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A general Monte-Carlo approach to consider a maximum admissible risk in decision-making procedures based on measurement results

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ABSTRACT
According to the standards, decision-making procedures generally consider both a threshold that should not be exceeded and the measurement uncertainty that is associated to the measurement result. However, the general indications given in the Standards, in their examples, refer to the particular case when the measurand distributes according to a normal PDF. But a generalization to other cases is not considered and is not straightforward.
In a previous paper, the Authors proposed a decision-making procedure which not only considers the measurement uncertainty and the threshold, but also considers a Maximum Admissible Risk. The proposed procedure leads to decisions taken with a risk of a wrong decision lower than the given Maximum Admissible Risk. In particular, closed-form formulas were derived under specific assumptions for the distributions of the measured values. Hence, the aim of this paper is to generalize the proposed decision rule and method for setting acceptance and rejection limits, by applying the Monte-Carlo method. In this way, it can be generally applied, even when the distribution associated to the measurement result is not a priori known in closed form.

Section: RESEARCH PAPER

Keywords: measurement uncertainty; threshold; tolerance limit; acceptance limit; decision making; risk of wrong decision; maximum admissible risk

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1. INTRODUCTION

When measurement results are employed in the decision-making procedures, the associated measurement uncertainty [1], [2] should be considered [3]-[8]. The present Standards in conformity assessment [9]-[10] define a decision rule, which is based on a given tolerance limit and the measurement uncertainty associated to the measured value. However, the application of this decision rule leaves a risk of taking a wrong decision; this risk is generally evaluated only *a posteriori*. Furthermore, very few practical indications are given to define the acceptance limits given the expected measurement uncertainty and the Maximum Admissible Risk (*MAR*) of incorrect assessment.

On the other hand, in the Authors' opinion, practical indications are necessary, particularly when dealing with critical measurements, such as the ones involving human health and the environment. For this reason, in a previous paper [11], the Authors have proposed a decision-making procedure which not only considers measurement uncertainty and the tolerance limits, but also considers the desired *MAR*. The procedure proposed in [11] yields a decision with an associated risk of wrong decision that is kept lower than the assumed *MAR*.

In that paper, the decision-making procedure has been explained in detail and, to apply it, closed-form formulas have been strictly evaluated, under specific assumptions.

Hence, the aim of this paper is to generalize the proposed decision-making procedure, so that it can be applied also when the assumptions required to employ closed-form formulas do not hold.

In particular, a more general procedure is needed when the equation representing the pdf associated to the measurement result is not known, or, if known, the cumulative probability function cannot be obtained in closed form. Therefore, under these assumptions, Monte-Carlo simulations [12] can provide the desired result.

Hence, this paper implements, in a numerical way, the same method proposed in [11], and considers also cases in which it cannot be implemented in closed form.

In next Section 2, the method proposed in [11] is briefly recalled, for the sake of clarity, although without entering into the mathematical details. Section 3 will then present the generalized method and different examples of applications are provided. The same case studies considered in [11] are here solved by applying the proposed general method, in order to validate it. Finally, an experimental validation is given in Section 4 and some conclusions are drawn in Section 5.

2. OVERVIEW OF THE PROPOSED DECISION RULE BASED ON THE MAXIMUM ADMISSIBLE RISK

In [11], a decision-making procedure was defined, based on:

- The given upper (T_U) or lower (T_L) tolerance limits.
- The measurement uncertainty and the probability density function (PDF) associated to the measurement result.
- The Maximum Admissible Risk (*MAR*).

In this section, the proposed method is briefly recalled, without entering into the mathematical details, for which the reader is addressed to [11].

Given the PDF $p(x)$ associated to the measurement result, the cumulative distribution function (CDF) is evaluated as:

$$F_X(x) = \int_{-\infty}^{x} p(t)\,dt\,. \tag{1}$$

It is well-known that, for every value x, the value taken by the CDF $F_X(x)$ represents the probability that variable X is lower than x; similarly, $1 - F_X(x)$ represents the probability that variable X is greater than x.

