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Using Mechanical Metamaterials in Guitar Top Plates: A Numerical Study

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Featured Application: The use of mechanical metamaterials in musical instruments could be an excellent way to engineer the wood of the instrument to obtain a particular sound.

Abstract: It has recently been shown that the mechanical properties of thin, rectangular wooden plates can be tuned by carving them with specific patterns of perforations, effectively realising a 2D wooden mechanical metamaterial. Such a material is of great interest for the construction of musical instruments, as it could allow a new degree of creative control for makers. Furthermore, issues with the shrinking supplies of tone-woods could be alleviated as wood samples that don not meet the desired requirements could simply be altered, instead of being discarded. In this work, we study the effect of the use of these metamaterials in the soundboards of classical guitars. By way of simulations, we evaluate their impact on the modal behaviour and on the sound pressure level of the instrument, as well as on its ability to sustain the load exerted by the strings. Our results show that the metamaterials can tune the instrument's response without compromising its structural integrity. We thus conclude that the use of wooden mechanical metamaterials in the soundboards of classical guitars is feasible and, in many ways, beneficial, not the least since it opens the door to using non-traditional woods with bespoke density and stiffness.

Keywords: musical instruments; modal analysis; finite element modelling; new materials for musical instruments

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1. Introduction

In the last few decades, scientists have focused on the theoretical, experimental, and numerical study of mechanical metamaterials, which exhibit unique mechanical properties derived from their structure rather than their composition [1]. An illustrative overview of different types of mechanical metamaterials and the state-of-the-art in this field can be found in [2–6]. In particular, a major challenge in the last ten years has been how to obtain lightweight mechanical metamaterials while maintaining their stiffness [3,7,8]. This is a topic that could deeply change the way we make musical instruments, as one is looking precisely for that combination of light weight and stiffness.

Traditional Western wooden instruments have been historically built using the materials available to craftspeople in their region. Norway Spruce (*Picea abies*) has been a favourite of makers since the middle ages, for everything from piano soundboards to violin top plates. Wood, though, presents a high degree of variability when it comes to its mechanical properties [9]. This results in unpredictable and inconsistent mechanical response across different samples of the same wood species [10]. Furthermore, the specific kind of woods that are most used in instrument making are prone to shortages due to deforestation or climate-change-related habitat shrinkage [11–13]. The possibility of deliberately engineering the vibrational behaviour of a material would be, therefore, of great interest for the design of musical instruments. Indeed, not only could it help deal with the issues of the availability and variability of wood, but it could constitute a new tool in the luthier's arsenal, allowing for the development of innovative instrument designs. Mechanical metamaterials, by their very nature, could constitute one such tool. However, very few studies have been conducted on their application to musical instruments [14,15] and even less to string instruments. Notably, in [16], one of us showed that the application of a tunable mechanical metamaterial to the soundboard of an acoustic guitar allows for controlled modification of its frequency response.

Further work [17] has demonstrated that the elastic behaviour of thin, rectangular wooden plates can be purposefully altered by carving patterns of holes. Inspired by simple geometries of widely studied mechanical metamaterials [18–21], the authors in [17] propose a novel method of manipulating the properties of thin wood plates. There, they studied the influence of elliptical and circular holes in rectangular wooden plates, both by simulations and experiments. They demonstrated that the mechanical parameters of the plate and, in particular, the longitudinal and radial moduli of elasticity can be controlled by changing the geometry of the holes, thus obtaining light, yet stiff plates and effectively creating a tunable metamaterial. This is a promising result for instrument making, as it could open the possibility for luthiers and manufacturers to directly control and tune the mechanical properties of the soundboards, resulting in greater consistency between instruments and allowing for innovative and better-performing instrument designs.

The objective of this article is to study the effect of those mechanical metamaterials when used on the top plate of a classical guitar. We set out to study how they would impact the sound of the instrument, as well as the structural integrity under the tension of the strings and the sound production. Our goal is to find out whether these mechanical metamaterials can produce differently sounding instruments and how much control they allow over the final response of the guitar.

2. Materials and Methods

The investigation was carried out via the finite element method (FEM), the standard method for the simulation of musical instruments in the literature [22–29]. The reference 3D model of the guitar body was based on an 1884 instrument by Antonio de Torres, as described in [30], and it was realised with Autodesk Fusion 360®. For the computation of the sound pressure level of the instrument, we coupled the FEM and boundary element method (BEM), as explained in [31].

