Luigi Bonatti* and Giulia Felice**

Endogenous growth and changing sectoral composition in advanced economies

ABSTRACT

We study how changing sectoral composition in employment and output shares affects aggregate growth by modeling a two-sector economy with a technologically "progressive" industry, which produces for consumption and investment, and a technologically "stagnant" industry producing only for consumption. Hence, unbalanced improvements in total factor productivity interact with changes in the composition of final demand in shaping the growth process. Within this endogenous growth framework, we show under what conditions on preferences Baumol's asymptotic stagnancy occurs. Beside studying the limiting behavior of the economy, numerical examples are presented to analyze the structural change going on along the transition path.

KEY WORDS: Balanced growth, structural change, non-homothetic preferences.

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*University of Trento <luigi.bonatti@economia.unitn.it>

**University of Milano <giulia.felice@unimi.it>

1 INTRODUCTION

There is a striking evidence that dramatic changes in the sectoral output and employment shares occur during any development and growth episode. In particular, a sharp increase in service-sector employment share to the detriment of manufacturing has taken place in industrialized economies during the last fifty years. This notwithstanding, growth theoreticians usually treat the economy as if its sectoral composition were constant for very long periods. In general, this literature does not provide an adequate framework for explaining structural change and its implication for aggregate growth. In contrast, this paper aims at modelling the changing sectoral composition that characterizes the economic dynamics of the advanced countries by developing a two-sector endogenous-growth framework.

Two main issues are addressed in this work. The first issue regards the pattern of structural change in the advanced countries: we intend to assess whether the increasing share of workers employed in the stagnant sector might affect negatively the aggregate growth rate of income and productivity in the advanced economies, as it is predicted by the Baumol's (1967) cost disease model and recently confirmed by Nordhaus (2006).[1] The second issue emerges at a more theoretical level as a consequence of the previous one and concerns the methodological problems arising when structural change is taken into account in growth modelling. The so-called structural change versus balanced growth path (BGP) debate has to do with these problems: on one side, there are those who disregard the employment sectoral changes focusing on the BGP, where--by definition--aggregate growth is steady and sectoral employment shares are stable; on the other side, there are those who seek to reconcile the stylized facts on structural change and steady aggregate growth in the long run. In dealing with these issues, an important conclusion is that when total factor productivity (TFP) grows unevenly across sectors, technological progress can be insufficient to generate perpetual growth, since final demand conditions may be determinant.

This model has two main features that are crucial for explaining the structural change which is peculiar to the growth process in the advanced economies. On the supply side, we assume that there is a "progressive" industry adopting an AK technology to produce both for consumption and for investment, and

[1] See the next section for a short discussion of Baumol's (1967) main point and a brief outline of Nordhaus' analysis.

a technologically “stagnant” industry, which produces only for consumption.[2] The stagnant industry uses an input (physical capital) that is produced by the progressive industry, thus benefiting indirectly by the possible improvements in TFP achieved in the latter. On the demand side, we consider both homothetic and non-homothetic consumers’ preferences, so as to analyze the consequences for the growth process of different hypotheses on the evolution of final demand. This formal set-up is especially suited to study how aggregate growth is affected by the interaction between the increase in total factor productivity, which has a stronger impact on the manufacturing sector, and the demand for services, which tends to increase—other things being equal—more than proportionally than total expenditure in consumption. To our knowledge, indeed, no other growth model—even among those recent theoretical contributions dealing with sectoral changes (see Echevarria, 1997; Laitner, 2000; Kongsamut et al. 2001; Ngai and Pissarides, 2004; Acemoglu and Guerrieri, 2005)—captures the joint effect of non-homothetic preferences and improvements in total factor productivity having an uneven impact on different industries, in a framework of endogenous growth.

With respect to the first issue, an important contribution of our paper consists in showing within an analytical set-up with optimizing agents and uneven technological progress that asymptotic stagnancy may occur when stagnant industries supply final goods or services and when preferences are non-homothetic, namely when what households spend on the stagnant good tends to increase more than proportionally than their total consumption expenditure. This result confirms and gives a solid micro-foundation to the Baumol’s (1967) thesis of asymptotic stagnancy. However, another important result of this paper amounts to showing that--even when stagnant industries supply final goods or services and preferences are non-homothetic--asymptotic stagnancy does not occur if a portion large enough of what households spend on consumption at

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[2] The “progressive” sector can be identified with manufacturing sector, with the possible inclusion of some service branches (transport, communications, financial services), which have experienced radical changes in their production processes because of the massive introduction of information and communication technologies (see also footnote 7 and 9). One may include in the “stagnant” sector the remaining branches of services. As examples of these stagnant activities, one may mention communal and personal services, and social activities: care of old people and child (medical and health), education, welfare, government, domestic activities, entertainment, eating, hotels, repair. A distinction along similar lines was proposed, but at the early stage of the information and communication technologies (ICT) revolution, by Baumol (1967) and Baumol et al. (1985).

any point in time is devoted to the progressive good. In this case, indeed, perpetual growth occurs even if stagnant industries supply final goods or services and even if what households spend on the products of the stagnant industries tends to increase more than proportionally than their total consumption expenditure.

With respect to the second issue, (i.e., the role of structural change in aggregate growth analysis), we study the asymptotic properties of an economy subject to structural changes in order to provide important insights on the direction towards which it will proceed in the very long run. However, we claim that it is essential for explaining what is currently going on in the advanced economies to focus on the transition path.[3] Indeed, the employment sectoral shares are constant by construction along a balanced growth path, while an important stylized fact regarding the industrialized countries is that these shares evolve in time. In this spirit, our model takes the existence of investment adjustment costs into account, thus adding an element of realism to the analysis and studying the transition path of the economy. Therefore, we present two numerical examples where we show that starting from an initial employment share of the progressive sector in overall employment greater than its long-run equilibrium share, the gradual shift of employment shares towards the stagnant sector is accompanied by rates of growth of output and capital stock that are higher in the stagnant sector than in the progressive one. Moreover, along this transition path, the relative price of the stagnant good is growing and the economy's GDP tend to grow at a higher rate than along the balanced growth path of the economy: the gradual shift of labor towards the stagnant sector is accompanied by a decline in the aggregate rate of growth. In other words, the pattern resulting from these numerical examples seems to be consistent with the stylized facts both in the case where preferences are assumed to be homothetic and in the case with non-homothetic preferences, although the latter case appears to be more

[3] Under this respect, we agree with Temple (2003), who emphasizes that one should focus not only on the BGP but also on the transitional dynamics, especially in the light of the fact that the transition can take decades. In Temple (2003) one can find an exhaustive and thoughtful discussion on the role of long run analysis in endogenous growth models. According to the author, the long run is a theoretical abstraction, useful as a tool of analysis, but to be considered with agnosticism especially for policy purposes. Besides that, Temple discusses the conditions under which perpetual growth is possible, pointing out that it is legitimate for growth models to generate perpetual growth or BGP along which growth has ceased. He argues that in the literature too much emphasis is given to growth effects over level effects, being the latter relevant when welfare issues are addressed.

relevant in the light of empirical estimates showing an income elasticity of demand greater than one for the services and lower than one for the manufactured goods.

To conclude, the model that we present in this paper is consistent with a world where i) learning by doing and technological spillovers are uneven across sectors and are a necessary but not a sufficient condition for perpetual growth, and ii) the composition of final demand and its evolution is relevant for long run growth.

This paper is organized as it follows. *Section 2* presents the main stylized facts about structural change and briefly reviews some theoretical and empirical contributions. *Section 3* presents the model. *Section 4* discusses the balanced growth paths of the economy. *Section 5* is devoted to the transition paths. *Section 6* concludes.

2 MOTIVATIONS

Stylized facts

We present some stylized facts that may help understanding the changes in sectoral composition that have occurred in the advanced countries in the last decades, together with some evidence on the characteristics of technology and demand for different groups of goods.

1. It is typically observed in industrialized economies a first phase of increase in manufacturing and services shares to the detriment of agriculture, followed by a second phase characterized by the sharp increase in the services share in overall employment to the detriment of manufacturing. Looking at Table 1 in the Appendix A, we see that starting, at the beginning of last century, from an employment share of 16% in Italy, 26.2% in Germany, 27.1% in France, 31.4% in the US, 43.1% in the UK, services have reached in 1990 respectively a share of 64.6% in France, 59,7% in Italy, 58.7% in Germany, and about 70% in the US and in the UK[4]. The services share in total expenditure remains constant or rises slightly as income grows, when expressed in real terms (constant prices), while it is

[4] In 1998, the services share in overall employment reached 70.7% in France, 64.1% in Italy, 62.1% in Germany, 71% in the UK and 73.8% in the US (see Table 3.2 in Oecd, 2000).

sharply increasing when measured in nominal terms (current prices). In 1998, the services share in nominal value added for the three countries is about 70% (Oecd, 2000, in particular Table 3.8). This evidence is also provided by Kravis, Heston and Summers (1983) and by Summers (1985) and, more recently, by Echevarria (1997), Appelbaum and Schettkat (1999) and Mattey, (2001).

2. The relative price of services increases with income. As mentioned before, the services share is growing more in nominal terms than in real ones. This is explained by the positive correlation between the price of services and GDP, as come out from the cross-sectional and longitudinal analysis presented by Kravis et al. (1983) and Summers (1985). According to Rowthorn and Ramaswami (1999), the relative price of services increases because their productivity grows more slowly.

We summarize what emerges from the empirical literature about the sectoral aspects of demand and technology in the following points:

a) Services are more labor intensive than manufacturing. The capital intensity (capital per hour worked) in 2000 is lower in most of the service sectors [5]. This is true despite the fact that the pace of capital accumulation appears to be faster in services than in manufacturing (see Glyn, 1997; Erdem and Glyn 2001).[6]

b) The recent empirical research reaches a general consensus in pointing out the negative productivity differential of most of the service branches compared with manufacturing ones (see Kravis et al., 1983; Summers, 1985; Maddison, 1991; Rowthorn and Ramaswamy, 1999; Inman in Oecd, 2000, Mohnen and ten Raa, 2001). One can see from Table 2 in Appendix A that the growth rates of productivity in services are lower than in manufacturing with the exception of branches like "Transport and Communications" and "Finance".[7] In the end, it is worth mentioning what shown by

[5] With the exceptions of "transport and communication" and of "financial services", as O'Mahony and Van Ark (2003) have pointed out.

[6] This being probably due to the faster capital accumulation in the branches "transport and communications" and "financial services", see the footnote above.

