



## Rapid assessment of seismic vulnerability of historic masonry structures through fragility curves approach and national database data

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### ABSTRACT

The research focuses on shared built heritage, which forms one of the most vulnerable parts of the building stock in historic centres. To prevent severe damage to this building category and to aid decision-making, the paper presents a rapid seismic vulnerability assessment procedure based on an engineering approach to safety assessment. The construction of fragility curves is developed on a probabilistic framework on the computed set of Safety Factors over a range of considered Peak-Ground-Accelerations. Input data necessary for the computations are extracted from CARTIS database. The methodology is implemented in a spreadsheet combined with a script in Visual Basic for Applications. Two case studies are used to demonstrate the applicability to a single building and at the territorial scale. Results show that the proposed methodology allows for a rapid testing of loss of structural performance given various scenarios, as well as contributing by prioritizing interventions in probabilistic terms.

### 1. Introduction

An important objective for small and medium-sized Italian towns is to avoid the depopulation of historic centres, in search of new, higher-performance buildings that are also seismically qualified. The assessment of the seismic vulnerability of existing masonry buildings as yet undamaged by an earthquake is an important issue to carry out proper prevention and to manage the possible safety of historic residential buildings throughout Italy. In particular in the case of shared built heritage, needs in the first instance, to be followed a procedure leading to a large-scale vulnerability classification, which should necessarily be simple and reliable. Furthermore, such a procedure should be able to provide elements in support of this possible classification, starting from data that can be easily found in the municipality archives and in filled template such as the CARTIS survey forms presented in (Zuccaro et al., 2015), without necessarily requiring additional data from in-situ surveys.

The seismic vulnerability of ordinary unreinforced masonry buildings is widely studied by the Italian scholars, but they often start from the post-earthquake assessment, as in (Sisti et al., 2019; Karantoni et al., 2014; Zuccaro et al., 2021; Rosti et al., 2018). The proposed models are often very elaborate and require detailed knowledge of the structure and significant processing times, (Saloustrous et al., 2015; Angjeliu et al.,

2020; Cusano et al., 2021; Montanino et al., 2022).

The aim of this research is to individuate a procedure to assess the seismic vulnerability of existing masonry residential buildings, which is reliable and can be easily extended on a territorial scale. The procedure may serve public administrations as a tool on which to base maintenance planning, with in a short timeframe. In fact, being able to assess in advance, with respect to the expected seismic event, the weaknesses of certain structures and their vulnerability to possible seismic actions would make it possible to: a) quantify the value exposed, b) inform citizens and owners of the extent of the potential risk, c) build a collective awareness of the need to secure the built environment, d) establish priorities for in-depth vulnerability investigations and for planning future interventions.

For this purpose, elements of deterministic methods already present in the literature are adopted and extended in the probabilistic framework. Focusing on two methods, which seem suitable for both single buildings and urban contexts:

- 1) The approach presented in (Borri et al., 2014, 2015; Borri and De Maria, 2016) proposes three types of simplified computations: simplified static assessment, simplified global horizontal load assessment, and simplified local mechanism assessment. The results of the three seismic evaluations demand imposed by the Italian Code

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(Italian, 2018), for the limit state of safeguarding human life (SLV) referred to the site under consideration, to the building type considered and to the class of use. This method is applied to each individual building and provides a safety factor ( $SF_{VG}$ ).

- 2) The approach proposed in Benedetti and Petrini (Benedetti and Petrini (1984) is based on the classification of existing buildings by means of a vulnerability index, which correlates the values of a damage parameter with the seismic input. The method provides a statistical estimate of the damage level in a considered range of accelerations.

In order to obtain a probabilistic prediction of damage as the acceleration changes, the first approaches have been implemented here and extended with the construction of fragility curves that correlate the decrease in the safety factor with the variation in recorded acceleration.

As a first application, the proposed method is applied to two case studies representing simple and widespread masonry residential buildings:

- a) a small historical residential building that represents the typical construction typology of small historic centres in the Apennine heights of central Italy in order to evaluate the propensity to damage before the 1997 earthquake and before the 2016 earthquake.
- b) the research was extended to the probabilistic assessment of the vulnerability of a medium-sized built-up area in Lombardy region, such as Desio (province of Monza and Brianza). It has recently been subject to an increase in its national seismic class and has never suffered seismic damage before.

The fragility curves, implemented here to evaluate the decrease in the factor of safety for the considered PGA interval, will be compared with a more established approach such as the vulnerability curves proposed in (Benedetti and Petrini, 1984) in order to verify their alignment.

## 2. Seismic vulnerability assessment of historic masonry residential buildings

### 2.1. Assessing the seismic vulnerability in historic centres

The Italian territory is a fragile territory, due to both the country's conformation and its hydrogeological and seismic character, as well as to a widespread presence of built heritage that requires urgent structural inspections and the execution of rapid safety measures where necessary. A problem that generally afflicts many small Italian urban centres is also the condition in which historic masonry residential buildings are to be found, often without regular maintenance, and where local intervention is carried out without proper design and without permissions, not being up to standard and therefore unsafe. This has led to a gradual shift of the population away from such dwellings in favour of newer and more high-performance houses in the suburbs. While by allowing people to continue to live in the historic dwellings in the city centre under safety conditions would have the advantage of constantly monitoring them and ensuring their regular maintenance, thus increasing their, sometimes centuries-old, life span. Historic dwellings have very often proved to be flexible and able to adapt to all modern requirements, including that of improving their energy efficiency (Cardani et al., 2022) while retaining the charm of local history and building traditions. It is therefore essential to find strategies for each municipality to adopt in this respect, preventing historic centres from losing their primary function, which is residential.

A method is required for municipalities, to be applied simply and clearly, to assess the risk level of historic dwellings and to identify buildings or building typologies that are more at risk from earthquakes than others. The latter, also because of the risk they may present to other neighbouring buildings and, therefore, to the inhabitants, as well as to the very same asset of historical-cultural value. The identified buildings

must subsequently be subjected to a more accurate assessment by professional engineers, according to the priority list identified by the method (Cardani, 2020).

