Data-Driven Controller Tuning for MIMO Systems: A Set-Membership Approach

Andres Cordoba-Pacheco and Fredy Ruiz, *Senior Member, IEEE*

***Abstract*—Over time, Single-Input Single-Output systems have received significant attention in the field of data-driven control. However, real-world applications often involve Multi-Input Multi-Output systems, where the challenges associated with multivariable control are considerably greater. This letter presents an innovative extension of the Set Membership Data-Driven approach from Single-Input Single-Output to Multi-Input Multi-Output systems. Exploiting unknown but bounded assumptions on process noise and fixed bases controllers parametrization, an efficient batch algorithm for controller tuning is developed, relying on low dimensional convex optimization problems. Through a comparative analysis with Virtual Reference Feedback Tuning, it is quantitatively demonstrated that the Set Membership Data-Driven approach significantly outperforms existing solutions, achieving reductions in Integral Square Error and Integral Absolute Error by up to 6% and 27%, respectively, thereby reducing coupling errors. Furthermore, the designed controllers exhibit faster rise and settling times, with improvements of up to 20% and 39%, eliminating the overshoots. These findings indicate that the SMDD approach effectively enhances decoupling and error minimization, making it a reliable solution for managing the complexities of MIMO systems.**

***Index Terms*—Data driven control, Identification for control, uncertain systems.**

I. Introduction

In the field of control systems, Single-Input Single-Output (SISO) systems have historically received significant attention. This is not only because their mathematical modeling and analysis are less complex, but also because their simplicity allows for testing new theories and control strategies without the complexity of multiple interacting variables [1].

However, many real-world applications involve multivariable systems, whose complexity poses significant challenges for analysis and control. In these systems, each input variable can affect multiple output variables, demanding an interactive approach to control tuning [2].

It is important to note that even SISO systems can be difficult to model accurately. Factors such as time-varying dynamics, time delays, parameter variations, and the intricacies of system identification can complicate the development of accurate models and effective control strategies. Given these challenges in SISO systems, the transition to Multi-Input Multi-Output (MIMO) systems, presents even greater difficulties due to their inherent inter-channel interactions and increased dimensionality [3].

In practice, comprehensive system behavior can often be inferred from input-output data. The literature typically outlines two main approaches for controller design in such scenarios. The first approach involves deriving a model from the data and then developing a controller based on this model. The second approach bypasses the modeling step and directly synthesizes the controller from the data [4].

With the advent of direct data-driven methods, the necessity for a precise dynamic model has shifted. These new approaches can tune the controller using only input-output data sets from the plant and a reference model, underscoring the impact of the data-driven methods [5]. There are two main types of data-driven approaches for offline tuning of controller parameters: iterative methods, which involve conducting experiments multiple times [6], and non-iterative methods, which require only a single experiment [7]. Techniques such as Virtual Reference Feedback Tuning (VRFT) and Fictitious Reference Iterative Tuning (FRIT) have facilitated the design of Two Degrees of Freedom (2DOF) controllers based on a desired reference model constrained by a stability margin [8]. Additionally, Errors-in-Variables and Output-Error approaches with Set-Membership formulations have been employed to design 2DOF controllers [9], and extensions of the VRFT method to continuous-time and deterministic settings have also been explored [10].

When analyzing MIMO systems, where matrix commutativity does not hold, data-driven tuning approaches used in SISO systems cannot be directly applied for controller design. However, some methods have been extended for MIMO cases. For example, the VRFT method has been adapted to MIMO systems, with controller parameters obtained through a least squares problem solution [11]. Similarly, Correlation-based Tuning (CbT) has been applied to multivariable systems to address load disturbance rejection [12]. Other methods for tuning controller parameters in MIMO systems include using

Received 15 March 2025; revised 14 May 2025; accepted 2 June 2025. Date of publication 11 June 2025; date of current version 25 June 2025. This work was supported by the Italian Ministry of University and Research under Grant "Learning-Based Model Predictive Control by Exploration and Exploitation in Uncertain Environments" (PRIN PNRR 2022 Fund) under Grant P2022EXP2W. Recommended by Senior Editor P. Tesi. *(Corresponding author: Andres Cordoba-Pacheco.)*

The authors are with the Dipartimento di Elettronica Informazione e Bioingegneria, Politecnico di Milano, 20133 Milan, Italy (e-mail: andresfelipe.cordoba@polimi.it; fredy.ruiz@polimi.it).