Therefore, if the tolerance limits T_U or T_L shall not be exceeded and the *MAR* is given, then, the following inequalities shall be satisfied:

$$\begin{cases} F_\mathrm{X}(T_\mathrm{U}) \geq 1 - \mathrm{MAR} & \text{when } x < T_\mathrm{U} \text{ is required} \\ F_\mathrm{X}(T_\mathrm{L}) \leq \mathrm{MAR} & \text{when } x > T_\mathrm{L} \text{ is required} \end{cases}. \tag{2}$$

Therefore, the values of the acceptance limits A_U or A_L (which ensure that the probability that the tolerance limits are exceeded is exactly equal to *MAR*) can be found by solving:

$$\begin{cases} A_\mathrm{U} | F_\mathrm{X}(T_\mathrm{U}) = 1 - \mathrm{MAR} & \text{if } x < T_\mathrm{U} \text{ is required} \\ A_\mathrm{L} | F_\mathrm{X}(T_\mathrm{L}) = \mathrm{MAR} & \text{if } x > T_\mathrm{L} \text{ is required} \end{cases}. \tag{3}$$

Of course, the solution of the above equations is strictly related to the CDF, that depends on the PDF associated with the measurement result.

In [11], the above equations have been solved in closed form under the following assumptions [11].

2.1. Normal PDF

Let us suppose that $p(x)$ is a normal PDF with standard deviation σ. Then, it follows [11]:

$$A_\mathrm{U,L} = T_\mathrm{U,L} \mp \sqrt{2}\,\sigma \cdot \mathrm{erfinv}(1 - 2 \cdot MAR)\,, \tag{4}$$

where erfinv is the *inverse error function*.

Therefore, to obtain the acceptance limit A_U or A_L, the tolerance limit is shifted to the left (or right) by quantity $\sqrt{2} \cdot \sigma \cdot \mathrm{erfinv}(1 - 2 \cdot MAR)$. In particular, according to the recommendations in [3]:

- The limit is shifted to the left when $x_\mathrm{m} \leq T_\mathrm{U}$ is required and guarded acceptance is applied.
- The limit is shifted to the left when $x_\mathrm{m} \geq T_\mathrm{L}$ is required and guarded rejection is applied.
- The limit is shifted to the right when $x_\mathrm{m} \geq T_\mathrm{L}$ is required and guarded acceptance is applied.
- The limit is shifted to the right when $x_\mathrm{m} \leq T_\mathrm{U}$ is required and guarded rejection is applied.

2.2. Uniform PDF

Let us suppose that $p(x)$ is a uniform PDF with a support width of $2 \cdot a$. Then, it follows [11]:

$$A_\mathrm{U,L} = T_\mathrm{U,L} \mp a \cdot (1 - 2 \cdot MAR)\,, \tag{5}$$

that is, the acceptance limit is obtained by shifting the tolerance limit to the right/left by quantity $a \cdot (1 - 2 \cdot MAR)$. The direction of the shift follows the same considerations as those given in Section 2.1.

2.3. Triangular PDF

Let us suppose that $p(x)$ is a symmetrical triangular PDF with a support width of $2 \cdot a$. Then, it follows [11]:

$$A_\mathrm{U,L} = T_\mathrm{U,L} \mp a \cdot \left(1 - \sqrt{2 \cdot MAR}\right) \tag{6}$$

that is, the acceptance limit is obtained by shifting the tolerance limit to the right/left by quantity $a \cdot \left(1 - \sqrt{2 \cdot MAR}\right)$. The direction of the shift follows the same considerations as those given in Section 2.1.

2.4. Trapezoidal PDF

Let us suppose that $p(x)$ is a symmetrical trapezoidal PDF with a support width of $2 \cdot a$ (*major* basis), and the ratio between the *minor* and the *major* bases is β. Then, it follows [11]:

$$A_\mathrm{U,L} = T_\mathrm{U,L} \mp a \cdot \left(1 - \sqrt{2 \cdot MAR \cdot (1 - \beta^2)}\right), \tag{7}$$

that is, the acceptance limit is obtained by shifting the tolerance limit to the right/left by quantity $a \cdot \left(1 - \sqrt{2 \cdot MAR \cdot (1 - \beta^2)}\right)$. The direction of the shift follows the same considerations as those given in Section 2.1.

3. APPLICATION OF THE MONTE-CARLO SIMULATIONS IN THE DECISION-MAKING PROCEDURE BASED ON THE MAXIMUM ADMISSIBLE RISK

In the previous section, the method proposed in [11] has been briefly recalled.

