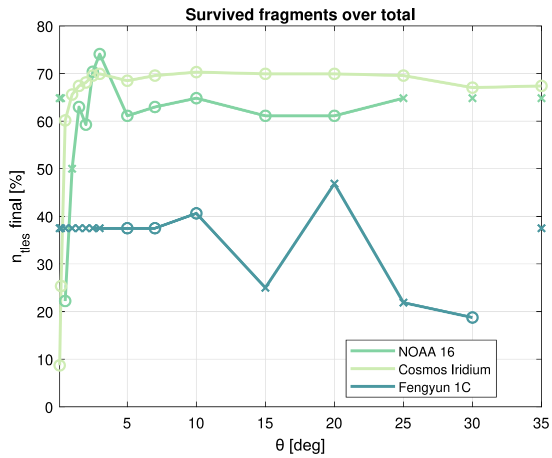


Fig. 8. Percentage of correctly identified fragments at the end of the reconstruction with respect to time filter threshold. The circles indicate that the parent was correctly found, while the crosses on the plot indicate that the parent was not correctly matched to the fragments.

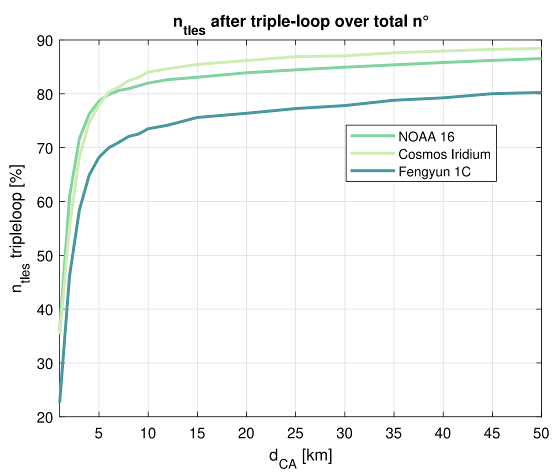


Fig. 9. Percentage of remaining objects after triple-loop with respect to close approach distance threshold.

2251-Iridium 33 exhibit the expected trend, i.e. with a narrow angular window only few fragments are identified. In the Fengyun 1C case instead the behaviour is more irregular, probably due to the quality of input TLEs. When the angular window is very wide, PUZZLE is not able to correctly match fragments and parents and at times it is not able to verify the fragmentation event (discontinuous line). This holds specifically when very few fragments are available, therefore when the threshold is too high, too many objects are considered in the analysis making it hard to recognise the breakup.

4.5. Distance margin

The distance margin influences the pruning of the close approaches that are deemed plausible for a fragmentation, as well as of the families of fragments that are likely due to a breakup event. For this reason, this parameter is expected to have a strong influence on the outcome of the breakup reconstruction. The margin was tested with values covering the range 1 km–50 km and the results right after the triple-loop and at the end of the PUZZLE method are shown in Figs. 9 and 10.

The results indicate that the pruning phase of the PUZZLE approach is heavily influenced by the value of the distance margin. The trend of the curves in Fig. 9 is similar to the one of the MOID, with a plateau-like

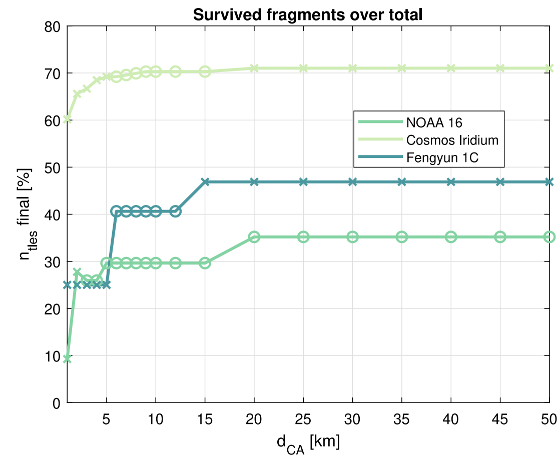


Fig. 10. Percentage of correctly identified fragments at the end of the reconstruction with respect to close approach distance threshold. The circles indicate that the parent was correctly found, while the crosses on the plot indicate that the parent was not correctly matched to the fragments.

