Discrete-Communication-based Bipartite Tracking of Networked Robotic Systems Via Hierarchical Hybrid Control

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***Abstract*—This paper investigates the bipartite tracking problem of networked robotic systems (NRSs) subject to input disturbances, discrete communications and signed directed graphs. Two new classes of hierarchical hybrid control algorithms (HHCAs), which involve both continuous and discontinuous signals in a uniform framework, are designed to solve the aforementioned problem in the model-independent control manner, i.e., without using the prior information of the system model. Besides, with the help of the Lyapunov statement and hybrid system theory, we establish several sufficient conditions for guaranteeing the convergence of the proposed hybrid algorithms. Finally, numerical examples are presented to illustrate the effectiveness of the proposed results.**

***Index Terms*—Discrete communication, bipartite tracking, networked robotic systems (NRSs), model-independent control, hierarchical hybrid control algorithm (HHCA).**

I. Introduction

Networked robotic system (NRS) is a group of artificial autonomous systems which uses network to communicate with each other in order to fulfill one or multi global tasks [1]. The NRS has many practical applications such as working at the destruction area to gather information, moving larger-size objects cooperatively and human-based recognition and rescue operations [2], [3]. Additionally, the distributed control approaches have recently attracted increasing attentions from various research communities, owing to their extensive applications in solving the problems of coordination tracking [4]-[6], formation control [7], [8], synchronization [9]-[11] and flocking [12]-[14]. It is worth pointing out that the above-mentioned literatures are mainly focused on the case that there are only cooperative interactions within the networked robotic or Euler-Lagrange systems.

In recent years, the distributed control approaches for multi-agent systems (MASs) with antagonistic interactions have aroused consideration interests [15]-[17], since both cooperative and competitive interactions coexist in some real world networks. For example, the phenomena of competition and antagonism among different individuals and groups are ubiquitous in nature, namely, animal groups compete with other ones for limited natural resources. Besides, the graph with negative edges is represented as signed graphs, i.e., there exist both the negative and positive elements in the adjacency matrix of communication graph. Increasing efforts have been devoted to the distributed control in the case of signed graphs since Altafini's pioneering work [15]. Considering a dynamic leader, the distributed bipartite tracking problem has been addressed for linear MASs in [18]. Taking the input saturation into consideration, the distributed bipartite consensus problem has been investigated for homogeneous generic linear agents in [19]. Considering nonidentical matching uncertainties, the bipartite consensus and tracking problem of MASs has been studied in [20]. Finite-time bipartite consensus for MAS on directed signed networks has been addressed in [21]. In [22], the bipartite synchronization with delayed velocity coupling of harmonic oscillator systems has been studied. Note that the above results mainly focus on first-, second-, and high-order integrators, as well as Lipschitz-type nonlinear systems. However, the bipartite tracking problem of NRSs remains unsolved, because all the above results cannot be directly extended to solve this challenging problem.

Note that the aforementioned results on distributed control approach are designed based on the system model, i.e., the information of system model are needed in the process of control design, which is called as model-based control. However, in practical applications, it is quite difficult to obtain the exact dynamical models, i.e., the model uncertainties are inevitable. Therefore, model-independent control has recently become a hot topic in practical control. Using model-independent approach, consensus problem has been studied for MASs described as Euler-Lagrange equations in [23]. In [24], based on model-independent control, consensus tracking problem of MASs has been addressed under fixed and iteration-varying topologies. However, there are a few researches on the bipartite tracking by using model-independent approach.

On the other hand, due to the capability limitation of digital sensors and controllers, it is difficult to ensure information transmission among the agents occurs in continuous time. There are many outstanding works to handle consensus problem for MASs with discrete communications [25]-[28]. In

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[25], the consensus problem has been investigated for second-order MASs. Motivated by the concepts of impulsive control and sampled control, a novel pulse-modulated intermittent control has been designed to solve consensus problem of MAS in [26]. Considering random switching network topologies and communication delays, leader-following consensus problem of MASs has been studied in [27]. Only using position states, the impulse bipartite consensus problem has been addressed in [28]. Taking communication delays and packet dropouts into consideration, the mean-square consensusability problem of discrete-time linear MAS has been investigated in [29]. By using coding-decoding communication protocol, the consensus problem of discrete-time MAS has been studied in [30]. However, the bipartite tracking problem of NRSs with discrete communications has not yet been completely solved, which partly motivates the current study.

Motivation by the above discussions, this paper aims to achieve bipartite tracking of NRSs with discrete communications and signed directed graphs. The considered signed directed graph is structurally balanced and has a spanning tree, which is less conservative than the assumption that the signed graph is undirected and connected. Besides, it is more challenging to analyze the closed-loop system due to the existence of input disturbances and discrete communications. Then, the main contributions of this paper are stated as follows. i) In contrast to bipartite tracking problem of traditional linear and Lipschitz-type dynamics [18]-[22], the case of NRS described as the Euler-Lagrange equation is studied. ii) Taking the external disturbances into consideration, discrete-communication-based bipartite tracking problem for NRS is successfully solved for the first time. iii) Based on discrete communications, two novel classes of hierarchical hybrid control algorithms are proposed to deal with both discontinuous and continuous signals in a uniform framework, which thus provides theoretical guidance for controlling and analyzing hierarchical hybrid systems.

The remainder of this paper is outlined as follows. Some preliminaries on signed directed graph are given in Section II. The convergence of the proposed algorithms is given in Section III. Finally, numerical examples and conclusion are provided in Sections IV and V, respectively.

Notations. $\mathbb{R}^{n\times n}$ is the $n\times n$ real matrix, especially, $\mathbb{R}^n$ represents the $n\times 1$ real matrix. $\|\cdot\|$ and $\|\cdot\|_\infty$ represent the Euclidean norm and supremum, respectively. $\mathrm{sign}(\cdot)$ is sign function. $\mathbf{1}=\{1,1,\cdots,1\}^T$ is the column vector of proper dimension. Let $\mathbb{Z}^+$ and $\mathbb{R}$ be the set of positive integer and real number, respectively. $|\cdot|$, $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$ represent the modulus, the real part and the imaginary part of a complex number, respectively. $\lambda_{\max}\{\cdot\}$, $\lambda_{\min}\{\cdot\}$ and $\det\{\cdot\}$ are the maximum, minimum eigenvalues and determinant of the given matrix. I_n stands for n-order identity matrix. $\mathrm{diag}(p_1,p_2,\cdots,p_n)$ denotes the diagonal matrix. The symbol $\otimes$ is the Kronecker product of two given matrices. $\max\{\cdot\}$ and $\min\{\cdot\}$ are the maximum and minimum values of the given vector.

II. Preliminaries

A. Graph Theory

Let a signed directed graph (i.e., digraph) $\mathcal{G}=\{\mathcal{V},\mathcal{E},\mathcal{A}\}$ describe the communications among the robots of the NRS, where $\mathcal{V}=\{1,2,\ldots,n\}$ is the robot set, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ stands for the edge set, and $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{n\times n}$ denotes the weighted adjacency matrix. A directed edge $\{i,j\}\in\mathcal{E}\Leftrightarrow a_{ij}\neq 0$ indicates that the ith robot can receive information from the jth one. Moreover, there are no self-loops, i.e., $a_{ii}=0$. $\mathcal{N}_i$ is the neighbor set of robot i. A directed path with length $r-1$ is denoted as $\{(i_1,i_2),(i_2,i_3),\cdots,(i_{r-1},i_r)\}$. A directed semipath is a sequence of robots $i_1,i_2,\cdots,i_d$ satisfying either $(i_{j-1},i_j)\in\mathcal{E}$ or $(i_j,i_{j-1})\in\mathcal{E}$, $\forall j\in\{2,3,\cdots,d\}$. A semicycle is a directed semipath beginning and ending with the same robot. The semicycle is positive if it contains an even number of negative edge weights; otherwise it is negative. $\mathcal{G}$ is structurally balanced if its semicycles are positive; it is structurally unbalanced otherwise. If $\mathcal{G}$ is signed directed and structurally balanced, then all robots can be divided into two sets $\mathcal{V}_1$ and $\mathcal{V}_2$, $\mathcal{V}=\mathcal{V}_1\bigcup\mathcal{V}_2$, $\emptyset=\mathcal{V}_1\bigcap\mathcal{V}_2$, such that $a_{ij}>0$ for all $i,j\in\mathcal{V}_s,(s\in\{1,2\})$ and $a_{ij}<0$ for all $i\in\mathcal{V}_s$, $j\in\mathcal{V}_q, s\neq q$ $(s,q\in\{1,2\})$. $\mathcal{G}$ has a spanning tree if there is a root robot containing a directed path to all other robots. Let $B=\mathrm{diag}\{b_1,b_2,\ldots,b_n\}$ denote the pinning matrix, where b_i is the communication weight between the robot i and the root robot. $b_i>0$ if the robot i can receive information from the root robot, otherwise $b_i=0$. The Laplacian matrix $L=[l_{ij}]\in\mathbb{R}^{n\times n}$ is defined as $l_{ij}=\sum_{j\in\mathcal{N}_i}|a_{ij}|$ if $i=j$; $l_{ij}=-a_{ij}$ if $i\neq j$. Besides, the communications only occur at the discrete time sequence $\{t_k|_{k=1}^{\infty}\}$ satisfying $t_0<t_1<\ldots<t_k<\ldots$, $\lim_{k\to\infty}t_k=\infty$, $t_k-t_{k-1}=h(\forall k\in\mathbb{Z}^+)$, where h stands for the sampling period.

