Wiener filtering in wall turbulence

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Years ago, a number of works demonstrated successful estimators [1] and controllers [2] of wall-bounded turbulence, starting from the linearized Navier–Stokes equations; their success is an indirect confirmation of the importance of linearity in the near-wall flow dynamics [3]. Following standard optimal control theory, the design was made in the time domain. Unfortunately, this approach results in large computational cost, and limits the informative content of the linear model of the turbulent system that can be included in the design process. Our proposed activity deals with the alternate path of designing a controller in the frequency domain, relying on the Wiener–Hopf theory. In the linearized Navier–Stokes equations, written as a linear time-invariant (LTI) system, the effect of non-linearity is normally accounted for by treating the non-linear terms as noise that enters the state equations as:

$$\begin{aligned} \dot{x} &= Ax + Bu + n \\ y &= Cx + d. \end{aligned} \tag{1}$$

where x is the state vector, u the input vector, y the output vector, n the state noise and d the measurement disturbance. The matrices A, B and C are respectively the state, input and output matrix.
In this work we will consider the problem of state estimation in a turbulent channel flow. Estimators developed here extend the ones of [1] and [4], and are the best possible LTI filters for the problem at hand, since no assumption other than linearity of the model is introduced. If $H(f)$ is the frequency response function of the system, the estimation problem can be represented via the block diagram plotted in figure 1 (left).
Differently from the time domain approach where the noise is assumed to be white in time, the Wiener theory allows to naturally account for the time structure of the forcing noise in the linearized equations, via the state-noise autocorrelation h of the non-linear terms defined – for each wavenumber pair – as

$$R_{\hat{r}\hat{r}}(y_1, y_2, \tau; \alpha, \beta) = E\left\{\hat{h}(y_1, t+\tau; \alpha, \beta)\hat{h}^H(y_2, t; \alpha, \beta)\right\} \tag{2}$$

Its nontrivial temporal structure is shown in figure 1 (right) calculated directly from a preliminary DNS simulation at low-Reynolds number ($Re_\tau = 100$).

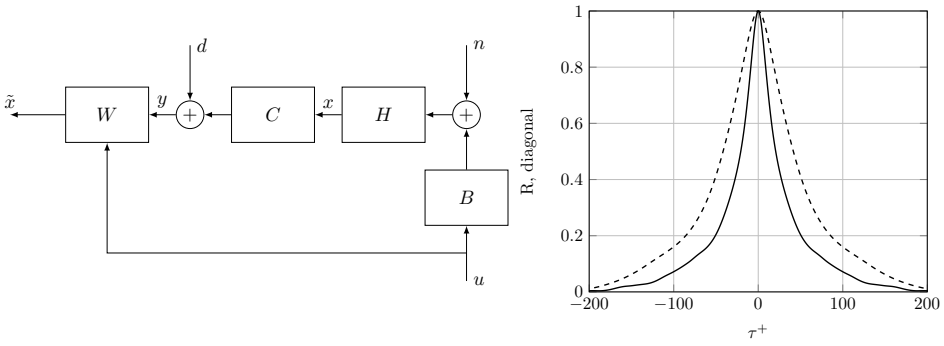

Figure 1: Left: block diagram representation of the Wiener estimation problem (W is the filter transfer function). Right: plot of the normalized terms of $R_{\hat{r}\hat{r}}$ for $\alpha = 1$ and $\beta = 2$ at an height of $y_1^+ = y_2^+ = 15$. Diagonal terms $|E\{\hat{h}_v(y_1, t+\tau)\hat{h}_v^H(y_2, t)\}|$ (solid) and $|E\{\hat{h}_\eta(y_1, t+\tau)\hat{h}_\eta^H(y_2, t)\}|$ (dashed). v and η represent the wall-normal velocity and vorticity.

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[1] Chevalier et al., *Journal of Fluid Mechanics* **552**, 167–187 (2006)
[2] Höpffner et al., *J. Fluid Mech.* **543**, 263–294, 2005
[3] Kim and Bewley, *Annual Review of Fluid Mechanics* **39**, 383–417 (2007)
[4] Amaral et al., *Journal of Fluid Mechanics* **927**, A17 (2021)

These filters will be employed in state estimation of wall turbulence, to assess the relevance of the temporal structure of state-noise correlation in providing satisfactory results and later coupled with feedback flow control algorithm, to assess the loss of performance compared to full-state controllers.