[7] Although the existence of a productivity bias in favour of manufacturing is widely accepted, it is not evident whether this differential will be preserved in the future, when ICT will increasingly affect the service sector. In this respect, a

Harrison (2003). Indeed, she presents evidence of low externality in production together with decreasing or constant internal returns to scale at firm level in the consumption sector, together with evidence of constant or increasing internal returns to scale and statistically significant externality in the investment sector.

c) The income elasticity of demand is estimated to be above unity for most of the service branches and for services as an aggregate. The same elasticity is sharply below unity for manufacturing branches and for the whole sector (see Curtis and Murthy, 1998; Rowthorn and Ramaswamy, 1999; Inman in Oecd, 2000; Möller, 2001).[8]

We presented the evidence above with reference to the standard industrial classification, that is to say with reference to the dichotomy manufacturing/services and, when possible, to some branches of the two macro sectors. However, our focus is on the distinction between "progressive" and "stagnant" sectors, no matters whether they belong to the services or to manufacturing industries.[9] As it is well known, this dichotomy was firstly introduced in Baumol's (1967) seminal work and in Baumol et al. (1985). In a recent paper, Nordhaus (2006) investigates Baumol's "disease" for the overall economy. He analyzes the impact of a differential in productivity growth on the dynamics of different sectors and of the overall economy.[10]

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consequence of the application of ICT is the possibility of separating production and consumption for many service activities, increasing their "stockability" and "transferability". For evidence and discussions on the impact of ICT on sectoral productivities, see Petit and Soete (1997), Greenhalgh and Gregory (2001), Mattey (2001), Triplett (2003), O' Mahony and van Ark (2003). Recent studies (Mohnen and ten Raa, 2000 and 2001) show that productivity differentials between services and manufacturing measured by labor productivity are higher than measured by TFP and that service branches are heterogeneous with respect to productivity growth and capital/labor substitution.

[8] Services as a whole appear to be highly price inelastic. Price rigidities are found by Curtis and Murthy (1998) and Möller (2001), although the evidence on the existence of price rigidities appears to be less univocal than the evidence on the existence of income elasticity.

[9] The distinction between services and manufactured goods is not so obvious and--starting from the origin of economics--it has been strictly related to the dichotomy between productive and unproductive activities. Being beyond the scope of our paper, we cannot go deeply into this debate. For a discussion on this issue showing its implications for growth theory, see Hill 1977, 1999, and Parrinello 2004.

[10] Nordhaus calculates labour productivity and TFP for the different industries using US data for the period 1948-2001.

Estimating some reduced forms of Baumol's relationships, he provides some evidence that confirms Baumol's predictions regarding the dynamics of sectoral output, and the evolution of employment shares and relative prices. In particular, he finds that the overall productivity growth has been negatively affected in the last decades by the increase in the share of the stagnant sectors.[11]

Theoretical literature

The evidence reported above raises two main questions: What is the role of the "stagnant" industries in the process of growth of aggregate income and productivity in the advanced economies? What are the implications of the fact that technological progress is uneven across industries for the predictions that can be derived from the BGP analysis, which is typically carried out in models with only one final good?

The role of stagnant activities in the growth process was considered by Baumol's (1967) seminal work. Baumol argues that in a world of unbalanced productivity growth, where the ratio between the output of the progressive sector and the output of the stagnant sector is held constant, the overall rate of economic growth will decline asymptotically toward zero. In Baumol's model, the ratio between the outputs of the two sectors can be constant either because demand exhibits price rigidities or because it is characterized by an income elasticity above unity for the good produced in the stagnant sector. Fuchs (1968), Kuznets (1971), and recently Ngai and Pissarides (2004) and Acemoglu and Guerrieri (2005) explain this constancy by invoking price rigidities, while Echevarria (1997) and by Kongsamut et al. (2001) explain it--in a slightly different context—by invoking the income elasticity of the stagnant good.

In particular, the pattern of structural change described by Baumol is driven by uneven exogenous technological progress, which in Ngai and Pissarides (2004) takes place in the presence of homothetic preferences and low substitutability between goods, while in Echevarria (1997) and Kongsamut et al. (2001) in the presence of non-homothetic preferences.[12] In Acemoglu and Guerrieri, Baumol's structural change is driven by different factor proportions across sectors and--in an extension of the basic model--by uneven

[11] Nordhaus' findings give support to the conclusions already reached by Peneder (2003).

[12] In Kongsamut et al. (2001) the case of uneven technological progress is not explicitly addressed, being beyond the scope of their paper.

endogenous technological change, in the presence of homothetic preferences and low substitutability in demand. Finally, Ngai and Pissarides (2004) show--in contrast with Baumol's predictions-- that perpetual growth can be an outcome of the model if it is introduced a capital good produced by the sector where productivity growth is higher.[13]

In the light of the previously reported evidence, our model gives micro-foundation to the second explanation proposed by Baumol, assuming non-homothetic preferences in a framework characterized by growth. In contrast with Ngai and Pissarides (2004), the introduction of a "progressive" capital good is not sufficient in our model to avoid asymptotic stagnancy. In the presence of non-homothetic preferences, the economy exhibits stagnation if the "stagnant" good expenditure share is increasing with income over time. Going beyond Baumol, stagnation can be avoided when preferences are non-homothetic if the composition of the consumer expenditure is "virtuous" at any point in time, i.e., if the expenditure share devoted to the "progressive" good is large enough at any point in time.[14] Again beyond Baumol, we show that asymptotic stagnancy is possible even when preferences are homothetic, if the share of the good produced in the stagnant sector in total consumer expenditure is large enough at any point in time.

As it is mentioned above, the evidence on structural change can be hardly reconciled with the predictions of growth models with one final good, unless it is shown that the shift of economic activities and employment from some sectors to others has no impact on the dynamics of the aggregate variables. In assessing this question, we also capture the relevance of the composition of final demand for long run

[13] In Baumol's model, labor is the only factor of production. In a recent contribution, Oulton (2001) shows that Baumol's stagnationist conclusion does not apply when the stagnant industries supply intermediate products. For similar conclusions, see Pugno (2006), which models an economy with endogenous growth and human capital accumulation, where the expenditure devoted to stagnant activities like education or health enhances growth through its impact on human capital accumulation.

[14] We postpone the analysis of the good produced in the stagnant sector as intermediate good to future work. Indeed, consensus has not yet been reached about the relative importance of the use of goods produced in stagnant industries as intermediate products (see Mohnen and ten Raa, 2001; Russo and Schettkat, 2001), while it is widely recognized the importance of physical capital as an input in most service industries, which is a feature that is captured by our model. For a recent review of the literature on the shift to services, see Schettkat and Yocarini (2006).

growth, usually disregarded by the literature.[15] This point is closely linked to the "conflict" between structural change and BGP, which can be summarized in the following way: in a world with unbalanced productivity growth, the sectoral composition of employment cannot change along a BGP. The changes in the employment shares of sectors characterized by different rates of TFP growth and/or by different factor intensities have an impact on the aggregate growth rate. In its turn, the latter is bound to be unsteady as far as the employment shares do not stabilize. Only when these shares stabilize, the economy exhibits a steady aggregate rate of growth, i.e., it reaches a BGP path. Thus, structural change intended as change in the sectoral composition of employment ceases by construction along a BGP.

The relevance of structural change with respect to aggregate dynamics was emphasized by the seminal work of Pasinetti (1984). In particular, in his work the use of models assuming the existence of only one final good as theoretical set-ups for the study of the aggregate dynamics is radically put under discussion. His fundamental appraisal was the basis for some further research, among which it is worth mentioning Reati (1998), Metcalfe (2000) and Montobbio (2001).

Nonetheless, the omission of structural change in most of recent growth models, and the priority given to BGP analysis, is commonly accepted. This could be attributed to the acceptance of the so-called "Kaldor facts"[16] as a good description of the behavior of aggregate variables in the long run by most growth theoreticians (including endogenous growth theoreticians). However, some recent papers seek to reconcile the Kaldor facts (in particular, steady aggregate growth) with the existence of structural change.[17] Some of these papers (Meckl, 2002, and Foellmi and Zweimüller, 2002) assume that technological progress is uniform across the sectors producing the final products, while some others (Ngai and Pissarides, 2004) assume that the intertemporal elasticity of substitution is unitary. In contrast with this approach, Echevarria (1997) and Kongsamut et al. (2001) argue that the long-run economic dynamics has to be analyzed out of the BGP. In particular, Kongsamut et al. find a knife-edge condition on parameters which must be satisfied for a generalized balanced growth path (GBGP) to exist, where the GBGP is characterized as a path along which

[15] See, as a notable exception, Aoki and Yoshikawa (2002).

[16] That is, *per capita* output grows at a rate that is roughly constant, the capital-output ratio is roughly constant, the real rate of return to capital is roughly constant, the share of labour and capital in national income are roughly constant.

[17] Which is sometimes labelled as "Kuznets facts".

the real interest rate is constant but the allocation of inputs across sectors evolves in time.[18] Consistently with the latter approach, we believe that--compared with the striking evidence regarding the changes in the sectoral composition of employment and output and in the structure of relative prices--the empirical evidence regarding the long-run steadiness of aggregate growth is much more controversial. Hence, our paper extends the analyses of Echevarria (1997) and Kongsamut et al. (2001) by allowing for endogenous growth and by introducing investment adjustment costs in order to examine the properties of the system along the transition path, thus avoiding the special assumptions on the parameters values on which their analysis rely. We also extend the non-homothetic approach of Meckl (2002) and Foellmi and Zweimüller (2002) by allowing for sector specific rates of technical progress.

The growth rates of GDP and productivity have decreased since the second half of the 1970s in industrialized economies, compared with their values in the previous decades. [19] We think that the puzzle of low and decreasing aggregate growth rates affecting the advanced economies in a world of accelerating technological progress can be solved if an endogenous growth framework is extended so as to take into account sector-specific rates of technological progress in the production of final goods and services, and income effect on the demand side of the economy. The study of the different BGP and transition paths that can be generated within this formal set-up shows how the existence of an engine of growth (which in our framework is given by a combination of learning by doing and technological spillovers) is a necessary but not sufficient condition for perpetual growth. Indeed, the composition of final demand emerges as an important determinant of long-run growth, since the boosting effects of learning by doing and technological spillovers on growth can be offset by a “not growth-enhancing” pattern of demand.

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[18] It is worth noting that the existence of the GBGP depends on very particular combinations of values of the parameters entering both the utility and the production function.

[19] GDP growth has decreased, in most of the industrialized economies, from yearly rates well above 4% in the decades before 1970, to rates about 2% in the post-1970 period. The trend of aggregate productivity appears to be similar. In the US both the rates have risen since the mid 90’s, while there is no evidence of a similar recovery in the EU countries (see Mc Guckin and Van Ark, 2003 and Oecd, 2002.

3 THE MODEL

We consider an economy in discrete time with an infinite time horizon. Consistently with the Baumol's well-known distinction, we assume that in this economy there is a "progressive" sector characterized by a combination of learning by doing and technological spillovers, and a "stagnant" sector with no learning by doing and technological spillovers. The good produced in the progressive sector (the "progressive good") is the numéraire of the system (its price is set to be one), and it can be both consumed and used for investment purposes. The good produced in the stagnant sector (the "stagnant good") can be only consumed. Finally, all markets are assumed to be perfectly competitive.