The choice was to explore data from an already available database which was used a starting point: the structural and seismic characterisation database, called CARTIS (Zuccaro et al., 2015). CARTIS is an inventory of building typologies financed by the Civil Protection Department and implemented by the Network of Italian University Laboratories of Earthquake Engineering (ReLUI) for seismic vulnerability assessment at a territorial scale. This database collects geometric-technical information on existing residential buildings in over 500 municipalities and continues to grow. Starting from a direct and punctual reading of the buildings (CARTIS EDIFICIO sheet), it then reports data in percentage on the different types of residential buildings that characterise the analysed historic centre.

### 2.2. The selected damage parameter: safety factor

In order to perform a rapid assessment of the residual capacity of a building in a seismic zone, it is necessary to construct a parameter that is sufficiently reliable and quantifiable on the basis of data available from shared databases. In (Angjeliu et al., 2022) the authors propose the damage parameter,  $SF$  (Safety Factor), as the minimum between two parameters defined as static safety factor  $SFS$  and global safety factor  $SFG$ . These two factors are quantifiable on the basis of geometric data of the structure, which can be found in cadastral and municipal databases, on the strength characteristics of the masonry, which can be quantified either empirically with a masonry quality analysis (Italian, 2018), or with non-invasive or moderately invasive instrumental tests (Binda and Tiraboschi, 1999; Binda et al., 2000; Binda and Cardani, 2015) and then associated with the local seismicity.

The safety factors  $SFS$  and  $SFG$  are calculated for each floor (in the following,  $SFS_i$ ,  $SFG_i$ ), while the final safety factor,  $SF$ , for the whole structure is evaluated in a conservative way as the minimum between all the computed values.

$$SF = \min[SFS_i; SFG_i] \quad [1]$$

### 2.3. Static safety factor

The static safety factor,  $SFS$ , depends on the gravity loads, the mechanical characteristics of the materials and on the confidence factor,  $CF$ , introduced in the Italian code NTC2018 (Borri et al., 2015), to establish the degree of building knowledge then taken into account. The greater the degree of building knowledge, the lower the value of the confidence factor,  $CF$ . For example, for historic buildings where it is difficult to find the original design project material data and often it is also difficult to proceed with diagnostic and survey campaigns, the confidence factor is imposed to 1.35, while for more modern buildings for which both complete project documentation and laboratory tests on the quality of materials are available, the expected confidence factor is 1.00.

The structure of the static safety factor, expressed as a percentage, is:

$$SFS_i = \left( \frac{f_m \cdot f}{\gamma_m \cdot CF} \right) \cdot \left( \frac{0.65}{\sigma_{0i}} \right) \cdot 100 \quad [2]$$

where:

- $f_m$  – the average compressive strength of masonry,
- $f$  – the reduction factor for existing cracking,
- $\gamma_m$  – the density of masonry,
- $CF$  – the confidence factor,
- $\sigma_{0i}$  – the level of average vertical stress at the  $i$ -th floor.

### 2.4. Global safety factor

The global safety factor,  $SFG$ , depends on the expected site peak

ground acceleration, PGA, the site characteristics, and the horizontal load capacity which is represented through the equivalent corresponding acceleration at each floor,  $a_{ULSi}$ :

$$SFG_i = \left( \frac{a_{ULSi}}{PGA * S} \right) * 100 \quad [3]$$

where:

$S$  – is the soil and terrain amplification factor.

Also, in this case the safety factor is expressed as a percentage. The acceleration  $a_{ULSi}$  which depends on the structural characteristics of the building is evaluated as:

$$a_{ULSi} = \frac{r * q * F_{ULSi}}{e^* * M * F_0} \quad [4]$$

where:

$r$  – is a corrective coefficient taking into account the building typology (1 for structures with one floor above ground, 2 for more than one floors above ground).

$q$  – is the behaviour factor taking into account the dissipative characteristics of the structure.

$F_{ULSi}$  – the ultimate limit state shear strength at the  $i$ -th floor, evaluated as the minimum value between the two main directions,  $x$  and  $y$ .

$e^*$  – is the mass participation factor,

$M$  – is the seismic mass of the structure.

$F_0$  – spectral amplification.

The shear strength  $F_{ULSi}$  in each of the main directions  $x$ , is evaluated as:

$$F_{ULSi} = \frac{0.8 * \xi_x * A_{xi} * \tau_{di} * 10000}{1.25} \quad [5]$$

where:

$\xi_x$  – is the factor describing the main failure mechanism (shear or flexure),

$A_{xi}$  – the area of the resisting element in the direction  $x$ ,

$\tau_{di}$  – the shear strength of the walls in the direction  $x$ , at the  $i$ -th floor.

The same considerations can be made also for the other direction,  $y$ .

The final global shear strength is evaluated as the minimum between the two directions,  $x$  and  $y$ :

$$F_{USLi} = \min[F_{USLi}; F_{USLy}] \quad [6]$$

### 3. Methodology

#### 3.1. Introduction to fragility curves

The vulnerability assessment on a territorial scale correlated to certain building typologies presents several uncertainties some of which are independent of the selected damage parameter. Hence, it is essential to approach this study from a probabilistic point of view. Whatever is the selected probabilistic tool, the problem is the sample size. A probabilistic forecast is reliable if the sample is large enough to be able to represent a “statistical truth”. Unfortunately, this is difficult to achieve. In alternative, simulation methodologies are adopted to obtain a dataset that can describe the experimental reality. However, it is important to be familiar with the phenomenon to be modelled in order to choose a modelling method that best describes its probable evolution over time.

With the aim of developing a reliable but relatively easy-to-use method, in this research we propose the application of fragility curves for modelling the variation of the safety factor  $SF$  as the induced action varies. The method guides in the development of curves that support a probabilistic forecast of the occurrence of the studied phenomenon when a considered condition changes, such as: the probability of reaching a damage threshold when the seismic input varies. Furthermore, it is necessary to note that as a probabilistic method it suffers from epistemic uncertainties, modelling, and sample size, as well as the

randomness of the uncertainty.

#### 3.2. Fragility curves

The construction of the fragility curves requires the identification of the damage parameter, which behaviour is to be observed when the imposed action, or the time of exposure to environmental aggressions varies. For instance: the loss of surface material as the number of aggressive cycles varies, the decrease of the resistant section as the service lifetime increases, the decrease of the safety factor as the severity of a catastrophic event increases.