behaviour at about 10–15 km. Hence, higher values of the threshold are not necessary to discard more objects as the majority is rejected with 10–15 km. The final results of the reconstruction (Fig. 10) also exhibit a dependence on the value of the distance margin, particularly with regard to the parent identification. For low values of the threshold, i.e. between 1 and 5 km, PUZZLE cannot identify and match the correct parent as too few objects are kept in the analysis. Similarly, when the tolerance is too high the identification fails because of too many objects.

4.6. Time margin

The time margin threshold sets the bounds for the estimation of the epoch of the event. The values for the sensitivity analysis spanned from 1 min up to 60 min. For this parameter, the results were evaluated based on the final reconstruction outcome, as done for the previous thresholds, and on the error in estimating the fragmentation epoch using the following metric:

$$err = \frac{T_{computed} - T_{real}}{T_{TLE} - T_{real}} \tag{9}$$

where $T_{computed}$ is the fragmentation epoch estimated with PUZZLE, T_{real} is the actual epoch of the fragmentation and T_{TLE} is the epoch of the TLEs. This allows to account for the influence of the epoch of the TLEs with respect to the fragmentation epoch. The results are presented in Figs. 11 and 12.

The error on the estimation of the epoch (Fig. 11) does not exhibit a significant dependence on the value of the time margin. For the Cosmos 2251-Iridium 33 and the Fengyun 1C cases the error is close to zero, while for NOAA 16 it is higher because the epoch estimated by PUZZLE is two hours earlier than the actual time of the event. As for the final outcome of the reconstruction, the behaviour of the curves does not demonstrate a consistent trend, particularly for Fengyun 1C. The cause for the erratic pattern is probably due to the quality of input TLEs. Nonetheless, for the other two test cases, the influence of the time margin on the successful output of PUZZLE is limited as the parent is correctly matched in all cases, regardless of the value of the threshold.

4.7. Cross sensitivity analysis

The sensitivity analysis of the pruning settings indicates that the MOID filter threshold and the distance margin have the most significant impact on the PUZZLE method. Indeed, these two parameters

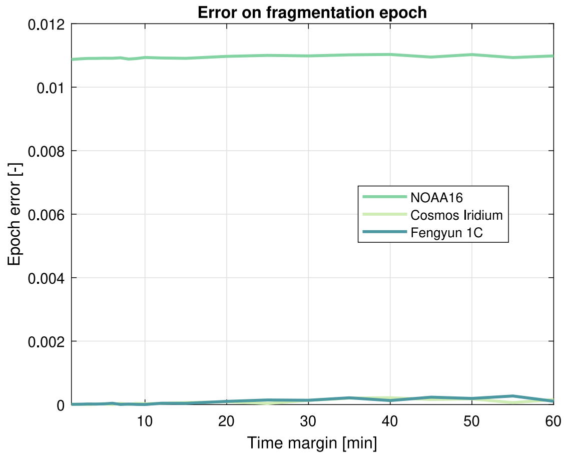


Fig. 11. Weighted error on the fragmentation epoch with respect to the time margin.

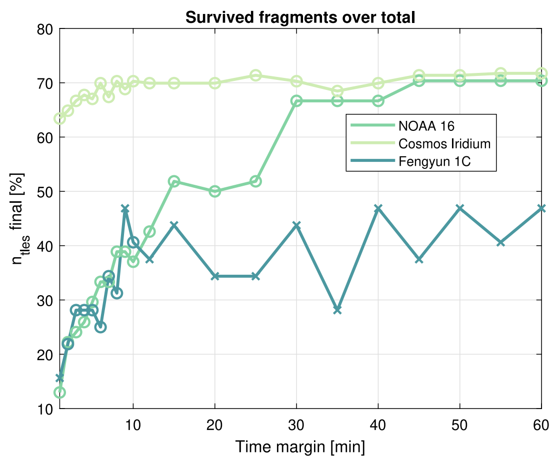


Fig. 12. Percentage of correctly identified fragments at the end of the reconstruction with respect to time margin threshold. The circles indicate that the parent was correctly found, while the crosses on the plot indicate that the parent was not correctly matched to the fragments.

influence the pruning process by discarding the majority of the objects, consequently impacting on the correct fragments-parent(s) match. The two filtering steps are applied sequentially, thus it is of interest to investigate the reciprocal influence of one threshold with respect to the other by making the two parameters vary simultaneously.