This procedure defines a clear relationship between tolerance limits, measurement uncertainty, acceptance limits and risk of exceeding the tolerance limits. Consequently, given a tolerance limit and two of the other quantities, the third one can be readily obtained. In [11] this procedure was applied to obtain the acceptance limit, having set uncertainty and *MAR*, under the assumption that the PDF of the distribution of values that can be reasonably attributed to the measurand was one of those considered in Section 2.

Considering only these PDFs is justified by the fact that they represent most of the practical situations for measurement results and their associated uncertainties.

However, since other situations cannot be *a priori* excluded, in order for the proposed decision-making procedure to be generally applied, a general method valid for whichever PDF should be defined.

3.1. The general method

In the method proposed in [11] and briefly recalled in Section 2, the key point is the evaluation of the cumulative distribution function. When the PDF associated to the measurement result is known, together with its mathematical equation, and this is integrable, it is possible to find the mathematical equation of the cumulative distribution function, so that a closed-form formula can be found for the evaluation of the acceptance limits A_U or A_L (as recalled in Section 2 [11]).

When the above conditions are not met, the proposed method does not lose validity, provided that a numerical method can be implemented.

In particular, numerical methods based on Monte Carlo simulations are already recommended [12] whenever a mathematical function representing the distribution of values that can be reasonably attributed to the measurand is not available, such as, for instance, when the measurand is not measured directly, but is determined through other quantities through a functional relationship [2] and the Central Limit Theorem cannot be applied [12].

By applying a Monte Carlo simulation, or by experimentally repeating the measurement procedure, if possible, N values are obtained for the measurand, and their PDF can be approximated by the histogram of the relative frequencies [12].

When N values are available, the best estimate of the measurand is generally supposed to be their mean value. Since we have to determine if the value of the measurand is lower or greater than a given tolerance limit, then let us suppose initially that the tolerance limit is exactly the mean value of the N measured values. This means that there is a 50 % probability to be below the tolerance limit and a 50 % probability to be above it.

The histogram of the N measured values allows one to evaluate its associated CDF in a numerical way as:

$$F_{\mathrm{X}}(x) = \sum_{-\infty}^{x} h(x) \tag{8}$$

where, for each class of the histogram, $h(x)$ represents the relative frequency of the class.

Let us now first consider the situation where the property of the measurand must be below the given tolerance limit T_U and let us suppose that the maximum admissible risk to be above the limit T_U is set to MAR. Since the CDF is a function that can assume all values – and only the values – between 0 and 1, MAR can be set between 0 and 1, where 0 corresponds to a percentage of 0 % and 1 corresponds to 100 % (100·MAR).

Under the above assumption, on the obtained curve $F_{\mathrm{X}}(x)$, the point corresponding to 1 – MAR is numerically found. This point, denoted as x_{MAR}, satisfies to:

$$x_{\mathrm{MAR}} | F_{\mathrm{X}}(x_{\mathrm{MAR}}) = 1 - MAR \tag{9}$$

and corresponds to that specific value for which the probability that the measurand is greater than it is exactly equal to MAR.

Let us now consider the opposite situation where the measurand must be above the given tolerance limit T_{L}. In this case, the point corresponding to MAR on the $F_{\mathrm{X}}(x)$ function is numerically found. This means to find the x_{MAR} value that satisfies to:

$$x_{\mathrm{MAR}} | F_{\mathrm{X}}(x_{\mathrm{MAR}}) = MAR \,, \tag{10}$$

which corresponds to that specific value for which the probability that the measurand is lower than it is exactly equal to MAR.

Once x_{MAR} is found, by applying either (9) or (10), the difference between this value and the tolerance limit is evaluated as:

$$\Delta = x_{\mathrm{MAR}} - T_{\mathrm{U,L}} \,. \tag{11}$$

It should be noted that Δ can be both positive or negative, depending on where, on the cumulative probability curve, the point x_{MAR} is found, to the right or the left of T_{U} or T_{L}.

Since the mean value of the considered histogram has been considered equal to the tolerance limit, thus meaning that this value corresponds exactly to 0.5 in the CDF, it can be stated that, if a risk lower than 0.5 is considered (as it seems to be the most likely situation to occur), then:

- If $x < T_{\mathrm{U}}$ is required, the desired value x_{MAR} will be at the right of T_{U}, so that Δ will be positive.
- If $x > T_{\mathrm{L}}$ is required, the desired value x_{MAR} will be at the left of T_{L}, so that Δ will be negative.