This model includes a traditional bracing design, with 7 fan braces glued on the underside of the lower bout of the soundboard, all 3 mm thick (see Figure 1a). A simplified model of the bridge, used in the static load simulations, is also included. The soundboard was then altered by cutting a pattern of elliptical holes in the underside, to a depth of 2 mm. We note that, unlike in the studies in [17], the plate here is not cut all the way through. Instead of holes, the metamaterial is composed of periodic cavities, of 2 mm in depth. This leaves a layer of solid material 0.5 mm thick on the outside of the guitar. Moreover, the material of the soundboard here is not homogeneous over its entire extent, as the cavities are cut only in the lower bout of the instrument. The hole patterns are generated by the juxtaposition of several copies of the same unitary cell (see Figure 1). The cell is rectangular, with a size of 26 × 12 mm for all simulations, but we distinguish between two kinds of holes based on the orientation: here and throughout the rest of this paper, when the long side is parallel to the longitudinal axis of the soundboard's wood, we talk about *longitudinal* holes, while when it is aligned with the radial axis, we talk about *radial* holes. The holes are centred with respect to the cell, and their size is varied while keeping the aspect ratio constant, by changing a dimensionless scaling parameter $p \leq 1$, which multiplies the sizes of the major and minor axes of the ellipses. With $p = 1$, the major semiaxis of the elliptical holes is 10 mm long, while the minor semiaxis is 2.5 mm long.