Assumption 1: The signed digraph $\mathcal{G}$ is structurally balanced and has a spanning tree.

Lemma 1: [16] Suppose that Assumption 1 holds. Then, there exists a diagonal matrix $\Phi=\mathrm{diag}\{\phi_1,\phi_2,\ldots,\phi_n\}$ such that $\Phi H\Phi$ is positive stable, where $H=L+B$, $\phi_i=1$ if $i\in\mathcal{V}_1$ and $\phi_i=-1$ if $i\in\mathcal{V}_2$.

B. System Formulation and Control Problem

Consider that the NRS contains n robots. The ith robot is described as [31]

$$M_i(x_i)\ddot{x}_i+\left(\tfrac{1}{2}\dot{M}_i(x_i)+S(x_i,\dot{x}_i)\right)\dot{x}_i+g_i(x_i)+d_i(t)=u_i, \tag{1}$$

where $t\geq 0$, $i\in\mathcal{V}$, $x_i,\dot{x}_i,\ddot{x}_i\in\mathbb{R}^p$ are the generalized position, velocity, and acceleration respectively, $M_i(x_i)\in\mathbb{R}^{p\times p}$ is the positive definite symmetric inertia matrix, $S(x_i,\dot{x}_i)\in\mathbb{R}^{p\times p}$ is a skew-symmetric matrix defined as follows:

$$S(x_i,\dot{x}_i)=\tfrac{1}{2}\left(\dot{M}_i(x_i)-\tfrac{\partial}{\partial x_i}\dot{x}_i^T M_i(x_i)\dot{x}_i\right),$$

$g_i(x_i)\in\mathbb{R}^p$ denotes the gravitational force, $d_i(t)\in\mathbb{R}^p$ stands for the ∞-norm bounded input disturbance, and $u_i\in\mathbb{R}^p$ is the control input. Besides, following [32], these dynamic items satisfy $\gamma_{mi}\leq\|M_i(x_i)\|\leq\gamma_{Mi}$, $2\gamma_{\dot{m}i}\|\dot{x}_i\|\leq\left\|\dot{M}_i(x_i)\right\|\leq$

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$2\gamma_{\dot{M}i}\|\dot{x}_i\|$, $\gamma_s\|\dot{x}_i\| \leq \|S(x_i,\dot{x}_i)\| \leq \gamma_S\|\dot{x}_i\|$, $\|d_i(t)\| \leq \gamma_{di}$ and $\|g_i(x_i)\| \leq \gamma_{gi}$, where $\gamma_{mi}, \gamma_{Mi}, \gamma_{\dot{m}i}, \gamma_{\dot{M}i}, \gamma_s, \gamma_S, \gamma_{di}, \gamma_{gi}$ are positive constants, $\forall x_i \in \mathbb{R}^p$.

The dynamics of the leader is presented as

$$\dot{x}_0(t) = v_0(t), \dot{v}_0(t) = a_0(t), \tag{2}$$

where $x_0, v_0, a_0 \in \mathbb{R}^p$ are the vectors of position, velocity and acceleration, respectively.

Assumption 2: The leader states $v_0(t)$ and $a_0(t)$ are uniformly essentially bounded, i.e., $\sup_{t\in[0,\infty)}\|v_0(t)\|_\infty \leq \sigma_1$, $\sup_{t\in[0,\infty)}\|a_0(t)\|_\infty \leq \sigma_2$, $\exists \sigma_1, \sigma_2 > 0$.

The control objective is to design a proper input u_i for solving the bipartite tracking problem.

Definition 1: The *bipartite tracking* is achieved for NRS if

$$\begin{cases} \lim\limits_{t\to\infty} \tanh(\|x_i - \phi_i x_0\|) \leq \rho_1, \\ \lim\limits_{t\to\infty} \|v_i - \phi_i v_0\| \leq \rho_2, \ \forall i \in \mathcal{V}, \end{cases} \tag{3}$$

where $\tanh(\cdot)$ is the hyperbolic tangent function, the robots only communicate with each other in the discrete time t_k, $\forall k \in \mathbb{Z}^+$, ρ_1 and $\rho_2 > 0$ can be sufficiently small by choosing appropriate control parameters.

Lemma 2: [33] For any $A \in \mathbb{R}^{n\times n}$, there exists a *small-value norm* $\|\cdot\|_A$ with $\|A\|_A < 1$ if A is *Schur stable*. In addition, $\varepsilon_1\|\cdot\|_\infty \leq \|\cdot\|_A \leq \varepsilon_2\|\cdot\|_\infty$ and $\varepsilon_3\|\cdot\| \leq \|\cdot\|_A \leq \varepsilon_4\|\cdot\|$, $\exists \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4 \in (0,\infty)$.

Lemma 3: [34] Let $R(\cdot)$ and $Q(\cdot)$ be polynomials satisfying

$$R(z) = (z-1)^d Q\left(\frac{z+1}{z-1}\right),$$

where $z \in \mathbb{R}\backslash\{1\}$. Then $Q(z)$ is *Schur stable* if and only if $R(z)$ (of degree d) is *Hurwitz stable*.

Remark 1: In some practical applications, both cooperation and competition are needed to accomplish specific tasks. For instance, multiple robots carry heavy objects, they need to move in opposite directions with each other to carry them. It is thus of significance to investigate bipartite consensus, where the agents converge to two states with the same modulus and different plus-minus signs. Furthermore, different from bipartite consensus, the modulus of the agreement states of the group/cluster consensus and multi-target tracking are not necessarily identical.

III. Main Results

A. Hierarchical Hybrid Control Algorithm

To solve the aforementioned problem, two classes of hierarchical hybrid control algorithms (HHCAs) are designed as

$$\begin{cases} u_i = K_{i1}(\hat{x}_i - x_i) + K_{i2}(\hat{v}_i - \dot{x}_i), \\ \dot{\hat{x}}_i = \hat{v}_i, \dot{\hat{v}}_i = 0, t \in (t_{k-1}, t_k], \\ \Delta\hat{x}_i = \alpha \sum\limits_{j\in\mathcal{N}_i} a_{ij}[\hat{x}_j - \mathrm{sign}(a_{ij})\hat{x}_i] \\ \qquad + \alpha b_i[\mathrm{sign}(\phi_i)x_0 - \hat{x}_i], \\ \Delta\hat{v}_i = \beta \sum\limits_{j\in\mathcal{N}_i} a_{ij}[\hat{v}_j - \mathrm{sign}(a_{ij})\hat{v}_i] \\ \qquad + \beta b_i[\mathrm{sign}(\phi_i)v_0 - \hat{v}_i], \end{cases} \tag{4}$$

and

$$\begin{cases} u_i = K_{i1}(\hat{x}_i - x_i) + K_{i2}(\hat{v}_i - \dot{x}_i), \\ \dot{\hat{x}}_i = \hat{v}_i, \dot{\hat{v}}_i = 0, t \in (t_{k-1}, t_k], \\ \Delta\hat{v}_i = \alpha \sum\limits_{j\in\mathcal{N}_i} a_{ij}[\hat{x}_j - \mathrm{sign}(a_{ij})\hat{x}_i] + \alpha b_i[\mathrm{sign}(\phi_i)x_0 - \hat{x}_i] \\ +\beta \sum\limits_{j\in\mathcal{N}_i} a_{ij}[\hat{v}_j - \mathrm{sign}(a_{ij})\hat{v}_i] + \beta b_i[\mathrm{sign}(\phi_i)v_0 - \hat{v}_i], \end{cases} \tag{5}$$

where $\hat{x}_i$ and $\hat{v}_i$ are the estimated states of x_0 and v_0, K_{i1} and K_{i2} are positive-definite diagonal matrices, α and β are the control gains, b_i is the pinning gain predefined before, $\Delta\hat{x}_i(t_k) = \hat{x}_i(t_k^+) - \hat{x}_i(t_k), \hat{x}_i(t_k^+) = \lim_{\eta\to 0^+}\hat{x}_i(t_k+\eta)$, $\Delta\hat{v}_i(t_k) = \hat{v}_i(t_k^+) - \hat{v}_i(t_k), \hat{v}_i(t_k^+) = \lim_{\eta\to 0^+}\hat{v}_i(t_k+\eta)$, and ϕ_i is defined in Lemma 1, $\forall i \in \mathcal{V}$. Besides, it is assumed that $\hat{x}_i$ and $\hat{v}_i$ are left continuous at t_k, $\forall i \in \mathcal{V}$, $k \in \mathbb{Z}^+$.