The evaluation of x_{MAR} on the CDF, by applying either (9) or (10), leads to easily evaluate the acceptance limits as:

$$A_{\mathrm{U,L}} = T_{\mathrm{U,L}} - \Delta = 2 \cdot T_{\mathrm{U,L}} - x_{\mathrm{MAR}} \tag{12}$$

This means that, in order not to exceed the tolerance limit $T_{\mathrm{U,L}}$ with a risk greater than MAR, the initial histogram should be shifted on the left/right by quantity Δ, i.e. the histogram should have a mean value equal to $A_{\mathrm{U,L}}$. Hence:

- If $x < T_{\mathrm{U}}$ is required and the maximum admissible risk that $x > T_{\mathrm{U}}$ is set to MAR, then this is surely satisfied whenever the measured value is lower than A_{U}.
- If $x > T_{\mathrm{L}}$ is required and the maximum admissible risk that $x < T_{\mathrm{L}}$ is set to MAR, then this is surely satisfied whenever the measured value is greater than A_{L}.

It can be readily checked that A_{U} and A_{L} represent the acceptance limits when guarded acceptance is required with a guard band that ensures that the probability of exceeding T_{U} (or T_{L}) is not greater than MAR for the considered PDF.

To verify the above statement, it is possible to find and draw the CDF associated to the shifted histogram and to evaluate its value in correspondence to $T_{\mathrm{U,L}}$. Of course, the CDF associated to the shifted histogram is simply the shifted CDF associated to the original histogram. In the shifted CDF, the acceptance limit A_{U} or A_{L}, is exactly the value for which the CDF is equal to 0.5. On the other hand, it can be verified that the CDF's value in T_{U} or T_{L} is exactly, respectively, 1 – MAR or MAR.

3.2. Comparison with the results obtained in closed-form

In this Section, the results obtained by applying the method described in Section 3.1 are shown. In particular, in order to validate the proposed general method, the same examples given in [11] are considered, where the concentration of a pollutant in water must be lower than a given tolerance limit, as summarized in Table 1.

In Table 1, T_{U} represents the upper tolerance limit, MAR the maximum admissible risk and A_{U} is the acceptance limit, as obtained with the closed-form formulas derived in [11] and recalled in Section 2. Furthermore, when considering the distribution of the measured values (the readers are addressed to

Table 1. Considered numerical example and values of the acceptance limits obtained with the closed-form formulas defined in [11].

T_{U}	MAR	pdf type	A_{U}
50 mg/l	0.05	Normal (σ = 5 mg/l)	41.8 mg/l
50 mg/l	0.05	Uniform (a = 10 mg/l)	41 mg/l
50 mg/l	0.05	Triangular (a = 10 mg/l)	43.2 mg/l
50 mg/l	0.05	Trapezoidal (a = 10 mg/l; β = 0.5)	42.7 mg/l

[11] for the details): σ is the standard deviation of the normal pdf; a is the semi-width of the considered pdf and β is the ratio between the two bases, in the case of the trapezoidal pdf.

For all examples given in Table 1, the following assumption is made, when applying the proposed general method based on Monte-Carlo simulations: N = 500,000 iterations are considered and a number of classes equal to $N/10$ is taken, to obtain the histogram of the simulated values, according to the assumed PDF (third column in Table 1).

3.2.1. The measurement results distribute according to a normal posterior PDF

When the measured values are supposed to distribute according to a normal PDF, the histogram of the relative frequencies is given in Figure 1 and the following procedure is applied.

- The cumulative probability function (blue line in Figure 2) is numerically evaluated by applying equation (8).
- It can be noted that, on this function, the probability of exceeding T_U is exactly 50 %.
- On this function, the point x_{MAR} corresponding to a probability 1 – MAR (0.95 in the considered example in Figure 2) is numerically obtained, as in equation (9).
- The difference $\Delta = x_{MAR} - T_U$ is evaluated, which corresponds to how far x_{MAR} is from T_U.
- The new acceptance limit A_U is found by applying equation (12).

By applying the above method, it follows A_U = 41.8 mg/l, which corresponds exactly to the value obtained when the closed-form formulas are applied [11] (see first row in Table 1).

This means that, when the measured value is equal to A_U, then there is a probability of exceeding T_U exactly equal to MAR (5 % the considered example). This means that every measured value lower than A_U will provide a probability of exceeding T_U lower than the required MAR.