To this aim, both the MOID filter threshold and the distance margin were tested with values ranging from 1 km to 30 km, considering the following constraint:

$$d_{CA} \geq d_{MOID} \tag{10}$$

as the MOID represents the minimum theoretical distance between two orbits, implying that the close approach distance cannot be smaller than the MOID. The analysis was carried out on the three test cases used for the previous sensitivity analysis and the results were evaluated based on the fraction of fragments correctly identified at the end of the simulation with respect to the amount in the initial set, and the correct parent match. The results are displayed in Figs. 13–15.

For all three test cases, the plots suggest that the MOID threshold is the strongest one, i.e. it is the parameter that affects the results the most. The quality of the outcome of the reconstruction changes according to the value of the MOID threshold, while it is not as affected by the value of the distance margin. Moreover, given a value for

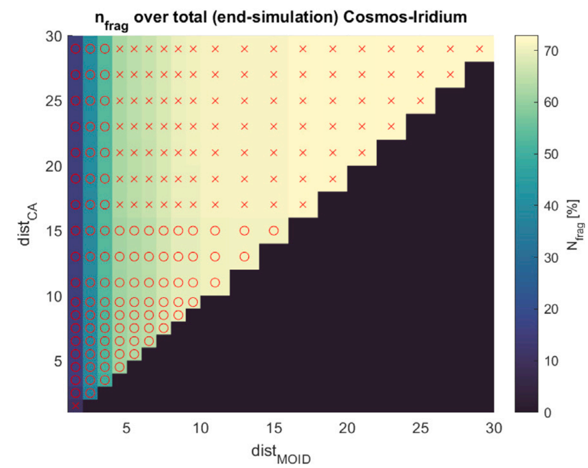


Fig. 13. Percentage of fragments correctly identified by the PUZZLE algorithm with respect to the MOID and CA thresholds for the Cosmos2251-Iridium33 test case. The red circles indicate that the parent was correctly identified, while the crosses indicate a wrong parent association.

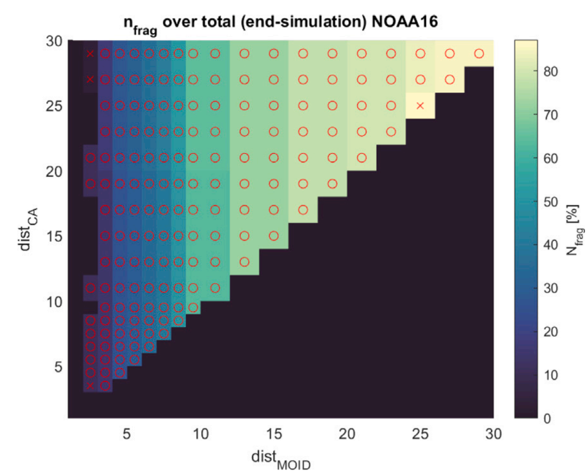


Fig. 14. Percentage of fragments correctly identified by the PUZZLE algorithm with respect to the MOID and CA thresholds for the NOAA 16 test case. The red circles indicate that the parent was correctly identified, while the crosses indicate a wrong parent association.