Having established different actions: load values, or damage detection times, it is necessary to proceed with the quantification of the damage parameter for each given action on buildings of the same typology. To clarify, in the case of seismic action for different levels of shaking,  $a^*$ , it is possible to proceed with the evaluation of the safety parameter on several buildings of the same typology. The results obtained will represent, in a deterministic form, the response of the parameter to that given action.

The response obtained for each imposed stress will certainly suffer from uncertainty, therefore the damage parameter will show different values with greater or lesser variation. For each imposed stress, this uncertainty leads to modelling the behaviour of the damage parameter in probabilistic terms (Fig. 1).

Probabilistic modelling requires the adoption of a probability density function (p.d.f.),  $f(x)$ , whose mathematical form can correctly interpret the physics of the phenomenon studied. In probabilistic modelling this is a delicate issue, since the choice must derive from a correct knowledge of the investigated phenomenon and its experimentally observed evolutionary phases, as well as from the knowledge of the mathematical formulation of the different probability distributions and their hazard rate function.

In the present paper the choice of distributions was based on elements reported in the literature (Garavaglia et al., 1996). In the following case studies, each choice will be further motivated.

The fragility curves describe the probability of reaching and exceeding a certain damage threshold as the induced action varies. The construction of the fragility curves will be based on the construction of the p.d.f. on each of the imposed action values (Fig. 1b). A damage threshold is established and identified on the abscissa of the constructed p.d.f. The area to the left of the threshold, underlying the p.d.f. measures the probability of reaching this damage threshold (probability that the damage is less than or at most equal to the threshold value) (Fig. 2a dashed area):

The area underlying the p.d.f. but to the right of the threshold, measures the probability of exceeding this threshold (probability that the damage is greater than the threshold value) (Fig. 2a coloured area).

The value of the area underlying the p.d.f. in the interval  $(-\infty; \text{threshold}]$  is the integral of the p.d.f. function itself in that interval and coincides with the value of the cumulative distribution function (C.D.F),  $F(x)$ , in the threshold value.

$$F(\bar{s}f, a^*) = \int_{-\infty}^{\bar{s}f} f(SF, a^*) \quad [7]$$

where:

- $SF$  is the damage index variable that can assume random values as the action varies,
- $a^*$  are the values that the considered action can assume,
- $\bar{s}f$  is the threshold value whose probability of attainment is to be predicted when varying  $a^*$ .

On the other hand, the value of the area underlying the p.d.f. in the interval  $(\text{threshold}; \infty)$  is the integral of the function itself in this second interval and coincides with the value of  $[1-F(x)]$  in the threshold value.

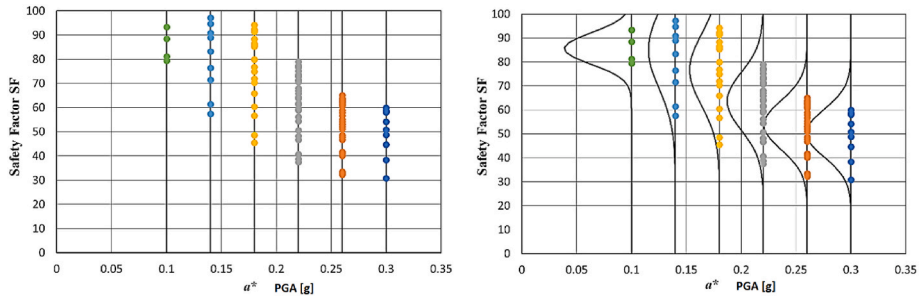


Fig. 1. Safety factors computed for buildings of the same typology within a range of considered accelerations, a) computed SF values b) probabilistic modelling of the previously computed values.

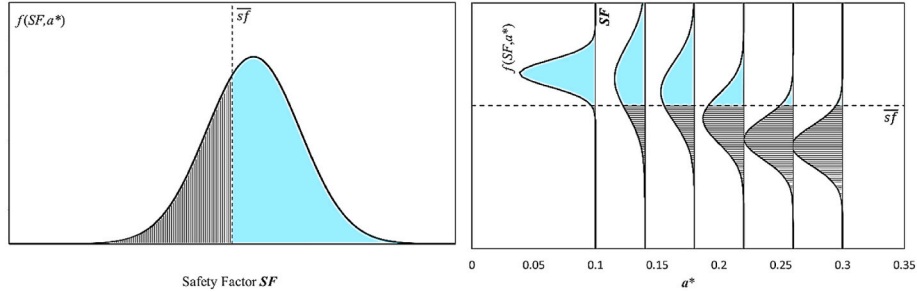


Fig. 2. Probability of reaching and exceeding the threshold  $\bar{s}f$ : in dashed - the probability of reaching  $\bar{s}f$ ; coloured - the probability of exceeding the threshold  $\bar{s}f$ : (a) p.d.f. for a value of  $a^*$ ; (b) p.d.f. for several values of  $a^*$ .

This function is called the survival function;

$$\mathfrak{S}(\bar{s}f, a^*) = [1 - F(\bar{s}f, a^*)] = \int_{\bar{s}f}^{\infty} f(SF, a^*) \quad [8]$$

Fig. 2b shows the areas described by eq. (8), relative to the threshold  $\bar{s}f$  and for different values of  $a^*$ . The p.d.f. and C.D.F. described in this section represent the basis for the construction of the experimental fragility curves.

By constructing the probability density for the chosen random variable,  $SF$ , and for each of the selected acceleration intervals (Fig. 2b), it is clear to understand how it is possible to construct the fragility curve linked to the experimental evidence, also named experimental fragility curve,  $F_{\bar{A}}(a^*)$ .

From eq (Angeliu et al., 2020). it is possible to obtain for each value of  $a^*$  and for each threshold  $\bar{s}f$  the experimental probability.

$$F(\bar{s}f, a^*) = Pr\{SF \leq \bar{s}f\} \quad [9]$$

which describes the probability of achieving  $\bar{s}f$  for a given value of  $a^*$ .