To verify the proposed numerical approach, let us consider the shifted CDF (cyan line in Figure 2), obtained by shifting to the left by quantity Δ the original CDF. It is worth noting that the cumulative distribution function is shifted to the left because, according to the example, an upper tolerance limit was assumed. In the case of a lower tolerance limit, the cumulative probability function should be shifted to the right.

It can be immediately seen (Figure 2) that, on the shifted CDF, the probability of exceeding T_U is exactly MAR (5 % in this example).

3.2.2. The measurement results distribute according to a uniform posterior PDF

When the measured values are supposed to distribute according to a uniform PDF with the characteristics given in Table 1, the histogram of the relative frequencies is given in Figure 3. Figure 4 shows the CDF (blue line) associated to the histogram of Figure 3, and the shifted CDF (cyan line) obtained as explained in the previous section. As described in detail in Section 3.2.1, this procedure allows one to find the acceptance limit A_U, which should not be exceeded to have a risk lower than MAR to exceed T_U. In particular, it follows A_U = 41 mg/l, which corresponds exactly to the value obtained when the closed-form formulas are applied [11] (see second row in Table 1).

3.2.3. The measurement results distribute according to a triangular posterior PDF

When the measured values are supposed to distribute according to a triangular PDF with the characteristics given in

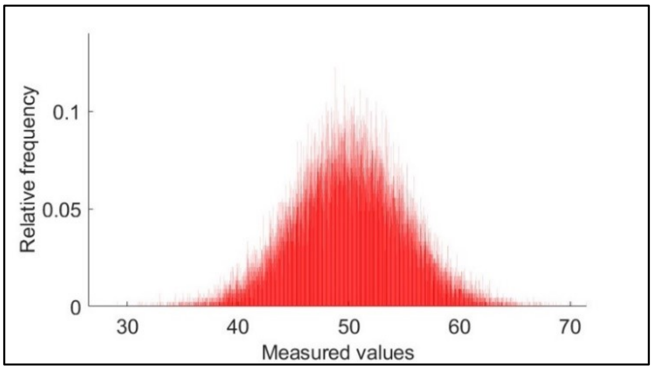

Figure 1. Histogram of the measured values in the case of Table 1, first row.

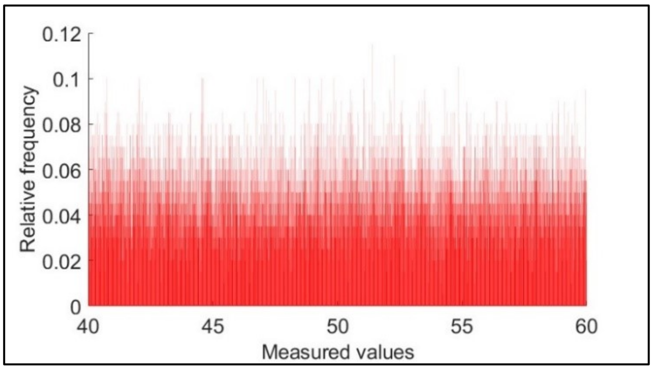

Figure 3. Histogram of the measured values in the case of Table 1, second row.

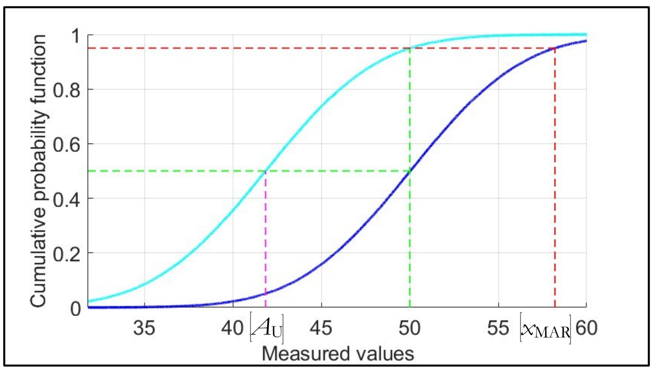

Figure 2. Cumulative distribution function (blue line) associated to the histogram of Figure 1 and shifted cumulative distribution function (cyan line).

