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A Virtual Testing Simulation Environment for the ESA's Micro Vibrations Measurement System

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Abstract

This paper presents the evaluation of various model reduction techniques as possible candidates for building a virtual testing simulation environment of the ESA's Micro Vibrations Measurement System (MVMS). The resulting tool would represent a key enabling technology for optimization of the tests to be carried out by the facility for the characterization of potential microvibration sources and environments. The present investigation involves both Component Mode Synthesis and state-space based methods. In particular, an enhanced version of the Craig-Bampton (CB) method with substructuring and a hybrid two-stage approach involving a preliminary CB reduction step followed by a balanced truncation are presented and discussed. The number of dominant vibration modes to be retained in each substructure is determined according to the Effective Interface Mass criterion. The different model reduction methods are compared in terms of performance and computational effort. It is shown that some preferable techniques can be identified for the specific purposes of the virtual testing environment of the MVMS.

Keywords: Virtual Testing, microvibrations, 6DOF vibration testing, Model Order Reduction

1 Introduction

Next-generation satellites for Earth observation and science missions will have to meet more and more stringent pointing requirements, with an unprecedented level of accuracy and resolution. Disturbances acting on on-board optical equipments and/or scientific instruments can introduce a lot of jitter and significantly degrade the image or data quality, with the risk of compromising the mission success [1]. A typical source of disturbance is mechanical vibrations of small amplitude, called spacecraft microvibrations, arising mainly from rotary mechanisms and devices such as cryo-coolers and reaction and momentum wheels [2]. The transmission and propagation of microvibrations to the sensitive receiver locations are promoted by the increasing flexibility of the satellite structure, potentially leading to severe performance degradation. Due to extremely low levels and a wide frequency bandwidth, one of the difficulties related to microvibrations is their comprehensive characterization and prediction through modeling and testing.

The Micro Vibrations Measurement System (MVMS) is a novel six-degrees-of-freedom (6-DOFs) microvibration facility developed for the European Space Agency (ESA) by the UK's National Physical Laboratory. It is designed to measure/impose microvibration accelerations, forces, and moments in the frequency range from 0.03 Hz to 100 Hz, thus allowing both the characterization of potential microvibration sources and the performance assessment of sensitive equipments when subjected to microvibrations [3]. The measurement and generation of a controlled microvibration environment are performed through closed-loop control systems, which should be properly designed and finely tuned to guarantee the required stability and accuracy. Due to potential dependency of the performance on the payload characteristics, a dedicated tuning of the control parameters is typically needed, causing significant time delay and requiring considerable effort before executing the actual test.

The availability of a virtual testing (VT) simulation environment would greatly help in designing controllers with increased performance and robustness and reducing time and costs to set up the test. A VT analysis implies modeling all the components involved in the microvibration test to simulate the functioning of the real facility [4]. It is understood that, in order to perform consistent VT simulations, a reliable digital replicate of MVMS is required. The finite element (FE) method is a natural choice to obtain a refined structural model, which could be complemented by adding suitable electromechanical relations to include the dynamics of the available actuators and sensors. However, the FE discretization is characterized by a huge number of degrees of freedom, resulting in a high-dimensional system. Model order reduction (MOR) techniques can be used to derive a low-order model able to represent the most significant dynamical behavior of the original refined model, thus facilitating both the numerical simulation and the controller design [5].

In this work, various MOR techniques applied to MVMS are evaluated and compared in terms of performance and computational issues, with the aim of identifying the best candidate for the VT simulation environment. Two different families of model reduction approaches are considered. The first one includes classical and more recent *Component Mode Synthesis* (CMS) or *dynamic substructuring* methods [6], formulated in the framework of structural dynamics. The second family regards *state-space based*

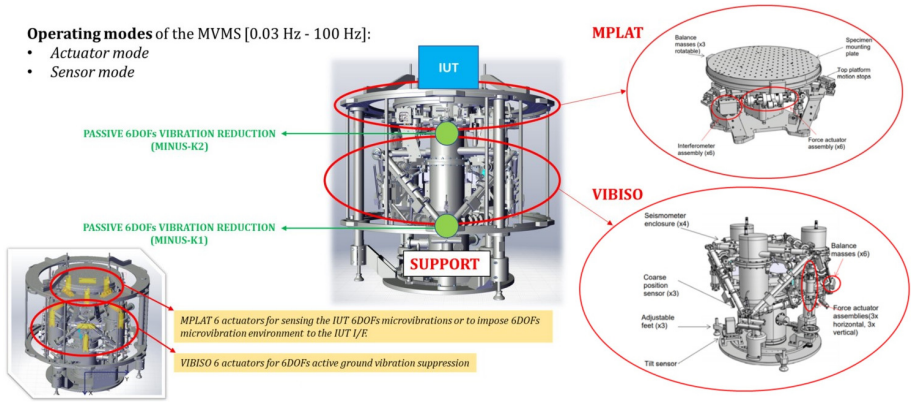

Fig. 1 General description of MVMS facility. The main components are highlighted and labeled (Image courtesy of ESA).

techniques, such as modal and balanced truncation (BT), developed in the systems and control community [5]. A hybrid technique consisting in a two-stage reduction [7] combining CMS methods with BT is implemented and presented as a viable tool to overcome the numerical difficulties associated with the direct application of BT to very high-order problems.

The present paper is organized as follows. First, the ESA's microvibration facility and its operating modes are presented in Section 2, along with a brief description of the high-fidelity finite element model of the structure. In Section 3, the selected MOR methodologies adopted in this work are outlined and reviewed with reference to the present specific application. The VT simulation environment is introduced in Section 4, with a focus on the application of MOR techniques to MVMS, which is illustrated and discussed through a set of significant results. Finally, some conclusions will be stated in Section 5.

2 ESA's Micro Vibrations Measurement System

2.1 Testing facility

The ESA's microvibrations facility is shown in Figure 1, along with a schematic representation of its parts and operating modes, that are described in the following.