From eq (Cusano et al., 2021). it is possible to obtain the experimental probability:

$$\mathfrak{S}(\bar{s}f, a^*) = Pr\{SF > \bar{s}f\} = 1 - F(\bar{s}f, a^*) \quad [10]$$

which describes the probability of exceeding  $\bar{s}f$  at a given value of  $a^*$ .

The areas calculated on different thresholds  $\bar{s}f$  provide the experimental fragility curves,  $F_{\bar{A}}(a^*)$ , for each of the established performance thresholds.

Fig. 3a shows the shape of an experimental fragility curve for a given threshold  $\bar{s}f$ . Each point of the diagram, corresponding to a certain value of  $a^*$ , represents the value of the dashed area in Fig. 2b in the same value of  $a^*$ .

The modelling of the experimental fragility curve with an appropriate theoretical probability distribution will allow the construction of the theoretical fragility curve able to describe the probability of reaching or exceeding a certain damage threshold as the imposed action varies (Fig. 3b).

The area above the threshold  $\bar{s}f$  is easily calculated using the survival function,

$$\mathfrak{S}_{sf}(SF, a^*) = Pr\{sf > SF\} = 1 - F_{sf}(SF, a^*) \quad [11]$$

where  $F_{sf}(SF, a^*)$  the cumulative distribution of  $sf$  at each acceleration  $a^*$ . It describes the probability that,  $sf$ , takes values greater than  $SF$ ,

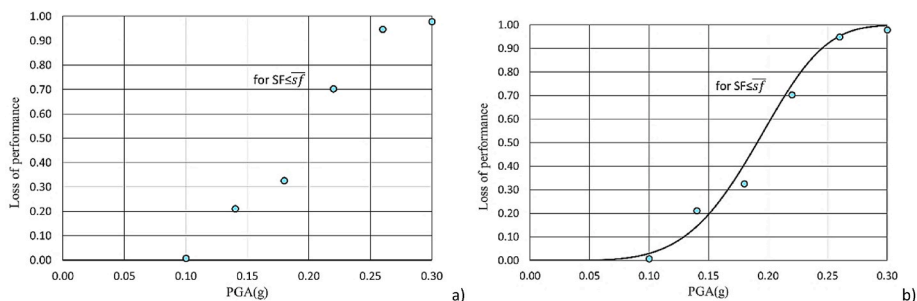


Fig. 3. Fragility curves: a) experimental fragility curve considering a threshold  $SF < \bar{s}f$ , b) theoretical modelling of the experimental fragility curve.

herein  $SF = \bar{s}f$ . The cumulative distribution function  $F_{sf}(SF, a^*) = \Pr\{sf \leq SF\}$  is computed as the area below the threshold  $\bar{s}f$ , which describes the probability that  $sf$  can assume values not exceeding  $SF$ , in our case  $SF = \bar{s}f$ .

The computation of  $F_{sf}(SF, a^*)$ , similar to the complementary  $\bar{F}_{sf}(SF, a^*)$ , is carried out by numerical integration of the probability density function  $f_{sf}(SF, a^*)$  in the intervals  $(-\infty; \bar{s}f]$  and  $(\bar{s}f; +\infty)$ . The areas calculated for each of the thresholds  $\bar{s}f$  give the experimental fragility curves,  $F_A(a^*)$ , for each of the established performance thresholds.

#### 4. Application in a single building

The validation of the proposed methodology is carried out by way of a small case study located in Campi Alto di Norcia, in the Province of Perugia, Umbria region (central Italy). The case study is ideal, given its simple structural shape as well as its documented state of damage and repairs after two earthquake events in 1997 and 2016.

##### 4.1. Description of the case study in Campi Alto di Norcia

The masonry building consists of a simple structural unit of about 60 m<sup>2</sup> on each floor. The structure is typical of constructions in sloping terrains where among the three storeys, the ground floor is built into the

ground on three sides while the first floor is built in to the ground only on one side and positioned below the back street level. The ground has been used as a basements and the other floors as a residence (Fig. 4). The building consists of a load-bearing masonry made in compact limestone. According to the definitions in the Italian Design Code (Italian, 2018), the masonry can be classified with a “roughly cut stone masonry (even irregularly shaped) with good texture”. A barrel vault is used on the ground floor which ends up against the rock, while the upper floor and the roof are made of timber structure.

The building once appeared to be part of a series of houses built in the steep terrain of Campi Alto; no longer standing since having collapsed in the past. For that reason, two buttresses of different size are still present in the South-East corner (Fig. 5), while the other corner of the facade features a good interlocking among the stones and also between the orthogonal walls. The buttresses were built probably to stabilize a wall, previously part of an adjacent structural unit, which is no longer present. Before the last restoration, the building presented iron tie rods in a mountain-valley direction and one parallel to the facade.

In 1997, Campi Alto di Norcia, as well as other towns located in Umbria region, was hit by a major shock of magnitude,  $M_w = 5.97$ , with a  $PGA = 0.2275$  g. After the 1997 earthquake the building was greatly damaged as the roof and parts of the upper walls were collapsed. Hence, the 1997 earthquake caused considerable damage to this structure (Fig. 5a).