Digital Object Identifier 10.1109/LCSYS.2025.3578572

Taylor expansions with linear least-squares methods [13] and decoupling matrices with measured data to estimate controllers based on the VRFT method [14]. Additionally, restricting the transfer function of the reference model to an identity matrix multiplied by a scalar transfer function has been exploited in [15] and [16].

A Set-Membership Data-Driven (SMDD) approach for controller tuning in SISO systems was proposed in [17], showing improved performance over existing methods, especially with small datasets. In contrast to VRFT, which typically identifies a single controller that best fits the data without providing explicit robustness guarantees, and to CbT, which relies on correlation-based estimators under stochastic assumptions, SMDD adopts a deterministic framework that assumes noise is unknown but bounded (UBB). This enables the computation of feasible parameter sets that are guaranteed to be consistent with the observed data, ensuring performance across the set. This letter introduces a new approach to extend the SMDD method to Multi-Input Multi-Output systems by exploiting the Nakamoto method for matrix product commutativity [18], as employed in previous works such as [19], [20], and by incorporating a Taylor expansion strategy as in [13]. Our aim is to address the inherent complexities of multivariable control systems by leveraging the unique characteristics of the SMDD framework to avoid statistical assumptions on the data. To validate the proposed tuning strategy, a detailed comparison is presented with the Virtual Reference Feedback Tuning (VRFT) methodology described in [11].

This letter is organized as follows. In Section II, the SMDD approach for MIMO systems is described. In Section III, the process for applying the SMDD in the MIMO system is summarized. Section IV explains the case of the study, along with the analysis of results. Finally, the conclusions end this letter in Section V.

II. Set-Membership MIMO Data-Driven Controller Tuning

The Set-Membership Data-Driven (SMDD) approach is a recent technique for tuning controllers directly from input-output data. It avoids plant model identification, making the tuning process faster, while meeting closed-loop performance requirements [17]. This section outlines the SMDD framework development for MIMO systems and summarizes the SMDD tuning procedure. The problem setup and necessary assumptions for controller tuning are outlined first.

The assumed control system structure is shown in Figure 1. Let $P(z) \in \mathbb{C}^{(n\times n)}$ denote the square MIMO plant, invertible over the frequency range of interest, $C(z,\theta) \in \mathbb{C}^{(n\times n)}$ the controller parameterized by vector θ, capable of decoupling the loop transfer function, and $M(z) \in \mathbb{C}^{(n\times n)}$ the reference model characterizing the desired behavior of the closed-loop system. I is the $n \times n$ identity matrix.

The available information about the plant consists of noisy input-output measurements $\mathcal{D} = \{u(k), y(k), k = 1, 2, \ldots, N\}$. The output noise $v(k)$ is an Unknown But Bounded (UBB) signal satisfying $||v(k)||_\infty \leq \epsilon$, where ϵ is the unknown maximum noise bound.

The aim of the controller tuning problem is to minimize the distance between the reference model $M(z)$ and the actual

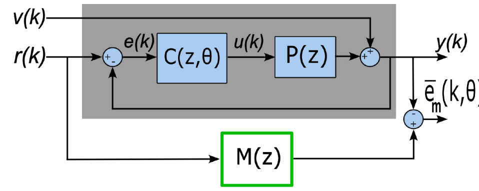


Fig. 1. Control System Tuning Structure.

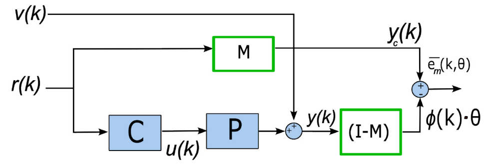


Fig. 2. Original matching error.

closed-loop transfer function,

$$\min_{C(z,\theta)\in\mathcal{C}} \left\| M(z) - (I + P(z)C(z,\theta))^{-1} P(z)C(z,\theta) \right\|_{\ell_1} \tag{1}$$

where $\mathcal{C}$ denotes the set of linear time-invariant (LTI) systems from which the controller is chosen, and $||.||_{\ell_1}$ is the induced ℓ_1 system norm.

Let's assume that there exists an optimal controller $C^\circ(z,\theta^\circ)$ that internally stabilized the system and sets the cost in (1) to zero, satisfying

$$\left\| M(z) - \left(I + P(z)C^\circ(z,\theta^\circ)\right)^{-1} P(z)C^\circ\left(z,\theta^\circ\right) \right\|_{\ell_1} = 0.$$

Finally, it is assumed that for each controller $C(z,\theta)$ in the class $\mathcal{C}$, close to the optimal $C^\circ(z,\theta^\circ)$, it holds that $(I + P(z)C(z,\theta))^{-1} \cong (I + P(z)C^\circ(z,\theta^\circ))^{-1}$, which is justified by the continuity of the closed-loop function with respect to the controller parameters, assuming that the system remains internally stable [4].