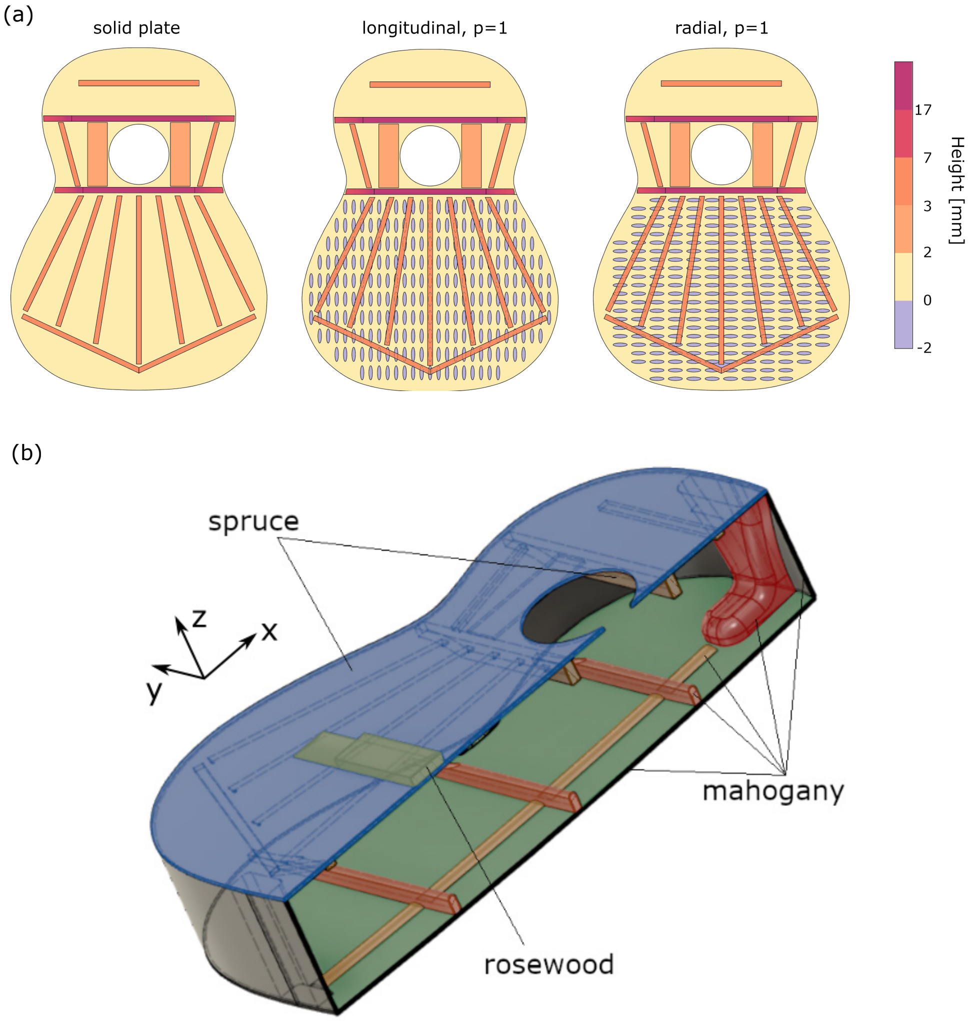

Figure 1. (**a**) Diagrams of the soundboards in three different configurations: solid, with longitudinal holes (p = 1), and with radial holes (p = 1). The figure shows the bracing and the hole patterns, and each surface is coloured according to its height with respect to the lower surface of the plate. (**b**) Three−dimensional cut view of the model of the instrument's body, showing all the individual components. The choice of material for the main parts is also reported.

All the materials were modelled as linear elastic and orthotropic, taking the mechanical properties for each particular wood (12% moisture content) from [32]. As detailed in Figure 1, three different woods were used: Engelmann spruce for the soundboard and the soundboard bracing ($E_L^{\text{top}} = 9.8$ (GPa), $E_R^{\text{top}} = 1.3$ (GPa), $\rho^{\text{top}} = 350$ (kg/m^3)), Honduran mahogany for the sides, back, and back braces ($E_L^{\text{back}} = 10.8$ (GPa), $E_R^{\text{back}} = 10.1$ (GPa), $\rho^{\text{back}} = 450$ (kg/m^3)), and rosewood for the bridge ($E_L^{\text{bridge}} = 15.8$ (GPa), $E_R^{\text{bridge}} = 1.64$ (GPa), $\rho^{\text{bridge}} = 609$ (kg/m^3)). Appropriate care was taken to correctly define the material axes of each component of the model. The simulations were performed using the COMSOL Multiphysics® simulation software, in particular the Solid Mechanics interface from the Structural Mechanics module. For the solid–air coupling, there are then 3 Multiphysics modules that deal with the boundaries between the domains: Solid Mechanics-Pressure Acoustic FEM, Solid Mechanics-Pressure Acoustic BEM, and Pressure Acoustic FEM-Pressure Acoustic BEM; this last module works on the sound-hole virtual surface where the portion of air inside the cavity meets the outer air. The numerical model here differs from the one without air in one way: whereas earlier, we ignored the effect of structural damping, here it is accounted for in the form of proportional, or Rayleigh damping. In particular, following [22], we used a stiffness-proportional damping matrix: $C = \beta K$, where C and K are the damping and stiffness matrices of the finite element model and $\beta = 2 \times 10^{-6}$ s. No mass-proportional damping term is included in order to avoid over-damping at low frequencies.

3. Results

3.1. Eigenfrequency Studies

We performed a series of eigenfrequency studies, where we compared and contrasted the eigenfrequencies and modal shapes of the unbraced soundboards first, then of the soundboards including the bracing, and finally, of the complete body of the guitar. These simulations were aimed at studying the effect of the metamaterials in these different configurations, with the goal of finding out whether their impact remains relevant when connecting the soundboards to the braces first, then to the rest of the body. In the following, we only show the results for the largest hole sizes ($p = 1$) in both orientations; simulations with $p < 1$ were also performed, but are omitted here for the sake of brevity, as they mostly yielded the same results shown here, albeit scaled down.

3.1.1. Unbraced Soundboards

At first, we computed the first 12 eigenfrequencies (and relative eigenmodes) of the plates detached from the rest of the body and without any bracing. These simulations were performed in simply supported boundary conditions, which were implemented by imposing a fixed constraint on the lower outer edge of the plates, thus preventing displacement at the boundary while still allowing for free bending rotation. The use of simply supported boundary conditions was meant to obtain a situation resembling that of the plate mounted to the rest of the body [33].