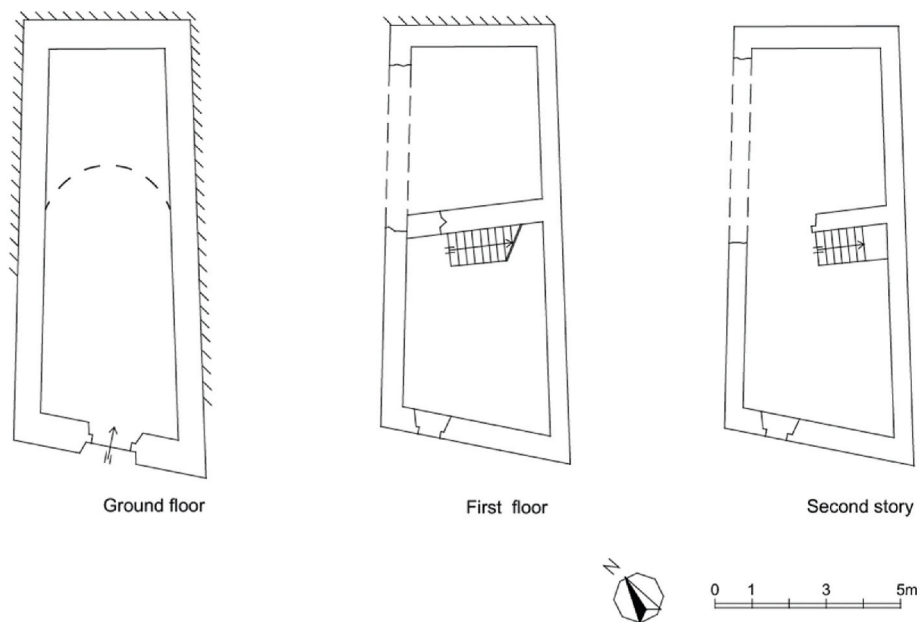


Fig. 4. Plans of the analysed historic masonry building in Campi Alto di Norcia (dashed lines for collapsed masonry portions after 1997 earthquake).



Fig. 5. Photos of the case study in Campi Alto di Norcia: a) after the 1997 earthquake; b) after 2000s restoration; c) and d) after the 2016 earthquake: overturning out-of-plane of the ground floor facade and rigid rotation of the renovated upper volume.

Around years 2000s, the building was repaired, seismic strengthened and inhabited again (Fig. 5b).

In October 2016 Campi Alto suffered a new strong earthquake with magnitude  $M_w = 6.61$  and  $PGA = 0.3025$  g, which affected a large part of central Italy. The structure object of the present study again suffered major damage despite the seismic strengthening of early 2000 (Fig. 5c and d), highlighting the vulnerabilities of the intervention (the addition of new brickwork in the two upper floors and the addition of reinforced concrete floors) but also the severity of the earthquake. The renovated part above the barrel-vaulted ground floor rotated rigidly, with the resulting effect of destroying both ground corners of the façade and the overturning out-of-plane of the lower part of the façade, corresponding to the ground floor only.

#### 4.2. The evaluation of the safety factor

The safety factor SF which is assumed as a damage parameter is computed based on the procedure explained in section 2. The input data for the case study are obtained from the 1997 post-earthquake damage survey datasheets (Cardani, 2004), the 2000 strengthening intervention project, visual surveys and data on the history of the building.

In Campi Alto di Norcia, the analysed subsoil is characterised by rocky or other rock-like geological formations hence according to the Italian Design Code (NTC, 2018) the site is classified as Class A (Italian, 2018). The area has slopes with an inclination between  $15^\circ$  and  $30^\circ$ , consequently, the site must be considered in topographic class T3 according to NTC 2018. Finally, the area is characterized by high seismicity, seismic zone 1, with a  $PGA > 0.25$  g for the probability of exceedance 10% in 50 years and maximum peak ground acceleration a (NTC2018) of  $PGA > 0.35$  g.

Clearly the evaluated Safety Factor, SF, is dependent on the expected site acceleration, PGA, and on the spectral amplification,  $F_0$ , characteristic of the area under examination. In the construction of a credible set of accelerations each associated with a probable amplification factor ( $PGA, F_0$ ) we started from the values established by the Italian law (Borri et al., 2015). Each couple ( $PGA, F_0$ ) was computed based on the series of the return periods 30-50-72-101-140-201-475-975 years and completing with linear interpolation the intervals between a return period and the other, as to as to have continuity. Thus, computing a set of values for  $F_0$  from 8 to 55 and the respective range of accelerations in the range from 0.02 g to 0.327 g.

#### 4.3. Fragility curves

The aim is to investigate the rapid assessment method to evaluate the loss of performance connected with recorded levels of PGA, and hence calibrate the method with reliable results. Two simulations were carried out on the first case study: a) to predict the propensity to damage of the building object of study during the 1997 earthquake, with the floor and wooden roof still present and with iron tie-rods; b) a second

investigation will then be carried out on the same building repaired after 1997 (with the insertion of new brickwork in the two upper floors and the addition of reinforced concrete floors), matching the results with the observed damaged after the 2016 earthquake.

Based on the procedure described in section 3, the experimental (ExFC) and theoretical (ThFC) fragility curves were constructed for the building object of study describing the probability of passing of certain thresholds of loss of the SF safety factor as a function of the varying PGA (Fig. 6, Table 1). The accelerations are chosen in the interval 0.02–0.327 g as representative of the analysed site.

The construction of the experimental fragility curve is obtained following the method explained in section 3 using the cumulative distribution function  $F_s(SF, a^*)$  and then modelled with a Gamma distribution. The choice of the Gamma distribution is derived from the analysis of the physical phenomenon studied. As the intensity of the earthquake increases, the probability of an imminent collapse certainly increases, therefore this tendency requires modelling with a probability function with an increasing hazard rate. The Gamma function has an increasing hazard rate tending towards an asymptotic value which seems to well describe this propensity.

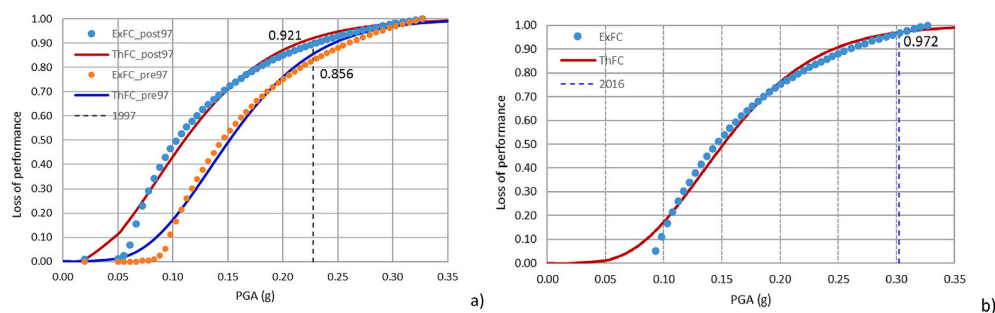
The prediction of the collapse of the structure in the considered acceleration range was evaluated for the 1997 earthquake (see the plot in red in Fig. 6a), and for the 2016 earthquake with seismic strengthening completed (Fig. 6b). The experimental ExFC and theoretical ThFC fragility curves for the two cases are computed for a probability of collapse measured in loss of performance close to 75%. The analysis shows that even before the 1997 event, the structure had a high predisposition to damage for events with accelerations greater than 0.2 g (see the plot in blue in Fig. 6a), which further deteriorates in the post-earthquake situation (see the plot in red in Fig. 6a). After the seismic strengthening which restored the collapsed parts, the computed safety factor SF (based on the inputs that we use in our fast seismic vulnerability assessment) results equal to the initial situation (Fig. 6b). Hence, showing a still rather high propensity to damage, so much that for acceleration values recorded in 2016, collapse is almost certain (as indeed happened). It is further clarified that given that the aim is to perform a fast seismic vulnerability assessment, at a methodological level (for the restored case) there is no distinction between rigid and flexible floors.