B. Analysis of Hierarchical Hybrid Control Algorithm (4)

Substituting HHCA (4) into system (1) yields that

$$\begin{cases} M_i(x_i)\ddot{x}_i + \Big(1/2\dot{M}_i(x_i) + S(x_i,\dot{x}_i)\Big)\dot{x}_i + g_i(x_i) \\ \quad = K_{i1}(\hat{x}_i - x_i) + K_{i2}(\hat{v}_i - \dot{x}_i) - d_i(t), \\ \hat{x}_i(t_{k+1}) = \hat{x}_i(t_k^+) + h\hat{v}_i(t_k^+), \ \hat{v}_i(t_{k+1}) = \hat{v}_i(t_k^+), \\ \hat{x}_i(t_k^+) = \hat{x}_i(t_k) - \alpha \sum\limits_{j\in\mathcal{N}_i} l_{ij}\hat{x}_j(t_k) - \alpha b_i\hat{x}_i(t_k) \\ \qquad + b_i\mathrm{sign}(\phi_i)x_0(t_k), \\ \hat{v}_i(t_k^+) = \hat{v}_i(t_k) - \alpha \sum\limits_{j\in\mathcal{N}_i} l_{ij}\hat{v}_j(t_k) - \alpha b_i\hat{v}_i(t_k) \\ \qquad + b_i\mathrm{sign}(\phi_i)v_0(t_k), \ \forall k \in \mathbb{Z}^+, \end{cases} \tag{6}$$

where $\hat{x}_i(t_1) = \hat{x}_i(t_0) + h\hat{v}_i(t_0)$, $\hat{v}_i(t_1) = \hat{v}_i(t_0)$, $\forall i \in \mathcal{V}$. Let $\bar{x}_i = \phi_i\hat{x}_i - x_0$, $\bar{v}_i = \phi_i\hat{v}_i - v_0$, $e_i = \phi_i x_i - x_0$, $\dot{e}_i = \phi_i\dot{x}_i - v_0$, and $X = \mathrm{col}(\bar{x}_i, \bar{v}_i) \in \mathbb{R}^{2np}$. System (6) can be formed as

$$\begin{aligned} M_i(x_i)\ddot{e}_i &+ \Big(1/2\dot{M}_i(x_i) + S(x_i,\dot{x}_i)\Big)\dot{e}_i \\ &= -\phi_i K_{i1}e_i - \phi_i K_{i2}\dot{e}_i + \phi_i K_{i1}\bar{x}_i + \phi_i K_{i2}\bar{v}_i \\ &\quad - \phi_i M_i(x_i)a_0 - \phi_i\Big(1/2\dot{M}_i(x_i) + S(x_i,\dot{x}_i)\Big)v_0 \\ &\quad - d_i(t) - g_i(x_i), \end{aligned} \tag{7}$$

$$X(t_{k+1}) = \Psi X(t_k) + \Delta_k, \tag{8}$$

where$\Psi = \begin{bmatrix} I_n - \alpha H_s & h(I_n - \beta H_s) \\ 0 & I_n - \beta H_s \end{bmatrix} \otimes I_p$, $H_s = \Phi H \Phi$, $H = L + B$, $\Delta_k = (\mathbf{1}\otimes\Delta_{k,1}, \mathbf{1}\otimes\Delta_{k,2})^T$, $\Delta_{k,1} = x_0(t_k) - x_0(t_{k+1}) + v_0(t_k)$, and $\Delta_{k,2} = v_0(t_k) - v_0(t_{k+1})$.

Theorem 1: Suppose that Assumptions 1-2 hold. By using HHCA (4) for system (1), if

$$\begin{aligned} &0 < \alpha, \beta < \min_{w_i\in\varpi(H_s)}\{\tfrac{2\mathrm{Re}(w_i)}{|w_i|^2}\}, \\ &\varepsilon^2\lambda_{\min}(K_{i1}) - 2\gamma_{M_i} > 0, \\ &\varepsilon\lambda_{\min}(K_{i2}) - \gamma_{M_i} - (\varepsilon+1)(\gamma_{\dot{M}i} + \gamma_S) > 0, \\ &\lambda_{\min}(K_{i1}) - \gamma_{\dot{M}i} - \gamma_S > 0, \ \exists \varepsilon \in (0,\infty), \end{aligned} \tag{9}$$

where $\varpi(H_s)$ denotes the spectrum of H_s, then the *bipartite tracking* problem is solved, namely, the inequalities (3) hold. Besides, ρ_1 and ρ_2 predefined in (3) are derived as

$$\begin{aligned} \rho_1 &= \tfrac{\eta_i}{\lambda_{\min}(K_{i1}) - \gamma_{\dot{M}i} - \gamma_S}, \\ \rho_2 &= \tfrac{\varepsilon\eta_i}{\varepsilon\lambda_{\min}(K_{i2}) - \gamma_{M_i} - (\varepsilon+1)(\gamma_{\dot{M}i} + \gamma_S)}, \end{aligned} \tag{10}$$

where $\eta_i = h\vartheta_1\left[\lambda_{\max}(K_{i1}) + \lambda_{\max}(K_{i2})\right] + \gamma_{M_i}\sigma_2 + (\gamma_{\dot{M}i} + \gamma_S)\sigma_1 + \gamma_{di} + \gamma_{gi}$, $\vartheta_1 = (\varepsilon_2/\varepsilon_3)\max(2\sigma_1, \sigma_2)(1 - \|\Psi\|_\Psi)^{-1}$, $\|\cdot\|_\Psi$ is a small value norm, ε_2 and ε_3 are given in Lemma 2, $\forall i \in \mathcal{V}$.

Proof: The emphasis of this proof lies on the convergence analysis of the closed-loop system (7)-(8). Firstly, let λ be an eigenvalue of matrix Ψ. Then, we have

$$\begin{aligned}&\det(\lambda I_{2np} - \Psi)\\ =&\det\left(\begin{bmatrix}\lambda I_n - I_n + \alpha H_s & -hI_n + \beta h H_s\\ 0 & \lambda I_n - I_n + \beta H_s\end{bmatrix}\otimes I_p\right)\\ =&\det\left([(\lambda I_n - I_n + \alpha w_i)(\lambda I_n - I_n + \beta w_i)]\otimes I_p\right)\\ =&\prod_{i=1}^{n}[\Gamma_i(\lambda)]^p,\end{aligned}$$

where $\Gamma_i(\lambda) = (\lambda I_n - I_n + \alpha w_i)(\lambda I_n - I_n + \beta w_i)$ and w_i is an eigenvalue of matrix H_s. Besides, H_s is positive stable according to Lemma 1, namely, $w_i > 0$, $\forall i \in \mathcal{V}$. It thus obtains that $\lambda \neq 1$. Taking the bilinear transformation $\lambda = (z+1)/(z-1)$ for $\Gamma_i(\lambda)$. Let

$$\begin{aligned}\tilde{\Gamma}_i(z) =&(z-1)^2\Gamma_i\left(\frac{z+1}{z-1}\right)\\ =&(\alpha w_i z + 2 - \alpha w_i)(\beta w_i z + 2 - \beta w_i).\end{aligned} \tag{11}$$

$\tilde{\Gamma}_i(z) = 0$ implies that $z_1 = 1 - 2/\alpha w_i$ and $z_2 = 1 - 2/\beta w_i$. Based on Hurwitz stability criterion, namely, $\tilde{\Gamma}_i(z)$ is Hurwitz stable if $\mathrm{Re}(z_1) < 0$ and $\mathrm{Re}(z_2) < 0$, then, the first inequality of (9) holds. Note that $\hat{x}_0(t_{k+1}) - \hat{x}_0(t_k) = \int_{t_k}^{t_{k+1}}\hat{v}_0(\eta)d\eta$ and $\hat{v}_0(t_{k+1}) - \hat{v}_0(t_k) = \int_{t_k}^{t_{k+1}}\hat{a}_0(\eta)d\eta$. It can be obtained from Lemma 2 that

$$\begin{aligned}&\|\Delta_{k,1}\|_\infty \leq 2h\sigma_1,\ \|\Delta_{k,2}\|_\infty \leq h\sigma_2.\\ &\|\Delta_k\|_\Psi \leq \varepsilon_2 h\max(2\sigma_1, \sigma_2).\end{aligned} \tag{12}$$