The unit is composed of three main components: a base support (BS), the VIBration ISOlation platform (VIBISO), and the vibration Measurement PLatform (MPLAT).

The base support interfaces with the ground by means of three adjustable feet positioned on the external part of the structure. A fourth point of contact with the floor is present at the center of BS. The base support is mechanically connected to VIBISO through a MINUS-K device which acts as a passive mechanical isolator. The function of BS is to sustain the upper MVMS components and to hold the seismometers and the fixed parts of the voice coil actuators, which form the set of sensors/actuators used

by VIBISO to complement and improve the passive isolation provided by the MINUS-K with an active control action. Indeed, one of the main factors that could endanger the accuracy of MVMS operations is the adverse effect that seismic vibrations could introduce on the test results. The aim of VIBISO is to actively isolate the upper part of the facility from vibrations transferred via the ground, thus ensuring a quiet environment for the measurement and generation of very small forces and torques.

The function of MPLAT is to measure/impose the microvibration environment from/to the test specimen. It interfaces VIBISO through a second MINUS-K device and an additional set of voice coil actuators. The MPLAT component allows the facility to operate in two modes: measurement mode and excitation mode. The purpose of the measurement mode is the assessment of the microvibrations level generated by the item under test (IUT). In particular, the strategy employed to carry out this evaluation consists in measuring the effects of the microvibrations transmitted by the IUT to the system via six position sensors, with the objective of counterbalancing them thanks to the action of six actuators located between MPLAT and VIBISO. The latter are in charge of MPLAT stability (avoiding its displacement or rotations). From the knowledge of the applied control actions, the microvibrations forces/moments coming from the IUT can be retrieved. On the other hand, the excitation mode is conceived for the evaluation of the response of the IUT to dynamic loads (e.g. the one corresponding to the mission, after applying the qualification factor). In this case, the facility acts as a multishaker, inducing vibrations in all six DOFs to the base of the IUT.

2.2 Finite Element Model

A refined FE model of the facility was developed in Nastran. It is shown in Figure 2 and presents a total of 103596 DOFs.

According to the topology of the structure, it is mainly composed of plate/shell and beam elements, with the addition of some concentrated masses. The MINUS-K devices are simply modeled as linear springs. The action of the actuators is expressed in the FE model by means of ideal forces applied to the FE nodes located in correspondence of the actuators' endpoints. No electromechanical coupling is included in this work since the electrical dynamics was supposed to fall beyond the bandwidth of interest of structural dynamics. According to a preliminary analysis, a mass lumping approximation was introduced instead of the consistent mass description without worsening significantly the accuracy of modal results.

The modal analysis on the full model revealed that the first 12 modes, with frequencies below 2 Hz, are mainly driven by the MINUS-K low stiffness, with no substantial elastic deformation of the rest of the structure. Indeed, the low-frequency dynamic behavior of MVMS is highly affected by the presence of the MINUS-K devices, which make the facility behave as a mechanical low-pass filter as shown later. The first 12 mode shapes of the structure are characterized by the reciprocal rigid body motion of the three MVMS structural segments, rather than by a generalized and diffused elastic deformation of the entire structure. The first two structural modes are bending modes at about 40 Hz involving mainly deformation of the base support frame. The successive modes are beyond 100 Hz. Table 1 lists the frequencies of the first 15 modes of the MVMS.

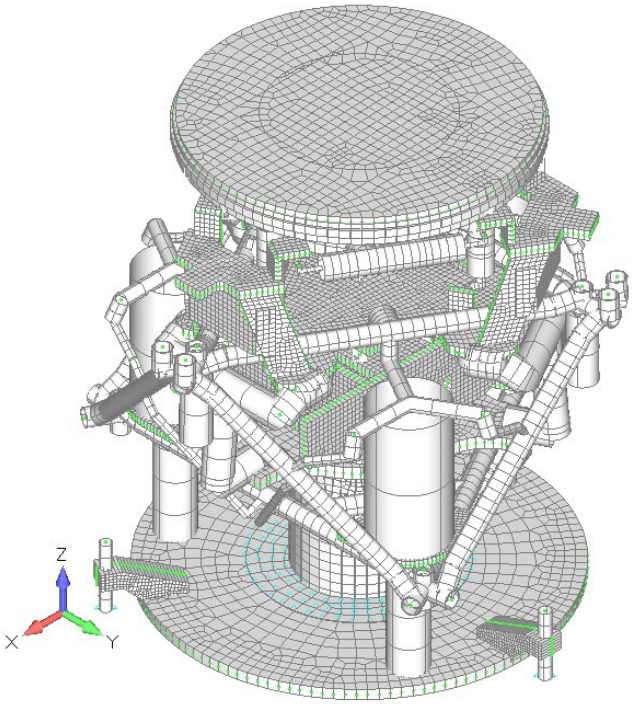

Fig. 2 Finite Element Model of the Micro Vibrations Measurement System (Image courtesy of ESA).

Table 1 First 15 natural frequencies of the MVMS facility.

Mode number	**Mode frequency**
1	0.396 Hz
2	0.397 Hz
3	0.454 Hz
4	0.601 Hz
5	0.637 Hz
6	0.640 Hz
7	0.928 Hz
8	1.139 Hz
9	1.141 Hz
10	1.347 Hz
11	1.845 Hz
12	1.946 Hz
13	42.39 Hz
14	42.44 Hz
15	129.1 Hz

3 Model reduction techniques

As already pointed out, the direct use of the previously described FE model is not suited for the purpose of building an efficient VT simulation environment of the system. It should be replaced by a low-order model through a proper model reduction. The reduction methods considered in this work are outlined in the following.

3.1 CMS methods

The key idea shared by techniques belonging to the CMS family is to divide the model of MVMS into parts called substructures, which are analyzed and reduced separately [8, 9]. As presented in Section 4, the peculiar structure of the ESA's microvibration facility allows for a natural choice of three substructures as the three main parts (BS, VIBISO and MPLAT) previously described. The assembly of the three reduced components is then performed to retrieve a low-order global model of the facility.