In addition to the collapse prediction, the fragility curves were also studied as the transition probability for a given threshold of values for the safety factor SF. The methodology used is the one developed by

**Table 1**

Parameter for curves in Fig. 6.

	ThFC pre 1997 state (plotted in blue in Fig. 6)	ThFC pre 1997 state (plotted in red in Fig. 6)
Alfa parameter	6.2088	3.2385
Beta parameter	0.0258	0.0379



**Fig. 6.** Residential building in Campi Alto di Norcia: fragility curve of the probability of reaching a value  $SF \leq \bar{s}f$  (for  $\bar{s}f = 26.68\%$ , Soil Category A, Topography T3). a) in blue the ThFC before the earthquake of 1997, in red the ThFC after the earthquake of 1997 (structure damaged); b) the ExFC and ThFC before the earthquake of 2016 (structure seismically improved). (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

Garavaglia et al. (2008). In this case, for modelling of  $SF \leq \bar{s}f$ , with a threshold value  $\bar{s}f$  between 20% and 70%, recorded at various accelerations (Sect. 3, Fig. 3) was chosen a normal distribution. The experimental fragility curves are constructed following the procedure presented in section 3, while modelling of these curves has been carried out with a Weibull distribution function. To describe the transition probability of a performance threshold as acceleration increases, it is judged here more appropriate to model the immediate transition risk with an increasing hazard rate function but faster than an asymptotic function. In fact, as acceleration increases, crossing the threshold is practically a safe event. The choice of a Weibull distribution follows precisely in this sense; it has an increasing hazard rate, but with an asymptote at infinity.

In Fig. 7 (Table 2) the experimental fragility curves defined on 12 intervals of PGA between 0.06 g and 0.30 g are reported. These curves define the probability of reaching a value, SF, less or equal to the assumed threshold  $\bar{s}f$ . The fragility curves were built for values of possible loss,  $\bar{s}f$ , between 20% and 80% for the two scenarios introduced above. The safety factors, SF, recorded for each step of the interval are modelled with a normal probability distribution.

From the two scenarios it emerges once again how the probability of loss of the SF results almost the same in the 1997 pre-event scenario and in the 2016 pre-event scenario for the same reasons already stated above. In the case of the 1997 seismic event the probability of expected loss is around 0.6 to reach a damage threshold  $\bar{s}f = 60\%$ . The maximum expected loss is for  $\bar{s}f = 70\%$  and for very severe accelerations close to 0.3, corresponding to the event of 2016. Finally, the results confirm that in both scenarios, the probabilities of loss of performance are severe, showing high values of percentages of loss.

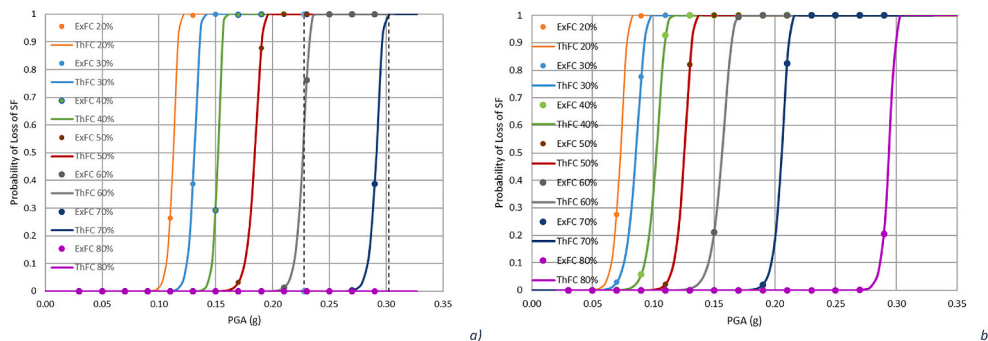


Fig. 7. Fragility curves (experimental with dots and theoretical modelling with lines) for the residential historic building in Campi Alto, describing the probability of loss of the SF at varying accelerations analysing the 1997 seismic event and the seismic event of 2016, b) after the seismic event of 1997.

Table 2  
Parameter for curves in Fig. 7.

Parameters for Fig. 7a							
ThFC	20%	30%	40%	50%	60%	70%	
Alfa parameter	33.393	35.913	51.707	37.314	51.957	69.932	
Beta parameter	0.114	0.133	0.153	0.186	0.228	0.293	
Parameters for Fig. 7b							
ThFC	20%	30%	40%	50%	60%	70%	80%
Alfa parameter	17.490	15.421	18.770	25.814	25.083	44.230	73.191
Beta parameter	0.075	0.088	0.105	0.127	0.159	0.207	0.289

#### 4.4. Comparison with the vulnerability index

The method proposed in (Benedetti and Petrini, 1984; Guagenti and Petrini, 1989) is applied to the single masonry building. The vulnerability index is computed as a weighted sum of the numerical values expressing the ‘seismic quality’ of elements (structural and non-structural) which have a significant role in the seismic response of the building.

