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## ANALYTICAL EVALUATION OF THE ELECTROMOTIVE FORCES IN DOUBLY SLOTTED SYNCHRONOUS MACHINES WITH DISTRIBUTED WINDINGS

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**Abstract** – An accurate, analytical evaluation of the e.m.f. waveforms in isotropic synchronous machines is developed, by employing a suited analytical field model. To this aim, the flux linkage of each armature coil is evaluated, as a function of time and rotor position, subsequently obtaining the phase flux linkage by suitably summing the coil contributions: the method, for now limited to unsaturated conditions, takes into account the actual winding features (field N° of slots/pole; armature N° of layers, N° of slots/(pole-phase), coil pitch), as like as the air-gap core geometry (air-gap and slot opening widths). Some comparisons with FEM simulations have been performed, obtaining very good agreement with the analytical results.

### Air-gap flux density and stator winding flux linkage and e.m.f.s

Called  $x$  and  $y$  the linear peripheral coordinates along the stator and rotor surfaces respectively, and called  $z$  the rotor origin position with respect to the stator origin, the coordinate along the rotor can be expressed as follows:

$$y = x - z. \quad (1)$$

In [1], an expression of the air-gap flux density  $b$  has been obtained, giving the  $b$  value in each position  $x$  along the stator, as a function of the rotor location  $z$  and of the time dependent stator and rotor m.m.f.s  $m_S$  and  $m_R$ :

$$b(x, x-z, t) = (\mu_0/g) \cdot \beta_S(x) \cdot \beta_R(x-z) \cdot [m_S(x, t) + m_R(x-z, t)] : \quad (2)$$

$g$  is the air-gap,  $\beta_S(x)$  and  $\beta_R(x-z)$  the “notch” field functions (giving the stator and rotor slotting effects) [1].

In case of a stator three-phase winding ( $p = 1, 2, 3$ , phase index), with  $N_t = N^\circ$  turns/coil,  $a = N^\circ$  of parallel paths, and with a rotor winding equipped with  $N_f = N^\circ$  turns/(field coil), the following expressions are valid:

$$m_S(x, t) = (N_t/a) \cdot \sum_{p=1,2,3} M_{fS} [x - (p-1) \cdot 2\tau/3] \cdot i_p(t) ; \quad m_R(x-z, t) = M_{fR}(x-z) \cdot N_f \cdot i_f(t) , \quad (3)$$

where:  $\tau$  is the pole pitch;  $M_{fS}(x)$  and  $M_{fR}(x-z)$  model the m.m.f. space distribution [1], due to the winding structures, while the time dependence is due to the phase and field currents waveforms ( $i_p(t)$ ,  $i_f(t)$ ).

Fig.1 shows an example of space waveform of the stator notch function  $\beta_S(x)$  (a similar waveform is valid for  $\beta_R(x-z)$ ), but with notches extended just along the slotted portion of the pole pitch  $\tau$ ;  $\tau_{sS}$  is the stator slot pitch.

Fig.2 illustrates the space waveform of the rotor m.m.f.  $M_{fR}(x-z)$ , expressed in p.u. referred to the peak value, for a field winding with  $c_p = 4$  slots/pole;  $\tau_{sR}$  is the rotor slot pitch. It should be noted that the stator and rotor m.m.f. variations are not step-wise, but slightly smoothed, in order to model the interpolar field behaviour [1].

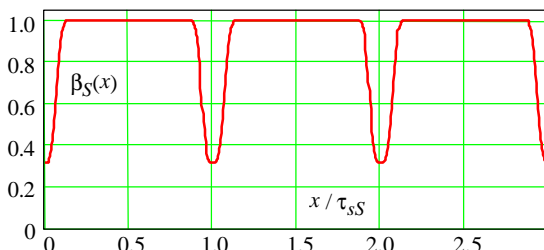


Fig.1 – Notch field function for the stator slotted surface (slot pitch/slot opening  $\approx 5.8$ ; slot opening/air-gap = 6).