The model reduction of each substructure starts from partitioning the vector of degrees of freedom into boundary DOFs (including interfaces between substructures and possible degrees of freedom of interest for the analysis) and internal DOFs. Accordingly, the equations of motion for the s-th substructure are written in the following form:

$$\begin{bmatrix} \mathbf{M}_{bb}^{(s)} & \mathbf{M}_{bi}^{(s)} \\ \mathbf{M}_{ib}^{(s)} & \mathbf{M}_{ii}^{(s)} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_b^{(s)} \\ \ddot{\mathbf{u}}_i^{(s)} \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{bb}^{(s)} & \mathbf{K}_{bi}^{(s)} \\ \mathbf{K}_{ib}^{(s)} & \mathbf{K}_{ii}^{(s)} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_b^{(s)} \\ \mathbf{u}_i^{(s)} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_b^{(s)} \\ \mathbf{f}_i^{(s)} \end{Bmatrix}, \tag{1}$$

where $s = 1, 2, 3$, $\mathbf{M}^{(s)}$ and $\mathbf{K}^{(s)}$ represent the FE mass and stiffness matrices, $\mathbf{u}$ and $\mathbf{f}$ represent the displacement and the externally applied force vectors, and subscripts b and i identify terms related to boundary and internal DOFs, respectively. It is noted that no damping is included at this stage and the model reduction is based on the undamped system.

Internal DOFs are expressed by a combination of static modes and a few dominant vibration modes. Depending on the boundary conditions imposed on the substructure for the computation of these modes, CMS methods can be divided into *fixed-interface* and *free-interface* techniques.

3.1.1 Craig-Bampton method

The classical Craig-Bampton (CB) method [10] is based on approximating the vector of internal displacements of each substructure by means of constraint modes $\mathbf{\Psi}_c^{(s)}$, associated with a unitary displacement of each boundary DOF while keeping the remaining boundary DOFs constrained, and fixed-interface normal modes $\mathbf{\Phi}_d^{(s)}$, representing a selected subset of dominant vibration modes obtained by restraining all the boundary DOFs. The transformation matrix $\mathbf{T}_{CB}^{(s)}$ for the s-th substructure is written as follows

$$\mathbf{u}^{(s)} = \begin{Bmatrix} \mathbf{u}_b^{(s)} \\ \mathbf{u}_i^{(s)} \end{Bmatrix} \approx \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{\Psi}_c^{(s)} & \mathbf{\Phi}_d^{(s)} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_b^{(s)} \\ \boldsymbol{\eta}^{(s)} \end{Bmatrix} = \mathbf{T}_{CB}^{(s)} \mathbf{q}_{CB}^{(s)}, \tag{2}$$

where $\mathbf{\Psi}_c^{(s)} = -\mathbf{K}_{ii}^{(s)-1}\mathbf{K}_{ib}^{(s)}$ and the matrix $\mathbf{\Phi}_d^{(s)}$ is computed by solving the following generalized eigenvalue problem:

$$(\mathbf{K}_{ii}^{(s)} - {\omega_j^{(s)}}^2 \mathbf{M}_{ii}^{(s)})\boldsymbol{\phi}_j^{(s)} = \mathbf{0}. \tag{3}$$

The reduced matrices of each substructure are obtained by projection as follows $\mathbf{M}_{CB}^{(s)} = {\mathbf{T}_{CB}^{(s)}}^T \mathbf{M}^{(s)} \mathbf{T}_{CB}^{(s)}$ and $\mathbf{K}_{CB}^{(s)} = {\mathbf{T}_{CB}^{(s)}}^T \mathbf{K}^{(s)} \mathbf{T}_{CB}^{(s)}$. The overall system is described by the following matrices:

$$\mathbf{M}_{CB} = \sum_{s=1}^{3} {\mathbf{L}^{(s)}}^T \mathbf{M}_{CB}^{(s)} \mathbf{L}^{(s)} \quad \mathbf{K}_{CB} = \sum_{s=1}^{3} {\mathbf{L}^{(s)}}^T \mathbf{K}_{CB}^{(s)} \mathbf{L}^{(s)}, \tag{4}$$

which are retrieved by resorting to a primal assembly procedure, where $\mathbf{L}^{(s)}$ represents the substructure Boolean localization matrix.

3.1.2 Rubin method

Differently from the CB technique, the Rubin method [11] belongs to the family of free-interface approaches. The displacement field is approximated by means of a linear superposition of free-interface vibration modes $\mathbf{\Theta}_d^{(s)}$, which may include also rigid-body modes, and residual attachment modes $\mathbf{\Psi}_{a,res}^{(s)}$. The latter include the contribution coming from the residual flexibility matrix.

The reduction matrix $\mathbf{T}_{RU}^{(s)}$ is obtained in two steps. First of all, the following transformation is imposed:

$$\mathbf{u}^{(s)} = \begin{Bmatrix} \mathbf{u}_b^{(s)} \\ \mathbf{u}_i^{(s)} \end{Bmatrix} \approx \begin{bmatrix} \mathbf{\Psi}_{a,res}^{(s)} & \mathbf{\Theta}_d^{(s)} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\eta}_b^{(s)} \\ \boldsymbol{\eta}^{(s)} \end{Bmatrix} = \mathbf{T}_{free}^{(s)} \mathbf{q}_{free}^{(s)}, \tag{5}$$

where the vector of generalized coordinates $\mathbf{q}_{free}^{(s)}$ does not contain the boundary displacements in physical coordinates, thus forbidding the implementation of primal assembly between substructures. In order to allow this operation, a further transformation is performed, which yields the following final transformation matrix $\mathbf{T}_{RU}^{(s)}$ for each substructure:

$$\mathbf{T}_{RU}^{(s)} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{\Psi}_{a,res,ib}^{(s)} \mathbf{G}_{res,bb}^{(s)^{-1}} & \mathbf{\Theta}_{d,i}^{(s)} - \mathbf{\Psi}_{a,res,ib}^{(s)} \mathbf{G}_{res,bb}^{(s)^{-1}} \mathbf{\Theta}_{d,b}^{(s)} \end{bmatrix}. \tag{6}$$