The obtained vulnerability index VI is calculated in three instants of time:

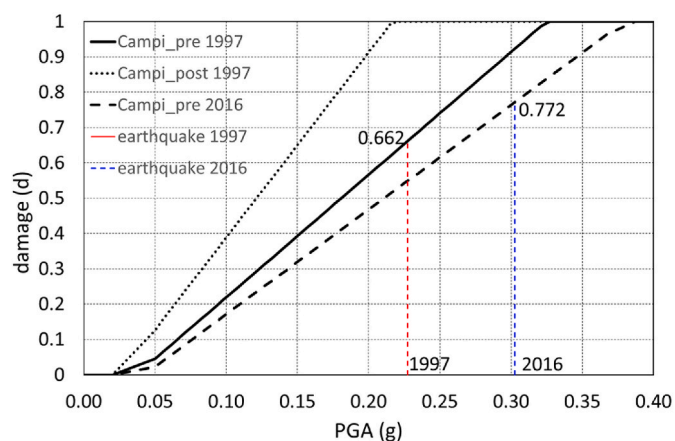
- Before 1997 earthquake,
- After the 1997 earthquake,
- Before the 2016 earthquake.

The computed vulnerability index VI parameters are reported in Table 3.

These data are reported in Fig. 8 in terms of two trilinear plots, expressing the damage in function of recorded PGA. About the situation

Table 3  
Vulnerability Index V.I. global parameter.

	Damage onset PGA	Collapse PGA	Vulnerability index
	$y_i$	$Y_c$	V.I.
Before the 1997 earthquake	0.037	0.325	33.99
After the 1997 earthquake	0.026	0.217	61.44
Before the 2016 earthquake	0.0423	0.379	23.85



**Fig. 8.** Building in Campi Alto di Norcia: damage as function of PGA before 1997, after 1997 and before 2016 shock, following the approach presented in (Benedetti and Petrini, 1984; Guagenti and Petrini, 1989). Red line (left side): expected damage for acceleration values equal to the shock in 1997. Blue line (right side): expected damage for acceleration value equal to that of 2016 event. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

before the 1997 earthquake, is computed an early damage acceleration of 0.037 g and a collapse acceleration of 0.325 g with a vulnerability index  $V.I. = 33.99$ .

After the 1997 seismic event the damage suffered by the structure increased the vulnerability (dotted line). The  $V.I.$  reached the value of 61.44 with an early damage acceleration of 0.026 g and a collapse acceleration of 0.217 g.

The strengthening interventions carried out in 2000s (dashed line in Fig. 8), show an improvement of the vulnerability index of the building by increasing the collapse acceleration to 0.379 ( $V.I. = 23.85$ ).

Compared to the acceleration recorded during 1997 earthquake,  $a_g = 0.2275$  g, it is predicted a damage close to 0.6. Considering the seismic event of 2016,  $a_g = 0.30256$  g, the expected damage remains high despite all the improvements around 2000s (Fig. 8). Nevertheless, if comparing the two graphs, it can be observed that without the improvements, the building would have suffered a damage close to 0.9.

Concerning the damaged structure, Fig. 8 shows a very similar behaviour recorded in Fig. 6a, the collapse would be reached for an acceleration close to 0.22.

The calculated vulnerability of the building object of study before the 1997 and before the 2016 earthquakes matches the damage observations. In particular, during 1997 event a localized damage was observed on the top of the building despite the lack of maintenance, however the loss in performance was serious. In fact, during the 2016 shock, near collapse consideration were observed.

The results show that the strengthening performed has improved the vulnerability index and lowered the probability of damage, but due to the strong intensity of the 2016 shock, a propensity to damage close to 0.78 is still computed (Fig. 8). However, regarding the interventions carried out, although with an improving aim, issues of incompatible with the type and materials of the original construction can be raised. This marks an important question on evaluating new strengthening techniques for the protection of built heritage.

## 5. Seismic vulnerability at the territorial scale

In this section the fragility curves are applied for the analysis of the seismic vulnerability of the residential buildings of a medium-sized town, which presents in its historical centre a typology of homogeneous buildings in terms of structure, height, and construction age. The aim is to show the flexibility of the method even on larger-scale investigation and always based on easily available datasets which, although

incomplete to face up to more refined analyses, can already provide reliable information on the possible structural response of the buildings in case of various seismic scenarios.

### 5.1. Description of the case study

The town of Desio, in the Lombardy region, is located in an area considered by Italian Design Code as a medium-low seismicity zone (NCT2018 seismic zone 3). According to the Italian Design Code, the acceleration values for a probability of exceeding 10% in 50 years for each seismic zone are defined as follows: a) zone 1:  $a_g > 0.25$  g, b) zone 2:  $0.15 > a_g > 0.25$ , c) zone 3:  $0.05 < a_g < 0.15$ , d)  $a_g < 0.05$ . The choice of this town is due to its change of seismic zone from the previous 4 (the lowest) to the zone 3. The building typology (Fig. 9) characterizing the historic center is widely diffused throughout the Lombardy region and therefore can be taken as an example that can also be extended to Lombard areas with higher seismicity such as the areas of Brescia or Garda Lake (seismic zone 2). In this regard, the structural behaviour of this building type was studied for acceleration values characteristic also of the Garda area (Table 4).

The historic center of the Municipality of Desio, in the province of Monza-Brianza, Northern Italy, is characterized by a very simple structural typology in brick masonry, with two floors and an average height between floors of 3 m (Fig. 9). This typology represents around 63% of the building stock, despite the size of the town (41'646 inhabitants) and its mainly industrial character.

The investigation of the seismic response in terms of global safety factor (SF) was based exclusively on data collected with the CARTIS form and on listed plans released by the Municipality of Desio.

The town of Desio recently passed from seismic zone 4 to seismic zone 3. The subsoil is of category C: medium-densified coarse-grained soils or medium-firm fine-grained soils, and topographic category T1:



**Fig. 9.** Town of Desio (MB): recurrent masonry building typology in Lombard urban context of the beginning of 20th century (in CARTIS database named MUR1 in the historic centre named CO1).

**Table 4**

Reference values of the return period for the region of Lombardy (moderate seismicity according to NCT2018 zone 2) used in the study for the definition of the action in the fragility curves.

Return Period (Years)	PGA(g)	$F_0$
30	0.042	2.551
50	0.057	2.483
475	0.158	2.483
975	0.206	2.485
2475	0.283	2.466



flat surface with slopes with an inclination of less than 15°. The geological and topographical parameters are  $S_S = 1.5$  and  $S_T = 1$ .