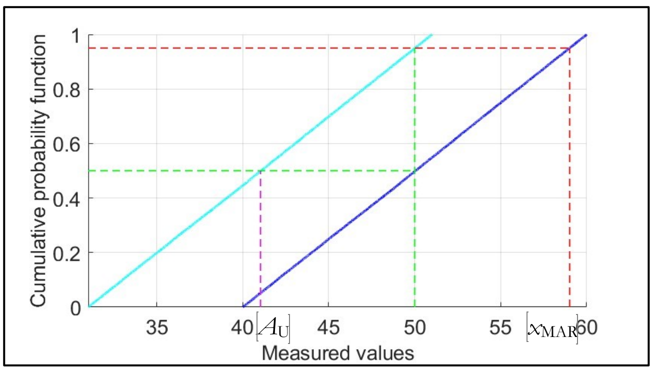

Figure 4. Cumulative distribution function (blue line) associated to the histogram of Figure 3 and shifted cumulative distribution function (cyan line).

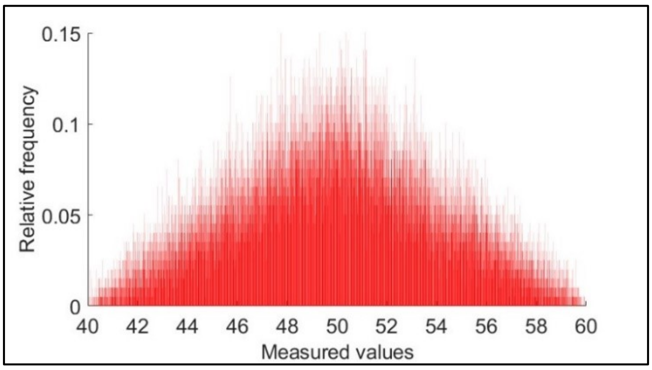

Figure 5. Histogram of the measured values in the case of Table 1, third row.

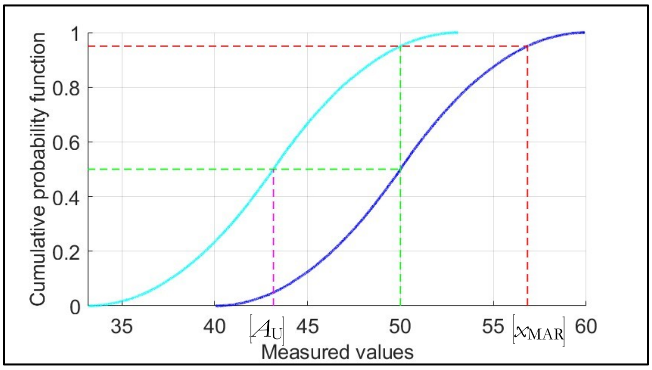

Figure 6. Cumulative distribution function (blue line) associated to the histogram of Figure 5 and shifted cumulative distribution function (cyan line).

Table 1, the histogram of the relative frequencies is given in Figure 5. Figure 6 shows the CDF (blue line), associated to the histogram of Figure 5, and the shifted CDF (cyan line) obtained as explained in Section 3.2.1. As described in detail in that section, this procedure allows one to find the acceptance limit A_U, which should not be exceeded to have a risk lower than MAR (0.05 % or 5 % in this example) to exceed T_U. In particular, it follows A_U = 43.2 mg/l, which corresponds exactly to the value obtained when the closed-form formulas are applied [11] (see third row in Table 1).

3.2.4. The measurement results distribute according to a trapezoidal posterior PDF

When the measured values are supposed to distribute according to a trapezoidal PDF with the characteristics given in Table 1, the histogram of the relative frequencies is given in Figure 7. Figure 8 shows the CDF (blue line), associated to the histogram of Figure 7, and the shifted CDF (cyan line) obtained as explained in Section 3.2.1. As described in detail in that section, this procedure allows one to find the acceptance limit A_U, which should not be exceeded to have a risk lower than MAR (5 % in this example) to exceed T_U. In particular, it follows A_U = 42.7 mg/l, which corresponds exactly to the value obtained when the closed-form formulas are applied [11] (see last row in Table 1).

4. EXPERIMENTAL VALIDATION

In this section, the proposed method is experimentally validated.

As a general example, let us consider the measurement of the RMS value of a sinusoidal voltage, whose amplitude is not strictly constant, but varies according to a given PDF.