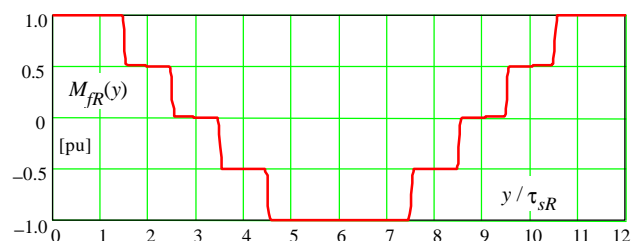


Fig.2 – Space waveform of the rotor m.m.f., referred to the peak value, for a field winding with  $c_p = 4$  slots/pole.

In what follows, flux linkage and e.m.f. will be evaluated for a group of  $q$  series connected coils under each pole. Be  $y_c$  the coil pitch (expressed in  $N^\circ$  of slot pitches), and  $x_{ip1}$  the position of the initial active side of the first stator coil of the  $p$ -th phase; the initial and final active side positions of the  $k$ -th coil of the same  $p$ -th phase are:

$$x_{ipk} = x_{ip1} + (k-1) \cdot \tau_{sS} ; \quad x_{fpk} = x_{ipk} + y_c \cdot \tau_{sS} \quad , \quad (4)$$

For an unskewed machine of stack axial length  $\ell$  and stator internal diameter  $D$ , the flux linkage of a group is:

$$\Psi_p(z,t) = \sum_{k=1}^q \Psi_{pk}(z,t) = (\mu_0/g) \cdot \ell \cdot N_t \cdot \sum_{k=1}^q \left\{ \int_{x_{ipk}}^{x_{fpk}} \beta_S(x) \cdot \beta_R(x-z) \cdot [m_S(x,t) + m_R(x-z,t)] \cdot dx \right\} . \quad (5)$$

In eq.(5) the integration is extended over the cylindrical surface positioned at the middle air-gap diameter.

The corresponding e.m.f.  $e_p$  of a group includes a motional and a transformer term; being  $z = z(t)$ , we have:

$$e_p(t) = \frac{d\Psi_p(z(t),t)}{dt} = \frac{\partial \Psi_p(z,t)}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial \Psi_p(z,t)}{\partial t} = e_{pm}(t) + e_{pt}(t) . \quad (6)$$

Being  $\Omega = \Omega(t)$  the rotational speed and  $k_m = \mu_0 \cdot \ell \cdot D \cdot N_t / (2 \cdot g)$ , by performing the  $z$ -derivative under the integral operator, the motional e.m.f.  $e_{pm}$  of a group becomes:

$$e_{pm}(t, z(t)) = k_m \cdot \Omega \cdot \sum_{k=1}^q \left\{ \int_{x_{ipk}}^{x_{fpk}} \left[ \frac{\partial \beta_R(x-z)}{\partial z} \cdot \beta_S(x) \cdot m_S(x,t) + \frac{\partial [\beta_R(x-z) \cdot m_R(x-z,t)]}{\partial z} \cdot \beta_S(x) \right] dx \right\} . \quad (7)$$

On the other hand, considering that  $\partial f(x-z)/\partial z = -\partial f(x-z)/\partial x$ , and performing a ‘‘per part’’ integration of the second term under integral, eq. (7) gives:

$$e_{pm}(t, z(t)) = -k_m \cdot \Omega \cdot \sum_{k=1}^q \left\{ \int_{x_{ipk}}^{x_{fpk}} \left[ \frac{\partial \beta_R(x-z)}{\partial x} \cdot \beta_S(x) \cdot m_S(x,t) - \frac{d\beta_S(x)}{dx} \cdot \beta_R(x-z) \cdot m_R(x-z,t) \right] dx + \left[ \beta_S(x) \cdot \beta_R(x-z) \cdot m_R(x-z,t) \right]_{x_{ipk}}^{x_{fpk}} \right\} . \quad (8)$$

The first two terms of eq.(8) depend on the space integral of the m.m.f.s distribution, of the notch field functions and of their derivatives, while the last term depends only on the actual values at the coil side extreme positions.

Fig.3 shows the waveforms of the notch field functions derivatives, for the stator and the rotor respectively; as can be observed, these derivatives are zero, except around the slot openings, where sharp, wide variations occur: this pulse-wise behaviour makes troublesome the numerical evaluation of the integrals.