Given the previous assumptions, $M(z)$ can be expressed as $M(z) = (I + P(z)C^\circ(z,\theta^\circ))^{-1} PC^\circ(z,\theta^\circ)$, and therefore, it is possible to simplify the model matching error in Eq.(1) by

$$\min_{C(z,\theta)\in\mathcal{C}} \|M(z) - (I - M(z))P(z)C(z,\theta)\|_{\ell_1} \tag{2}$$

The matching problem can be represented by the block diagram in Figure 2.

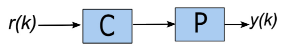
Differently from the SISO case, the limitation caused by matrix non-commutativity does not allow for separating the plant in the cost function to transform the problem into a data-driven estimation one. To overcome this challenge, the approach described in [18] is exploited to address this problem. The methodology considers performing n experiments where n different signals are applied, resulting in n output signals, that are later summed up, as illustrated in Figure 3.

The following Lemma shows that for some specific input signals $u^{<i>}(k)$, it is possible to commute the product between $P(z)$ and $C(z)$. This is known as the Nakamoto principle, as presented in [18].

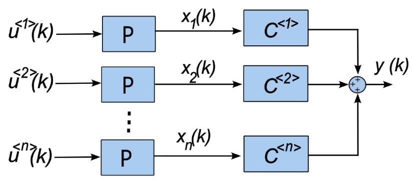
Lemma 1: Let $P(z)$ and $C(z)$ be square systems such that $P(z) \cdot C(z) = L(z)$, with $L(z)$ diagonal.

For the input signal

$$r(k) = \left[r'(k), r'(k), \ldots, r'(k)\right]^{\mathsf{T}}.$$



(a) Output $y(k)$



(b) Output $y(k)$ Nakamoto method

Fig. 3. Commutation blocks.

Let the output of the series connection of $P(z)$ and $C(z)$ be

$$y(k) = P(z) \cdot C(z) * r(k) \tag{3}$$

and

$$y^s(k) = \sum_{i=1}^{n} y^{<i>}(k), \tag{4}$$

where

$$y^{<i>}(k) = C^{<i>}(z) \cdot P(z) * u^{<i>} \tag{5}$$

with

$$C^{<i>}(z) = diag(C_i(z)),$$

where $C_i(z)$ is the i^{th} column of $C(z)$, given by

$$C(z,\theta) = \begin{bmatrix} C_{1,1}(z,\theta) & \cdots & \left[\begin{matrix} C_{1,i}(z,\theta) \\ C_{2,i}(z,\theta) \\ \vdots \\ C_{n,i}(z,\theta) \end{matrix}\right] & \cdots & C_{1,n}(z,\theta) \\ C_{2,1}(z,\theta) & \cdots & & \cdots & C_{2,n}(z,\theta) \\ \vdots & \ddots & & \ddots & \vdots \\ C_{n,1}(z,\theta) & \cdots & & \cdots & C_{n,n}(z,\theta) \end{bmatrix}$$

and $u^{<i>}(k)$ is a signal formed as $r'(k)$ in the i^{th} component and zero elsewhere,

$$u^{<i>}(k) = \left[0, \ldots, r'(k), \ldots, 0\right]^{\mathsf{T}}.$$

then,

$$y(k) = y^s(k)$$

Proof: Starting from Eq.(3) and applying the superposition principle, where the total response can be determined by summing up the individual responses,

$$y(k) = \sum_{i=1}^{n} P(z)C(z) * u^{<i>}(k)$$

where the product $C(z) * u^{<i>}(k)$ picks out the i^{th} column multiplied by $r'(k)$,

$$C(z) * u^{<i>}(k) = C_i(z) * r'(k).$$

Then, the output becomes

$$y(k) = \sum_{i=1}^{n} P(z)C_i(z) * r'(k)$$

since $L(z)$ is assumed diagonal, the product of $P(z)C_i(z)$ results in a vector with only one nonzero entry in the i^{th} position, which is

$$P(z)C_i(z) = \left[0, 0, \ldots, \textstyle\sum_{j=1}^{n} P_{i,j}(z)C_{j,i}(z), 0, \ldots, 0\right]^{\mathsf{T}}.$$

Thus, considering the previous equation that all the products are scalar, it is possible to commute $P_{i,j}(z)$ and $C_{j,i}(z)$ and, then, the total output can be expressed as