The computed eigenfrequencies for the plates with longitudinal ($p = 1$) and radial ($p = 1$) holes, as well as the solid plate are reported in Figure 2a, which also shows the percentage variations for the two metamaterial plates with respect to the solid case. Clearly, most of the eigenfrequencies were lowered, if slightly, when introducing the metamaterial. There seems to be a clear separation between the case of the longitudinal holes and that of the radial holes; however, the eigenfrequencies were indeed lowered far more in the latter case, with those for modes 5 and 12 showing a decrease of $\sim$8%, whereas the variation with the longitudinal holes did not exceed $\sim$5%.

3.1.2. Braced Soundboards

Here, the plates were simulated detached from the rest of the instrument, but the bracing was included. The first 12 eigenfrequencies for the cases of the solid plate, the longitudinal holes with $p = 1$, and the radial holes with $p = 1$ are shown in Figure 2. The figure also shows the percentage variation in the frequencies relative to those of the solid plates for all the simulations that were performed. The differences between these three cases did not seem particularly significant, with the largest being the 6% increase in the frequency of the first mode for the longitudinal $p = 1$ case. Some of the frequencies were clearly increased or decreased with varying hole sizes, but overall, we cannot identify collective upwards or downwards trends.

3.1.3. Complete Body

We now move on to the modal analysis of the complete body of the instrument. The first 12 eigenfrequencies in this case are reported in Figure 2c, together with the percentage variations relative to the case with the solid soundboard. The variation with respect to the solid case was even smaller here, remaining in all cases below 3%, and clear overall trends cannot be identified here either. The difference between the two configurations of the metamaterial was minimal and probably due to the decrease in the radial stiffness. This seems to imply, however, that the effects of the metamaterial are independent of the hole orientation.

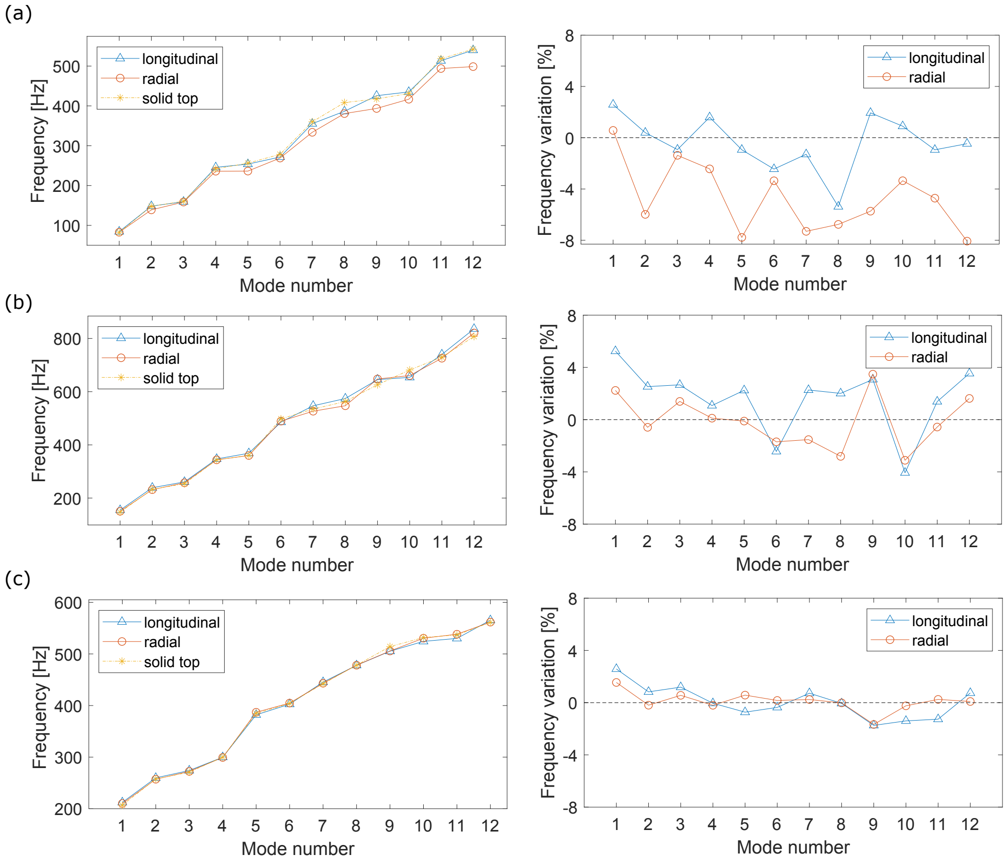

Figure 2. Eigenfrequencies values and percentage variations with respect to the case of the solid top plate. (**a**) Unbraced plates in simply supported boundary conditions, with longitudinal ($p = 1$) holes and radial ($p = 1$) holes. (**b**) Braced plates in simply supported boundary conditions, with longitudinal ($p = 1$) holes and radial ($p = 1$) holes. (**c**) Complete body, with longitudinal ($p = 1$) holes and radial ($p = 1$) holes.