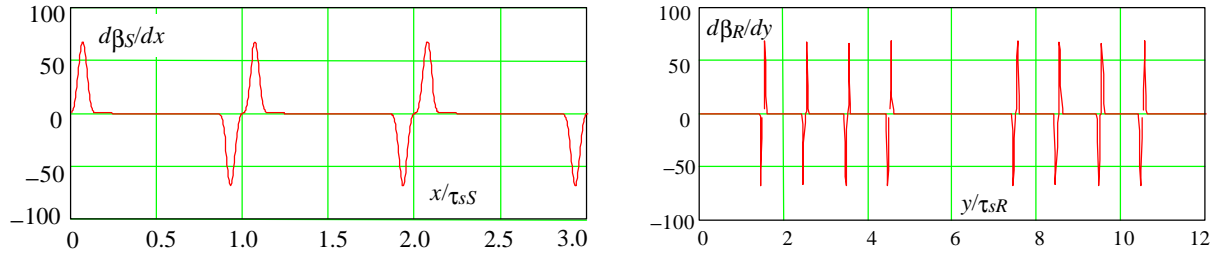


Fig.3 – Notch field function derivatives for the stator and the rotor slotted surfaces ( $c_p = 4$  rotor slots/pole; same stator and rotor slot openings; stator slot pitch/slot opening  $\approx 5.8$ ; rotor slot pitch/slot opening  $\approx 7.7$ ; slot opening/air-gap = 6).

As regards the transformer e.m.f.  $e_{pt}$  of a group, by time derivating under the space integral operator, it becomes:

$$e_{pt}(t, z(t)) = (\mu_0/g) \cdot \ell \cdot N_t \cdot \sum_{k=1}^q \left\{ \int_{x_{ipk}}^{x_{fpk}} \beta_S(x) \cdot \beta_R(x-z) \cdot \left[ \frac{\partial m_S(x,t)}{\partial t} + \frac{\partial m_R(x-z,t)}{\partial t} \right] \cdot dx \right\} . \quad (9)$$

The integration of the eq.s (8) and (9) appears cumbersome, because of the heavy expressions of the involved quantities, and because a different integration solution seems to be required for each instantaneous rotor position  $z(t)$ ; moreover, the time dependence of the m.m.f.s seems to complicate the evaluation: on the other hand, it is possible to extract the time dependent factors out of the integrals, leaving inside just the space dependent terms:

$$e_{pm}(t, z(t)) = -k_m \cdot \Omega \cdot (N_t/a) \cdot \{ J_1(z(t)) \cdot i_1(t) + J_2(z(t)) \cdot i_2(t) + J_3(z(t)) \cdot i_3(t) \} + k_m \cdot \Omega \cdot \left\{ J_f(z(t)) - \sum_{k=1}^q \left[ \beta_S(x) \cdot \beta_R(x-z) \cdot M_{fR}(x-z) \right]_{x_{ipk}}^{x_{fpk}} \right\} \cdot N_f \cdot i_f(t) \quad , \quad \text{with} \quad (10)$$

$$J_p(z) = \sum_{k=1}^q \left\{ \int_{x_{ipk}}^{x_{fpk}} \left[ \frac{\partial \beta_R(x-z)}{\partial x} \cdot \beta_S(x) \cdot M_{fS} [x - (p-1) \cdot 2\tau/3] \right] dx \right\}, \quad p = 1, 2, 3, \quad (11)$$

$$J_f(z) = \sum_{k=1}^q \left\{ \int_{x_{ipk}}^{x_{fpk}} \left[ \frac{d\beta_S(x)}{dx} \cdot \beta_R(x-z) \cdot M_{fR}(x-z) \right] dx \right\}; \quad (12)$$

$$e_{pt}(t, z(t)) = \sum_{u=1,2,3} L_{pu}(z) \cdot di_u/dt + L_{pf}(z) \cdot di_f/dt, \quad p = 1, 2, 3, \quad \text{with} \quad (13)$$