Households

For simplicity and without loss of generality, it is assumed that the population is constant and that each household contains one adult working member of the current generation. Thus, there is a fixed and large number (normalized to be one) of identical adults who take account of the welfare and resources of their actual and perspective descendants. Indeed, following Barro and Sala-i-Martin (1995), this intergenerational interaction is modeled by imaging that the current generation maximizes utility and incorporates a budget constraint over an infinite future. That is, although individuals have finite lives, the model considers immortal extended families ("dynasties"). Again for simplicity and without loss of generality, it is assumed that all households--being the firms' owners--are entitled to receive an equal share of the firms' net cash flow ("net profits").[20]

Households decide in each period what fraction of their labor income and gross returns on wealth to spend on consumption rather than on buying corporate bonds. Simultaneously, they decide how to allocate their consumption expenditure over the progressive good and the stagnant one. Hence, the representative household's problem amounts to deciding a contingency plan for C_{Mt}, C_{St} and B_{t+1} in order to maximize:

$$\sum_{t=0}^{\infty} \theta^t C_{Mt}^{\eta} (C_{St} + \varepsilon)^{\gamma}, \quad 0<\theta<1,\ 0<\eta<1,\ 0<\gamma<1,\ \varepsilon\geq 0, \tag{1}$$

subject to

$$B_{t+1} + C_{Mt} + P_t C_{St} \leq W_t + (1+r_t)B_t + \pi_{Mt} + \pi_{St},\ B_0 \text{ given.} \tag{2}$$

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[20] As in Barro and Sala-i-Martin (1995, p. 120), we assume that the firms' net cash flow is paid out as dividends to the shareholders.

In (1) and (2), C_{Mt} and C_{St} are, respectively, the progressive good and the stagnant good consumed by the representative household in period t; B_t are corporate bonds with maturity in period t and issued in t-1; θ is a time-preference parameter; ε can be interpreted as the amount of stagnant good that is produced at home; P_t is the price of the stagnant good (or the relative price, namely the units of progressive good that are necessary to buy one unit of the stagnant good); W_t is the wage rate (the quantity of labor supplied by each household is assumed to be fixed and set to be one); r_t is the one-period market rate of interest, and π_{Mt} and π_{St} are the net cash flows generated in period t, respectively, by the firms producing the progressive good and by the firms producing the stagnant good. It is worth to note that in the special case where ε=0 the period-utility function is Cobb-Douglas, while for ε>0 preferences are not homothetic: in the latter case, the elasticity of the demand for the stagnant good with respect to the household's consumption expenditure is more than one, while the elasticity of the demand for the manufactured good with respect to the household's consumption expenditure is less than one.[21]

Firms producing the progressive good

The progressive good is denoted by Y_{Mt} and is produced by a large number (normalized to be one) of identical firms according to the technology

$$Y_{Mt} = A_t L_{Mt}^{\alpha} K_{Mt}^{1-\alpha},\ 0 < \alpha < 1, \tag{3}$$

where A_t is a variable measuring the state of technology, K_{Mt} is the capital installed in the progressive sector (capital can be interpreted in a broad sense, inclusive of all reproducible assets) and L_{Mt} is labor employed in the progressive sector. It is assumed that A_t is a positive function of the stock of capital existing in the

[21] The intuition underlying the case with ε>0 is that the stagnant good is produced by a traditional technology, so that it can always be produced at home (personal care, social activities, education, entertainment, child and old people care, repairing and food preparing are examples of stagnant activities mentioned in the previous section). As income exceeds a certain threshold, the households start buying the stagnant good on the marketplace; then—as income grows further--the expenditure devoted to the stagnant good grows more than proportionally. This is reflected in the non-homotheticity of preferences, which is consistent with the empirical evidence reported in the previous section. As the income grows steadily at a positive rate, the effect of ε vanishes asymptotically, and the share of the stagnant good on total expenditure tends to grow proportionally with income.

progressive sector: $A_t = K_{Mt}^{\alpha}$. This assumption combines the idea that learning by doing works through each firm's capital investment and the idea that knowledge and productivity gains spill over instantly across all firms (see Barro and Sala-i-Martin, 1995). Therefore, in accordance with Frankel (1962), it is supposed that although A_t is endogenous to the economy, each firm takes it as given, since a single firm's investment decisions have only a negligible effect on the aggregate stock of capital.

The period net cash flow π_{Mt} of the firm producing the progressive good is given by:

$$\pi_{Mt} = Y_{Mt} - W_t L_{Mt} - (1 + r_t) B_{Mt}, \tag{4}$$

where B_{Mt} are the bonds with maturity in period t and issued by a firm producing the progressive good in t-1 to finance its investment expenditure in that period.

Firms producing the stagnant good

The stagnant good is denoted by Y_{St} and is produced by a large number (normalized to be one) of identical firms according to the technology

$$Y_{St} = L_{St}^{\beta} K_{St}^{1-\beta}, \; 0 < \beta < 1, \tag{5}$$

where K_{St} is the capital installed in the stagnant sector and L_{St} is labor employed in the stagnant sector. By definition, in the stagnant sector no advance in total factor productivity takes place: the intuition is that in this sector the production process is intrinsically time intensive and there is no room for learning by doing or spillovers leading to productivity gains. The good produced by this sector is not used in production, while the good produced by the "progressive" sector is used as capital in both sectors, this being one of the reasons of the narrow opportunity of learning in the stagnant sector.[22] However, the stagnant sector can in some way internalize the increases in efficiency which characterize the progressive sector by accumulating K_{St}.

The period net cash flow π_{St} of a firm producing the stagnant good is given by:

$$\pi_{St} = P_t Y_{St} - W_t L_{St} - (1 + r_t) B_{St}, \tag{6}$$

where B_{St} are the bonds with maturity in period t and issued by a firm producing the stagnant good in t-1 to finance its investment expenditure in that period.

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[22] As mentioned above, in some recent contributions (see e.g. Harrison, 2003), disaggregate analysis provides evidence of externalities being at work, showing that sectors differ in their degree of externalities and/or internal returns.

Investment

The process of installing new capital and adapting the existing production facilities to the new machinery and equipment reduces the progressive good available for consumption purposes and for adding to the stock of capital. One may think of this adjustment cost indifferently as if the producers of the progressive good must divert resources from production in order to assist the capital users in installing the new capital, or as if some amount of the progressive good is used up in the process of installing the new capital. Since firms finance their investment costs $c(I_{it},K_{it})$ by issuing debt, one has:

$$c(I_{it},K_{it})=I_{it}+\frac{I_{it}^2}{K_{it}}=B_{it+1},\ \ i=M,S, \tag{7}$$

where investment costs are assumed to be the sum of gross investment I_{it} and adjustment costs, that are a quadratic function of I_{it} and a decreasing function of K_{it}.[23]

The capital stock installed in each sector evolves according to

$$K_{it+1}=I_{it}+(1-\delta)K_{it}\ ,\ 0\leq\delta\leq1,\ K_{i0}\text{ given, } i=M,S, \tag{8}$$

where δ is a capital depreciation parameter.

Firms' objective

The representative firm chooses sequences $\{L_{it}\}_{t=0}^{\infty}$ and $\{I_{it}\}_{t=0}^{\infty}$ in order to maximize the present value of its net cash flows between 0 and infinity, discounted in accordance with the market rate of interest:

$$\sum_{t=0}^{\infty}\frac{\pi_{it}}{\prod_{v=1}^{t}(1+r_v)},\ \ i=M,S, \tag{9}$$

[23] This functional form for the adjustment costs was introduced by Lucas (1967). In order to assess the importance of these costs, one may think that part of the value added generated by the industries producing capital goods derives from their role in assisting the capital users in the process of installing the new capital goods and adapting them to their specific productive set-ups. The fact that the adjustment costs are assumed to be decreasing in K captures the idea that adapting a new piece of machinery to the production process is less costly if this additional unit of capital represents a smaller fraction the existing stock of capital. In an endogenous growth framework, this assumption is essential for preserving the possibility of perpetual growth**.** Capital adjustment costs are needed in an AK framework in order to study the transitional dynamics of the system in the case of perpetual growth and homothetic preferences.

subject to (5), (6) and (7), where $\prod_{v=1}^{0}(1+r_v)=1$.

Markets equilibrium

Equilibrium in the product markets requires, respectively,

$$Y_{Mt}= C_{Mt}+I_{Mt}+\frac{I_{Mt}^2}{K_{Mt}}+I_{St}+\frac{I_{St}^2}{K_{St}} \tag{10}$$

and

$$Y_{St}= C_{St}. \tag{11}$$

Equilibrium in the labor market requires

$$1=L_{Mt}+L_{St}. \tag{12}$$

Equilibrium in the asset market requires

$$B_{Mt}+B_{St}=B_t. \tag{13}$$

4 BALANCED GROWTH PATHS

The intersectoral efficiency condition (see (A1)) implies that one can have a path along which all the variables grow at a constant rate and the real interest rate is constant, i.e. a balanced growth path (BGP), only if the employment shares stabilize. This confirms that the BGP analysis can be hardly reconciled with the study of structural change, namely with the study of the evolution in time of the sectoral employment shares.

Homothetic Preferences

As preferences are homothetic, the equilibrium trajectory of the economy is governed by a system of three difference equations in L_{Mt}, $X_{Mt}\equiv\frac{I_{Mt}}{K_{Mt}}$ and $X_{St}\equiv\frac{I_{St}}{K_{St}}$ (see equations (A19)-(A21) in Appendix B). Therefore, a BGP can be characterized by setting $L_{Mt+1}=L_{Mt}=L_M$, $X_{Mt+1}=X_{Mt}=X_M$ and $X_{St+1}=X_{St}=X_S$ in the system (A19)-(A21). In other words, along a BGP, the employment shares and the investment-capital ratios stabilize.

If a BGP exists, it is characterized by $\overset{\circ}{X}_S=\overset{\circ}{X}_M$, $\overset{\circ}{L}_M=f(\overset{\circ}{X}_M)$, $\overset{\circ}{X}_M=g(\overset{\circ}{X}_M)$ ("°" denotes the BGP value of a variable when ε=0), where

$$f(\mathrm{X_M}) = \left\{ \left[\frac{(\mathrm{X_M} + 1 - \delta)^{1-\eta-\gamma(1-\beta)}}{\theta} - 1 + \delta \right] \frac{(1 + 2\mathrm{X_M})}{(1-\alpha)} - \frac{(\mathrm{X_M})^2}{(1-\alpha)} \right\}^{1/\alpha}, \tag{14}$$

$$g(\mathrm{X_M}) = -\frac{1}{2} + \left[\frac{1}{4} + z(f(\mathrm{X_M})) \right]^{1/2}, \tag{15}$$

$$z(f(\mathrm{X_M})) = \frac{(1-\alpha)[(\beta\gamma + \alpha\eta)(f(\mathrm{X_M}))^{1+\alpha} - \alpha\eta(f(\mathrm{X_M}))^{\alpha}]}{\gamma[\alpha(1-\beta) + (\beta-\alpha) f(\mathrm{X_M})]}. \tag{16}$$