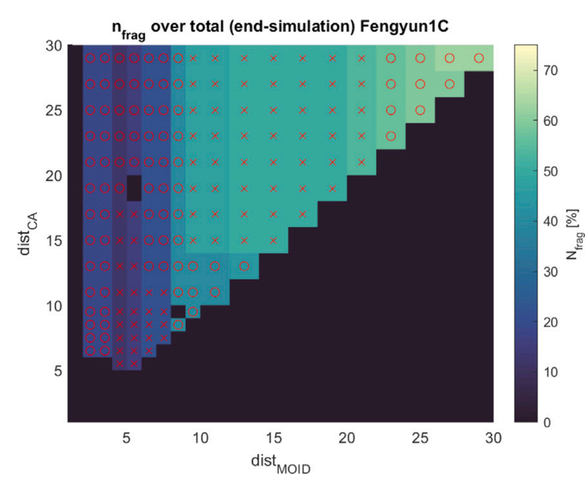


Fig. 15. Percentage of fragments correctly identified by the PUZZLE algorithm with respect to the MOID and CA thresholds for the Fengyun 1C test case. The red circles indicate that the parent was correctly identified, while the crosses indicate a wrong parent association.

the MOID threshold, the fraction of correctly recognised fragments does not significantly change with the value of the close approach margin. Therefore, the figures indicate that setting d_{CA} equal to d_{MOID} results in outcomes comparable to those obtained with a higher close approach threshold. Moreover, increasing the threshold leads to higher computational time, thus setting $d_{CA} = d_{MOID}$ is the most efficient trade-off. As for the identification of the parent(s), no clear trend can be identified except that when the thresholds are strict, often too few objects are kept in the analysis therefore it is challenging to match the correct parent. Conversely, when the threshold values are high, sometimes too many objects are kept in the analysis making the parents matching more uncertain and more likely to be wrong. For the cases of NOAA 16 (Fig. 14) and Fengyun 1C (Fig. 15) there are some gaps in the plots at small values of the MOID threshold, due to PUZZLE stopping the reconstruction because there were not enough elements to verify the fragmentation.

4.8. Considerations on sensitivity analysis

The sensitivity analysis allowed to define optimal values of the pruning thresholds of PUZZLE to enhance the fragmentation reconstruction for LEO breakup events. However, the fragmentation reconstruction process is strongly dependent on the available data right after the event, thus each breakup has to be analysed in a case-by-case scenario. For this reason, the optimal values for the parameters are defined with a range to improve the flexibility and adaptability to different cases. The selected values, looking at the results, are the following:

- for the apogee-perigee filter 5 km, with a range from 5 to 10 km;
- for the MOID filter 10 km, with a range from 5 to 20 km;
- for the time filter 10 deg, with a range from 5 to 15 deg;
- for the distance margin 10 km, with a range from 5 to 20 km;
- for the time margin 10 min with a range from 5 to 20 min.

The selection of these values took into account the successful fragmentation reconstruction, i.e. high number of correctly recognised fragments and the right parent match. Furthermore, when multiple threshold values yielded similar results, the lowest value was chosen to minimise computational effort for PUZZLE.

5. Application of PUZZLE to real fragmentation events

The PUZZLE approach presented in this paper is applied to the fragmentation reconstruction of Fengyun 1C anti-satellite test, considering the standard version of the method, i.e. without the uncertainty assessment, both with large thresholds for pruning filters and with optimal tolerances, and the version with uncertainty quantification. The objective of this section is to assess the accuracy of the approach by applying it to a well-known fragmentation event and evaluate the effectiveness of two versions of the method and of the optimisation of the tolerances.

5.1. Original PUZZLE method without optimisation

The test case proposed here involves the reconstruction of the fragmentation of Fengyun 1C, already used in Section 4.

For this analysis, an initial set of 54 TLEs is considered, of which 45 TLEs belong to 27 generated fragments. The TLEs were taken from SpaceTrack 10 days after the fragmentation event. The initial objects considered in the analysis and their orbits are presented in Fig. 16.

The reconstruction is first carried out with the original PUZZLE version, without the optimisation of the pruning tolerances. Therefore, the pruning thresholds are set to large values, i.e. 50 km for apogee-perigee, MOID and close encounter distance, 50 min for the time margin and 50 deg for the time filter. The main results of the reconstruction are reported in Table 1.