$$L_{pu}(z) = (\mu_0/g) \cdot \ell \cdot (N_t^2/a) \cdot \sum_{k=1}^q \left\{ \int_{x_{ipk}}^{x_{fpk}} \beta_S(x) \cdot \beta_R(x-z) \cdot M_{fS} [x - (u-1) \cdot 2\tau/3] \cdot dx \right\}, \quad p, u = 1, 2, 3, \quad (14)$$

$$L_{pf}(z) = (\mu_0/g) \cdot \ell \cdot N_t \cdot N_f \cdot \sum_{k=1}^q \left\{ \int_{x_{ipk}}^{x_{fpk}} \beta_S(x) \cdot \beta_R(x-z) \cdot M_{fR}(x-z) \cdot dx \right\}, \quad p = 1, 2, 3. \quad (15)$$

Equations (10)-(15) suggest the following remarks:

- the space dependent functions  $J_p(z)$ ,  $J_f(z)$ ,  $L_{pu}(z)$ ,  $L_{pf}(z)$  ( $p, u = 1, 2, 3$ ) can be evaluated off line just once, for a suited number of rotor positions  $z$ , subsequently interpolating the calculated points; thus, when the time dependence is to be taken into account, these space quantities can be considered as known functions;
- $J_p(z)$ ,  $J_f(z)$ ,  $L_{pu}(z)$ ,  $L_{pf}(z)$  are able to correctly model the local stator and rotor slotting field effects (including the effects of partial slot facings [1]), and the actual field and armature winding structures: this property ensures an accurate modelling of all the e.m.f. harmonic contributions, including the well known toothing e.m.f. harmonics, particularly noisy in synchronous machines;
- of course, the adopted approach is rigorously valid just supposing perfectly unsaturated operation, because it implies the application of the superposition principle;
- moreover, it should be noted that, in general, no closed forms can be found for  $J_p(z)$ ,  $J_f(z)$ ,  $L_{pu}(z)$ ,  $L_{pf}(z)$ .

### **Closed form e.m.f. formulation for ideally unslotted structures and stepped m.m.f.s**

The previous equations can be greatly simplified, leading to closed form expressions of the e.m.f.s, if the following simplifying hypotheses are adopted:

- the slot openings are considered very small, compared with the air-gap width: this hypothesis implies that:
$$\beta_S(x) = 1; \quad d\beta_S(x)/dx = 0; \quad \beta_R(x-z) = 1; \quad \partial\beta_R(x-z)/\partial x = 0 \quad \text{for every } x \text{ and } z; \quad (16)$$
- the m.m.f. space distribution is assumed step-wise, with a sharp variation occurring in each slot axis position, whose amplitude depends on the instantaneous total current included in the considered slot; if  $\sigma(x)$  indicates the step function:

$$\sigma(x) = 1 \text{ for } x \geq 0, \quad \sigma(x) = 0 \text{ for } x < 0, \quad (17)$$

the stepped m.m.f.s expressions consist of superposed  $\sigma(x)$  and  $\sigma(y)$  terms, suitably amplified and displaced; for example, in a stator, single-layer, winding, we would have [1]:

$$M_{fS\sigma}(x) = \sum_{j_S=1}^q M_{cS\sigma} \left[ x - \left( \frac{q-1}{2} - (j_S-1) \right) \cdot \tau_{sS} \right], \quad M_{cS\sigma}(x) = \sigma[\cos(\pi \cdot x/\tau)] - 1/2, \quad (18)$$

while the rotor m.m.f. is modelled as follows:

$$M_{fR\sigma}(y) = \sum_{j_R=1}^{c_p} M_{cR\sigma} \left[ y - \left( \frac{c_p-1}{2} - (j_R-1) \right) \cdot \tau_{sR} \right], \quad M_{cR\sigma}(y) = \sigma[\cos(\pi \cdot y/\tau)] - 1/2, \quad (19)$$

where  $c_p$  is the number of rotor slots/pole,  $\tau_{sS}$  and  $\tau_{sR}$  the stator and rotor slot pitches respectively.