Considering (A15) and (A16), note that $\overset{\circ}{\mathrm{X}}_\mathrm{S} = \overset{\circ}{\mathrm{X}}_\mathrm{M}$ entails $\overset{\circ}{\mu}_\mathrm{M} = \overset{\circ}{\mu}_\mathrm{S}$, where $\mu_{\mathrm{it}} \equiv \frac{\mathrm{K_{it+1}} - \mathrm{K_{it}}}{\mathrm{K_{it}}}$, i=M,S (along a BGP, the capital stock grows at the same rate in the two sectors). Note also that $\overset{\circ}{\mu}_\mathrm{M} = \overset{\circ}{\mu}_\mathrm{S} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$ whenever $\overset{\circ}{\mathrm{X}}_\mathrm{M} = \overset{\circ}{\mathrm{X}}_\mathrm{S} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \delta$ (the steady-state rate of growth of capital is positive if and only if the steady-state ratio between gross investment and capital stock is larger than the capital-depreciation parameter). By inspecting (15), one can also check that $\overset{\circ}{\mathrm{X}}_\mathrm{S} = \overset{\circ}{\mathrm{X}}_\mathrm{M} = \delta$ entails $\mathrm{Z} \equiv z(f(\mathrm{X_M}))\Big|_{\mathrm{X_M} = \delta} = \delta + \delta^2$. Moreover, for parameter values consistent with $\frac{dg(\mathrm{X_M})}{d\mathrm{X_M}}\Big|_{\mathrm{X_M} = \delta} > 1$ and the existence of $\overset{\circ}{\mathrm{X}}_\mathrm{M}$ in a neighborhood of δ, one can verify that $\overset{\circ}{\mathrm{X}}_\mathrm{M} = \overset{\circ}{\mathrm{X}}_\mathrm{S} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \delta$ whenever $\mathrm{Z} \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \delta + \delta^2$ (see Appendix C). Since Z increases with γ and decreases with η,[24] this implies that a larger γ--which shifts consumer demand towards the stagnant good--

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[24] $\frac{\partial \mathrm{Z}}{\partial \gamma} = \frac{\alpha\eta \left[f(\mathrm{X_M})\big|_{\mathrm{X_M} = \delta} \right]^{\alpha} \left[1 - f(\mathrm{X_M})\big|_{\mathrm{X_M} = \delta} \right]}{\gamma^2 (1-\alpha)^{-1} \left[\alpha - \alpha\beta - (\alpha - \beta) f(\mathrm{X_M})\big|_{\mathrm{X_M} = \delta} \right]} \geq 0$, $\frac{\partial \mathrm{Z}}{\partial \eta} = \frac{\alpha \left[f(\mathrm{X_M})\big|_{\mathrm{X_M} = \delta} \right]^{\alpha} \left[f(\mathrm{X_M})\big|_{\mathrm{X_M} = \delta} - 1 \right]}{(1-\alpha)^{-1} \gamma \left[\alpha - \alpha\beta - (\alpha - \beta) f(\mathrm{X_M})\big|_{\mathrm{X_M} = \delta} \right]} \leq 0$. As a numerical example, let α=0.6, β=0.7, δ=0.1, θ=0.8488549 and ε=0. Given these parameter values, one has $\overset{\circ}{\mathrm{X}}_\mathrm{M} = 0.101 > \delta$ and $\mathrm{Z} = 0.1047399 < \delta + \delta^2$ if γ=0.3039671 and η=0.7, while one has $\overset{\circ}{\mathrm{X}}_\mathrm{M} = 0.099 < \delta$ and $\mathrm{Z} = 0.1151969 > \delta + \delta^2$ if γ=0.3088881 and η=0.6973335.

makes less likely that the steady-state rate of growth of the capital stock is positive, while a larger η tends to have the opposite effect.

Considering (3), the production function in the progressive sector, and the fact that $A_t = K_{Mt}^{\alpha}$, one has $\overset{\circ}{\rho}_M = \overset{\circ}{\mu}_M$; while--considering (5)--one has $\overset{\circ}{\rho}_S = \left(1+\overset{\circ}{\mu}_S\right)^{1-\beta} -1$, where $\rho_{it} \equiv \frac{Y_{it+1} - Y_{it}}{Y_{it}}$, i =M,S. Together with $\overset{\circ}{\mu}_M = \overset{\circ}{\mu}_S$, this entails $\overset{\circ}{\rho}_M \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \overset{\circ}{\rho}_S$ whenever $\overset{\circ}{X}_M = \overset{\circ}{X}_S \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \delta$ (along a BGP, the output of the progressive sector grows at a higher rate than the output of the stagnant sector if and only if the steady-state ratio between gross investment and capital stock is larger than the capital-depreciation parameter). Note also that the GDP of this economy grows along a BGP at the same rate as the capital stock: $\rho^{\circ} = \overset{\circ}{\mu}_M = \overset{\circ}{\mu}_S$, where $\rho_t \equiv \frac{GDP_{t+1} - GDP_t}{GDP_t}$ and

$$GDP_t = Y_{Mt} + P_t Y_{St} = K_{Mt} L_{Mt}^{\alpha} + \frac{\alpha K_{Mt}(1 - L_{Mt})}{\beta L_{Mt}^{1-\alpha}}. \tag{17}$$

This implies the following proposition:

Proposition 1. With homothetic preferences (ε=0), the economy displays perpetual growth (ρ°>0) whenever the parameter values are such that $\overset{\circ}{X}_M > \delta$. In particular, a smaller share of stagnant good in total consumption expenditure (smaller γ) and a larger share of progressive good in total consumption expenditure (larger η) can contribute to generate a positive steady-state rate of growth.

Finally, considering (A4) and (A18), one has $\omega^{\circ} = \left(1+\overset{\circ}{\mu}_M\right)^{\beta} -1$, where $\omega_t \equiv \frac{P_{t+1} - P_t}{P_t}$. Note that $\omega^{\circ} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$ whenever $\overset{\circ}{X}_M = \overset{\circ}{X}_S \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \delta$, which amounts to state that the relative price of the stagnant good grows (at a constant rate) along a BGP whenever the economy displays perpetual growth. This fact reflects both the presence of uneven growth of total factor productivity and of different factor intensities between sectors: given that along a BGP the employment shares are constant and the capital stock tends to grow at the same rate in both sectors, the intersectoral efficiency condition (A1) implies that P_t adjusts along a BGP so as to equalize the marginal productivity of labor of the two sectors in each t.

It is worth to emphasize that, if at any point in time a relatively large share of total expenditure is devoted to the progressive good, the firms producing this good are stimulated to invest. This ignites a virtuous circle, since--in the presence of the AK technology that characterizes the progressive sector--higher investment in this sector accelerates the growth of aggregate productivity and the decline in the relative price of the progressive good. In its turn, this decline boosts the demand for the progressive good, thus reinforcing the process. Along a BGP, the relative price of the progressive good continues to decrease at a constant rate, while the output of the stagnant sector grows slower than the output of the progressive sector. Hence, the share of the stagnant good in total output vanish asymptotically, although an increasing amount of the stagnant good is produced, since the stagnant sector uses as capital the good produced in the progressive sector, thus benefiting by the advances in total factor productivity occurring in the latter.

In general, the AK sector of the economy is more likely to dominate in the long run, thus allowing for perpetual growth, when preferences are homothetic and consumer demand does not shift from the progressive good to the stagnant good as income grows. However, along a BGP, the employment shares and the nominal output shares remain constant. Together with the fact that the share of the progressive good in total output increase along a BGP, this prediction is not consistent with the empirical evidence concerning the advanced economies in the last decades that we reported in the second section. For this reason, in the next section, we analyze the transitional dynamics of the economy.

Non-homothetic preferences

As preferences are non homothetic, the equilibrium trajectory of the economy is governed by a system of four difference equations in L_{Mt}, X_{Mt}, X_{St} and $Q_t \equiv \frac{(1-L_{Mt})}{K_{St}}$ (see equations (A22)-(A25) in Appendix B). Therefore, a BGP can be characterized by setting $L_{Mt+1}=L_{Mt}=L_M$, $X_{Mt+1}=X_{Mt}=X_M$, $X_{St+1}=X_{St}=X_S$ and $Q_{t+1}=Q_t=Q$ in the system (A22)-(A25). In particular, if a BGP exists, it exhibits:

$$K_M^* = \frac{\beta K_S^* L_M^*(1-\alpha)}{\alpha(1-L_M^*)(1-\beta)}, K_S^* = \left\{\left[(L_M^*)^\alpha - \delta - \delta^2\right]\frac{\beta\gamma(1-L_M^*)^{\beta-1}}{\alpha\eta\varepsilon(L_M^*)^{\alpha-1}} - \frac{(1-L_M^*)^\beta}{\varepsilon} - \frac{\gamma(\delta+\delta^2)(1-\beta)(L_M^*)^{-\alpha}}{\varepsilon\eta(1-\alpha)(1-L_M^*)^{-\beta}}\right\}^{\frac{1}{(\beta-1)}},$$

$L_M^* = f(X_M)\Big|_{X_M=\delta}$, $X_M^* = X_S^* = \delta$, $Q^* = \frac{(1-L_M^*)}{K_S^*}$ whenever $Z \geq \delta+\delta^2$, and $X_S^* = X_M^* > \delta$,

$L_M^* = f(X_M^*)$, $X_M^* = g(X_M^*)$, $Q^*=0$ whenever $Z<\delta+\delta^2$, (“*” denotes the BGP value of a variable when

ε>0), where $f(X_M)$, $g(X_M)$ and $Z \equiv z(f(X_M))\big|_{X_M=\delta}$ are given, respectively, by (14), (15) and (16). In other words, even in the presence of non-homothetic preferences, one may have perpetual growth if and only if $Z<\delta+\delta^2$. Indeed, it is only when $Z<\delta+\delta^2$ that the economy may converge asymptotically to a BGP characterized by $\mu_M^* = \mu_S^* > 0$, thus allowing $K_{St}\to\infty$ and $Q_t\to 0$ as $t\to\infty$ (see the Appendix C).[25]

In summary, if the preferences are non homothetic, two long-run outcomes are possible depending on the parameter values. In one case ($Z\geq\delta+\delta^2$), the fact that the share of the stagnant good in total expenditure is increasing *over time* prevents the virtuous circle of investment and productivity growth in the progressive sector from taking place. Hence, Baumol's prediction is confirmed: perpetual growth does not occur. Indeed, the potential for unbounded growth of the AK technology adopted by the progressive sector is offset by the evolution of consumer demand. In the other case ($Z<\delta+\delta^2$), a smaller share of the stagnant good in total consumption expenditure at *any point in time* (small γ and/or large η) may permit to avoid asymptotic stagnancy.

We could say that there are two different engines at work: the expenditure share for the different goods at *any point in time* and the evolution of the shares *over time*. The parameters values for which the "stagnation" equilibrium is generated are such that the two engines work in the same direction (higher relative expenditure share for the stagnant good at any point in time and increasing expenditure shares devoted to the stagnant sector as income grows *over time*). On the contrary, the parameters values implying perpetual growth are such that the two engines contrasts each other (lower relative expenditure share devoted

[25] As a numerical example of an economy populated by households with non-homothetic preferences which displays unbounded growth, let α=0.6, β=0.7, δ=0.01, ε=0.1, γ=0.9, η=0.9 and θ=0.8073. Given these parameter values, one can check that there exists a unique BGP characterized by $L_M^* \approx 0.48$, Q*=0 and $X_M^* = X_S^* \approx 0.0264$, thus entailing $\mu_M^* = \mu_S^* = \rho_M^* \approx 0.0164$, $\rho_S^* \approx 0.0048$ and $\omega^* \approx 0.0114$. Furthermore, one can show that in a neighborhood of this BGP the economy converges asymptotically to it. Indeed, by linearizing (A22)-(A25) around $(L_M^*, X_M^*, X_S^*, Q^*)$, one can verify that the characteristic roots of the linearized system are: $\zeta_1\approx$1.60656, $\zeta_2\approx$0.981288+0.0328022i, $\zeta_3\approx$0.981288-0.0328022ii and $\zeta_4\approx$0.983855.

to the stagnant good at any point in time and increasing expenditure shares devoted to the stagnant sector as income grows *over time*). Thus:

Proposition 2. With non-homothetic preferences ($\varepsilon>0$), the economy may display asymptotic perpetual growth ($\rho^*>0$) whenever the parameter values are such that $Z<\delta+\delta^2$ and long run stagnancy ($\rho^*=0$) whenever the parameter values are such that $Z\geq\delta+\delta^2$. In particular, a smaller share of the stagnant good in total consumption expenditure (small γ and/or large η) may permit to avoid asymptotic stagnancy.