3.1.3 Enhanced Craig-Bampton method

The Enhanced Craig-Bampton (ECB) method was developed recently [12] with the aim of improving the performance of the CB method. The method was originally derived in the case without substructuring by expressing the reduction matrix as the summation of the $\mathbf{T}_{CB}$ matrix of the overall structure and an additional contribution

$\mathbf{T}_{add,ECB}$ accounting for the residual flexibility associated with the truncated fixed-interface normal modes. The additional matrix is given by

$$\mathbf{T}_{add,ECB} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{G}_{res}\left[\mathbf{M}_{ib} + \mathbf{M}_{ii}\boldsymbol{\Psi}_c\right] & \mathbf{0} \end{bmatrix} \boldsymbol{\Xi} = \bar{\mathbf{T}}_{add,ECB}\boldsymbol{\Xi}, \tag{7}$$

where $\mathbf{G}_{res}$ contains the static part of the residual flexibility and $\boldsymbol{\Xi} = \mathbf{M}_{CB}^{-1}\mathbf{K}_{CB}$. In case of substructuring, Kim et al. [13] suggested performing primal assembly from the partitioning of the overall system matrices. Therefore, the system ECB-reduced matrices are built without the necessity of reducing the substructures models individually. However, the above procedure requires the availability of the partitioned structural matrices of the overall system, which could be unfeasible in some situations. To overcome this issue, a different approach to perform the assembly is here implemented. Starting from the substructures matrices, CB reductions are performed to obtain $\mathbf{M}_{CB}^{(s)}$, $\mathbf{K}_{CB}^{(s)}$ and $\mathbf{T}_{CB}^{(s)}$. Similarly, $\bar{\mathbf{T}}_{add,ECB}^{(s)}$ matrices can be also obtained for each substructure. Localization matrices are then exploited to obtain the system assembled matrices $\mathbf{M}_{CB}$, $\mathbf{K}_{CB}$, $\mathbf{T}_{CB}$ and $\bar{\mathbf{T}}_{add,ECB}$. The matrix $\boldsymbol{\Xi}$ is now correctly retrieved from the system CB matrices. Finally, the system ECB reduction matrix can be obtained as $\mathbf{T}_{ECB} = \mathbf{T}_{CB} + \bar{\mathbf{T}}_{add,ECB}\,\boldsymbol{\Xi}$, and the matrices $\mathbf{M}_{ECB}$ and $\mathbf{K}_{ECB}$ can be computed by projection.

3.2 State-space based methods

Model reduction techniques belonging to this family have their roots in dynamical systems theory and control design. The starting point is the description of the dynamics of MVMS using the state-space form:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \boldsymbol{\mathcal{A}}\mathbf{x}(t) + \boldsymbol{\mathcal{B}}\mathbf{w}(t) \\ \mathbf{y}(t) &= \boldsymbol{\mathcal{C}}\mathbf{x}(t) + \boldsymbol{\mathcal{D}}\mathbf{w}(t) \end{aligned}, \tag{8}$$

where $\boldsymbol{\mathcal{A}}$ is the state matrix, $\boldsymbol{\mathcal{B}}$ and $\boldsymbol{\mathcal{C}}$ are the input and output matrices, respectively, $\boldsymbol{\mathcal{D}}$ is the direct feedthrough matrix, $\mathbf{x}(t)$ represents the state vector, $\mathbf{y}(t)$ the output vector and $\mathbf{w}(t)$ the input vector. Basically, state-space based reduction methods rely on transforming the original state-space representation of the system into a convenient realization where dominant states can be identified according to a prescribed criterion. The resulting reduced state-space form is obtained by keeping in the model only dominant elements and discarding the remaining ones [14]. It is noted that, differently from the substructuring approach adopted in CMS techniques, state-space based reduction methods are typically applied to the model of the entire system.

3.2.1 Modal truncation

Modal truncation relies on transforming the original state-space model into its *diagonal form*, also known as *modal decomposition form*, by solving the eigenvalue problem $\boldsymbol{\mathcal{A}}\boldsymbol{\Phi}_{eig} = \boldsymbol{\Phi}_{eig}\boldsymbol{\Lambda}$, where $\boldsymbol{\Lambda} = Diag\{\lambda_i\}$ is the diagonal matrix of eigenvalues and $\boldsymbol{\Phi}_{eig}$ is the corresponding eigenvector matrix. The transformation is given by $\mathbf{x}(t) =$

$\boldsymbol{\Phi}_{eig}\mathbf{x}_m(t)$, where the subscript m identifies the state vector in modal form. It follows that

$$\begin{aligned} \dot{\mathbf{x}}_m(t) &= \boldsymbol{\Lambda}\mathbf{x}_m(t) + \boldsymbol{\mathcal{B}}_m\mathbf{w}(t) \\ \mathbf{y}(t) &= \boldsymbol{\mathcal{C}}_m\mathbf{x}_m(t) + \boldsymbol{\mathcal{D}}\mathbf{w}(t) \end{aligned}, \tag{9}$$

where $\boldsymbol{\mathcal{B}}_m = \boldsymbol{\Phi}_{eig}^{-1}\boldsymbol{\mathcal{B}}$ and $\boldsymbol{\mathcal{C}}_m = \boldsymbol{\mathcal{C}}\boldsymbol{\Phi}_{eig}$. The state-space model corresponding to this modal realization is partitioned as follows

$$\begin{aligned} \begin{Bmatrix} \dot{\mathbf{x}}_{m,1} \\ \dot{\mathbf{x}}_{m,2} \end{Bmatrix} &= \begin{bmatrix} \boldsymbol{\Lambda}_1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Lambda}_2 \end{bmatrix} \begin{Bmatrix} \mathbf{x}_{m,1} \\ \mathbf{x}_{m,2} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{\mathcal{B}}_{m,1} \\ \boldsymbol{\mathcal{B}}_{m,2} \end{bmatrix} \mathbf{w} \\ \mathbf{y} &= \begin{bmatrix} \boldsymbol{\mathcal{C}}_{m,1} & \boldsymbol{\mathcal{C}}_{m,2} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_{m,1} \\ \mathbf{x}_{m,2} \end{Bmatrix} + \boldsymbol{\mathcal{D}}\mathbf{w} \end{aligned}, \tag{10}$$

where the set of states $\mathbf{x}_{m,1}$ are those to be retained in the reduced model. The kept states are associated with the dominant eigenvalues, which are selected according to the classical dominancy criterion [15], considering the proximity of the poles to the imaginary axis. A remarkable property of the modal truncation is the existence of an approximation error upper bound [16], which can be computed before performing the model reduction.