By adding the subscript  $\sigma$  to all the quantities evaluated according to the previous hypotheses (and indicating with  $M_{fS\sigma}(x)$  the m.m.f. of a generic stator winding - one or two layers, integer or shorted pitch -), it follows:

$$J_{p\sigma}(z) = 0 \quad p = 1, 2, 3; \quad J_{f\sigma}(z) = 0; \quad (20)$$

$$e_{p\sigma}(t, z(t)) = -k_m \cdot \Omega \cdot \left\{ \sum_{k=1}^q \left[ M_{fR\sigma}(x-z) \right]_{x_{ipk}}^{x_{fpk}} \right\} \cdot N_f \cdot i_f(t) + \sum_{u=1,2,3} L_{pu\sigma}(z) \cdot \frac{di_u}{dt} + L_{pf\sigma}(z) \cdot \frac{di_f}{dt}, \quad (21)$$

$$\text{with } L_{pu\sigma}(z) = (\mu_0/g) \cdot \ell \cdot (N_t^2/a) \cdot \sum_{k=1}^q \left\{ \int_{x_{ipk}}^{x_{fpk}} M_{fS\sigma} [x - (u-1) \cdot 2\tau/3] \cdot dx \right\}, \quad p, u = 1, 2, 3, \quad (22)$$

$$L_{pf\sigma}(z) = (\mu_0/g) \cdot \ell \cdot N_t \cdot N_f \cdot \sum_{k=1}^q \left\{ \int_{x_{ipk}}^{x_{fpk}} M_{fR\sigma}(x-z) \cdot dx \right\}, \quad p = 1, 2, 3. \quad (23)$$

In the case of no-load operation with constant values of the rotational speed  $\Omega$  and of the field current  $I_f$ , from eq. (21), the expression of the no-load e.m.f.  $e_{0\sigma}(t)$  induced in a group of  $q$  stator coils reduces to:

$$e_{0\sigma}(t, z(t)) = -k_m \cdot \Omega \cdot \left\{ \sum_{k=1}^q [M_{fR\sigma}(x-z)]_{x_{ipk}}^{x_{fpk}} \right\} \cdot N_f \cdot I_f, \quad (24)$$

in which  $M_{fR\sigma}(y)$  must be evaluated by eq.(19), considering the initial and final active side positions (see eq.(4)).

In case of shorted pitch coils, the active side positions are expressed by:

$$x_{ipk} = x_{ip1} + (k-1) \cdot \tau_{sS}; \quad x_{fpk} = x_{ipk} + y_{csh} \cdot \tau_{sS} \quad \text{with} \quad y_{csh} = 3 \cdot q - \varepsilon. \quad (25)$$

Choosing as initial position  $x_{ip1}$  of the first active side the following one:

$$x_{ip1} = \tau_{sR} \cdot (c_p - 1) / 2, \quad (26)$$

equation (24) becomes:

$$e_{0\sigma} = k_m \cdot N_t \cdot \Omega \cdot \left\{ \sum_{k=1}^q \left[ \sum_{j_R=1}^{c_p} \left( \sigma \left[ \cos \left( \frac{\pi}{\tau} \cdot ((x_{ipk} + (k-1) \cdot \tau_{sS} - z) + (j_R - 1) \cdot \tau_{sR}) \right) \right] + \right. \right. \right. \left. \left. \left. - \sigma \left[ \cos \left( \frac{\pi}{\tau} \cdot ((x_{ipk} + (k-1 + y_{csh}) \cdot \tau_{sS} - z) + (j_R - 1) \cdot \tau_{sR}) \right) \right] \right) \right] \right\} \cdot N_f \cdot I_f \quad (27)$$

Equation (27) can be simplified if angular peripheral variables will be used instead of linear ones, according to the following correspondences:

$$\xi = \pi \cdot x / \tau; \quad \vartheta = \pi \cdot y / \tau; \quad \zeta = \pi \cdot z / \tau; \quad \alpha_S = \pi \cdot \tau_{sS} / \tau; \quad \alpha_R = \pi \cdot \tau_{sR} / \tau; \quad (28)$$

by some manipulations, the final expression of the no-load e.m.f. becomes:

$$e_{0\sigma}(\zeta(t)) = \frac{\partial \Psi}{\partial \zeta} \cdot \frac{d\zeta}{dt} = k_m N_t N_f I_f \Omega \cdot \sum_{j_S=1}^q \left[ \sum_{j_R=1}^{c_p} \left( \sigma \left\{ \cos [(j_S - 1) \cdot \alpha_S + (j_R - 1) \cdot \alpha_R - \zeta] \right\} + \right. \right. \left. \left. - \sigma \left\{ \cos [(j_S - 1 + 3 \cdot q - \varepsilon) \cdot \alpha_S + (j_R - 1) \cdot \alpha_R - \zeta] \right\} \right) \right]. \quad (29)$$

