Keplerian map in the restricted N-body problem

Lorenzo Giudici[(1)], Camilla Colombo[(1)]

(1) Politecnico di Milano, Department of Aerospace Science and Technology, Via La Masa 34, 20156 Milano, Italy

lorenzo1.giudici@polimi.it; camilla.colombo@polimi.it

Abstract: The concept of Keplerian map was firstly introduced by Petrosky and Broucke [1] and by Chirikov and Vecheslavov [2], who had the idea of applying the perturbation techniques to the circular restricted three-body problem. In astrodynamics, the theory was mainly applied to the modelling of distant flybys [3][4][5], for which semi-analytical equations were derived by applying the first Picard iteration to the Gauss planetary equations. The use of the theory for such a narrow range of application was induced by a strong dependency of its accuracy on the distance between the mass-less particle and the disturbing body [4].

This paper aims at extending the field of application of the theory. The disturbing potential of the Keplerian map is here computed as the scalar function, whose gradient equals the differential acceleration with respect to the reference Keplerian motion, as only conservative forces are involved. This alternative derivation avoids any assumption on the mass-parameter of the system, improving the accuracy of the method. The perturbing term is differentiated with respect to the Keplerian elements. In this context, it is demonstrated the need of using time, and not anomaly, as independent variable, coherently with the Lagrangian brackets derivation; the resulting additional terms, obtained through derivative chain rule, according to the non-linear relation true anomaly – time, lead to major improvements on the model accuracy.

The accuracy and efficiency of the model are verified in two very different scenarios. Firstly, the method is adopted for computing the variation of the Keplerian elements of a near-Earth asteroid caused by Earth's gravity field. The initial conditions are properly tuned to highlight any dependence of the accuracy on the distance asteroid – Earth. Close encounters well-inside the Earth's sphere of influence are also considered in the validation. Secondly, the proposed model is tested in the many-body scenario, propagating the orbital elements of the JUICE spacecraft under the simultaneous gravitational attraction of Jupiter and all the Galilean moons.

A mission design based on the Keplerian map is eventually proposed. It aims at optimising a low-thrust trajectory for a CubeSat leaving a Halo orbit, around the Sun-Earth Lagrangian point L_2, along an unstable manifold, and targeting a near-Earth asteroid. In the optimisation process the Keplerian map is extensively used; in particular, a direct transcription method based on the Keplerian map theory is adopted, to transform the continuous optimal control problem into a non-linear programming problem.

Keywords: Keplerian map; N-body problem; Disturbing potential; Trajectory design.

References:

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