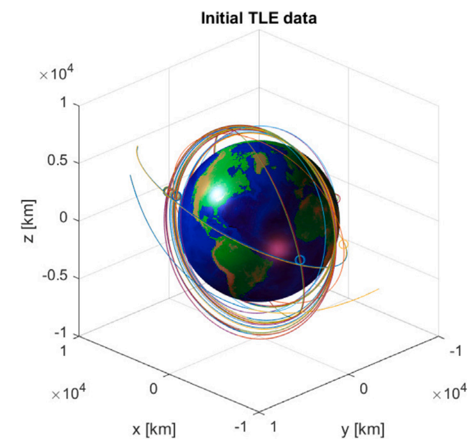


Fig. 16. Objects considered in Fengyun 1C analysis.

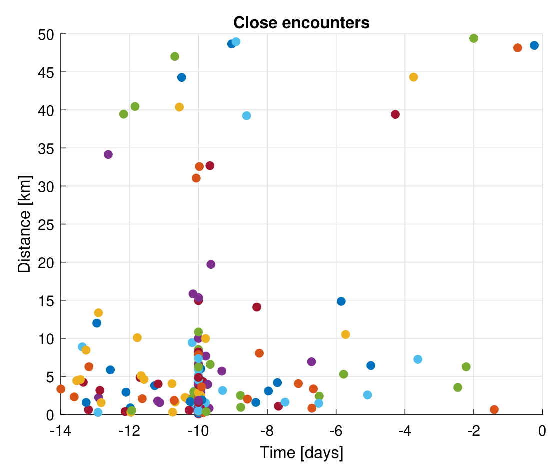


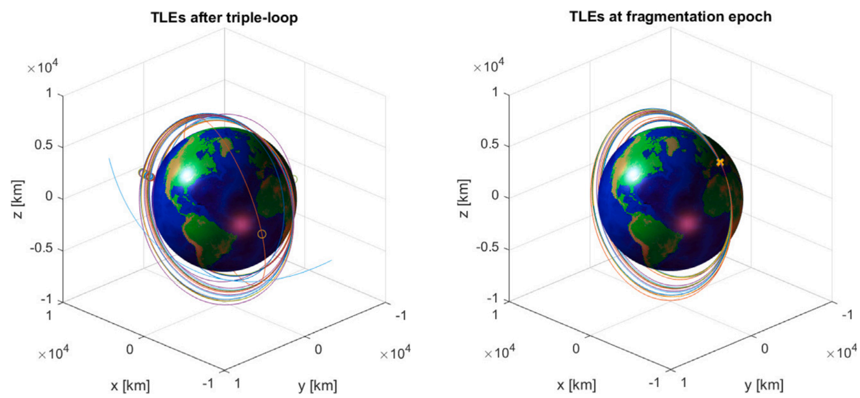
Fig. 17. Close encounter distance between couples of objects with respect to time.

Figs. 17 to 18(a) and 18(b) present the results of PUZZLE throughout the steps, from the results after the triple-loop (Fig. 18(a)) to the estimation of the fragmentation epoch (Fig. 17) and the TLEs of the objects involved in the event propagated to the estimated epoch (Fig. 18(b)).

PUZZLE accurately estimates the epoch of the breakup. Indeed, as visible in Fig. 17 which reports the close approaches between every pair of objects in time, a concentration of close encounters occurs approximately 10 days before the TLEs epoch. This aligns with the reported fragmentation date. As for the family identification process, the accuracy of the method decreases. PUZZLE reconstructs two families of objects instead of one, matching them to two different parents. One of them is the actual parent object, while the other is a Fengyun 1C debris. The failed attempt to identify the family of fragments is due to the fact that the large tolerances keep in the analysis more objects than just the generated fragments, leading to a wrong estimation of the family of objects.

5.2. PUZZLE with optimised tolerances

The reconstruction of the event is repeated with the standard values of the pruning thresholds defined in Section 4. Within the provided threshold ranges, the following values were picked: 5 km for the apogee-perigee filter, 10 km for the MOID and distance margin filters, 10 deg for the time filter and 10 min for the time margin. The main results of the reconstruction are reported in Table 2.