One can conclude that--with non-homothetic preferences--the asymptotic behavior of the economy can be different depending on the parameter values: the economy may display perpetual growth if the share of total consumption expenditure devoted to the progressive good is not too small, while it exhibits stagnancy if a relatively large portion of what households spend on consumption is devoted to the stagnant good. Hence, the composition of final demand plays a crucial role in the determination of the limiting behavior of the economy.

The result that--when the consumer demand for the stagnant good tends to grow faster than the demand for the progressive good over time (non-homothetic preferences)—long run stagnancy may emerge, in spite of the learning by doing and the technological spillovers which take place in the progressive sector of the economy, gives micro-foundation to the well-known Baumol's hypothesis. In this case ($Z\geq\delta+\delta^2$), the existence of learning by doing and technological spillovers in some industries is not sufficient to have perpetual growth, and the ratio between the outputs of the two sectors is constant along a BGP. Moreover, even when the economy is stagnant, everything that (other things being equal) induces the households to devote a larger fraction of their consumption expenditure to the progressive good (higher η or ε, lower γ) leads to larger K_M^* and K_S^*, thus boosting Y_M^* and Y_S^*.

In contrast, when at *any point in time* final demand is not too much unbalanced towards the product of the stagnant industry, a virtuous circle may be ignited, whereby growing market production of both the progressive good and the stagnant makes progressively less relevant the fixed amount of stagnant good that is produced at home. This result goes beyond Baumol's prediction, since it shows that unbounded growth can be possible even when preferences are non homothetic.[26] However, also in the case in which the

[26] This result is robust with respect to non-homothetic preference specifications different from the one we have used in

presence of non-homothetic preferences is consistent with unbounded growth ($Z<\delta+\delta^2$), a BGP is such that the share of the stagnant good in total output tends to vanish asymptotically and its share in nominal output tends to remain constant forever. Again, these features—together with the fact that by construction the employment shares are constant along a BGP—are at odds with the evidence regarding the advanced economies. Thus, even if it is important to study the limiting behavior of an economic system subject to structural change, relevant insights on its properties are provided by focusing on the transition path along which the employment shares adjust to the changes generated by the evolution in total factor productivity and in final demand.

5 THE TRANSITION PATH: NUMERICAL EXAMPLES

Homothetic preferences

As a numerical example, let α=0.6, $\beta=\gamma$=0.7, δ=0.008, ε=0, η=0.3 and θ=0.851797. Given these parameter values,[27] one can show that there exists a unique BGP[28] characterized by $\mathrm{L}^{\circ}_{\mathrm{M}} \approx 0.28$ and $\mathrm{X}^{\circ}_{\mathrm{M}} = \mathrm{X}^{\circ}_{\mathrm{S}} \approx 0.0096$, thus entailing $\mu^{\circ}_{\mathrm{M}} = \mu^{\circ}_{\mathrm{S}} = \rho^{\circ}_{\mathrm{M}} \approx 0.0016$, $\rho^{\circ}_{\mathrm{S}} \approx 0.00049$ and $\omega^{\circ} \approx 0.0011$. Furthermore, by linearizing (A19)-(A21) around $(\mathrm{L}^{\circ}_{\mathrm{M}}, \mathrm{X}^{\circ}_{\mathrm{M}}, \mathrm{X}^{\circ}_{\mathrm{S}})$, one can show that the linearized system is saddle-path

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this paper (contact the authors for more details on this point).

[27] The values of the parameters η and γ entering the utility function have been chosen looking at the expenditure shares for the two sector as reported by Mattey (1997), Oecd (2000), Business Statistic of the US (2002). These expenditure shares include government expenditure. The parameter α—entering the production function of the progressive sector--is consistent with the evidence reported in the Survey of Current Business (2003) for US. A larger value is assigned to the corresponding parameter entering the production function of the stagnant sector (β), so as to account for the evidence showing that this sector is more labor intensive (see O'Mahony and Van Ark, 2003; particularly Table II.6). These parameters values are in line with those chosen by Kongsamut et al. (2001) in their examples.

[28] The existence and the uniqueness of the BGP are guaranteed by the following facts: i) both $f(\mathrm{X_M})$ and $g(\mathrm{X_M})$ are continuous and monotonically increasing in $\mathrm{X_M}$ for $0 \le \mathrm{X_M} \le \overline{\mathrm{X}}$, where $\overline{\mathrm{X}}$ is that value of $\mathrm{X_M}$ such that $f(\mathrm{X_M})=1$; ii) $g(\mathrm{X_M})-\mathrm{X_M}<0$ at $\mathrm{X_M}=0$ and $g(\mathrm{X_M})-\mathrm{X_M}>0$ at $\mathrm{X_M} = \overline{\mathrm{X}}$, and iii) $g'>1$ for $\underline{\mathrm{X}} \le \mathrm{X_M} \le \overline{\mathrm{X}}$, where $\underline{\mathrm{X}} > 0$ is that value of $\mathrm{X_M}$ such that $g(\mathrm{X_M})=0$.

stable, since the characteristic roots are: $\sigma_1 \approx 0.8923$, $\sigma_2 \approx 1.3062+0.1421i$ and $\sigma_3 \approx 1.3062-0.1421i$. The unique path converging to $(\overset{\circ}{L}_M, \overset{\circ}{X}_M, \overset{\circ}{X}_S)$ is governed by

$$L_{Mt} - \overset{\circ}{L}_M = Ze_1\sigma_1^t, \tag{18}$$

$$X_{Mt} - \overset{\circ}{X}_M = Ze_2\sigma_1^t, \tag{19}$$

$$X_{St} - \overset{\circ}{X}_S = Ze_3\sigma_1^t, \tag{20}$$

where $\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \approx \begin{bmatrix} 0.3965 \\ 0.1386 \\ 0.4725 \end{bmatrix}$ are the characteristic vectors associated with the stable root σ_1, Z is a constant to be determined, $X_{S0} = -\frac{1}{2} + \left\{\frac{1}{4} + \frac{K_{M0}}{K_{S0}}\left[L_{M0}^{\alpha}\left(1+\frac{\alpha\eta}{\beta\gamma}\right) - \frac{\alpha\eta}{\beta\gamma}L_{M0}^{\alpha-1} - X_{M0} - X_{M0}^2\right]\right\}^{1/2}$ is obtained from (A17) and $\frac{K_{M0}}{K_{S0}}$ is given.

Recalling that $\mu_{Mt} = X_{Mt} - \delta$ and $\mu_{St} = X_{St} - \delta$, equations (18)-(20) tell us that--whenever $L_{M0} > \overset{\circ}{L}_M$--both μ_{Mt} and μ_{St} are larger along the transition path than along the BGP. Moreover--along the transition path--μ_{St} tends to be larger than μ_{Mt}: along this path, the capital stock tends to grow at a faster rate in the stagnant sector than in the progressive sector when the share of the progressive sector on total employment tends to decline. Finally, the combined effect of a declining share of the progressive sector on total employment and of $\mu_{St}>\mu_{Mt}$ may imply that for some t $\rho_{St} \geq \rho_{Mt}$.

Obviously, the value that L_{M0} must assume along the path converging to the BGP depends on the initial condition $\frac{K_{M0}}{K_{S0}}$. In particular, it is apparent that $L_{M0} = \overset{\circ}{L}_M$ and $X_{M0} = \overset{\circ}{X}_M = X_{S0} = \overset{\circ}{X}_S$ whenever $\frac{K_{M0}}{K_{S0}} = \left(\frac{K_M}{K_S}\right)^{\circ} \approx 0.6$ (the system is at its steady state starting from period 0 if the initial value of the ratio between the capital stock installed in the progressive sector and the capital stock installed in the stagnant sector is equal to its steady-state value, which is approximately equal to 0.6). One can also check that

$$\left.\frac{\partial L_{M0}}{\partial \frac{K_{M0}}{K_{S0}}}\right|_{\frac{K_{M0}}{K_{S0}} = \left(\frac{K_M}{K_S}\right)^{\circ}} > 0 \tag{21}$$

(as the initial value of the ratio between the capital stock installed in the progressive sector and the capital stock installed in the stagnant sector tends to be larger than its steady-state value, also the initial value of the share of the progressive sector on total employment tends to be larger than its steady-state value). Finally, one can easily check that

$$\left.\frac{\partial(X_{S0}-X_{M0})}{\partial\frac{K_{M0}}{K_{S0}}}\right|_{\frac{K_{M0}}{K_{S0}}=\left(\frac{K_M}{K_S}\right)^{\circ}}>0 \tag{22}$$

(as the initial value of the ratio between the capital stock installed in the progressive sector and the capital stock installed in the stagnant sector tends to be larger than its steady-state value, the initial value of the gross investment-installed capital ratio tends to be larger in the stagnant sector than in the progressive sector).

For instance, take $\frac{K_{M0}}{K_{S0}}\approx 0.6592>\left(\frac{K_M}{K_S}\right)^{\circ}$. Given this initial condition, one has $L_{M0}\approx 0.3>L_M^{\circ}$ and $X_{S0}\approx 0.03347>X_{M0}\approx 0.016626>X_M^{\circ}=X_S^{\circ}$. Furthermore--in a neighborhood of the BGP--one has:

$$\omega_0=\omega^{\circ}+(X_{M0}-X_M^{\circ})(1+X_S^{\circ}-\delta)^{\beta-1}-(1-\beta)(X_{S0}-X_S^{\circ})(1+X_S^{\circ}-\delta)^{\beta-2}(1+X_M^{\circ}-\delta)-$$
$$-\left[\frac{(1-\beta)}{(1-L_M^{\circ})}+\frac{(1-\alpha)}{L_M^{\circ}}\right](1+X_S^{\circ}-\delta)^{\beta-1}(1+X_M^{\circ}-\delta)(L_{M1}-L_{M0})\approx 0.00492, \tag{23}$$

$$\rho_{M0}=\rho_M^{\circ}+X_{M0}-X_M^{\circ}+\frac{\alpha(1+X_M^{\circ}-\delta)(L_{M1}-L_{M0})}{L_M^{\circ}}\approx 0.0039356, \tag{24}$$

$$\rho_{S0}=\rho_S^{\circ}+(1-\beta)(1+X_S^{\circ}-\delta)^{-\beta}(X_{S0}-X_S^{\circ})-\frac{\beta(1+X_S^{\circ}-\delta)^{1-\beta}(L_{M1}-L_{M0})}{(1-L_M^{\circ})}\approx 0.009727, \tag{25}$$

$$\rho_0-\rho^{\circ}=X_{M0}-X_M^{\circ}-\alpha\left[\frac{1-\alpha-L_M^{\circ}(\beta-\alpha)}{\alpha L_M^{\circ}+(L_M^{\circ})^2(\beta-\alpha)}\right](1+X_M^{\circ}-\delta)(L_{M1}-L_{M0})\approx 0.0097298. \tag{26}$$

We have from equation (23) that the relative price of the stagnant good tends to grow along a transition path characterized by a declining employment level in the progressive sector. In addition, one can see by comparing (24) and (25) that along such a path the output of the stagnant sector may grow at a higher rate than the output of the progressive sector. Finally, equation (26) shows that along this transition path the economy's GDP may increase at a higher rate than along the BGP: the economy's rate of growth tends to decline over time as the share of the two factors of production used in the progressive sector shrinks.