The polynomial $\tilde{\Gamma}_i(z)$ is Schur stable, which implies that $\rho(Q) < 1$. It thus obtains that $\lim_{k\to\infty}\Psi^k = 0$. From Eq.(7), it obtains that $X(t_{k+1}) = \Psi^k X(t_0) + \sum_{i=0}^{k-i}Q^{k-i}\Delta_i$, which implies that

$$\begin{aligned}\lim_{t\to\infty}\|X(t)\|_\Psi =&\lim_{k\to\infty}\left\|\Psi^{k+1}X(t_0) + \sum_{i=0}^{k}\Psi^{k-i}\Delta_i\right\|_\Psi\\ \leq&\lim_{k\to\infty}\left\|\Psi^{k+1}X(t_0)\right\|_\Psi + \lim_{k\to\infty}\left\|\sum_{i=0}^{k}\Psi^{k-i}\Delta_i\right\|_\Psi\\ \leq&\|\Delta_k\|_\Psi\lim_{k\to\infty}\sum_{i=0}^{k}\|\Psi\|_\Psi^{k-i}.\end{aligned}$$

It follows that $\lim_{t\to\infty}\|X(t)\|_\Psi \leq \varepsilon_2 h\max(2\sigma_1, \sigma_2)(1 - \|\Psi\|_\Psi)^{-1}$. From Lemma 2, it follows that $\varepsilon_3\|\cdot\| \leq \|\cdot\|_A$, which implies $\lim_{t\to\infty}\|X(t)\| \leq h(\varepsilon_2/\varepsilon_3)\max(2\sigma_1, \sigma_2)\times(1 - \|\Psi\|_\Psi)^{-1}$. It thus concludes that $\lim_{t\to\infty}\|\bar{x}_i\| \leq h\vartheta_1$ and $\lim_{t\to\infty}\|\bar{v}_i\| \leq h\vartheta_1$, $\forall i \in \mathcal{V}$, where $\vartheta_1 = (\varepsilon_2/\varepsilon_3)\max(2\sigma_1, \sigma_2)(1 - \|\Psi\|_\Psi)^{-1}$.

The second step is to analyze the convergence of subsystem (7). Let $\xi_i = \phi_i[K_{i1}\bar{x}_i + K_{i2}\bar{v}_i - M_i(x_i)a_0 - \left(1/2\dot{M}_i(x_i)+S(x_i,\dot{x}_i)\right)v_0] - d_i(t) - g_i(x_i)$. Then, the system (7) becomes

$$\begin{aligned}&M_i(x_i)\ddot{e}_i + \left(1/2\dot{M}_i(x_i)+S(x_i,\dot{x}_i)\right)\dot{e}_i\\ =&-\phi_i K_{i1}e_i - \phi_i K_{i2}\dot{e}_i + \xi_i,\end{aligned} \tag{13}$$

where ξ_i is bounded with $\|\xi_i\| \leq \eta_i + (\gamma_{\dot{M}i} + \gamma_S)\|\dot{e}_i\|$, $\eta_i = \lambda_{\max}(K_{i1})\|\bar{x}_i\| + \lambda_{\max}(K_{i2})\|\bar{v}_i\| + \gamma_{M_i}\sigma_2 + (\gamma_{\dot{M}i} + \gamma_S)\sigma_1 + \gamma_{di} + \gamma_{gi}$ and $\vartheta_1 = (\varepsilon_2/\varepsilon_3)\max(2\sigma_1, \sigma_2)(1 - \|\Psi\|_\Psi)^{-1}$.

For $i \in \mathcal{V}_1$, $\phi_i = 1$. Then consider the following Lyapunov function candidate for system (7)

$$\begin{aligned}V_i =&\frac{1}{2}\dot{e}_i^T M_i(x_i)\dot{e}_i + \frac{1}{\varepsilon}\tanh^T(e_i)M_i(x_i)\dot{e}_i\\ &+\frac{1}{2}e_i^T K_{i1}e_i + \frac{1}{\varepsilon}K_{i2}\ln(\cosh(e_i)),\end{aligned} \tag{14}$$

where $\tanh(\cdot)$ is defined in Definition 1, $\cosh(\cdot)$ is the hyperbolic cosine function, $\ln(\cdot)$ denotes the natural logarithm function, and ε is a positive constant. Note the fact that

$$\begin{aligned}&\frac{1}{4}\dot{e}_i^T M_i(x_i)\dot{e}_i + \frac{1}{\varepsilon}\tanh^T(e_i)M_i(x_i)\dot{e}_i + \frac{1}{2}e_i^T K_{i1}e_i\\ =&\frac{1}{4}(\dot{e}_i + \frac{2}{\varepsilon}\tanh(e_i))^T M_i(x_i)(\dot{e}_i + \frac{2}{\varepsilon}\tanh(e_i))\\ &-\frac{1}{\varepsilon^2}\tanh^T(e_i)M_i(x_i)\tanh(e_i) + \frac{1}{2}e_i^T K_{i1}e_i\\ \geq&\tanh^T(e_i)(\frac{1}{2}K_{i1} - \frac{M_i(x_i)}{\varepsilon^2})\tanh(e_i).\end{aligned} \tag{15}$$

It then follows that

$$\begin{aligned}V_i \geq&\frac{1}{4}\dot{e}_i^T M_i(x_i)\dot{e}_i + \frac{1}{\varepsilon}K_{i2}\ln(\cosh(e_i))\\ &+\tanh^T(e_i)(\frac{1}{2_i}K_{i1} - \frac{M_i(x_i)}{\varepsilon^2})\tanh(e_i),\end{aligned} \tag{16}$$

where $\varepsilon^2\lambda_{\min}(K_{i1}) - 2\gamma_{M_i} > 0$. Thus, V_i is positive definite.

Taking the time derivative of V_i along system (7) yields that

$$\begin{aligned}\dot{V}_i =&\dot{e}_i^T(-K_{i1}e_i - K_{i2}\dot{e}_i + \xi_i) + \frac{1}{\varepsilon}\dot{e}_i^T\sec\mathrm{h}(e_i^2)M_i(x_i)\dot{e}_i\\ &+\frac{1}{\varepsilon}\tanh^T(e_i)\left(\frac{1}{2}\dot{M}_i(x_i) - S(x_i,\dot{x}_i)\right)\dot{e}_i\\ &+\frac{1}{\varepsilon}K_{i2}\dot{e}_i^T\tanh(e_i) + \frac{1}{\varepsilon}\tanh^T(e_i)(-K_{i1}e_i - K_{i2}\dot{e}_i + \xi_i).\end{aligned}$$

Note the fact that $\dot{e}_i^T\mathrm{sech}(e_i^2)M_i(x_i)\dot{e}_i \leq \dot{e}_i^T M_i(x_i)\dot{e}_i$, $\tanh^T(e_i)\tanh(e_i) \leq \tanh^T(e_i)e_i \leq e_i^T e_i$, and

$$\begin{aligned}&(\dot{e}_i^T + \frac{1}{\varepsilon}\tanh^T(e_i))\xi_i\\ \leq&(\dot{e}_i^T + \frac{1}{\varepsilon}\tanh^T(e_i))(\eta_i + (\gamma_{\dot{M}i} + \gamma_S)\|\dot{e}_i\|)\\ \leq&\eta_i\|\dot{e}_i\| + (\gamma_{\dot{M}i} + \gamma_S)\|\dot{e}_i\|^2 + \frac{\eta_i}{\varepsilon}\tanh^T(e_i)\\ &+\frac{(\gamma_{\dot{M}i} + \gamma_S)}{2\varepsilon}\tanh^T(e_i)\tanh(e_i) + \frac{(\gamma_{\dot{M}i} + \gamma_S)}{2\varepsilon}\|\dot{e}_i\|^2.\end{aligned}$$

It then follows that

$$\begin{aligned}\dot{V}_i \leq&\dot{e}_i^T(K_{i2}\dot{e}_i + \frac{M_i(x_i)}{\varepsilon})\dot{e}_i + (\dot{e}_i^T + \frac{1}{\varepsilon}\tanh^T(e_i))\xi_i\\ &-\frac{1}{\varepsilon}\tanh^T(e_i)K_{i1}\tanh(e_i) + \frac{(\gamma_{\dot{M}i} + \gamma_S)}{2\varepsilon}\|\dot{e}_i\|^2\end{aligned}$$

$$
\begin{aligned}
&+\frac{(\gamma_{\dot{M}i}+\gamma_S)}{2\varepsilon}\tanh^T(e_i)\tanh(e_i) \qquad (17)\\
&\leq -\left[(\lambda_{\min}(K_{i2})-\frac{\gamma_{M_i}}{\varepsilon}-(1+\frac{1}{\varepsilon})(\gamma_{\dot{M}i}+\gamma_S))\left\|\dot{e}_i\right\|\right.\\
&\quad -\eta_i]\left\|\dot{e}_i\right\|-[(\frac{\lambda_{\min}(K_{i1})}{\varepsilon}-\frac{(\gamma_{\dot{M}i}+\gamma_S)}{\varepsilon})\left\|\tanh(e_i)\right\|\\
&\quad -\frac{\eta_i}{\varepsilon}]\left\|\tanh(e_i)\right\|.
\end{aligned}
$$

It provides that $\dot{V}_i \leq 0$ if $\tanh(\|e_i\|) > \rho_1$ and $\|\dot{e}_i\| > \rho_2$, where ρ_1 and ρ_2 are presented in (10). It then follows that $\lim_{t\to\infty}\tanh(\|e_i\|) \leq \rho_1$ and $\lim_{t\to\infty}\|\dot{e}_i\| \leq \rho_2$. For $i \in \mathcal{V}_2$, it can similarly analyze the convergence of $\dot{e}_i$ and $\tanh(e_i)$. This completes the proof. ■

Remark 2: As shown in Theorem 1, by using HHCA (4), the position and velocity error states respectively converge to two neighbourhoods of the origin with the convergence radiuses ρ_1 and ρ_2. These radiuses can be arbitrarily close to zero by choosing proper control parameters K_{i1}, K_{i2} for the robot i and an appropriate sampling period h.