3.2. Displacement Under Tension

To quantify the effect of the holes in the stability of the guitar, static load tests were carried out, simulating the tension of the strings. The load was applied to six circular areas with a diameter of 2 mm located in the back surface of the bridge, corresponding to the points where the strings are mounted on a real guitar. Assuming a typical range of tensions of 50–80 N for nylon guitar strings [33], we used a total force of 480 N in the positive x (vertical) direction.

These load tests were computed with various choices of hole size for the top plate, with both longitudinal and radial orientation, as well as with the solid top. Figure 3a shows the normal displacement fields for the longitudinal ($p = 1$) and radial ($p = 1$) holes together with the original solid top, used as a reference. The soundboard exhibits the expected warped configuration, with a slight bulge behind the bridge and a slight dip in front. This deformation is accentuated with the introduction of the metamaterial, and increases with the hole size. Moreover, the soundboard is more deformed, if slightly, in the case of the radial holes. This is consistent with the results in [17], where the introduction of elliptical holes was found to be correlated with higher effective values of Young's modulus in the direction parallel to the major axis.

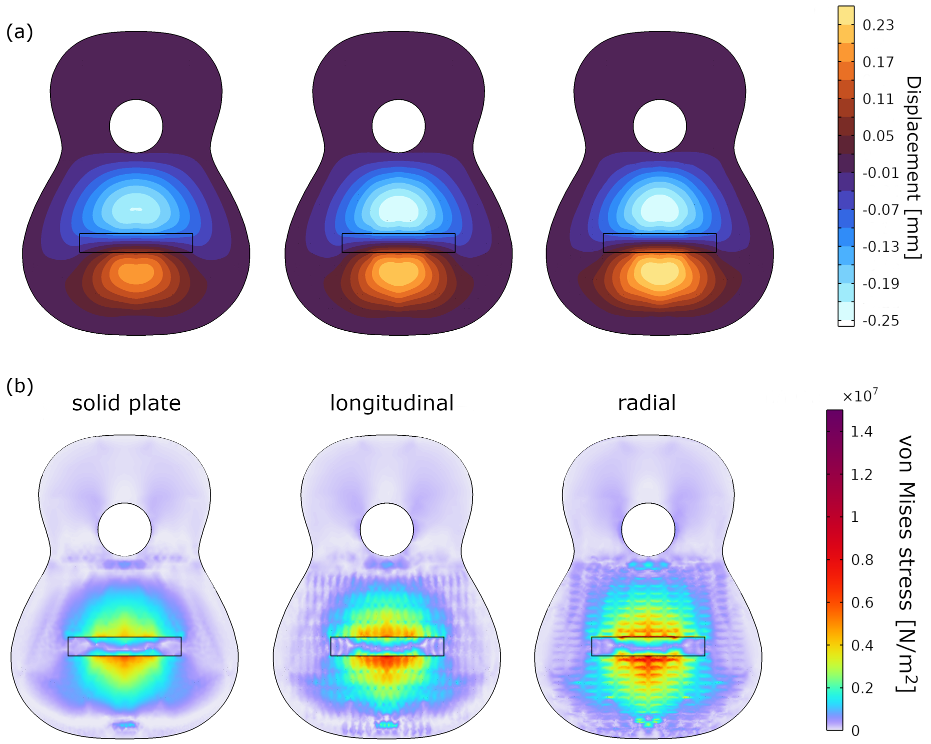

Figure 3. Effect of the static load of the strings in the guitar top plate for solid, longitudinal ($p = 1$), and radial ($p = 1$) holes. The complete guitar was simulated, but we only show the displacement of the top surface. (**a**) z−displacement shows a slight increase for both hole patterns, but the difference between patterns is not very noticeable. Notice, however, that the displacement field for the radial holes is larger, indicating a decreased stiffness of the top compared to the longitudinal pattern. (**b**) von Mises stress on the top plate under the same load. In this case, the stress distribution between the two hole patterns is clearly different, the radial holes showing a greater concentration of stresses on the bridge area, which could lead to premature failure of the instrument. The longitudinal holes, on the other hand, albeit having more concentrated stresses than the solid top case, are able to spread the load in a more even way longitudinally by the "channels" of solid wood left by the pattern.