Non-homothetic preferences

As a numerical example, let α=2/3, β=γ=0.8, δ=0.05, ε=0.1, η=0.2 and θ=0.93. Given these parameter values, one has Z≥δ+δ^2, and the unique BGP is characterized by $L_M^* \approx 0.25859$, $X_M^* = X_S^* = 0.05$, $K_M^* \approx 0.33453$ and $K_S^* \approx 0.47958$, thus entailing $\frac{K_M^*}{K_S^*} \approx 0.6976$. Furthermore, by linearizing (A22)-(A25) around $(L_M^*, X_M^*, K_M^*, K_S^*)$, one can show that the linearized system is saddle-path stable, since the characteristic roots are: ξ_1≈0.9973, ξ_2≈0.8883, ξ_3≈1.21163+0.1742i and ξ_4≈1.21163-0.1742i. The unique path converging to $(L_M^*, X_M^*, K_M^*, K_S^*)$ is governed by

$$L_{Mt} - L_M^* = H_1 d_{11} \xi_1^t + H_2 d_{12} \xi_2^t, \tag{27}$$

$$X_{Mt} - X_M^* = H_1 d_{21} \xi_1^t + H_2 d_{22} \xi_2^t, \tag{28}$$

$$K_{Mt} - K_M^* = H_1 d_{31} \xi_1^t + H_2 d_{32} \xi_2^t, \tag{29}$$

$$K_{St} - K_S^* = H_1 d_{41} \xi_1^t + H_2 d_{42} \xi_2^t, \tag{30}$$

where $\begin{bmatrix} d_{11} \\ d_{21} \\ d_{31} \\ d_{41} \end{bmatrix} \approx \begin{bmatrix} 0.00426 \\ 0.00163 \\ 0.00169 \\ 0.99998 \end{bmatrix}$ and $\begin{bmatrix} d_{12} \\ d_{22} \\ d_{32} \\ d_{42} \end{bmatrix} \approx \begin{bmatrix} 0.06748 \\ 0.01922 \\ 0.08193 \\ 0.99416 \end{bmatrix}$ are the characteristic vectors associated—respectively--with ξ_1 and with ξ_2, H_1 and H_2 are two constants to be determined, K_{M0} and K_{S0} are given.

Given (27)-(30), one can ascertain that initial conditions such that $K_{S0} < K_S^*$ and $\frac{K_{M0}}{K_{S0}} > \frac{K_M^*}{K_S^*}$ are consistent with a saddle path displaying a declining level of employment in the progressive sector and a positive (but declining) rate of growth of the economy's GDP. For instance, take K_{S0}≈0.42 and $\frac{K_{M0}}{K_{S0}} \approx 0.880$. Given these initial conditions, one has $L_{M0} \approx 0.2863 > L_M^*$ and X_{S0}≈0.0921>X_{M0}≈0.057>$X_M^* = X_S^*$. Furthermore--in a neighborhood of the BGP--one has:

$$\omega_0 = \omega^* + (X_{M0} - X_M^*)(1 + X_S^* - \delta)^{\beta-1} - (1-\beta)(X_{S0} - X_S^*)(1 + X_S^* - \delta)^{\beta-2}(1 + X_M^* - \delta) - \\ - \left[\frac{(1-\beta)}{(1-L_M^*)} + \frac{(1-\alpha)}{L_M^*} \right](1 + X_S^* - \delta)^{\beta-1}(1 + X_M^* - \delta)(L_{M1} - L_{M0}) \approx 0.0045, \tag{31}$$

$$\rho_{M0} = \rho_M^* + X_{M0} - X_M^* + \frac{\alpha(1 + X_M^* - \delta)(L_{M1} - L_{M0})}{L_M^*} \approx -0.0007, \tag{32}$$

$$\rho_{S0} = \rho_S^* + (1-\beta)(1 + X_S^* - \delta)^{-\beta}(X_{S0} - X_S^*) - \frac{\beta(1 + X_S^* - \delta)^{1-\beta}(L_{M1} - L_{M0})}{(1 - L_M^*)} \approx 0.0120, \tag{33}$$

$$\rho_0 - \rho^* = X_{M0} - X_M^* - \alpha\left[\frac{1 - \alpha - L_M^*(\beta - \alpha)}{\alpha L_M^* + (L_M^*)^2(\beta - \alpha)}\right](1 + X_M^* - \delta)(L_{M1} - L_{M0}) \approx 0.0114. \tag{34}$$

As in the numerical example with homothetic preferences, one can see from (31) that the relative price of the stagnant good tends to grow along a transition path characterized by a declining employment level in the progressive sector. By comparing (32) and (33), one can verify that along such a path the output of the stagnant sector may grow at a higher rate than the output of the progressive sector. Again, equation (34) shows that along this transition path the economy’s GDP may increase at a higher rate than along the BGP: as along the transition path considered in the Cobb-Douglas case, the economy’s rate of growth tends to decline over time as the share of the two factors of production used in the progressive sector shrinks.

6 CONCLUSIONS

The massive reallocation of resources among sectors and, in particular, the reallocation from manufacturing to services in the industrialized economies which have characterized the latest decades, has induced us to develop a model that can account for these impressive evidence. This formal set-up has permitted to study how aggregate growth is affected by the interaction between the increase in total factor productivity, which has a stronger positive impact on what we labeled the “progressive” sector, and the demand for stagnant goods, which tends to increase—other things being equal—more than proportionally than total expenditure in consumption.

Indeed, we have presented two numerical examples where it is shown that starting from an initial employment share of the progressive sector in overall employment greater than its long-run equilibrium share, the gradual shift of employment shares towards the stagnant sector is accompanied by rates of growth of output and capital stock that are higher in the stagnant sector than in progressive one. Moreover, along this transition path, the relative price of the stagnant good is growing and the economy’s GDP tend to grow at a higher rate than along the balanced growth path of the economy: the gradual shift of labor towards the stagnant sector is accompanied by a decline in the aggregate rate of growth. In other words, the pattern resulting from these numerical examples seems to be consistent with the stylized facts.

In addition, we have shown within this analytical framework that positive long-term growth is possible even if what households spend on stagnant goods tends to increase more than proportionally than their total consumption expenditure, namely when their preferences are non homothetic: perpetual growth can take place if a large portion of what households spend on consumption is devoted to the progressive good at any point in time. This implies that tastes and attitudes of households may have relevant consequences for the long-term growth performances of an economy by affecting the composition of consumers' demand. More in general, one may conclude that every factor affecting the composition of final demand can influence long-run growth. Such conclusion suggests two interesting extensions of this paper: introducing heterogeneity of agents with respect to income distribution and/or public demand for final products in order to model their effects on growth via their impact on the composition of final demand. Indeed, it is our conviction that--for a better understanding of the growth process in the advanced economies--more effort should be done to study the interaction between uneven technological progress and the evolution of final demand.

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APPENDIX A

Table 1. Sectoral employment shares.

	Agricultu	Manu	Service		Agricultu	Manu	Service
Franc				**German**			
1901	41.4	31.5	27.1	**1907**	33.9	39.9	26.2
1949	29.6	33.1	37.3	**1950**	22.1	44.7	33.2
1960	22.0	36.9	41.1	**1960**	13.8	47.7	38.5
1970	13.3	38.7	47.9	**1970**	8.5	48.4	43.1
1980	8.6	35.4	56.0	**1980**	5.2	42.8	52.0
1990	6.1	29.2	64.6	**1990**	3.5	39.1	57.4
Italy				**Japan**			
1901	61.7	22.3	16.0	**1906**	61.8	16.2	22.0
1951	43.9	29.5	26.7	**1950**	48.3	22.6	29.0
1960	32.2	36.2	31.6	**1960**	32.6	29.7	37.6
1970	19.6	38.4	42.0	**1970**	17.4	35.7	46.9
1980	13.3	36.9	49.2	**1980**	10.4	35.3	54.2
1990	8.7	31.6	59.7	**1990**	7.2	34.1	58.7
UK				**US**			
1901	13.0	43.9	43.1	**1900**	40.4	28.2	31.4
1951	5.0	47.4	47.6	**1950**	12.8	31.5	55.7
1961	3.7	48.4	47.9	**1960**	8.6	30.6	60.8
1970	3.2	44.1	52.7	**1970**	4.4	33.0	62.6
1980	2.6	37.2	60.3	**1980**	3.5	29.9	66.6
1990	2.1	28.7	69.2	**1990**	2.8	25.7	71.5

***Sources*: OECD, *Job Study*, 1994.**

Table 2. Annual labour productivity growth by sector.