Remark 3: Comparing with the existing works [1], [35], [36], in which the control algorithms are constructed based on the structures of the system models, the presented HHCAs are designed without employing any information of the system model and are thus model-independent.

C. Analysis of Hierarchical Hybrid Control Algorithm (5)

Substituting HHCA (5) into system (1) yields that

$$
\begin{cases}
M_i(x_i)\ddot{e}_i+\left(\frac{1}{2}\dot{M}_i(x_i)+S(x_i,\dot{x}_i)\right)\dot{e}_i\\
\qquad\qquad = -\phi_i K_{i1}e_i-\phi_i K_{i2}\dot{e}_i+\xi_i,\\
X(t_{k+1}) = \Upsilon X(t_k)+\Delta_k,
\end{cases} \qquad (18)
$$

where $\xi_i = \phi_i[K_{i1}\bar{x}_i + K_{i2}\bar{v}_i - \left(1/2\dot{M}_i(x_i)+S(x_i,\dot{x}_i)\right)v_0 - M_i(x_i)a_0] - d_i(t) - g_i(x_i)$,

$$
\Upsilon = \begin{bmatrix} I_n-\alpha hH_s & h(I_n-\beta H_s)\\ -\alpha H_s & I_n-\beta H_s \end{bmatrix}\otimes I_p,
$$

H_s and Δ_k are predefined in Section III-B.

Theorem 2: Suppose that Assumptions 1 and 2 hold. By employing HHCA (5) for system (1), if

$$
\begin{aligned}
&\alpha>0,\ 0<\beta<\min_{w_i\in\varpi(H_s)}\tfrac{2\mathrm{Re}(w_i)}{|w_i|^2},\\
&0<h<\min_{w_i\in\varpi(H_s)}\tfrac{\beta^2(4\mathrm{Re}(w_i)|w_i|^2-2\beta|w_i|^4)}{\alpha(4\mathrm{Im}^2(w_i)+\beta^2|w_i|^4)},\\
&\varepsilon^2\lambda_{\min}(K_{i1})-2\gamma_{M_i}>0,\\
&\varepsilon\lambda_{\min}(K_{i2})-\gamma_{M_i}-(\varepsilon+1)(\gamma_{\dot{M}i}+\gamma_S)>0,\\
&\lambda_{\min}(K_{i1})-\gamma_{\dot{M}i}-\gamma_S>0,
\end{aligned} \qquad (19)
$$

then the *bipartite tracking* problem is solved, namely, the inequalities (3) hold. Besides, ρ_1 and ρ_2 predefined in (3) are derived as

$$
\begin{aligned}
\rho_1 &= \tfrac{\theta_i}{\lambda_{\min}(K_{i1})-\gamma_{\dot{M}i}-\gamma_S},\\
\rho_2 &= \tfrac{\varepsilon\theta_i}{\varepsilon\lambda_{\min}(K_{i2})-\gamma_{M_i}-(\varepsilon+1)(\gamma_{\dot{M}i}+\gamma_S)},
\end{aligned} \qquad (20)
$$

where $\theta_i = h\vartheta_2\left[\lambda_{\max}(K_{i1})+\lambda_{\max}(K_{i2})\right]+\gamma_{M_i}\sigma_2+(\gamma_{\dot{M}i}+\gamma_S)\sigma_1+\gamma_{di}+\gamma_{gi}$, $\vartheta_2 = (\varepsilon_2/\varepsilon_3)\max(2\sigma_1,\sigma_2)(1-\|\Upsilon\|_\Upsilon)^{-1}$, and $\|\cdot\|_\Upsilon$ is a small value norm, $\forall i \in \mathcal{V}$.

Proof: The emphasis of this proof lies on the convergence analysis of the closed-loop system (18). Let λ be an eigenvalue of matrix Υ. Then,

$$
\begin{aligned}
&\det(\lambda I_{2np}-\Upsilon)\\
&=\det\left(\begin{bmatrix}\lambda I_n-I_n+\alpha hH_s & h(\beta H_s-I_n)\\ \alpha H_s & \lambda I_n-I_n+\beta H_s\end{bmatrix}\otimes I_p\right)\\
&=\prod_{i=1}^{n}\left[\lambda^2+\lambda(\alpha hw_i+\beta w_i-2)+1-\beta w_i\right]^p,
\end{aligned}
$$

where w_i is an eigenvalue of matrix H_s. Let $\Gamma_i(\lambda) = \lambda^2 + \lambda(\alpha hw_i + \beta w_i - 2) + 1 - \beta w_i$. Owing to the fact $w_i > 0$, it obtains that $\lambda \neq 0$. Taking the bilinear transformation $\lambda = (z+1)/(z-1)$ for $\Gamma_i(\lambda)$. Let

$$
\begin{aligned}
\tilde{\Gamma}_i(\lambda) =&(z-1)^2\Gamma_i(\frac{z+1}{z-1})\\
=&(z+1)^2+(\alpha hw_i+\beta w_i-2)(z+1)(z-1)\\
&+(1-\beta w_i)(z-1)^2\\
=&\alpha hw_iz^2+2\beta w_iz+4-\alpha hw_i-2\beta w_i,\\
\Gamma_i'(\lambda) =&\frac{\tilde{\Gamma}_i(\lambda)}{\alpha hw_i}\\
=&z^2+\frac{2\beta}{\alpha h}z+\frac{4}{\alpha h}w_i'-1-\frac{2\beta}{\alpha h}\\
=&z^2+\frac{2\beta}{\alpha h}z+\frac{4}{\alpha h}\mathrm{Re}(w_i')-1-\frac{2\beta}{\alpha h}+\frac{4}{\alpha h}\mathrm{Im}(w_i'),
\end{aligned}
$$

where $w_i' = 1/w_i$.

Based on chapter 1.4 from [37], $\Gamma_i'(\lambda)$ is *Hurwitz stable* if $\frac{2\beta}{\alpha h} > 0$ and

$$
\left(\tfrac{2\beta}{\alpha h}\right)^2\left(\tfrac{4}{\alpha h}\mathrm{Re}(w_i')-1-\tfrac{2\beta}{\alpha h}\right)-\left(\tfrac{4}{\alpha h}\mathrm{Im}(w_i')\right)^2>0.
$$

By Lemma 3, polynomial $\Gamma_i(\lambda)$ is *Schur stable* if and only if polynomial $\Gamma_i'(\lambda)$ is *Hurwitz stable*, $\forall i \in \mathcal{V}$. Then $\Gamma_i(\lambda)$ is *Schur stable* if (19) holds.

By the similar analysis of Theorem 1, it yields that $\lim_{t\to\infty}\|\bar{x}_i\| \leq h\vartheta_2$ and $\lim_{t\to\infty}\|\bar{v}_i\| \leq h\vartheta_2$, $\forall i \in \mathcal{V}$, where $\vartheta_2 = (\varepsilon_2/\varepsilon_3)\max(2\sigma_1,\sigma_2)(1-\|\Upsilon\|_\Upsilon)^{-1}$. It thus follows that ξ_i in system (18) is bounded with $\|\xi_i\| \leq \theta_i + (\gamma_{\dot{M}i}+\gamma_S)\|\dot{e}_i\|$, $\theta_i = h\vartheta_2\left[\lambda_{\max}(K_{i1})+\lambda_{\max}(K_{i2})\right]+\gamma_{M_i}\sigma_2+(\gamma_{\dot{M}i}+\gamma_S)\sigma_1+\gamma_{di}+\gamma_{gi}$ and $\vartheta_2 = (\varepsilon_2/\varepsilon_3)\max(2\sigma_1,\sigma_2)(1-\|\Upsilon\|_\Upsilon)^{-1}$. Following the proof of Theorem 1, if $\varepsilon^2\lambda_{min}(K_{i1}) - 2\gamma_{M_i} > 0$, $\varepsilon\lambda_{\min}(K_{i2}) - \gamma_{M_i} - (\varepsilon+1)(\gamma_{\dot{M}i}+\gamma_S) > 0$ and $\lambda_{\min}(K_{i1}) - \gamma_{\dot{M}i} - \gamma_S > 0$, it thus can be concluded that

$$
\begin{aligned}
\lim_{t\to\infty}\tanh(\|e_i\|) &\leq \rho_1 = \tfrac{\theta_i}{\lambda_{\min}(K_{i1})-\gamma_{\dot{M}i}-\gamma_S},\\
\lim_{t\to\infty}\|\dot{e}_i\| &\leq \rho_2 = \tfrac{\varepsilon\theta_i}{\varepsilon\lambda_{\min}(K_{i2})-\gamma_{M_i}-(\varepsilon+1)(\gamma_{\dot{M}i}+\gamma_S)},
\end{aligned}
$$

where $\theta_i = h\vartheta_2\left[\lambda_{\max}(K_{i1})+\lambda_{\max}(K_{i2})\right]+\gamma_{M_i}\sigma_2+(\gamma_{\dot{M}i}+\gamma_S)\sigma_1+\gamma_{di}+\gamma_{gi}$ and $\vartheta_2 = (\varepsilon_2/\varepsilon_3)\max(2\sigma_1,\sigma_2)(1-\|\Upsilon\|_\Upsilon)^{-1}$. This ends the proof. ■