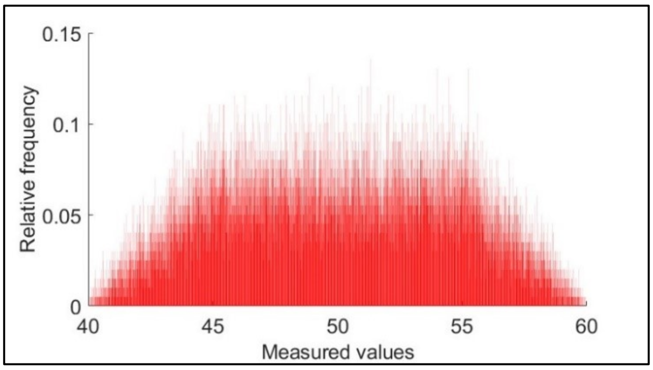

Figure 7. Histogram of the measured values in the case of Table 1, last row.

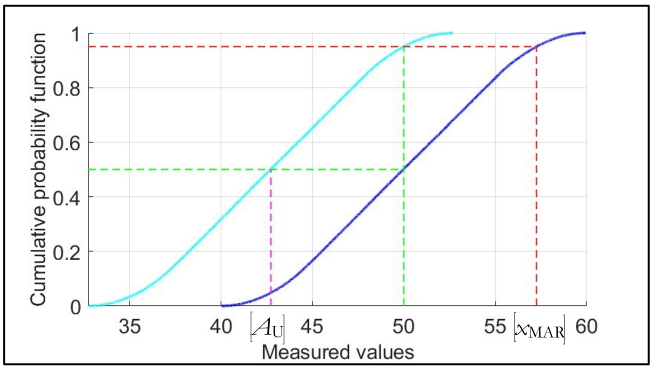

Figure 8. Cumulative probability function (blue line) and shifted cumulative probability function (cyan line) associated to the histogram of Figure 7.

The aim of this experiment is to define an acceptance limit, having set a tolerance limit for the admissible variability of the voltage signal amplitude, and a MAR of wrong decision.

As a PDF of the variability of the sinusoidal amplitude, an asymmetrical trapezoidal probability distribution is assumed, and 25,000 different random values are extracted.

A voltage signal is then generated, using the 25,000 extracted values as the rms values of the signal as follows: for every rms value, 4 periods are generated, so that, in total, we have 100,000 periods of the voltage signal (25,000 groups of 4 periods).

The signal is acquired, and the RMS value is evaluated for every group, by considering, in particular, only the two central periods (and avoiding the first and last periods which could be affected by transient phenomena, due to the change in the parameters). Figure 9 shows the obtained histogram of the cumulative frequencies associated to the evaluated RMS values.

In Table 2, the considered values of the tolerance limits $T_{U,L}$ and the value of MAR are reported. According to the procedure

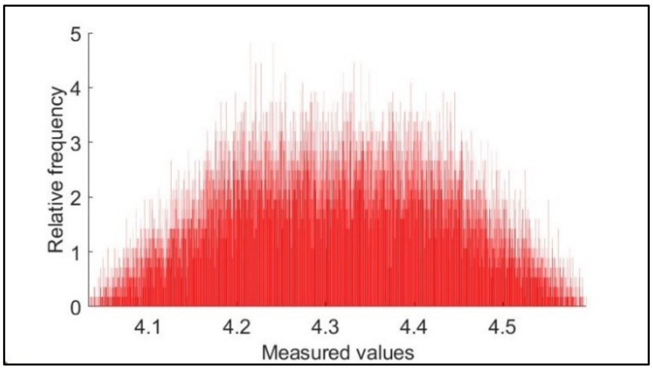

Figure 9. Histogram associated to the voltage RMS values.

Table 2. Considered values of the tolerance limit and the maximum admissible risk for the application in Section 4.

$T_{U,L}$	MAR
4.31 V	5 %

discussed in Section 3.1, the sinusoidal signal is generated, at first, in such a way that the considered tolerance limit $T_{U,L}$ is the same as the mean value of the 25,000 measured values.

Let us first suppose that the voltage RMS value must be lower than T_U. Then, by applying the proposed method, Figure 10 can be drawn and the acceptance limit A_U can be found as A_U = 4.11 V (dashed pink line in Figure 10). In particular, the blue line in Figure 10 corresponds to the CDF - numerically evaluated as in (8) - associated to the histogram in Figure 9. As expected, the obtained CDF shows that the probability of exceeding T_U is 0.5 (green dashed lines).