(a) Objects after triple-loop. (b) Objects at fragmentation epoch.

Fig. 18. Results of the application of PUZZLE to Fengyun 1C breakup.

Table 1
Fengyun 1C fragmentation reconstruction results.

Initial size of TLE set	54
Search window	14 days
Number of discarded outliers	1
Number of objects survived after triple-loop	30
Estimated fragmentation epoch	11/01/2007 22:23:43 UTC
Number of involved objects	21
Identified parent(s)	25 730 29 731

Table 2
Fengyun 1C fragmentation reconstruction results with optimised thresholds.

Initial size of TLE set	54
Search window	14 days
Number of discarded outliers	1
Number of objects survived after triple-loop	24
Estimated fragmentation epoch	11/01/2007 22:25:28 UTC
Number of involved objects	7
Identified parent	Fengyun 1C DEB NORAD: 29722

Table 3
Fengyun 1C fragmentation reconstruction results with uncertainties quantification.

Initial size of TLE set	54
Size of TLE set after uncertainties	288
Search window	14 days
Number of objects survived after triple-loop	269
Estimated fragmentation epoch	11/01/2007 22:26:18 UTC
Number of involved objects	9
Identified parent	Fengyun 1C NORAD: 25730

Fig. 19 represents the close encounter plot. Despite the presence of less points on the plot with respect to the reconstruction in Section 5.1, consequence of a stricter triple-loop pruning process, the graph clearly shows a concentration of points 10 days prior to the TLEs epoch. Indeed, the fragmentation epoch is perfectly captured by PUZZLE.

The family identification module here recognises the presence of just one family, however the cluster of fragments is matched to a Fengyun 1C debris object (NORAD 29722) rather than to the correct parent. Nonetheless, this represents an improvement with respect to the unoptimised method. Indeed, it is assumed that, at the breakup epoch, fragments and parent share the same initial position hence the erroneous association is deemed plausible.

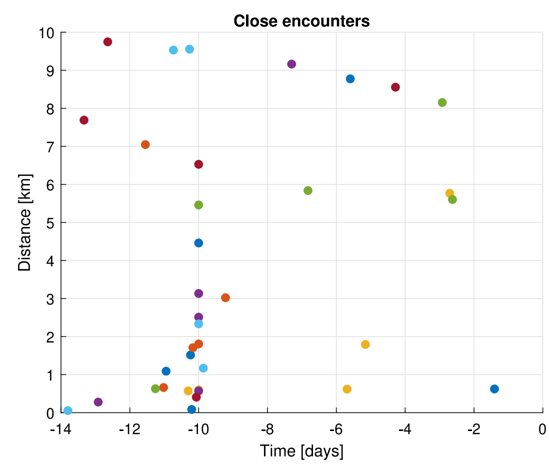


Fig. 19. Close encounter distance between couples of objects with respect to time.

5.3. PUZZLE with uncertainties

The same test case is used to test the PUZZLE method with the quantification of uncertainties. The increased accuracy due to the introduction of uncertainties is the reason why this approach is well suited when only few TLEs are available. The tolerances for the apogee-perigee filter, MOID filter and close approach were set to 10 km. The value of the uncertainty was set to 0.01, Cartesian coordinates were selected and 7 kernels were used for the GMM, leading to 288 TLEs to be analysed.

With the addition of uncertainties, PUZZLE is able to identify the correct fragmentation epoch and the parent of the breakup event. The main results of the approach are reported in Table 3. Thanks to the addition of synthetic TLEs, PUZZLE is able to recognise the occurrence of a breakup and to correctly characterise it.

Fig. 20 shows the entire TLE set under analysis after the addition of synthetic TLEs, while Fig. 21 displays the close encounter distance with respect to their epoch for each pair of objects. The grey dots represent the close encounters due to synthetic TLEs. Clearly, there is a concentration of close approaches corresponding to the epoch of the breakup event.