In order to further assess the capacity of the metamaterial instruments to withstand the tension of the strings, we can also look at the distribution of stresses in the soundboard. Unfortunately, we cannot develop a way to predict failure, as failure theories in anisotropic materials require the definition of a number of constants to be determined experimentally [34].

Figure 3b shows the von Mises stress for the case of the solid plate, of the longitudinal holes ($p = 1$), and of the radial holes ($p = 1$). Contrary to the displacement, the stress distribution between the two hole patterns is clearly different, with the radial holes showing a greater concentration of stresses on the bridge area. This could potentially lead to premature failure of the instrument for the radial hole patterns. The longitudinal holes, on the other hand, albeit having more concentrated stresses than the solid top case, are able to spread the load in a more even way longitudinally by the "channels" of solid wood left by the pattern, therefore actually making a stronger top than the radial pattern for an equivalent density. This difference of stiffness for the same equivalent density should imply a higher radiation coefficient, and this is the subject of the next subsection.

3.3. Sound Pressure Level

We performed frequency-domain simulations to obtain the Sound Pressure Level (SPL) of the complete body of the instrument. These were obtained by imposing a normal harmonic load on a circular area with a 2 mm radius on the top surface of the soundboard, located under the bridge, in the middle of the lower bout.

The frequencies analysed in the second study went from 5 to 500 Hz with a step of 5 Hz and from 510 to 1000 Hz with a step of 10 Hz. To obtain the SPL frequency response plot, the SPL for each frequency was computed as the average of the SPL evaluated over the surface of a sphere, with a 1m radius, built around the guitar.

Figure 4a shows the results for the solid top and the the two metamaterial ones. There are clear differences in the Helmholtz resonance, which occur at roughly the same frequency since the volume variation in the air cavity is insignificant (<2%), as well as in the 400 Hz range and a further large difference at 650Hz, but with a smaller absolute pressure level, so probably less audible. To stress the difference, the variation in pressure level is shown in Figure 4b.