3.2.2 Balanced truncation

Balanced truncation is focused on preserving as much as possible in the low-order model the full-order input/output behavior [14]. To this aim, model reduction is achieved by truncating states with low degree of reachability and observability, since they would provide a small contribution to the input/output behavior of the system. To define the degree of reachability and observability of the states, the infinite reachability gramian $\mathbf{G}_r^{\infty}$ and the infinite observability gramian $\mathbf{G}_o^{\infty}$ are adopted. According to a classical approach, they can be computed by solving the following Lyapunov equations:

$$\begin{aligned} \boldsymbol{\mathcal{A}}\mathbf{G}_r^{\infty} + \mathbf{G}_r^{\infty}\boldsymbol{\mathcal{A}}^T + \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{B}}^T &= \mathbf{0}, \\ \boldsymbol{\mathcal{A}}^T\mathbf{G}_o^{\infty} + \mathbf{G}_o^{\infty}\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{C}}^T\boldsymbol{\mathcal{C}} &= \mathbf{0}. \end{aligned} \tag{11}$$

To perform an effective truncation, the system is first transformed into its *balanced realization*, in which the states can be ordered according to their contribution to the input–output behavior. A reduced model is then obtained on the basis of this balanced realization by discarding the states with the smallest influence. Like in the modal truncation, also in this case it is possible to define an error bound associated with the approximation [14]. The approach described thus far is sought for a good approximation in the entire frequency range. When a good approximation is required only in a specific bandwidth, an efficient frequency-limited balanced truncation could be adopted [17]. However, this approach has been discarded since it is limited to applications where time-domain analyses are not requested.

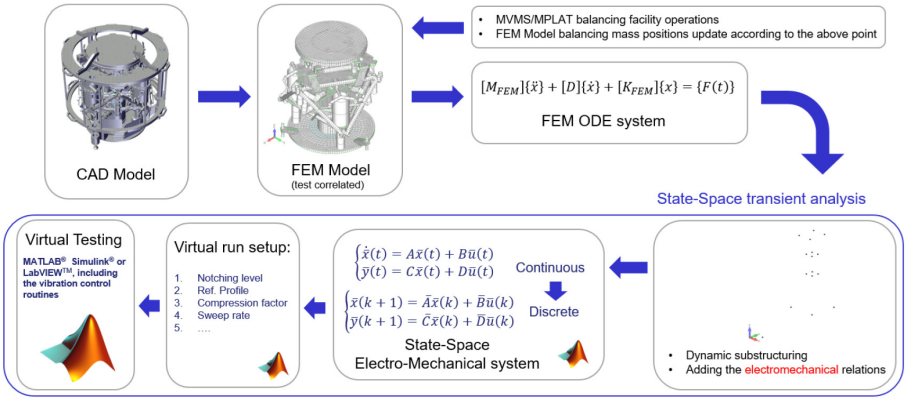

Fig. 3 Overview of the computational methodology of MVMS Virtual Testing (Image courtesy of ESA).

4 The MVMS virtual testing environment

4.1 Computational methodology

The VT environment of the facility is aimed at performing efficient simulation and prediction of its dynamical behavior and providing an efficient tool to design and tune its control systems. To this aim, the final numerical framework will be based on a reduced model starting from the previously described high-fidelity FE model of the MVMS structure. An overview of the computational methodology is depicted in Figure 3, where it is also highlighted that the VT analysis is carried out within the Matlab/Simulink environment by exploiting its numerous simulation tools available for state-space models. Illustrating step by step the flux presented in this figure, the CAD model of the facility was used to obtain the corresponding FE model, whose properties were correlated towards test results. The facility has a mass balancing system aiming at nullifying or reducing possible in-plane offsets of the IUT. It follows that the balancing masses configuration had to be updated in the FE model. The governing equations in terms of FE ordinary differential equations system are thus obtained, leading to the corresponding state-space system writing. At this point, in this example, a state-space dynamic substructuring is considered. Electromechanical relations can be added on top of that in a later stage. After passing from continuous to discrete state-space, the transient analysis can be run, where all the test-setup parameters are included, whence the name of VT analysis. With appropriate differences, the present contribution can be considered analogous to the current efforts in developing reliable and efficient Virtual Shaker Testing approaches, where a multidisciplinary simulation environment is built to closely represents the real vibration test for optimization of the test execution and improvement of the control system [18].

In the following the implementation and performance evaluation of the selected different MOR strategies is presented, with the aim of identifying the possible best candidate for the purposes of the VT environment.

4.2 Application of CMS techniques

With regard to the application of CMS methods, the MVMS structure is divided into three substructures corresponding to BS, VIBISO and MPLAT and having the endpoints of the springs modeling the MINUS-K devices as interfaces. It is noted that, strictly speaking, the Craig-Bampton method and its enhanced version, as well as the Rubin method, can be also directly applied to the entire structure without introducing any substructure. However, the direct approach was discarded in this work as it involves greater computational effort without guaranteeing superior performance compared to substructuring.

Even if the employment of substructuring leads to several advantages, certain complexities arise, for example in the choice of the number of normal modes to be retained in each substructure. A crude approach would be to adopt the same number of modes. However, one substructure could contribute much more significantly to the overall dynamic response than the others. This is particularly evident in the present case, where the three substructures have different characteristics in terms of size, shape or stiffness and mass properties.