6. Conclusions

With the increasing number of fragmentation events occurring in space, a prompt reconstruction of breakups is essential for risk assessment operations and to gain information on the causes of the event.

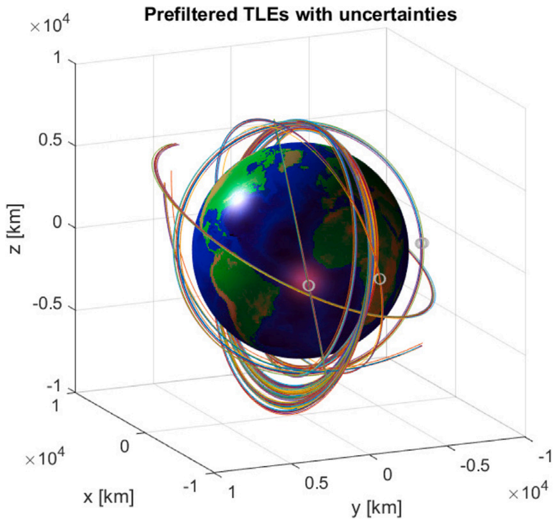


Fig. 20. Objects considered in Fengyun 1C analysis, including synthetic TLEs.

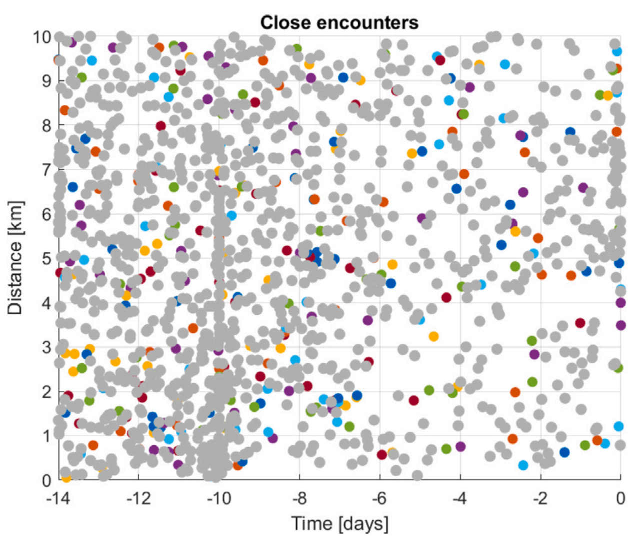


Fig. 21. Close encounter distance between couples of objects with respect to time in Fengyun 1C analysis. The grey dots are the close encounters due to synthetic TLEs.

Many approaches require some *a priori* information on the objects involved in the fragmentations, which is not always possible. As a result, the PUZZLE approach was devised to work in general conditions, starting from the TLEs of objects in publicly available catalogues.

This paper aimed at presenting the PUZZLE method for the reconstruction of fragmentation events in the short-term and at extending it to account for uncertainties by including additional TLEs in the analysis generated from the original ones. Through a series of filters, PUZZLE discards outliers from the analysis and is capable of selecting the objects generated by a breakup event, estimating the event epoch and matching them to the correct parent exploiting a backward propagation with SGP4. The potential issue related to the unreliability of TLE data was addressed by introducing the possibility of generating additional TLEs with a given uncertainty with respect to the state of the original ones. Then, a selection of the optimal TLE set was carried out with an optimisation algorithm to perform a correct matching between families of objects and parents. The application of the method to well-known fragmentation events gave promising results. Indeed, the original PUZZLE approach was able to perfectly reconstruct the CZ6 A breakup and the introduction of uncertainty proved to be efficient in the reconstruction of the NOAA 16 explosion even when the tolerances were very strict, which would otherwise lead to discarding the fragments involved in the event.

CRediT authorship contribution statement

Francesca Ottoboni: Writing – original draft, Validation, Software, Methodology, Formal analysis, Data curation, Conceptualization. **Andrea Muciaccia:** Writing – review & editing, Supervision, Conceptualization. **Camilla Colombo:** Supervision, Project administration, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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