Fig.4 shows some no-load stepped e.m.f. waveforms, all referred to the reference e.m.f.  $E_r = k_m \cdot N_t \cdot N_f \cdot I_f \cdot \Omega \cdot q \cdot c_p$ : as expected, the higher are  $q$  and  $c_p$ , the more the no-load e.m.f. waveform is similar to a sinusoid.

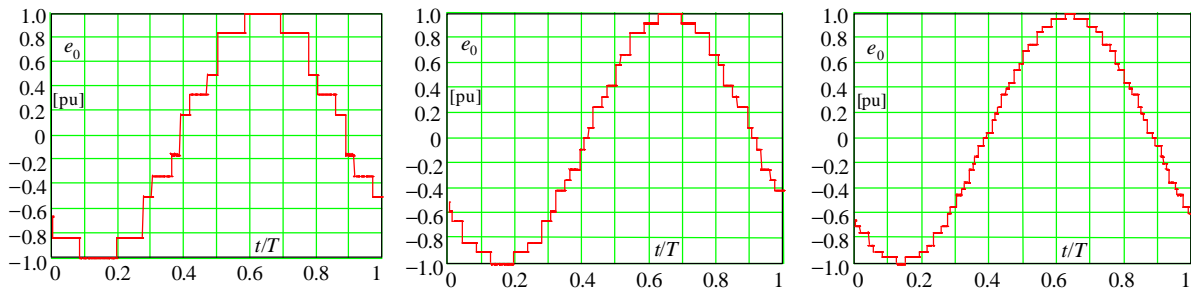


Fig.4 – No-load e.m.f. waveforms (referred to  $E_r = k_m \cdot N_t \cdot N_f \cdot I_f \cdot \Omega \cdot q \cdot c_p$ ) of the group of  $q$  series connected coils under one pole, for a double layer stator winding with shorted pitch coils; model hypotheses: no slotting effects, stepped m.m.f. distributions;

left:  $q = 2, \varepsilon = 1, c_p = 3$ ;

middle:  $q = 4, \varepsilon = 2, c_p = 3$ ;

right:  $q = 4, \varepsilon = 2, c_p = 5$ .

## Comparison among analytical and FEM evaluation of the e.m.f. waveforms

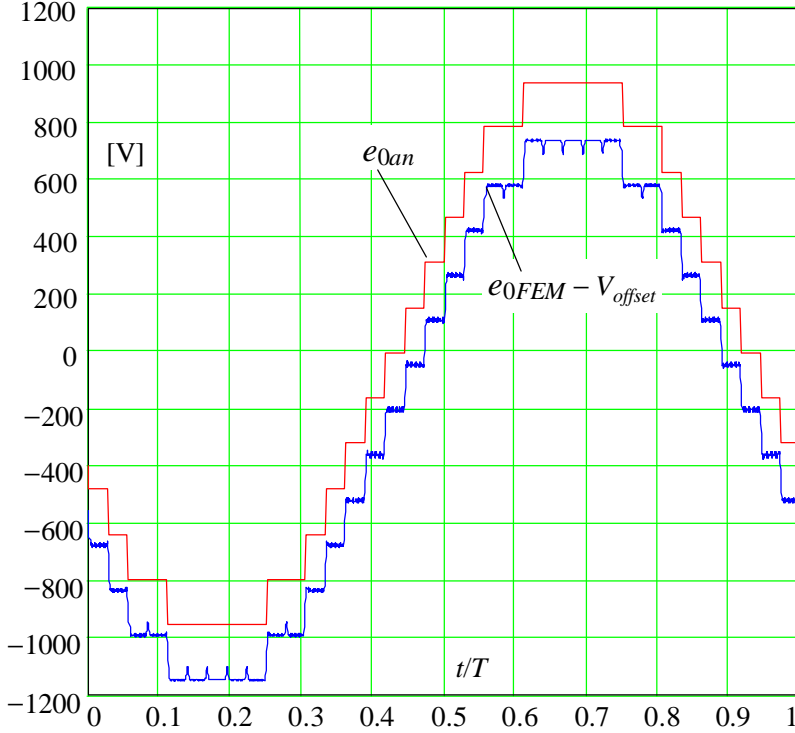
### Case of ideally unslotted machine, with stepped m.m.f.s