The acceptance limit A_U can be obtained from (12), after having shifted the CDF to the left by (11). The shifted CDF is represented by the cyan plot in Figure 10, from which it can be readily checked that the probability of exceeding T_U is now 0.05.

A new set of 25,000 sinusoidal signals was then generated, in such a way that the mean value of the measured values corresponds to A_U. The number of measured values exceeding T_U was found to be the 5 % of the total number of measured values, thus confirming the validity of the proposed method.

For the sake of completeness, let us now suppose, on the other hand, that the voltage RMS value must be greater than T_L. Then, by applying the proposed method, Figure 11 can be drawn and the acceptance limit A_L can be found as A_L = 4.51 V (dashed pink line in Figure 11).

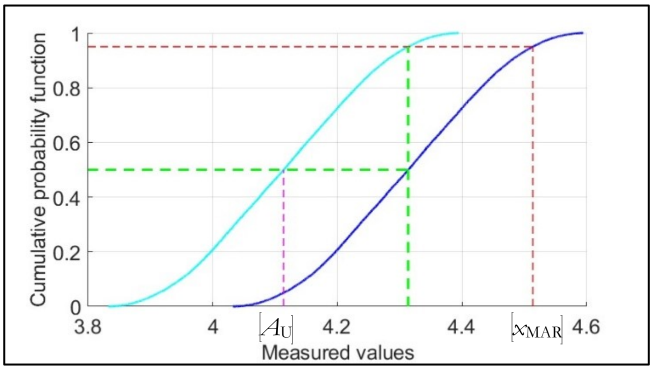

Figure 10. CDF (blue line) associated to the histogram of Figure 9 and shifted CDF (cyan line), when the voltage RMS value must be lower than T_U.

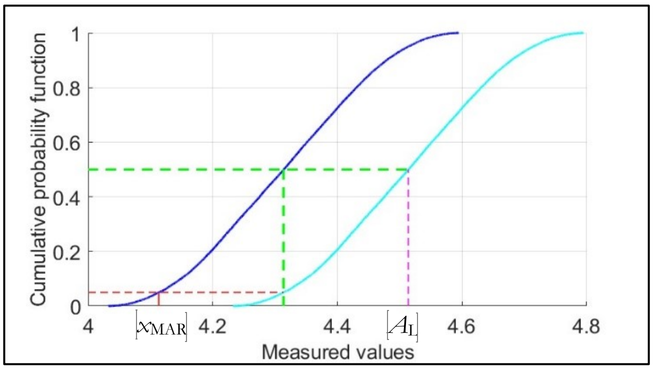

Figure 11. Cumulative probability function (blue line) and shifted cumulative probability function (cyan line) associated to the histogram of Figure 9, when the voltage RMS value must be greater than T_L.

In this case, the CDF is shifted to the right by (11), so that the A_L value is obtained from (12). A similar experimental validation as the one above mentioned was implemented and the validity of the method was confirmed in this case too.

5. CONCLUSIONS

This paper has generalized the method proposed in [11] aimed at providing a method for proving conformity capable of ensuring that, given a measurement uncertainty, the risk of a wrong declaration of conformity is kept below a desired *MAR* value.

The method proposed in [11] was applied to the case when the known PDFs representing the distribution of values that can be reasonably attributed to the measurand can be expressed by mathematical integrable functions, so that both the corresponding CDFs and the acceptance limits can be derived in closed form. This way, when the measurement result lies inside the acceptance interval, the risk of exceeding the tolerance limits remains lower than the desired *MAR*.

However, cases exist, especially when the measurand cannot be measured directly [12], in which the PDF representing the distribution of values that can be reasonably attributed to the measurand cannot be expressed in terms of a known mathematical function. In such cases, a Monte Carlo simulation is recommended to estimate such distribution of values [12].

This paper has extended the method proposed in [11] to this case and has proved that suitable acceptance limits can be obtained from a Monte Carlo simulation too, capable of ensuring that, if the measurement result remains within the obtained acceptance interval, the risk of exceeding the tolerance limits is not greater than the desired *MAR* also in this case.

The proposed method has been validated both in simulation, by comparison with the case studies discussed in [11], and experimentally, by considering conformity of the amplitude of a sinusoidal signal to given tolerance limits for its variability.

The obtained results proved that the method can be usefully employed.

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