In order to maximize the performance of the reduction, the allocation of the number of normal modes to be retained among the various substructures is established here by resorting to the *Effective Interface Mass* (EIM) criterion [19]. This technique allows the modes of the substructure to be classified according to their contribution to the forces at the interface of the substructure when the system is excited by a boundary acceleration. Each vibration mode makes a contribution to a quantity called *cumulative effective interface mass*, which takes on values ranging from 0 to 1 depending on the number of modes considered (a unitary value is reached when the complete set of modes is considered). By selecting a specific threshold for the cumulative EIM, it is possible to retrieve the corresponding number of modes which provide the selected contribution to the dynamics of the substructure. Starting from the computation of the number of modes for each substructure, it is possible to deduce which substructures require a larger amount of modes to be properly represented and thus provide a greater contribution to the dynamics of the structure. The resulting modes allocation is taken as a starting point for all the CMS reductions, with some adjustments for the Rubin method to consider all the rigid body modes and some free interface modes in the floating substructures. The resulting combination of normal modes for BS/VIBISO/MPLAT is 15/37/4, respectively, according to Craig-Bampton and Enhanced Craig-Bampton methods, while it is given by 15/30/11 when the Rubin method is adopted. The number of boundary DOFs is equal to 60 in all cases.

Figure 4 illustrates the relative error between the first 70 natural frequencies of the MVMS computed from the full-order FE model and those computed with the reduced models obtained from the three CMS techniques here considered. The frequency ranges corresponding to the different modes are reported in the bottom part of the figure. It is noted that the relative error is small for the first 12 modes and the difference among the three techniques is negligible. This result is expected since these modes are mainly driven by the MINUS-K devices, involving only the elongation of the springs with no substantial elastic deformation of the rest of the structure. It is also observed that the modal elastic behavior in the frequency region up to about

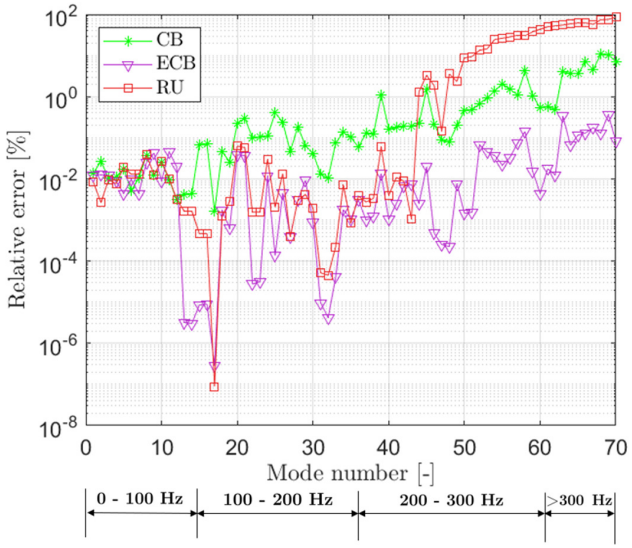

Fig. 4 Relative error between the first 70 natural frequencies computed from the full-order FE model and those computed with the reduced models obtained from Craig-Bampton (CB), Rubin (RU) and Enhanced Craig-Bampton (ECB) methods. The frequency ranges associated with the eigenmodes are also reported.

250 Hz is more accurately approximated by the Rubin and ECB reduction techniques than the classical CB method. For some modal frequencies, the difference in the relative error is of two or even three orders of magnitude. The ECB technique maintains superior performance also concerning the higher frequency modes, while a significant worsening is experienced with the Rubin method.

The performance of the reduced models is also compared by evaluating the degree of consistency between mode shapes by means of the Modal Assurance Criterion (MAC). The results are reported in Figs. 5, 6, and 7 for CB, RU and ECB techniques, respectively. It is shown that, compared to the classical Craig-Bampton method, the Rubin method seems to correctly reproduce a slightly lower number of mode shapes, although the accuracy in matching the full-order model natural frequencies was higher. It is confirmed that the ECB method presents superior performance also in terms of MAC.

It is worth noting that the good performance of the Enhanced Craig-Bampton reduction is not achieved without cost. Indeed, the ECB technique requires much more computational effort than the classical Craig-Bampton method. However, thanks to the exploitation of substructuring and the adoption of the EIM criterion by carefully tuning the number of dominant modes, the computational burden associated with the application of the ECB method can be limited. Therefore, especially when reduced mass and stiffness matrices are desired, this method is the best option to consider.

4.3 Application of state-space based techniques

The application of state-space based reduction methods involves the construction of the state-space model of the entire structure, without the possibility of resorting to

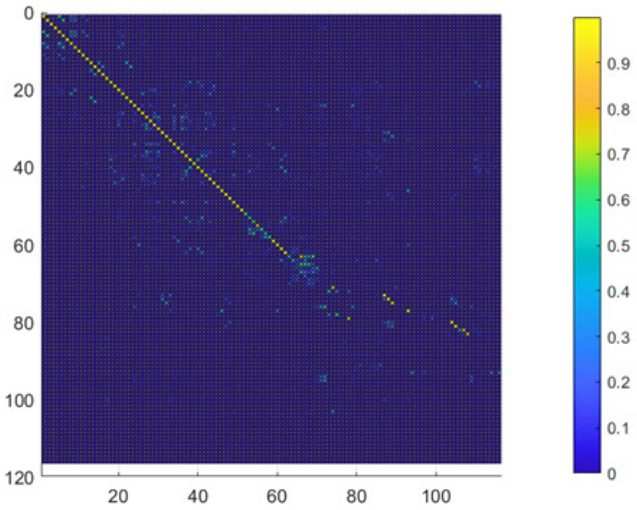

Fig. 5 MAC results considering the full-order model and the reduced-order model obtained with CB method.