	EU-15			US		
	1979-	1990-	1995-	1979-	1990-	1995-
Total Economy	2.2	2.3	1.7	1.4	1.1	2.3
Agr., Forestry, Fish.	5.2	4.8	3.3	6.4	1.7	9.1
Mining, quarrying	2.9	13.1	3.5	4.4	5.1	-0.2
Manufacturing	3.4	3.5	2.3	3.4	3.6	3.8
Elect., gas, water	2.7	3.6	5.7	1.1	1.8	0.1
Construction	1.6	0.8	0.7	-0.8	0.4	-0.3
Distributive trade	1.3	1.9	1.0	1.8	1.5	5.1
Transport	2.8	3.8	2.3	3.9	2.2	2.6
Communications	5.2	6.2	8.9	1.4	2.4	6.9
Financial services	2.2	1.0	2.8	-0.7	1.7	5.2
Business services	0.7	0.7	0.3	0.1	0.0	0.0
Community, social, personal	-0.3	0.4	0.3	1.2	0.9	-0.4
Public Ad., Education, Health	0.6	1.1	0.8	-0.4	-0.8	-0.6
Source: Table 1.4b in O'Mahony-Van Ark (2003)						

APPENDIX B

DERIVATION OF THE GENERAL EQUILIBRIUM PATHS

Labor allocation across sectors and equilibrium relative price

Given that labor is homogeneous and perfectly mobile across sectors, the firms' optimality condition with respect to the choice of labor implies that in equilibrium

$$\alpha A_t K_{Mt}^{1-\alpha} L_{Mt}^{\alpha-1} = W_t = \beta P_t K_{St}^{1-\beta} L_{St}^{\beta-1}. \tag{A1}$$

One can use (A1) to obtain the employment shares of the two sectors:

$$L_{Mt} = K_{Mt}\left(\frac{\alpha A_t}{W_t}\right)^{1/(1-\alpha)}, \tag{A2}$$

$$L_{St} = K_{St}\left(\frac{\beta P_t}{W_t}\right)^{1/(1-\beta)}. \tag{A3}$$

One can also use (A1)—together with (12) and the fact that $A_t = K_{Mt}^{\alpha}$ --to obtain the relative price:

$$P_t = \frac{\alpha K_{Mt}(1-L_{Mt})^{1-\beta}}{\beta K_{St}^{1-\beta} L_{Mt}^{1-\alpha}}. \tag{A4}$$

Households' optimal behavior

One can solve the intertemporal problem of the representative household by maximizing

$\sum_{t=0}^{\infty}\theta^t\left\{C_{Mt}^{\eta}(C_{St}+\varepsilon)^{\gamma} + \chi_t[W_t + (1+r_t)B_t + \pi_{Mt} + \pi_{St} - B_{t+1} - C_{Mt} - P_t C_{St}]\right\}$ with respect to C_{Mt}, C_{St}, B_{t+1} and χ_t,

and then by eliminating the multiplier χ_t, thus obtaining:

$$C_{Mt} = \frac{\eta P_t}{\gamma}(C_{St}+\varepsilon) \tag{A5}$$

and

$$\theta^{-1}\left(\frac{P_{t+1}}{P_t}\right)^{1-\eta}\left(\frac{C_{St+1}+\varepsilon}{C_{St}+\varepsilon}\right)^{1-\eta-\gamma} = 1 + r_{t+1}. \tag{A6}$$

Therefore, along an optimal path a household must satisfy (A5), (A6) and the transversality condition

$$\lim_{t\to\infty}\theta^t B_t \eta\left(\frac{\eta P_t}{\gamma}\right)^{\eta-1}(C_{St}+\varepsilon)^{\gamma+\eta-1} = 0. \tag{A7}$$

Intertemporal optimal behavior of the firms producing the progressive good

By using (3), (7) and (A2), one can rewrite (4) as

$\pi_{Mt} = (1-\alpha)K_{Mt}\left[A_t\left(\frac{\alpha}{W_t}\right)^{\alpha}\right]^{\frac{1}{(1-\alpha)}} - (1+r_t)\left(I_{Mt-1} + \frac{I_{Mt-1}^2}{K_{Mt-1}}\right)$. Hence, one can solve the intertemporal problem of the representative firm producing the progressive good by maximizing

$$\sum_{t=0}^{\infty}\frac{(1-\alpha)K_{Mt}\left[A_t\left(\frac{\alpha}{W_t}\right)^{\alpha}\right]^{\frac{1}{(1-\alpha)}} - (1+r_t)\left(I_{Mt-1} + \frac{I_{Mt-1}^2}{K_{Mt-1}}\right) + \lambda_{Mt}[I_{Mt} + (1-\delta)K_{Mt} - K_{Mt+1}]}{\prod_{v=1}^{t}(1+r_v)}$$ with respect to I_{Mt}, K_{Mt+1} and λ_{Mt}, and then by eliminating the multiplier λ_{Mt}, thus obtaining (8) and

$$\frac{(1-\alpha)\left[A_{t+1}\left(\frac{\alpha}{W_{t+1}}\right)^{\alpha}\right]^{\frac{1}{(1-\alpha)}} + X_{Mt+1}^2 + (1-\delta)(1+2X_{Mt+1})}{1+2X_{Mt}} = 1+r_{t+1}, \quad X_{Mt} \equiv \frac{I_{Mt}}{K_{Mt}}. \tag{A8}$$

Therefore, along an optimal path a manufacturing firm must satisfy (8), (A8) and the transversality condition

$$\lim_{t\to\infty}\frac{(1+2X_{Mt})K_{Mt}}{\prod_{v=1}^{t}(1+r_v)} = 0. \tag{A9}$$

Intertemporal optimal behavior of the firms producing the stagnant good

By using (5), (7) and (A3), one can rewrite (6) as

$\pi_{St} = (1-\beta)K_{St}\left[P_t\left(\frac{\beta}{W_t}\right)^{\beta}\right]^{\frac{1}{(1-\beta)}} - (1+r_t)\left(I_{St-1} + \frac{I_{St-1}^2}{K_{St-1}}\right)$. Hence, one can solve the intertemporal problem of the representative firm producing the stagnant good by maximizing

$$\sum_{t=0}^{\infty}\frac{(1-\beta)K_{St}\left[P_t\left(\frac{\beta}{W_t}\right)^{\beta}\right]^{\frac{1}{(1-\beta)}} - (1+r_t)\left(I_{St-1} + \frac{I_{St-1}^2}{K_{St-1}}\right) + \lambda_{St}[I_{St} + (1-\delta)K_{St} - K_{St+1}]}{\prod_{v=1}^{t}(1+r_v)}$$ with respect to I_{St}, K_{St+1} and λ_{St}, and then by eliminating the multiplier λ_{St}, thus obtaining (8) and

$$\frac{(1-\beta)\left[P_{t+1}\left(\frac{\beta}{W_{t+1}}\right)^{\beta}\right]^{\frac{1}{(1-\beta)}}+X_{St+1}^{2}+(1-\delta)(1+2X_{St+1})}{1+2X_{St}}=1+r_{t+1},\quad X_{St}\equiv\frac{I_{St}}{K_{St}}\ . \tag{A10}$$

Therefore, along an optimal path a firm producing the stagnant good must satisfy (8), (A10) and the transversality condition

$$\lim_{t\to\infty}\frac{(1+2X_{St})K_{St}}{\prod_{v=1}^{t}(1+r_{v})}=0\ . \tag{A11}$$

General equilibrium path

Considering (5), (11) and (12), one can obtain

$$C_{St}=K_{St}^{1-\beta}(1-L_{Mt})^{\beta}\ . \tag{A12}$$

One can use (A1), (A4), (A6), (A12) and the fact that $A_t=K_{Mt}^{\alpha}$ to write (A8) as

$$\frac{(1-\alpha)L_{Mt+1}^{\alpha}+X_{Mt+1}^{2}+(1-\delta)(1+2X_{Mt+1})}{1+2X_{Mt}}=\theta^{-1}\left[\frac{K_{Mt+1}(1-L_{Mt+1})^{1-\beta}K_{St}^{1-\beta}L_{Mt}^{1-\alpha}}{K_{Mt}(1-L_{Mt})^{1-\beta}K_{St+1}^{1-\beta}L_{Mt+1}^{1-\alpha}}\right]^{1-\eta}\left[\frac{K_{St+1}^{1-\beta}(1-L_{Mt+1})^{\beta}+\varepsilon}{K_{St}^{1-\beta}(1-L_{Mt})^{\beta}+\varepsilon}\right]^{1-\eta-\gamma}. \tag{A13}$$

Similarly, one can use (12), (A1), (A4), (A6) and (A12) to write (A10) as

$$\frac{\dfrac{\alpha(1-\beta)K_{Mt+1}(1-L_{Mt+1})}{\beta K_{St+1}L_{Mt+1}^{1-\alpha}}+X_{St+1}^{2}+(1-\delta)(1+2X_{St+1})}{1+2X_{St}}=\theta^{-1}\left[\frac{K_{Mt+1}(1-L_{Mt+1})^{1-\beta}K_{St}^{1-\beta}L_{Mt}^{1-\alpha}}{K_{Mt}(1-L_{Mt})^{1-\beta}K_{St+1}^{1-\beta}L_{Mt+1}^{1-\alpha}}\right]^{1-\eta}\left[\frac{K_{St+1}^{1-\beta}(1-L_{Mt+1})^{\beta}+\varepsilon}{K_{St}^{1-\beta}(1-L_{Mt})^{\beta}+\varepsilon}\right]^{1-\eta-\gamma}. \tag{A14}$$

Considering (8), one can obtain:

$$\frac{K_{St+1}}{K_{St}}=X_{St}+1-\delta\ , \tag{A15}$$

$$\frac{K_{Mt+1}}{K_{Mt}}=X_{Mt}+1-\delta\ . \tag{A16}$$

Finally, one can use (3), (A4), (A5), (A12) and the fact that $A_t=K_{Mt}^{\alpha}$ to write (10) as

$$K_{Mt}L_{Mt}^{\alpha}=\frac{\alpha\eta K_{Mt}(1-L_{Mt})}{\beta\gamma L_{Mt}^{1-\alpha}}\left[1+\frac{\varepsilon(1-L_{Mt})^{-\beta}}{K_{St}^{1-\beta}}\right]+K_{Mt}(X_{Mt}+X_{Mt}^{2})+K_{St}(X_{St}+X_{St}^{2})\ . \tag{A17}$$

The system (A13)-(A17) governs the general equilibrium path of the economy. Moreover, equation (A17) can be used to obtain:

$$\frac{K_{St}}{K_{Mt}} = n(L_{Mt}, X_{Mt}, X_{St}, Q_t) = \frac{L_{Mt}^{\alpha} - \frac{\alpha\eta}{\beta\gamma L_{Mt}^{1-\alpha}}\left(1 - L_{Mt} + \varepsilon Q_t^{1-\beta}\right) - (X_{Mt} + X_{Mt}^2)}{(X_{St} + X_{St}^2)}, \; Q_t \equiv \frac{(1-L_{Mt})}{K_{St}}. \quad \text{(A18)}$$

The general equilibrium path when preferences are homothetic

In the case in which the period-utility function is Cobb-Douglas (ε=0), equation (A18) is such that $\frac{K_{St}}{K_{Mt}} = n(L_{Mt}, X_{Mt}, X_{St})$. Using (A16) and (A18), one can rewrite (A13)-(A15) as a system of three difference equations in L_{Mt}, X_{Mt} and X_{St} governing the general equilibrium path of the economy:

$$\Psi(L_{Mt+1}, X_{Mt+1}, X_{St+1}, L_{Mt}, X_{Mt}, X_{St}) = \frac{(1-\alpha)L_{Mt+1}^{\alpha} + X_{Mt+1}^2 + (1-\delta)(1+2X_{Mt+1})}{1+2X_{Mt}} -$$

$$-\theta^{-1}\left[\frac{(X_{Mt}+1-\delta)(1-L_{Mt+1})L_{Mt}^{1-\alpha}}{(1-L_{Mt})L_{Mt+1}^{1-\alpha}}\right]^{1-\eta-\gamma}\left[\frac{(X_{Mt}+1-\delta)^{\beta}L_{Mt}^{1-\alpha}[n(L_{Mt},X_{Mt},X_{St})(1-L_{Mt+1})]^{1-\beta}}{L_{Mt+1}^{1-\alpha}[n(L_{Mt+1},X_{Mt+1},X_{St+1})(1-L_{Mt})]^{1-\beta}}\right]^{\gamma} = 0, \quad \text{(A19)}$$

$$\Phi(L_{Mt+1},X_{Mt+1},X_{St+1},L_{Mt},X_{Mt},X_{St}) = \frac{\frac{\alpha(1-\beta)(1-L_{Mt+1})}{\beta n(L_{Mt+1},X_{Mt+1},X_{St+1})L_{Mt+1}^{1-\alpha}} + X_{St+1}^2 + (1-\delta)(1+2X_{St+1})}{1+2X_{St}} -$$

$$-\theta^{-1}\left[\frac{(X_{Mt}+1-\delta)(1-L_{Mt+1})L_{Mt}^{1-\alpha}}{(1-L_{Mt})L_{Mt+1}^{1-\alpha}}\right]^{1-\eta-\gamma}\left[\frac{(X_{Mt}+1-\delta)^{\beta}L_{Mt}^{1-\alpha}[n(L_{Mt},X_{Mt},X_{St})(1-L_{Mt+1})]^{1-\beta}}{L_{Mt+1}^{1-\alpha}[n(L_{Mt+1},X_{Mt+1},X_{St+1})(1-L_{Mt})]^{1-\beta}}\right]^{\gamma} = 0, \quad \text{(A20)}$$

$$\Lambda(L_{Mt+1}, X_{Mt+1}, X_{St+1}, L_{Mt}, X_{Mt}, X_{St}) = \frac{(X_{Mt}+1-\delta)n(L_{Mt+1}, X_{Mt+1}, X_{St+1})}{n(L_{Mt}, X_{Mt}, X_{St})} - X_{St} - 1 + \delta = 0. \quad \text{(A21)}$$