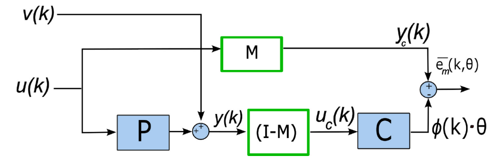


Fig. 4. Modified matching error.

$$y(k) = \begin{bmatrix} \sum_{j=1}^{n} C_{j,1}(z)P_{1,j}(z) \\ \sum_{j=1}^{n} C_{j,2}(z)P_{2,j}(z) \\ \vdots \\ \sum_{j=1}^{n} C_{j,n}(z)P_{n,j}(z) \end{bmatrix} * r'(k). \tag{6}$$

On the other hand, from the definition in Eq.(5), the product $P(z) * u^{<i>}(k)$ picks out the i^{th} column of $P(z)$ with scalar input $r'(k)$,

$$P(z) * u^{<i>}(k) = P_i(z) * r'(k)$$

where $P_i(z)$ is the i^{th} column of $P(z)$. Therefore,

$$y^{<i>}(k) = C^{<i>}(z) \cdot P_i(z) * r'(k)$$

considering that $C^{<i>}(z)$ is defined as a diagonal matrix, it follows that

$$y^{<i>} = \begin{bmatrix} C_{i,1}(z)P_{1,i}(z) \\ C_{i,2}(z)P_{2,i}(z) \\ \vdots \\ C_{i,n}(z)P_{n,i}(z) \end{bmatrix} * r'(k)$$

and eq.(4) becomes

$$y^s(k) = \begin{bmatrix} \sum_{i=1}^{n} C_{i,1}(z)P_{1,i}(z) \\ \sum_{i=1}^{n} C_{i,2}(z)P_{2,i}(z) \\ \vdots \\ \sum_{i=1}^{n} C_{i,n}(z)P_{n,i}(z) \end{bmatrix} * r'(k), \tag{7}$$

The claim follows equating (6) and (7). ■

The previous result shows that, by decomposing the system into individual components and leveraging the assumption of the diagonal structure of $L(z)$, the matrix commutativity problem is effectively bypassed. One disadvantage of using this method is the amount of experiments to carry out; however, it is an effective way to handle non-commutative systems.

Based on Lemma 1, the block diagram of the model matching error can be redrawn as in Figure 4, and the model matching error becomes,

$$e^i_m(\theta,k) = M * u(k) - \sum_{j=1}^{n} C_{j,i}(\theta)(I - M) * y_j(k) + d(\theta,k), \tag{8}$$

resulting in n equations, one for each output, where to find the controller, it can be stated that

$$u^j_c(k) = (I - M) * y_j(k),$$

the corresponding measured output is given by

$$y_c(k) = M * u(k)$$

and the UBB signal at the output, caused by the noise $v(k)$, is

$$d(\theta,k) = C(z,\theta)(I - M) * v(k)$$

bounded as

$$||d(k)||_\infty \leq n \cdot ||C(z,\theta)(I-M)||_{\ell_1}\epsilon \doteq \delta_p$$

which depends on the number of experiments.

Considering that the controller is parameterized as a linear combination of fixed basis functions,

$$C_{j,i}(z,\theta) = \sum_{l=1}^{m} \theta_{j,i,l}\beta_{j,i,l}(z) \tag{9}$$

where m denotes the highest complexity of the controller. Note that the choice of the basis functions is fundamental under the assumption of the existence of the controller $C^\circ(z,\theta^\circ)$. The framework supports general fixed bases, such as Laguerre, Legendre, or Hermite functions.

Let $\phi_{j,i,l}(k)$ be defined as

$$\phi_{j,i,l}(k) = \beta_{j,i,l}(z) * u_c^i(k) \tag{10}$$

resulting in

$$e_m^i(\theta,k) = y_c^i(k) - \sum_{j=1}^{n}\sum_{l=1}^{m} \theta_{j,i,l}\phi_{j,i,l}(k). \tag{11}$$

From now on, the controller tuning problem can be analyzed as an identification task, where the Feasible Parameter Set (FPS) includes all parameters θ that define a controller compatible with both the experimental dataset $\mathcal{D}$ and the desired reference model $M(z)$. The FPS is defined as

$$FPS = \left\{ \begin{array}{c} \theta_i \in R^m : |y_c^i(k) - \sum_{j=1}^{n}\sum_{l=1}^{m} \theta_{j,i,l}\phi_{j,i,l}(k)| \leq \delta_p^i, \\ \forall \quad k = \{1,2,\ldots,N\} \end{array} \right\} \tag{12}$$

where δ_p^i is the noise bound for output i.