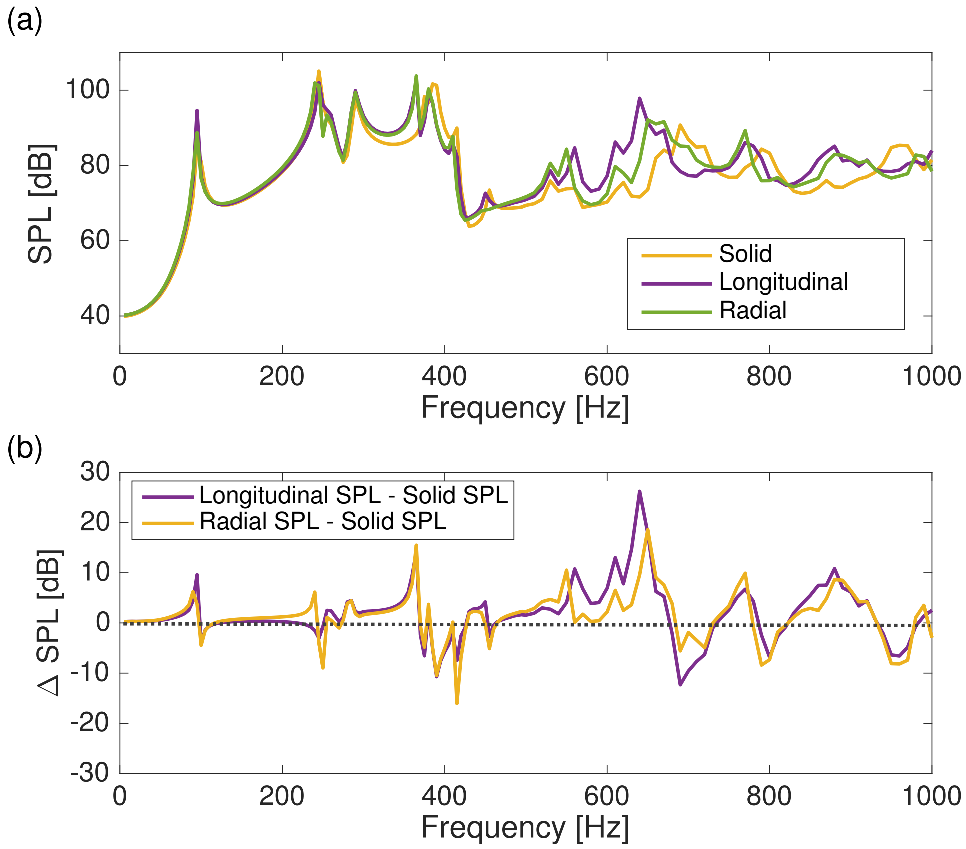

Figure 4. Sound pressure level of the complete body of the instrument for the solid top and two metamaterial configurations. These were obtained by imposing a normal harmonic load on a circular area with a 2 mm radius on the top surface of the soundboard, located under the bridge, in the middle of the lower bout. (**a**) Absolute value of the SPL. (**b**) Relative difference between the guitar with a solid top and the guitar with a metamaterial top plate.

4. Discussion

In this article, we showed by means of simulations that the use of mechanical metamaterials for the top plate of a classical guitar is a feasible option regarding its load-bearing capacity, as well as its dynamic response. The results are extremely relevant as they present a way to, in principle, compensate the intrinsic material variation that characterises wood by modifying the pattern of the metamaterial to suit particular needs. We showed that, for all practical purposes, tops made with metamaterials behave in a way that is indistinguishable from solid wood. By doing so, the variability of wood becomes a treatable feature, much like other design variables of the guitar, and not a given. Using the material as a design parameter could open a realm of possibilities, mostly when used with artificial intelligence approaches [35], such that one can start engineering musical instruments as opposed to crafting them.

Our results show that the dynamic response, measured via the eigenfrequencies of the instrument, is not very sensitive to the orientation of the metamaterial, i.e., to its longitudinal stiffness. This is consistent with existing studies on the impact of the variation of the mechanical properties of different components on the vibrational response of the guitar. In [36], the density of the top plate is the second-most relevant feature, whereas its stiffness has little relevance. Our findings confirm this, given that, with the patterns we used, the density difference between the two orientations of the holes is negligible, whereas their effective radial and longitudinal stiffnesses will vary considerably. A guitar is not only an instrument that needs to vibrate though, it is also necessary that it bears the load of the strings without breaking or deforming too much so as to influence its playability. Indeed, the orientation of the holes does seem to have a clearer impact on the structure's load-bearing capacity, as we found with our static load simulations. Our findings indicate that metamaterial soundboards with longitudinal holes are better suited to withstand the tension of the strings. Given this advantage, therefore, this choice of orientation seems to be overall preferable.

In terms of sound production, though, the picture is more nuanced. There are several studies in the literature that have related the frequency response of an instrument to its perceived quality [33,37,38], and this could even influence the quality of the instrument [39]. The results presented there showed that the inclusion of metamaterials would have both positive and negative effects in the sound quality, which need to be balanced by the guitar maker to achieve a desired sound. The introduction of parametric metamaterials allows one such a way to fine-tune the frequency response of the instrument.

The implications of these results for contemporary guitar making, and instrument making in general, are far fetching. The constant and sustained habitat reduction that tone-wood suffers due to global warming [12] is not going to be reversed anytime soon: making do with lower-grade woods may be the only possibility to continue making traditional instruments. These results present a straightforward way to use different woods in the construction of guitars by tuning their density and stiffness to a desired, perhaps optimal, value.

Author Contributions: Conceptualisation, S.G. and C.E.; methodology, S.G.; software, M.L. and G.L.; investigation, M.L.; writing—review and editing, M.L., C.E. and S.G.; visualisation, M.L.; supervision, S.G.; funding acquisition, C.E., F.A. and A.S. All authors have read and agreed to the published version of the manuscript.

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