The general equilibrium path when preferences are non homothetic

In the case in which ε>0, one can use (A18) to rewrite (A13)-(A16) as a system of four difference equations in L_{Mt}, X_{Mt}, X_{St} and Q_t governing the general equilibrium path of the economy:

$$\Omega(L_{Mt+1}, X_{Mt+1}, X_{St+1}, Q_{t+1}, L_{Mt}, X_{Mt}, X_{St}, Q_t) = \frac{(1-\alpha)L_{Mt+1}^{\alpha} + X_{Mt+1}^2 + (1-\delta)(1+2X_{Mt+1})}{1+2X_{Mt}} -$$

$$-\theta^{-1}\left\{\left[\frac{(X_{Mt}+1-\delta)L_{Mt}^{1-\alpha}}{L_{Mt+1}^{1-\alpha}}\right]\left[\frac{(1-L_{Mt+1})+\varepsilon Q_{t+1}^{1-\beta}}{(1-L_{Mt})+\varepsilon Q_t^{1-\beta}}\right]\right\}^{1-\eta-\gamma}\left[\frac{(X_{Mt}+1-\delta)^{\beta}L_{Mt}^{1-\alpha}[n(L_{Mt},X_{Mt},X_{St},Q_t)(1-L_{Mt+1})]^{1-\beta}}{L_{Mt+1}^{1-\alpha}[n(L_{Mt+1},X_{Mt+1},X_{St+1},Q_{t+1})(1-L_{Mt})]^{1-\beta}}\right]^{\gamma} = 0, \quad \text{(A22)}$$

$$\Gamma(L_{Mt+1},X_{Mt+1},X_{St+1},Q_{t+1},L_{Mt},X_{Mt},X_{St},Q_t) = \frac{\frac{\alpha(1-\beta)(1-L_{Mt+1})}{\beta n(L_{Mt+1},X_{Mt+1},X_{St+1},Q_{t+1})L_{Mt+1}^{1-\alpha}} + X_{St+1}^2 + (1-\delta)(1+2X_{St+1})}{1+2X_{St}} -$$

$$-\theta^{-1}\left\{\left[\frac{(X_{Mt}+1-\delta)L_{Mt}^{1-\alpha}}{L_{Mt+1}^{1-\alpha}}\right]\left[\frac{(1-L_{Mt+1})+\varepsilon Q_{t+1}^{1-\beta}}{(1-L_{Mt})+\varepsilon Q_t^{1-\beta}}\right]\right\}^{1-\eta-\gamma}\left[\frac{(X_{Mt}+1-\delta)^{\beta}L_{Mt}^{1-\alpha}[n(L_{Mt},X_{Mt},X_{St},Q_t)(1-L_{Mt+1})]^{1-\beta}}{L_{Mt+1}^{1-\alpha}[n(L_{Mt+1},X_{Mt+1},X_{St+1},Q_{t+1})(1-L_{Mt})]^{1-\beta}}\right]^{\gamma} = 0, \quad \text{(A23)}$$

$$\Theta(L_{Mt+1},X_{Mt+1},X_{St+1},Q_{t+1},L_{Mt},X_{Mt},X_{St},Q_t) = (X_{Mt}+1-\delta) - \frac{(X_{St}+1-\delta)n(L_{Mt},X_{Mt},X_{St},Q_t)}{n(L_{Mt+1},X_{Mt+1},X_{St+1},Q_{t+1})} = 0, \quad \text{(A24)}$$

$$\Sigma(L_{Mt+1},Q_{t+1},L_{Mt},X_{St},Q_t) = (1-L_{Mt+1})Q_t - (X_{St}+1-\delta)(1-L_{Mt})Q_{t+1} = 0 \,. \qquad \text{(A25)}$$

APPENDIX C

PROOFS

1) Proof that $X^{\circ}_M = X^{\circ}_S \left\{\begin{matrix} > \\ = \\ < \end{matrix}\right\} \delta$ *whenever* $Z \left\{\begin{matrix} < \\ = \\ > \end{matrix}\right\} \delta + \delta^2$

The existence of X°_M in a neighborhood of δ and the fact that $\left.\dfrac{dg(X_M)}{dX_M}\right|_{X_M=\delta} > 1$ entail $g(X_M) \left\{\begin{matrix} > \\ = \\ < \end{matrix}\right\} X_M$ whenever $X_M \left\{\begin{matrix} > \\ = \\ < \end{matrix}\right\} X^{\circ}_M$, where X_M is in a neighborhood of δ. Hence, one has:

$$\left. g(X_M)\right|_{X_M=\delta} - \delta \left\{\begin{matrix} > \\ = \\ < \end{matrix}\right\} 0 \;\text{ whenever }\; \delta \left\{\begin{matrix} > \\ = \\ < \end{matrix}\right\} X^{\circ}_M \,. \qquad \text{(A26)}$$

In its turn, it is apparent by inspecting (15) that

$$\left. g(X_M)\right|_{X_M=\delta} - \delta \left\{\begin{matrix} > \\ = \\ < \end{matrix}\right\} 0 \;\text{ whenever }\; Z \equiv \left. z(f(X_M))\right|_{X_M=\delta} \left\{\begin{matrix} > \\ = \\ < \end{matrix}\right\} \delta + \delta^2 . \qquad \text{(A27)}$$

From (A26) and (A27) it follows that $X^{\circ}_M = X^{\circ}_S \left\{\begin{matrix} > \\ = \\ < \end{matrix}\right\} \delta$ whenever $Z \left\{\begin{matrix} < \\ = \\ > \end{matrix}\right\} \delta + \delta^2$ (see Fig. 1 and Fig. 2).

FIGURE 1

Case in which Z<δ+δ²

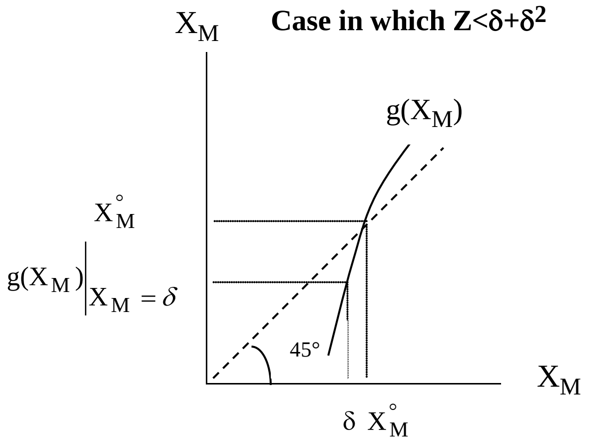
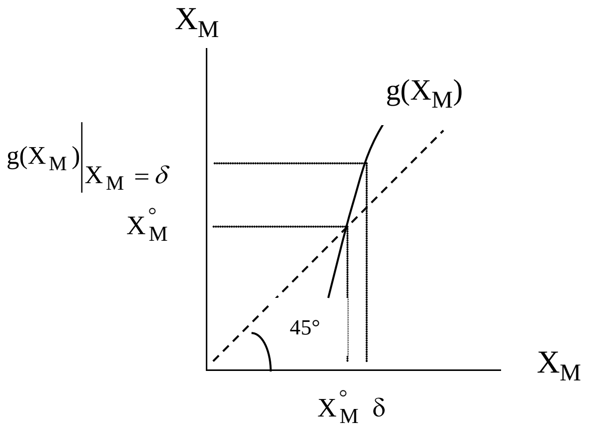


FIGURE 2

Case in which Z>δ+δ²



2) Proof that in the presence of non-homothetic preferences the economy may converge asymptotically to a BGP characterized by perpetual growth if and only if Z< δ+ δ²

Given the definition of Q_t $\left(Q_t \equiv \frac{(1-L_{Mt})}{K_{St}}\right)$ and the fact that an economy which converges asymptotically to a BGP characterized by perpetual growth (i.e., to a BGP characterized by $\mu^*_S = \mu^*_M > 0$) has $K_{St}\rightarrow\infty$, it is necessarily the case that $\mu^*_S = \mu^*_M > 0$ entails $\lim_{t\to\infty} Q_t = Q^* = 0$. Given Q*=0, a BGP characterized by $\mu^*_S = \mu^*_M > 0$ must be such that $X^*_S = X^*_M$, $L^*_M = f(X^*_M)$ and $X^*_M = g(X^*_M)$. In its turn, this implies that $\mu^*_S = \mu^*_M > 0$ if and only if Z<δ+δ² (see point 1) in this Appendix).

One can also easily verify that there cannot exist a BGP characterized by $X^*_S = X^*_M$, $L^*_M = f(X^*_M)$, $X^*_M = g(X^*_M)$ and Q*=0 whenever Z≥δ+δ². In this case, indeed, one would have $X^*_S = X^*_M \le \delta$ (see point 1) in this Appendix), entailing $\mu^*_S = \mu^*_M \le 0$, which is inconsistent with the fact that both $\lim_{t\to\infty} Q_t = Q^* = 0$ and $\lim_{t\to\infty} L_{Mt} = L^*_M = f(X^*_M)$ must hold (since $L^*_M = f(X^*_M)$ is generally different from 1).

Finally, to see that there cannot exist a BGP characterized by fixed levels of K_{Mt} and K_{St} whenever Z<δ+δ², consider that $(K^*_S)^{\beta-1} = \left[(L^*_M)^{\alpha} - \delta - \delta^2\right]\frac{\beta\gamma(1-L^*_M)^{\beta-1}}{\alpha\eta\varepsilon(L^*_M)^{\alpha-1}} - \frac{(1-L^*_M)^{\beta}}{\varepsilon} - \frac{\gamma(\delta+\delta^2)(1-\beta)(L^*_M)^{-\alpha}}{\varepsilon\eta(1-\alpha)(1-L^*_M)^{-\beta}} < 0$ if Z<δ+δ², where $L^*_M = f(X_M)\big|_{X_M=\delta}$.