III. SMDD Tuning Methodology

This section describes the tuning procedure for applying the SMDD approach in MIMO systems. The methodology is summarized in the following steps:

1) Collect a data set $\mathcal{D}$ through experimental testing in an open/close-loop operation of the system. Following Lemma 1, the data is obtained from n experiments, where the responses to each input $u^{<i>}(k)$ are summed up to build the complete system output $y(k)$.
2) Select a diagonal reference model $M(z)$ to ensure a decoupling system with the required closed-loop characteristics. This selection should consider factors such as delay times, settling time, system responsiveness, signal resolution, etc.

$$M(z) = \begin{bmatrix} M_{1,1} & 0 & \cdots & 0 \\ 0 & M_{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & M_{n,n} \end{bmatrix} \tag{13}$$

with $M_{j,i}$ of the form $z^{-k_0}\frac{N(z)}{D(z)}$,where k_0 is the delay time, $N(z)$ and $D(z)$ are the numerator and denominator polynomials, respectively. Define the controller structure as a set of basis functions $\beta_{j,i,l}$ (P, PI, PD, PID structures, etc).

3) Obtain the set of noise bounds δ_p^i that guarantees a non-empty FPS, i.e.,

$$\delta_p^i = \underset{\delta,\theta\in\Theta}{\arg\min} \quad \delta$$

subject to

$$\left\| y_c^i(k) - \sum_{j=1}^{n}\sum_{l=1}^{m} \theta_{j,i,l}\phi_{j,i,l}(k) \right\|_\infty \leq \delta$$

$$\delta \geq 0 \tag{14}$$

$$\forall \quad k = \{1,2,\ldots,N\}$$
$$i = \{1,2,\ldots,n\}$$

δ_p^i allows to evaluate if the reference model is achievable with the selected basis functions with acceptable error.

4) For each component of θ determine its uncertainty intervals and estimate the Chebyshev center of the FPS, $\hat{\theta}$, i.e.,

$$\overline{\theta_{j,i,l}} = \underset{\theta\in FPS}{\arg\max} \quad \theta_{j,i,l}$$

subject to

$$\left\| y_c^i(k) - \sum_{j=1}^{n}\sum_{l=1}^{m} \theta_{j,i,l}\phi_{j,i,l}(k) \right\|_\infty \leq \delta_p^i\alpha \tag{15}$$

$$\forall \quad k = \{1,2,\ldots,N\}$$
$$i = \{1,2,\ldots,n\}$$

$$\underline{\theta_{j,i,l}} = \underset{\theta\in FPS}{\arg\min} \quad \theta_{j,i,l}$$

subject to

$$\left\| y_c^i(k) - \sum_{j=1}^{n}\sum_{l=1}^{m} \theta_{j,i,l}\phi_{j,i,l}(k) \right\|_\infty \leq \delta_p^i\alpha \tag{16}$$

$$\forall \quad k = \{1,2,\ldots,N\}$$
$$i = \{1,2,\ldots,n\}$$

being α a tolerance gap typically around 10% [21]; and

$$\hat{\theta} = \frac{\overline{\theta} + \underline{\theta}}{2} \tag{17}$$

5) Implement the controller with the estimated parameters $\hat{\theta}$, taking into account that $\overline{\theta_{j,i,l}} - \underline{\theta_{j,i,l}}$ is an indicator of uncertainty.

IV. Results and Discussion

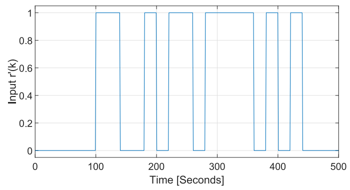
A comparative analysis is performed between the SMDD and VRFT tuning methods for the MIMO system described in [11]. The performance of both controllers is evaluated under various conditions to assess their effectiveness in managing the complexities of MIMO systems. Additionally, details of the experimental setup, performance metrics, and the outcomes of the comparative analysis are provided.

A set of noisy input-output data $\mathcal{D}$ with $N = 500$ is obtained from the plant taken from [11], where the output noise is bounded as $||v(k)||_\infty \leq 1e-4$,

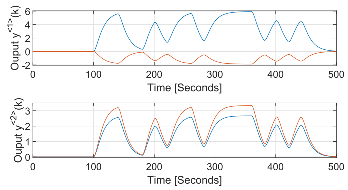
$$P(z) = \begin{bmatrix} \frac{0.09516}{z-0.9048} & \frac{0.03807}{z-0.9048} \\ \frac{-0.02974}{z-0.9048} & \frac{0.04758}{z-0.9048} \end{bmatrix} \tag{18}$$

and the reference model is given by,

$$M(z) = \begin{bmatrix} \frac{0.2}{z-0.8} & 0 \\ 0 & \frac{0.4}{z-0.6} \end{bmatrix}. \tag{19}$$



(a) Input $r'(k)$



(b) Outputs $y^{<1>}(k)$ (top) and $y^{<2>}(k)$ (bottom).