Remark 4: As shown in Theorem 2, by using HHCA (5), the tracking errors converge to two neighbourhoods of the origin with the convergence radiuses ρ_1 and ρ_2. These radiuses can be arbitrarily close to zero by choosing proper control gains α, β, the eigenvalues of H_s, and an appropriate sampling period h. This is thus different from the results of Theorem 1.

Remark 5: The results of Lipschitz-type dynamics can not be directly extended to the case of NRS, due to its high nonlinearity and coupling. The proposed HHCAs contain a

TABLE I
THE CONTROL PROCESS OF THE CONTROL INPUT IN EQS.(4) AND (5)

Algorithm: main input: Main output:	The hierarchical hybrid control algorithm (4) x_0, v_0, K_{i1}, K_{i2}, σ_1, σ_2, u_i x_i, $\dot{x}_i$	The hierarchical hybrid control algorithm (5) x_0, v_0, K_{i1}, K_{i2}, σ_1, σ_2, u_i x_i, $\dot{x}_i$
Step 1	Initialize system parameters Let the initial values $x_i,\, \dot{x}_i \in L_\infty \bigcap L_2$	Initialize system parameters Let the initial values $x_i,\, \dot{x}_i \in L_\infty \bigcap L_2$
Step 2	Design the distributed impulsive estimators as shown in Eq.(4) such that the estimated errors $\bar{x}_i$ and $\bar{v}_i$ satisfying $\lim\limits_{t\to\infty} \|\bar{x}_i\| \leq h\vartheta_1$and $\lim\limits_{t\to\infty} \|\bar{v}_i\| \leq h\vartheta_1$, $\forall i \in \mathcal{V}$.	Design the distributed impulsive estimators as shown in Eq.(5) such that the estimated errors $\bar{x}_i$ and $\bar{v}_i$ satisfying $\lim\limits_{t\to\infty} \|\bar{x}_i\| \leq h\vartheta_2$ and $\lim\limits_{t\to\infty} \|\bar{v}_i\| \leq h\vartheta_2$, $\forall i \in \mathcal{V}$.
Step 3	For the HHCA (4), design the control input by selecting proper K_{i1} and K_{i2} such that the Lyapunov function V_i presented in (14) satisfying $\dot{V}_i \leq 0,\ \forall i \in \mathcal{V}$.	For the HHCA (5), design the control input by selecting proper K_{i1} and K_{i2} such that the Lyapunov function V_i presented in (14) satisfying $\dot{V}_i \leq 0,\ \forall i \in \mathcal{V}$.
Step 4	Consequently, it follows that $\tanh(\|e_i\|)$ and $\|\dot{e}_i\|$ are upper bounded by ρ_1 and ρ_2 as $t \to \infty$, $\forall i \in \mathcal{V}$.	Consequently, it follows that $\tanh(\|e_i\|)$ and $\|\dot{e}_i\|$ are upper bounded by ρ_1 and ρ_2 as $t \to \infty$, $\forall i \in \mathcal{V}$.

distributed impulsive estimator layer and a model-independent control layer. Specifically, the estimator layer is used to estimate the final state of each robot, while the control one is employed to solve the local tracking problem. Combing two layers can address system uncertainties, input disturbances and discrete-communication at the same time. Besides, the control process of HHCAs is shown in Table I.

Remark 6: Considering the dynamic uncertainties of NRS, system (1) can be rewritten as $M_{io}(x_i)\ddot{x}_i + (1/2\dot{M}_{io}(x_i){+}S_o(x_i,\dot{x}_i))\dot{x}_i + g_{io}(x_i) = u_i + \rho(t)$, where $M_i(x_i){=}M_{io}(x_i) + \Delta M(x)$, $1/2\dot{M}_i(x_i){+}S(x_i,\dot{x}_i) = 1/2\dot{M}_{io}(x_i){+}S_o(x_i,\dot{x}_i) + 1/2\Delta\dot{M}(x) + \Delta S(x,\dot{x})$, $g_i(x_i) = g_{io}(x_i) + \Delta g(x)$ and $\rho(t) = -\Delta M(x)\ddot{x}_i - (1/2\Delta\dot{M}(x) + \Delta S(x,\dot{x}))\dot{x}_i - \Delta g(x) - d_i(t)$. Following [38], $\rho(t)$ is bounded with $\|\rho(t)\| \leq b_{01} + b_{02}\|x\| + b_{03}\|\dot{x}\|^2$, where b_{01}, b_{02} and b_{03} are positive constants. Then, the practical convergence property can still be guaranteed by using the presented control algorithms.

Remark 7: The kinematics of NRS in task-space is defined as $X_i = \hbar(x_i)$ and $\dot{X}_i = J(x_i)\dot{x}_i$, where $X_i \in \mathbb{R}^c$ denotes the generalized position in task space, $\hbar(x_i) \in \mathbb{R}^{p\times p}$ is the mapping from the joint space to the task space, $J_i(x_i) = \partial\hbar(x_i)/\partial x_i \in \mathbb{R}^{c\times p}$ is the known Jacobian matrix, which is nonsingular and bounded. It can be found that the result of joint space can be extended to the case of task space through kinematics changes. Besides, it has gained outstanding results in robotic manipulator control due to the fundamental role of joint space, such as [33], [38]. Therefore, it is meaningful to study joint space control of NRS.

Remark 8: Different from the constant gains, the matrix-weighted gains allows us to adjust the gains of different entries of the controlled state. From the application point of the control algorithms, the control algorithms with diagonal-matrix-weighted gains are easier to analyze.

Remark 9: Different from [39], in which the uncertain discrete nonlinear system is studied, the presented algorithms can deal with the cases of Euler-Lagrange system, which can be employed to describe many real-world systems. Different from [33], in which the traditional consensus tracking problem is investigated, we address the bipartite consensus tracking problem, which fills a gap in the field of multi-agent control.

IV. SIMULATION RESULTS

In this section, the simulation experiments of using the HHCAs (4) and (5) are carried out in the case of two different classes of networked robotic systems, namely, multi planar robotic systems and multi mobile robotic systems.

A. Bipartite Tracking of Multi Planar Robotic Systems

Consider the multi planar robotic systems containing seven two-DOF robot manipulators. The structure of robot manipulator is described by [40]. Moreover, the physical parameters of robotic manipulators are listed in Table II, where m_k, l_k, r_k and J_k are the mass, length, center of links mass and the moment of inertia of link k, for $k \in \{1, 2\}$, respectively. Besides, the robots communicate with each other through the

TABLE II
THE PHYSICAL PARAMETERS OF THE SEVEN ROBOTIC MANIPULATORS.

i^{th} robot	$m_k(kg)$	$l_k(m)$	$r_k(m)$	$J_k(kg\cdot m^2)$
1, 2	1.3, 1.8	2.0, 2.0	1.00, 1.00	0.4333, 0.600
3, 4	1.6, 1.3	2.1, 1.9	1.05, 0.95	0.5880, 0.3911
5, 6	2.0, 1.5	1.8, 2.2	0.90, 1.10	0.5400, 0.6050
7	1.9, 1.6	2.1, 2.0	1.05, 1.00	0.6983, 0.5333

signed digraph as shown in Fig.1, whose adjacency weight is chosen as $a_{ij} \in \{1, 0, -1\}$. It can be seen from Fig.1 that the assumption 1 holds, $B = \mathrm{diag}(0, 0, 1, 1, 0, 0, 0)$, $\mathcal{V}_1 = \{1, 2, 3\}$, and $\mathcal{V}_2 = \{4, 5, 6, 7\}$. Then the Laplancian matrix L is obtained as

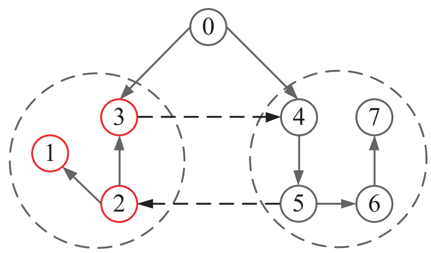

Fig. 1. The signed digraph of robots.