This first, idealised machine comparison is aimed to show the correspondence among analytical results, connected with the use of eq. (29), and numerical results, obtained by suited simulations, performed by means of FEM transient tools [2]: fig.5 shows the no-load e.m.f. waveforms for a machine equipped with a single-layer

stator winding of  $q = 3$  slots/(pole-phase) and integer pitch coils ( $\epsilon = 0$ ), with a field winding with  $c_p = 4$  slots/pole (see Table I; the FEM e.m.f. curve have been vertically displaced by  $V_{offset} = 200$  V, for visibility).

The numerical FEM curve is fairly similar to the analytical one, except for the following aspects:

- a high frequency ripple arises in the FEM waveform, that can be totally ascribed to numerical noise;
- some notches can be observed in the FEM waveform, occurring because of the small, but non-zero, slot openings adopted in the FEM model (thus,  $\beta_S(x)$  and  $\beta_R(y)$  are not exactly equal to 1 everywhere).



N° of poles	2
N° of stator slots	18
N° of stator parallel paths	1
N° of stator phases	3
N° of winding layers	1
N° of rotor slots / pole	4
mech. angle among rotor slots	30°
stator internal diameter D [m]	1
air-gap [mm]	5
stator, rotor slot openings [mm]	5

Fig.5 – No-load e.m.f. simulated waveforms for the machine whose data are given in Table I: almost closed slot openings have been considered for the FEM model; the FEM curve is vertically displaced by  $V_{offset} = 200$  V, in order to improve each curve visibility:  $e_{0an}(t)$  = analytical evaluated waveform (see eq. (29), for  $\epsilon = 0$ );  $e_{0FEM}(t)$  = F.E.M. simulation result.

#### Case of a doubly slotted machine, with non-stepped m.m.f.s

In this case, equations (10) and (13) should be used, in which the notch field functions and their derivatives are included, and the stator and rotor m.m.f. space distributions  $M_{fS}(x)$  and  $M_{fR}(y)$  take into account the tanh-wise level variations, modelling the interpolar field behaviour [1].

In the particular case of the steady-state, no-load, operation, just the motional e.m.f. exists; considering the  $q$  series connected coils of the group under one pole of the phase  $p$ , its e.m.f. is given by (see eq.s (10), (12)):

$$e_0(t, z(t)) = k_m \cdot \Omega \cdot \left\{ \sum_{k=1}^q \left\{ \int_{x_{ipk}}^{x_{fpk}} \left[ \frac{d\beta_S(x)}{dx} \cdot \beta_R(x-z) \cdot M_{fR}(x-z) \right] dx \right\} + \left[ \beta_S(x) \cdot \beta_R(x-z) \cdot M_{fR}(x-z) \right]_{x_{ipk}}^{x_{fpk}} \right\} \cdot N_f \cdot I_f \quad (30)$$

Fig.6 shows the comparison among the no-load phase-to-neutral e.m.f. evaluated analytically (by means of eq. (30), multiplied by the N° of poles  $N_p = 2$ ), and by a transient FEM simulation, performed for the same machine geometry described in Table I, and in the same operating conditions, except for the stator and rotor slot openings, here assumed equal to 30 mm. The following remarks can be made:

- the analytical and FEM results are very close, confirming the correctness of the developed model;
- the waveform disturbances due to the stator and rotor slotting effects are very important: they correspond to the classical, well known e.m.f. tooth harmonics, whose amplitude can be reduced by using skewing only (not considered here, just with the aim to show and estimate the evaluation accuracy of this effect);

- apart from the slotting effects, the envelope e.m.f. waveform presents voltage levels near to those evaluated in the ideal case of fig.5.