Fig. 5. Dataset $\mathcal{D}$, input-output.

To apply the SMDD approach, the controller structure is chosen as a PID, which is a combination of three basis functions,

$$\beta_{j,i}(z) = \left\{1 \; \frac{z}{z-1} \; \frac{z-1}{z}\right\}. \tag{20}$$

This choice aligns with [11] for comparative purposes.

The framework is developed in a simulation environment using MATLAB/Simulink. A PseudoRandom Binary Sequence (PRBS) was selected as the input to obtain sufficient frequency information. The applied input $r'(k)$ and the outputs obtained from the plant are shown in Figure 5.

The Tuning process proceeds with obtaining the set of PID parameters compatible with the data and the reference model. First, the noise bounds are estimated as in step 3). Then, the central controller is derived solving the optimization problems in step 4). The PID controller corresponding to the central estimate parameters is

$$C^*(z) = \begin{bmatrix} \frac{1.636z^2-2.814z+1.204}{z^2-z} & \frac{-2.564z^2+4.411z-1.888}{z^2-z} \\ \frac{1.114z^2-1.95z+0.853}{z^2-z} & \frac{6.462z^2-11.31z+4.943}{z^2-z} \end{bmatrix} \tag{21}$$

On the other hand, a controller $C_P(z)$ obtained through the VRFT method from [11] is identified from a single experiment, where the PRBS signal $r'(k)$ is applied to both inputs in an open-loop configuration. The resulting controller is

$$C_P(z) = \begin{bmatrix} \frac{1.944z^2-3.441z+1.523}{z^2-z} & \frac{-3.17z^2+5.631z-2.506}{z^2-z} \\ \frac{1.006z^2-1.812z+0.8195}{z^2-z} & \frac{6.186z^2-11.1z+5.001}{z^2-z} \end{bmatrix} \tag{22}$$

Once the controllers are obtained, their performance is compared. Key performance indicators (KPIs) such as rise time, overshoot, and settling time are identified to evaluate the performance and responsiveness of the MIMO system, where both controllers are subject to identical testing procedures with the same reference signal, as shown in Figure 6.

As detailed in Figure 7 and Figure 8, both controllers achieve a stable response that effectively tracks the reference signals. However, upon closer examination, both controllers exhibit overshoots and are not well decoupled. In this case, the SMDD approach demonstrates better performance than the VRFT methods. To further evaluate performance, the Integral Square Error (ISE) and Integral Absolute Error (IAE) are calculated to assess the cross-coupling response. The results are presented in Table I, as well as in Figure 9. The SMDD approach demonstrates superior performance, reducing the ISE by up to 99.4% and the IAE by approximately 93% for output $y_1(k)$. Similarly, for output $y_2(k)$, the ISE is reduced by around 5.5% and the IAE by 26.8%. Additionally, the ℓ_1 norm between the reference model and the obtained closed-loop response is also reported, showing that the proposed method leads to a controller that is 3 times closer to the reference model than the result of VRFT.

Table II describes the step-response characteristics of the closed-loop systems, where it is evident that the SMDD approach outperforms the VRFT method. For output $y_1(k)$, the rise time with the SMDD approach was reduced by 20.2%, indicating a faster response. This is further supported by a 21.6% reduction in settling time compared to VRFT tuning, with no overshoot observed. For output $y_2(k)$ the improvements are even more pronounced, with a 25.7% reduction in rise time and a 39.5% reduction in settling time.

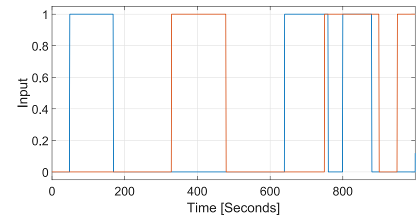


Fig. 6. Input $r(k)$, $r_1(k)$ Blue $r_2(k)$ red.