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}.$$

Define $\Phi = \text{diag}(1, 1, 1, -1, -1, -1, -1)$, then the eigenvalues of matrix $H_s = \Phi(L + B)\Phi$ are $w_1 = w_2 = w_3 = 1, w_4 = 0.382, w_5 = 1.5 + 0.866\mathbf{i}, w_6 = 1.5 - 0.866\mathbf{i}$, $w_7 = 2.618$. Let $K_{i1} = K_{i2} = 200I_2$, $\varepsilon = 1$, $d_i(t) = -0.5[\cos(t), \sin(t)]^T$, then inequality (9) holds. Let the leader trajectory be

$$\begin{cases} x_0 = [6.0 + 2.4\sin(0.5t), 5.0 + 2.4\cos(0.5t)]^T, \\ v_0 = [2.4 \times 0.5\cos(0.5t), -2.4 \times 0.5\sin(0.5t)]^T, \\ a_0 = [-2.4 \times 0.5^2\sin(0.5t), -2.4 \times 0.5^2\cos(0.5t)]^T. \end{cases}$$

The elements of $\hat{x}(0)$, $\hat{v}(0)$, $x(0)$ and $\dot{x}(0)$ are randomly selected from $[-10, 10]$.

Example 1. For HHCA (4), the tracking errors convergence to the neighborhoods of the origin if condition (9) holds, i.e., $\alpha, \beta < \min(2\text{Re}(w_i)/|w_i|^2) = 0.7639$. It is illustrated from Fig.2 that the control problem is solved if $\alpha = \beta = 0.75$. However, it cannot be solved if $\alpha = \beta = 0.764$ as shown in Fig.3. Figs.4 and 5 present that the evolution of position errors and velocity errors for $\alpha = \beta = 0.75$ and different sampling periods h. It can be seen that smaller h leads to smaller tracking errors.

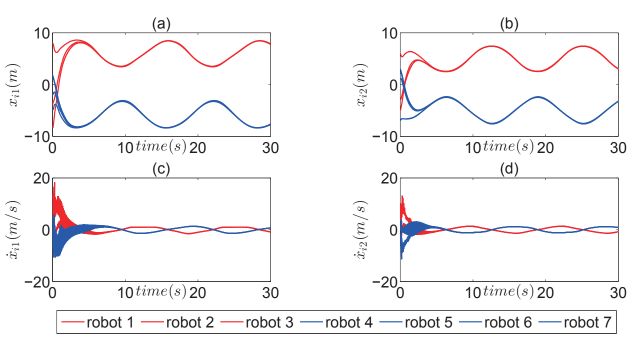

Fig. 2. The evolution of x_i and $\dot{x}_i$ for $\alpha = \beta = 0.75$.

Example 2. For HHCA (5), the bipartite tracking can be achieved if condition (19) holds. It can be obtained from (19) that $\beta < \min(2\text{Re}(w_i)/|w_i|^2) = 0.764$. Besides, choose $\beta =$

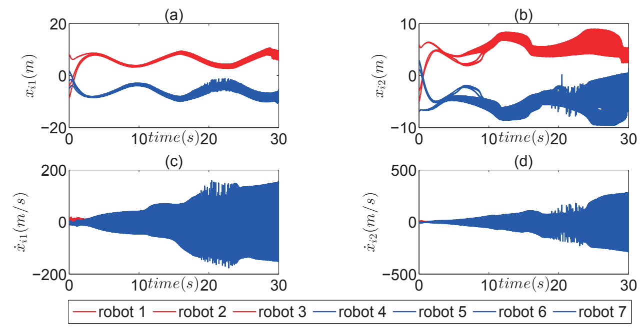

Fig. 3. The evolution of x_i and $\dot{x}_i$ for $\alpha = \beta = 0.764$.

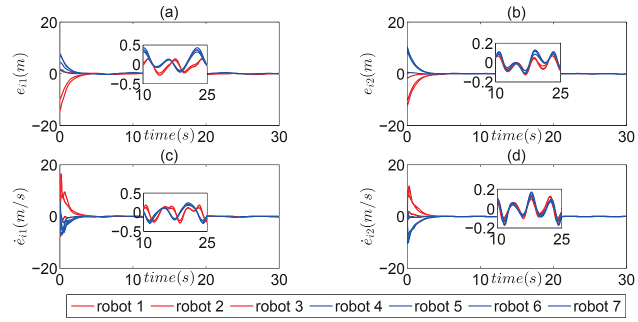

Fig. 4. The evolution of e_i and $\dot{e}_i$ for $\alpha = \beta = 0.75$ and $h = 0.01$.

0.5 and $\alpha = 1$. It then follows that

$$\min_{w_i \in \varpi(H_s)} \frac{\beta^2(4\text{Re}(w_i)|w_i|^2 - 2\beta|w_i|^4)}{\alpha(4\text{Im}^2(w_i) + \beta^2|w_i|^4)} = 0.428.$$

According to Theorem 2, the bipartite tracking with HHCA (5) can be achieved if $h < 0.428$. Fig.6 shows that the control problem is addressed if $h = 0.4$. However, the tracking errors are unstable if $h = 0.43$ as shown in Fig.7.

B. Bipartite Tracking of Multi Mobile Robotic Systems

Consider the multi mobile robotic systems containing eight identical mobile robots. Following [41], the ith robot is modelled as

$$M_i\ddot{q}_i + \beta_i\dot{q}_i = u_i, i \in \{1, 2, \cdots, 8\}, \tag{21}$$

where M_i and β_i are mass and damping constants, respectively. Assume that $M_i = 1$ and $\beta_i = 0.5$,

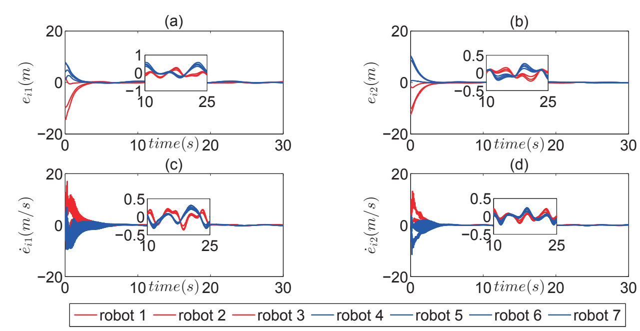

Fig. 5. The evolution of e_i and $\dot{e}_i$ for $\alpha = \beta = 0.75$ and $h = 0.05$.

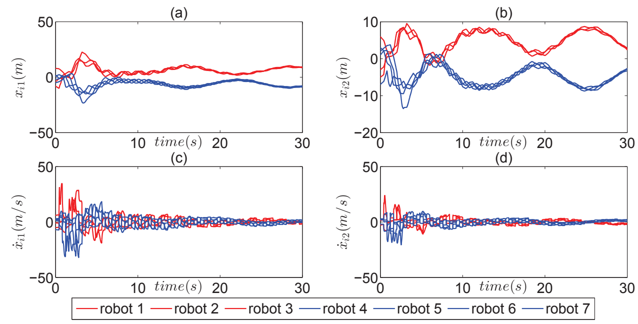

Fig. 6. The evolution of x_i and $\dot{x}_i$ for $h = 0.4$.

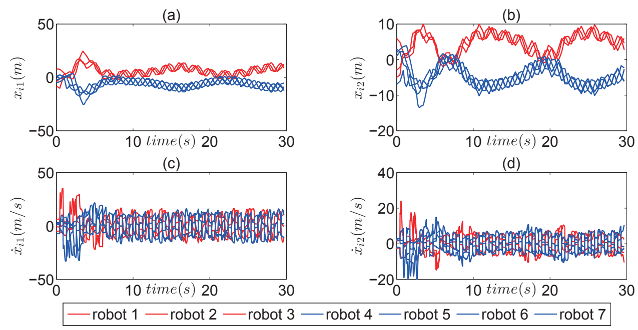

Fig. 7. The evolution of x_i and $\dot{x}_i$ for $h = 0.43$.

$\forall i \in \{1, 2, \cdots, 8\}$. Besides, the robots communicate with each other through the signed digraph as shown in Fig.9, whose adjacency weight is chosen as $a_{ij} \in \{1, 0, -1\}$. In Fig.9, $B = \text{diag}(0, 1, 0, 0, 0, 1, 0, 0)$, $\Phi = \text{diag}(1, 1, 1, 1, -1, -1, -1, -1)$, and the Laplancian matrix L is

$$L = \begin{bmatrix} 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

The eigenvalues of matrix $H_s = \Phi(L+B)\Phi$ are $w_1 = w_4 = 3$, $w_2 = w_3 = w_5 = w_6 = w_7 = 1$. Let $K_{i1} = K_{i2} = 10I_2$ and $\varepsilon = 1$, then inequality (9) holds. Let the trajectory of the leader

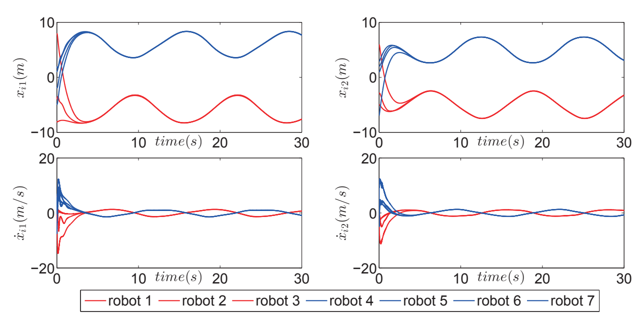

Fig. 8. The evolution of x_i and $\dot{x}_i$ for $\alpha = \beta = 0.75$.