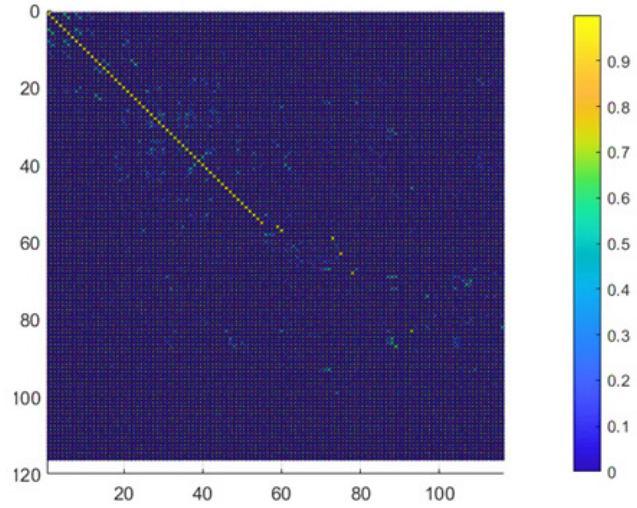

Fig. 6 MAC results considering the full-order model and the reduced-order model obtained with RU method.

substructuring. To build such a representation for our structural system, it is necessary to have the physical damping matrix of the model [6]. Modal damping being the approach chosen in this work, the derivation of the above-mentioned matrix for the MVMS FE model is not computationally feasible, as it would be necessary to calculate the entire set of eigenmodes. Furthermore, as presented above, the implementation of the balanced reduction technique requires the solution of two Lyapunov equations. This is computationally costly, with practical limitations to dynamic systems with a

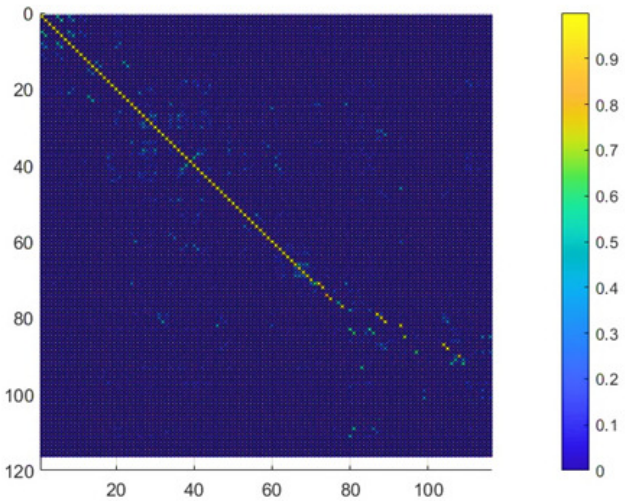

Fig. 7 MAC results considering the full-order model and the reduced-order model obtained with ECB method.

number of states in the order of a thousand [5]. Various techniques have recently been proposed in the literature to reduce the computational burden, but the accuracy of the results is highly dependent on the specific case under consideration. Their evaluation is postponed to future developments.

In order to overcome the above issues, a hybrid approach consisting in a two-stage reduction is here adopted, building on some ideas taken from the work of Spanos et al. [7]. The two-stage approach is summarized in Figure 8. First, a preliminary classical Craig-Bampton reduction is performed to derive a reduced model with a number of DOFs still considerably large, but not enough to prevent the application of the state-space based techniques. This reduction can be profitably implemented with substructuring to limit the computation effort. Then, the modal damping matrix is derived and the state-space realization is built. In this work, the input of the system is represented by the base acceleration, while the IUT longitudinal acceleration is selected as output. Other input-output pairs can be selected as well without substantially changing the procedure. It is worth noting that the imposed base motion requires to represent the corresponding inertial loads and to project the latter onto the CB subspace. The final step involves the reduction of the CB model through Modal Truncation or Balanced Truncation. To carry out a consistent comparison, the number of states retained corresponds to twice the size of the reduced structural matrices obtained with CMS techniques.

Concerning the BT method, in addition to the classical approach in computing the gramians using Eq. (11), a different formulation for the infinite observability gramian based on the so-called stochastic balancing technique is also considered in this work [20]. Due to the limited number of structural modes in the frequency range of interest, the two approaches provide very similar results in the present case, both in terms of performance and computational cost.

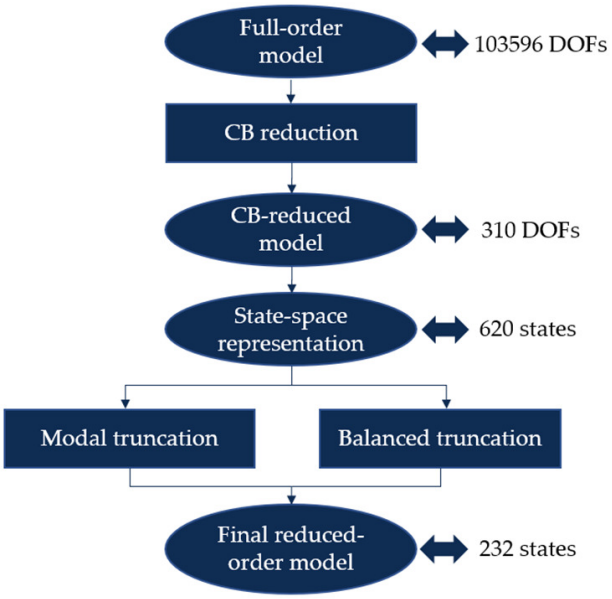

Fig. 8 Two-stage reduction scheme; at each step the number of DOFs/states characterizing the model is reported.