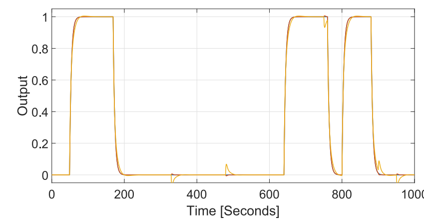


Fig. 7. Output $y_1(k)$: $M(z)$ (Blue) *SMDD* (Red) *VRFT* (yellow).

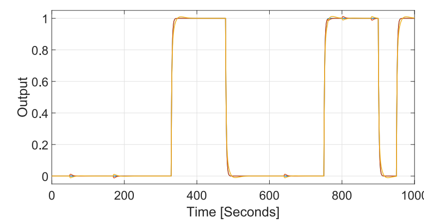


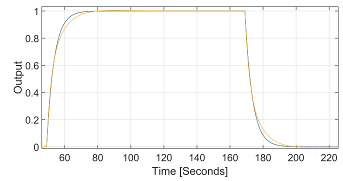
Fig. 8. Output $y_2(k)$: $M(z)$ Blue *SMDD* Red *VRFT* Yellow.

TABLE I
DECOUPLING PERFORMANCE $r_i(k)$ TO $y_j(k)$

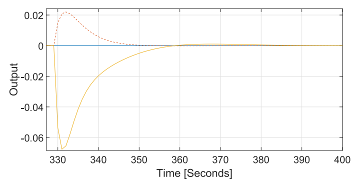
Method	$r_2(k)$ to $y_1(k)$		$r_1(k)$ to $y_2(k)$		ℓ_{ℓ_1} norm of (1)
	ISE	IAE	ISE	IAE	
VRFT	0.02	0.61	$8.15e-4$	0.14	9.55
SMDD	$1.44e-4$	0.04	$7.7e-4$	0.1	3.41

TABLE II
TRACKING PERFORMANCE $r_i(k)$ TO $y_i(k)$

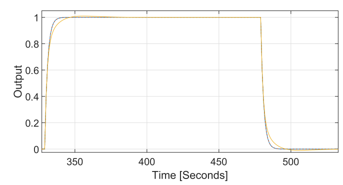
Characteristic	$r_1(k)$ to $y_1(k)$		$r_2(k)$ to $y_2(k)$	
	$VRFT$	$SMDD$	$VRFT$	$SMDD$
Rise Time (s)	12.34	9.85	5.82	4.32
Settling Time (s)	22.41	17.56	12.76	7.72
Overshoot (%)	0.44	0	0.97	0



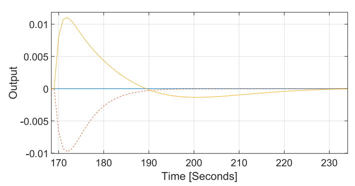
(a) Output $y_1(k)$



(b) Cross-coupling $y_1(k)$



(c) Output $y_2(k)$



(d) Cross-coupling $y_2(k)$

Fig. 9. Output $y(k)$: $M(z)$ (blue), *SMDD* (red), *VRFT* (yellow).

This comparative analysis was conducted using the methodology described in Section II, providing strong evidence that the SMDD approach outperforms the VRFT method in this evaluated case. The significant reductions in all KPIs indicate that the SMDD approach not only handles decoupling errors better than VRFT but also increases responsiveness and stability in the studied MIMO system.

V. Conclusion

Throughout this letter, an innovative and successful extension of the SMDD approach from SISO to MIMO systems is presented, addressing the complexities inherent in multivariable control. The proposed methodology allows to estimate controller parameters for linearly parametrized structures by exploiting information from properly designed experiments (one experiment for each control input). The estimation process is solved through linear programming and directly provides information about the uncertainty on each controller parameter.

Numerical results demonstrate that the SMDD approach significantly outperforms the VRFT method in terms of Integral Square Error and Integral Absolute Error, reducing them by up to 5.5% and 26.8%, respectively. This improvement leads to better decoupling and error minimization. Additionally, achieves faster rise times and settling times for both outputs, with reductions of up to 20%, and completely eliminates overshoots.

Future work will focus on further validating these findings across different MIMO system configurations, reducing the number of experiments, improving noise handling, and evaluating in real-world applications.