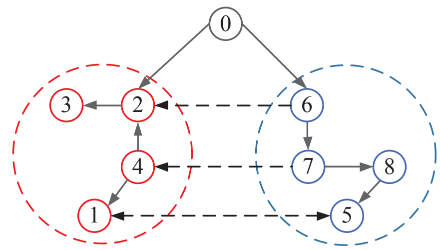

Fig. 9. The signed digraph of robots.

be chosen as

$$\begin{cases} x_0 = [7.0 + 2.5\sin(0.4t), 4.0 + 2.5\cos(0.4t)]^T, \\ v_0 = [2.5 \times 0.4\cos(0.4t), -2.5 \times 0.4(0.4t)]^T, \\ a_0 = [-2.5 \times 0.4^2\sin(0.4t), -2.5 \times 0.4^2\cos(0.4t)]^T. \end{cases}$$

The elements of $\hat{x}(0)$, $\hat{v}(0)$, $x(0)$ and $\dot{x}(0)$ are randomly selected from $[-10, 10]$.

Example 1. From HHCA (4), the tracking errors convergence to the neighborhoods of the origin if condition (9) holds, i.e., $\alpha, \beta < \min(2\text{Re}(w_i)/|w_i|^2) = 0.667$. It is illustrated from Fig.10 that the control problem is solved if $\alpha = \beta = 0.66$. However, it cannot be solved if $\alpha = \beta = 0.668$ as shown in Fig.11. Figs.12 and Fig.13 present that the evolution of position errors and velocity errors for $\alpha = \beta = 0.5$ and different sampling periods h, it can be seen that smaller h leads to smaller tracking errors.

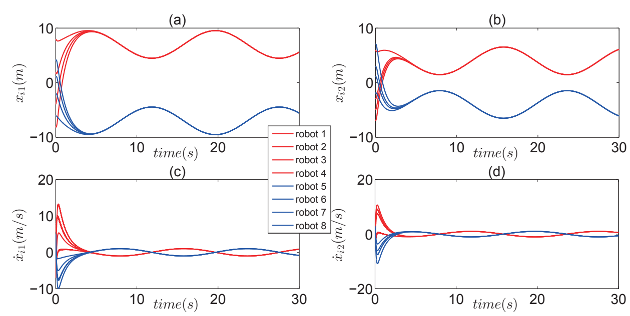

Fig. 10. The evolution of x_i and $\dot{x}_i$ for $\alpha = \beta = 0.66$.

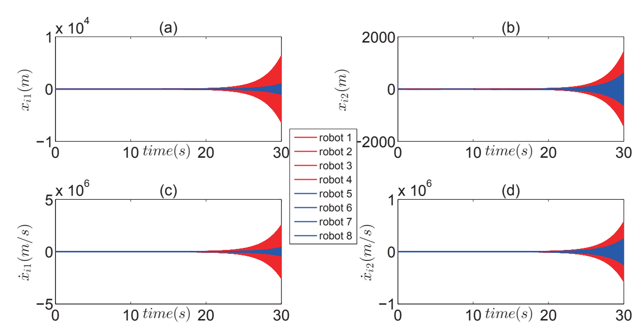

Fig. 11. The evolution of x_i and $\dot{x}_i$ for $\alpha = \beta = 0.668$.

Example 2. For HHCA (5), the bipartite tracking can be achieved if the condition (19) holds. It can be concluded from (19) that $\beta < \min(2\text{Re}(w_i)/|w_i|^2) = 0.667$. Besides, choose

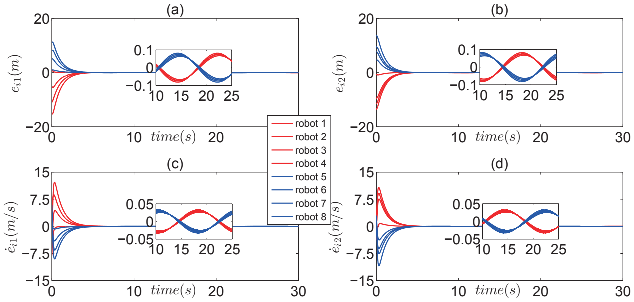

Fig. 12. The evolution of e_i and $\dot{e}_i$ for $\alpha = \beta = 0.5$ and $h = 0.01$.

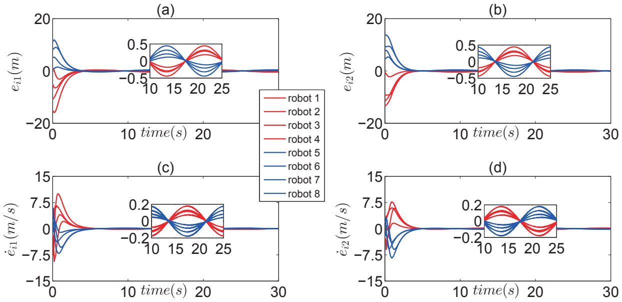

Fig. 13. The evolution of e_i and $\dot{e}_i$ for $\alpha = \beta = 0.5$ and $h = 0.1$.

$\beta = 0.5$ and $\alpha = 1$. It then follows that

$$\min_{w_i \in \varpi(H_s)} \frac{\beta^2(4\mathrm{Re}(w_i)\,|w_i|^2 - 2\beta\,|w_i|^4)}{\alpha(4\mathrm{Im}^2(w_i) + \beta^2\,|w_i|^4)} = 0.333.$$

According to Theorem 2, the bipartite tracking with HHCA (5) can be achieved if $h < 0.333$. Fig.14 shows that the control problem is addressed if $h = 0.30$. However, it cannot be solved if $h = 0.34$ as shown in Fig.15.

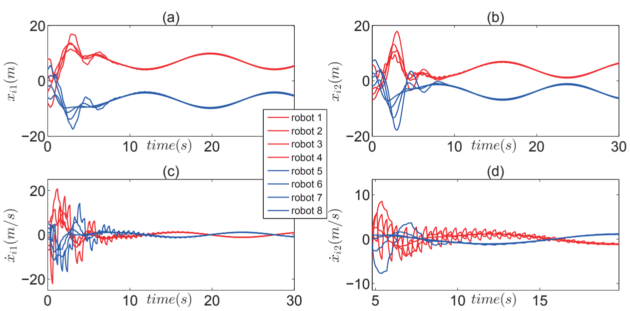

Fig. 14. The evolution of x_i and $\dot{x}_i$ for $h = 0.3$.

Remark 10: It can be seen from Lemma 1 that different values of ϕ_i lead to different tracking trajectories, but they are completely controllable, i.e., plus-minus signs of tracking trajectory are determined by the value of ϕ_i if Assumption 1 holds. For the comparison purpose, let $\Phi = \mathrm{diag}(-1, -1, -1, 1, 1, 1, 1, 1)$, it can be seen from Fig.8 that agents 1,2 and 3 track the opposite values of leader, this is the opposite result of what Fig.2 shows.

V. Conclusion

This paper has provided two classes of HHCAs to solve bipartite tracking problem for NRS with input disturbances, discrete communications and signed directed graphs. Without using the prior information of the system model, the proposed algorithms can address the continuous and discontinuous signals in a uniform framework. Some sufficient conditions on the communication graph, the control gains and the sampling periods are established to ensure the achievement of bipartite tracking for NRS by using Lyapunov-based stability theory. Finally, the performance of the proposed algorithms has been illustrated through numerical examples. In further work, bipartite tracking problems for NRS with switching communication graphs and stochastic noise will be taken into consideration.

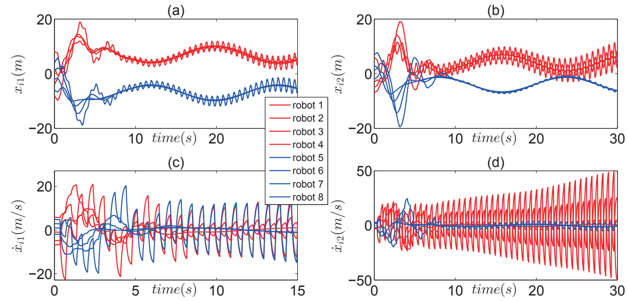

Fig. 15. The evolution of x_i and $\dot{x}_i$ for $h = 0.34$.

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