An overall evaluation of the performance of the two-stage hybrid approach, including a comparison with the three CMS techniques, is reported in Figure 9, which shows the transmissibility function resulting from the full-order model and the reduced-order models when the MVMS is subjected to a seismic ground acceleration in the longitudinal direction and the acceleration of the item under test is measured in the same direction. The number of fixed-interface normal modes retained in the preliminary CB reductions associated to the hybrid approach (Hybrid CB + MT and Hybrid CB + BT) is determined by resorting to the EIM criterion. The resulting combination for BS/VIBISO/MPLAT is 85/125/40, respectively. The number of boundary DOFs is equal to 60 in all cases, leading to a CB-reduced model with a total of 310 DOFs as the starting point for the modal and balanced truncation. All the methods provide an overall good approximation of the system's response in the low-frequency resonance region. Again, the lack of a notable difference among the methods is due to the peculiar dynamic behavior of MVMS associated with the low-pass filtering action induced by the MINUS-K devices. Although the full potential of the BT method cannot be appreciated, better focus shows that balanced reduction is able to approximate the full-order model with greater accuracy than modal truncation, both in the high-frequency asymptotic behavior and in the resonance region away from resonance peaks. This worsening is due to the complete truncation of residual flexibility in the MT technique. To thoroughly characterize the model reduction methods, an index of computational burden associated with the generation of the reduced-order models is reported in Table 2. This parameter is defined as the ratio between the computational time of each method and the one of the most efficient technique (corresponding to CB in this case). As it can be noticed, Rubin and ECB require a significantly longer time than

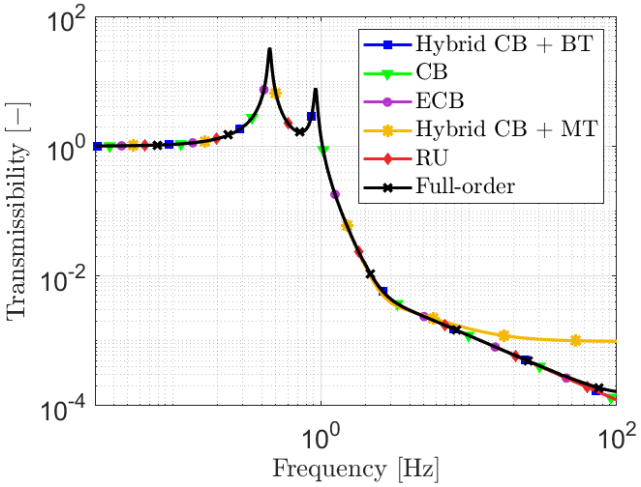

Fig. 9 Transmissibility function of the facility in the longitudinal direction. Comparison of the response resulting from the reduced models with respect to the full-order model.

the other techniques. This increment in computational effort is due to the derivation of the residual flexibility matrix.

Table 2 Index of computational burden associated with the derivation of the reduced-order model from the full-order system with the methods considered in this work.

Method	**Index of computational burden**
Craig-Bampton	1
Rubin	15.6
Enhanced Craig-Bampton	22.1
Hybrid Craig-Bampton + balanced truncation	1.7
Hybrid Craig-Bampton + modal truncation	1.08

The difference in the various approximations is more visible when the previous analysis is performed with a significantly lower number of DOFs. The results are reported in Figure 10, which shows the relative error in the transmissibility function between the full-order and the reduced-order models, when 64 DOFs, corresponding to 128 states, are retained. It is clear that the Enhanced Craig-Bampton and the Hybrid CB + BT method give the best approximation, with significant improvements with respect to classical CB and RU methods over wide portions of the selected frequency range. The two-stage approach relying on the modal truncation is totally unsuitable in this case, which reveals its inefficiency in providing good approximation with a limited number of modes.

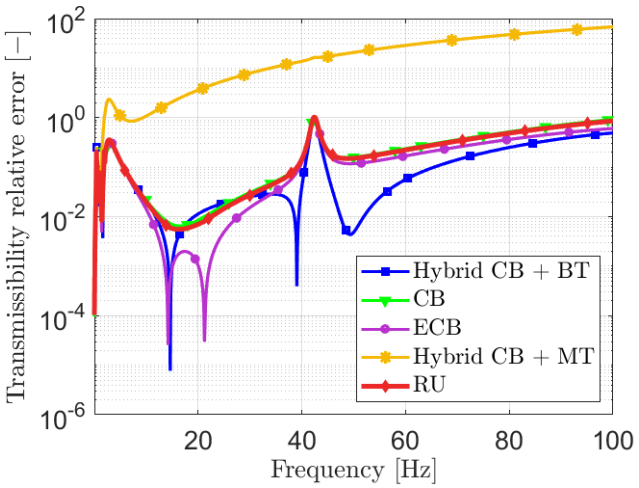

Fig. 10 Relative error between the transmissibility of the full-order model and each of the reduced-order models, retaining a lower number of states.

5 Conclusions

An investigation of various model reduction techniques as possible candidates for a virtual testing simulation environment of the ESA's Micro Vibrations Measurement System is presented in this work. A virtual testing approach would allow for optimized test execution and a better fine-tuning of the control system of the facility. The resulting tool should be reliable but also efficient, thus requiring to reduce the order of the high-fidelity finite element model of the MVMS. Both Component Mode Synthesis and state-space based methods (modal and balanced truncation) are considered and evaluated. In particular, in addition to the classical Craig-Bampton and Rubin techniques, the recently proposed Enhanced Craig-Bampton method is implemented and applied to this case by using a specific substructuring assembly procedure. The computational issues associated with the direct application of state-space based techniques to the high-order FE model are overcome by resorting to a hybrid two-stage approach, where a preliminary reduction is performed by using the Craig-Bampton method with substructuring to obtain an intermediate model onto which the state-space realization and reduction are applied. In all cases, the number of normal modes to be retained in each substructure is determined according to the Effective Interface Mass criterion. Results show that the ECB and the hybrid approach with balanced truncation give superior performance compared to more classical methods. Although the computational effort of the ECB technique is high, the application of substructuring with an optimized selection of fixed-interface normal modes makes it manageable. If a state-space representation is required without the need to manage the reduced structural matrices, the hybrid approach CB + BT is the most convenient and accurate. Furthermore, the computational burden associated with this technique is significantly lower than the one of ECB. Given the resulting performance, the replacement of CB with ECB in the two-stage approach is an interesting future development. An investigation

could also be carried out to understand the dependence of performance on the choice of the combination of truncated DOFs in the preliminary reduction and neglected states with balanced truncation, with the aim of determining an optimality criterion.

Declarations

The authors have no conflicts of interest to declare that are relevant to the content of this article.

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