References

[1] N. Jain and A. G. Alleyne, “Comparison of SISO and MIMO control techniques for a diagonally dominant vapor compression system,” in *Proc. Amer. Control Conf.*, 2009, pp. 1580–1585.
[2] Ujjwal, H. P. Singh, and N. Verma, “Self-tuning of multivariable plant,” in *Proc. IEEE Int. Conf. Technol., Eng., Manag. Soc. Impact Using Mark., Entrep. Talent (TEMSMET)*, 2020, pp. 1–6.
[3] M. Li, T. Chen, G. Zhi, R. Cheng, K. Yang, and S. Jin, “Data-driven MIMO feedforward control approach based on measured step response,” *IEEE/ASME Trans. Mechatronics*, vol. 29, no. 2, pp. 1441–1452, Apr. 2024.
[4] S. Formentin, K. van Heusden, and A. Karimi, “A comparison of model-based and data-driven controller tuning,” *Int. J. Adapt. Control Signal Process.*, vol. 28, no. 10, pp. 882–897, 2014.
[5] X. Gong, X. Wang, and B. Cao, “On data-driven modeling and control in modern power grids stability: Survey and perspective,” *Appl. Energy*, vol. 350, Nov. 2023, Art. no. 121740.
[6] H. Hjalmarsson, M. Gevers, S. Gunnarsson, and O. Lequin, “Iterative feedback tuning: Theory and applications,” *IEEE Control Syst. Mag.*, vol. 18, no. 4, pp. 26–41, Aug. 1998.
[7] A. Karimi, K. van Heusden, and D. Bonvin, “Non-iterative data-driven controller tuning using the correlation approach,” in *Proc. Eur. Control Conf. (ECC)*, 2007, pp. 5189–5195.
[8] A. Sakaki, T. Kinoshita, and T. Yamamoto, “Design of a data-driven two-degree-of-freedom control system considering robustness,” in *Proc. 15th Int. Conf. Control, Autom., Robot. Vis. (ICARCV)*, 2018, pp. 389–393.
[9] F. Valderrama, F. Ruiz, and D. Patino, “Evaluation of set-membership approaches for data-driven tuning of two-degree-of-freedom controllers,” in *Proc. Amer. Control Conf. (ACC)*, 2019, pp. 5668–5673.
[10] S. Formentin, M. C. Campi, A. Carè, and S. M. Savaresi, “Deterministic continuous-time virtual reference feedback tuning VRFT with application to PID design,” *Syst. Control Lett.*, vol. 127, pp. 25–34, May 2019.
[11] L. Campestrini, D. Eckhard, L. A. Chía, and E. Boeira, “Unbiased MIMO VRFT with application to process control,” *J. Process Control*, vol. 39, pp. 35–49, Mar. 2016.
[12] R. W. P. da Silva and D. Eckhard, “Multivariable correlation-based tuning for load disturbance rejection,” *IFAC-PapersOnLine*, vol. 53, no. 2, pp. 3977–3982, 2020.
[13] M. Saeki, “Data-driven tuning of model-reference controllers for stable MIMO plants,” *Automatica*, vol. 131, Sep. 2021, Art. no. 109786.
[14] K. Liu, Y. Liu, F. Song, and J. Tan, “Data-driven calibration of decoupling matrix for MIMO precision motion stages,” *ISA Trans.*, vol. 150, pp. 223–231, Jul. 2024.
[15] S. Formentin, S. M. Savaresi, and L. Del Re, “Non-iterative direct data-driven controller tuning for multivariable systems: Theory and application,” *IET Control Theory Appl.*, vol. 6, pp. 1250–1257, Jun. 2012.
[16] T. E. Passenbrunner, S. Formentin, S. M. Savaresi, and L. Del Re, “Direct data-driven control of internal combustion engine test benches using closed-loop experiments,” in *Proc. IEEE 51st IEEE Conf. Decis. Control (CDC)*, 2012, pp. 3765–3770.
[17] F. Valderrama and F. Ruiz, “Limited-complexity controller tuning: A set membership data-driven approach,” *Eur. J. Control*, vol. 58, pp. 82–89, Mar. 2021.
[18] M. Nakamoto, “A direct method to calculate control parameters by using input and output data,” *Trans. Soc. Instrum. Control Eng.*, vol. 42, no. 8, pp. 863–868, 2006.
[19] K. Yanagawa, K. Yubai, D. Yashiro, and S. Komada, “Output complementary sensitivity shaping for MIMO systems without multiple experiments,” in *Proc. 37th Chin. Control Conf. (CCC)*, 2018, pp. 182–187.
[20] M. Ishizaki, K. Yubai, and J. Hirai, “A design of H_∞ model-matching multivariable controller by spectral analysis using model-free controller tuning,” in *Proc. 12th Int. Conf. Control, Autom. Syst.*, 2012, pp. 1611–1614.
[21] M. Lauricella and L. Fagiano, “Set membership identification of linear systems with guaranteed simulation accuracy,” *IEEE Trans. Autom. Control*, vol. 65, no. 12, pp. 5189–5204, Dec. 2020.