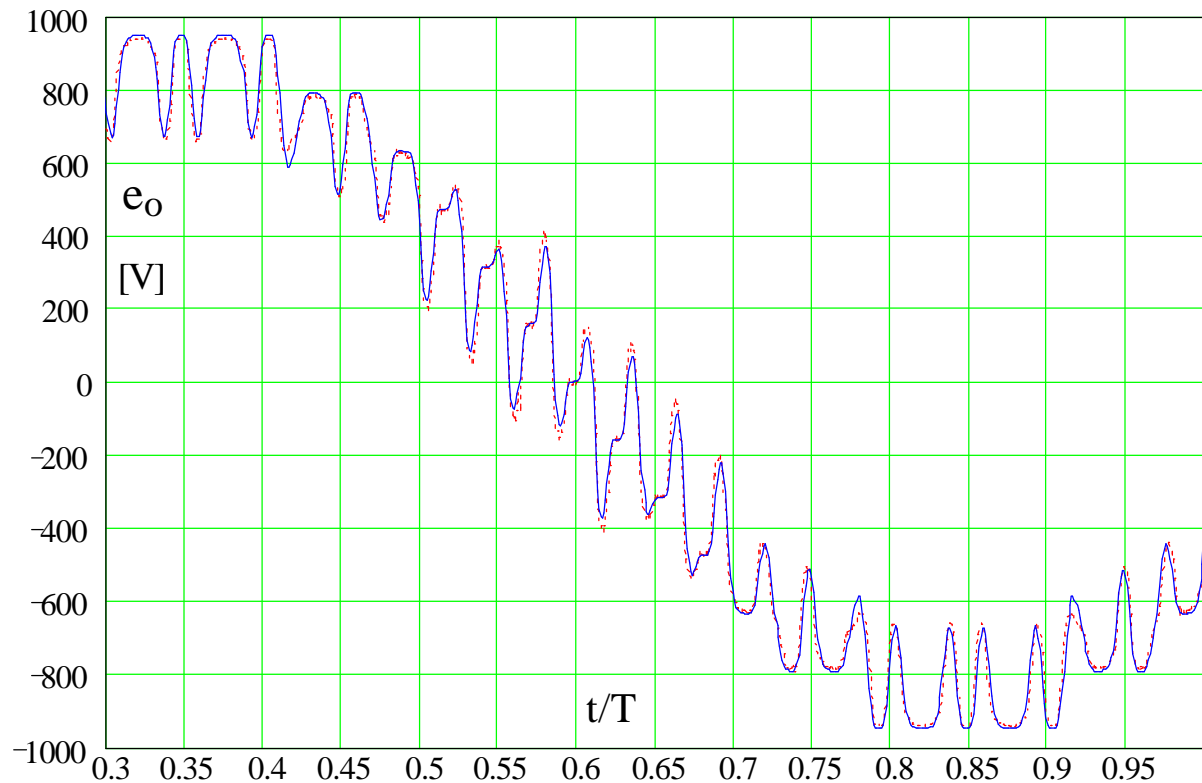


Fig.6 – No-load e.m.f. simulated waveforms for the machine whose data are given in Table I, except for the stator and rotor slot openings, here equal to 30 mm: operating conditions corresponding to the same considered in fig.5 simulation; solid line = analytical evaluated waveform (see eq. (30)); dotted line = FEM transient simulation waveform.

## Conclusion

An analytical method for the evaluation of the e.m.f. waveform at the stator winding terminals of an isotropic synchronous machine has been developed, taking into account the stator and rotor slotting effects and the actual time dependent current waveforms and winding turns space distribution: its good level of accuracy has been verified by means of comparisons with transient FEM simulations.

The described method is well suited to the analytical, accurate, evaluation of e.m.f. waveforms, in many modelling and design problems concerning transient and steady-state analysis of slotted electrical machines.

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