The choice of this municipality may seem inappropriate for the purposes of verifying the seismic vulnerability of the existing structures, but the procedure here presented intends to verify the seismic response (in terms of reduction of the safety factor, or loss of performance) of certain masonry structural typologies (Fig. 9). It is important to state that these typologies are often common in many areas of the Italian territory and therefore it may be interesting to detect, from a probabilistic point of view, the probable seismic behaviour in the considered range of PGA.

5.2. Construction of the theoretical and experimental fragility curves for single case studies

For the Municipality of Desio, ten buildings were studied, all homogeneous in terms of construction typology. The data necessary for the application of the proposed procedure were obtained from the CARTIS database, from the plans of the buildings made available by the Municipality. The data regarding the geology and seismicity of the territory were reported in the previous section 5.1.

Then, for each of the 10 buildings, the safety factor SF (the lowest between the static factor  $SF_{S_i}$  and global factor  $SF_{G_i}$ ) was computed for all the acceleration values as well as the consequent loss of performance in the structural response. Based on seismic parameters (see section 5.1) was chosen the acceleration ranges which values range from 0.06 g to 0.30 g with amplitude 0.02.

The next step was the construction of the fragility curves describing the probability of exceed selected thresholds of loss of the SF factor as the PGA varied (Fig. 10, Tables 5 and 6). The values of the safety factor, SF, computed for each interval are modelled with a normal p.d.f. (see Sec. 3). The construction of experimental fragility curves is obtained using the c.d.f.  $F(\bar{s}f, a^*)$  according to eq. (9). The experimental curves were then modelled with Weibull-type probability distributions, as introduced in 4.3.

The results obtained for the sample of 10 buildings were then used to construct the fragility curves of the investigated typology. As shown in

Table 5  
Parameter for curves in Fig. 10a.

ThFC	20%	30%	40%	50%
Alfa parameter	51.995	37.712	67.962	68.904
Beta parameter	0.188	0.217	0.252	0.296

Table 6  
Parameter for curves in Fig. 10b.

ThFC	20%	30%	40%	50%
Alfa parameter	15.956	12.981	11.888	11.434
Beta parameter	0.277	0.289	0.304	0.322

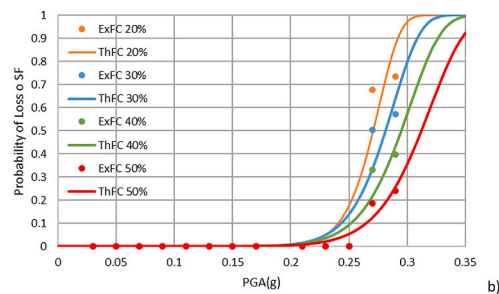
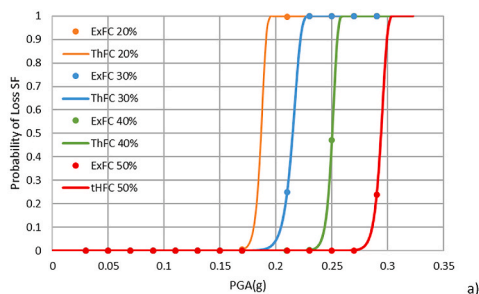


Fig. 10. Example of experimental (dots) and theoretical (lines) fragility curves constructed for 2 (a and b) of the 10 analysed buildings in Desio town (see parameters in Tables 5 and 6).

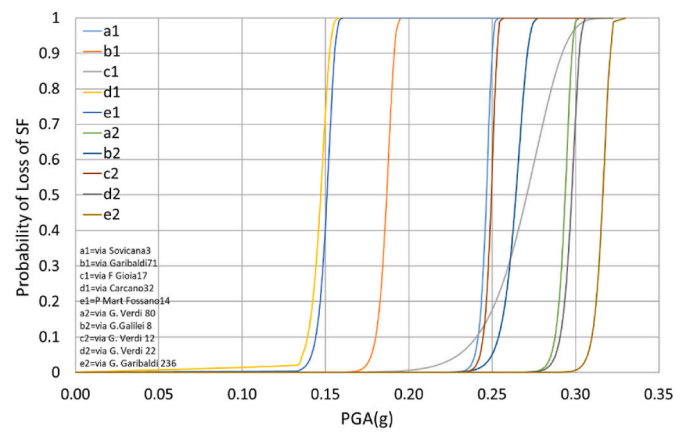


Fig. 11. Fragility curves construction for the full sample of 10 buildings in Desio town, analysed with a threshold  $\bar{s}f \leq 20\%$ .

Fig. 11 the probability to reach the threshold  $\bar{s}f = 20\%$  could happen for a PGA > 0.12, which is much greater than the seismic demand in Desio.

As far as the Desio area is concerned, which has an expected PGA of 0.059 g with a return period of 975 years, it is concluded that all the buildings studied can be considered a reliable typology in a medium-low intensity seismic area.

Considering the typology as a typology spread over almost all the Lombardy territory and evaluating it on a medium-high seismicity zone such as that of Garda (ag/g close to 0.21 for a return period of 975 years) we can see that some samples could show a probable loss of performance close to 30% but hardly higher than that as noted in Fig. 10, however still quite reliable for the Lombardy area.

5.3. Construction of the theoretical and experimental fragility curves for the complete sample

In this section, the data obtained in the previous section are assembled to compose a single statistical sample representative of the investigated masonry building typology, on which we proceed with the calculation of the fragility curves, this time representative of the entire sample (Fig. 12). The results in Fig. 12, show that the probability of loss of performance can reach values up to 30% for a threshold  $\bar{s}f \leq 30\%$  (blue line in Fig. 12) and PGA values around 0.20 g. The probability of exceeding a loss of performance threshold greater than  $\bar{s}f \leq 40\%$  (green line in Fig. 12) is low for PGA value below 0.30 g. This probability can become significant for PGA values greater than 0.35 g, but according to the seismic hazard map for the Lombard territory such values are not to be expected. Also, in this case the performance of this building typology results satisfactory in terms of probability of loss of the safety factor for an area with medium-low seismicity such as Desio as well as for an area with medium-high seismicity such as the Garda area. Further comments are made in the next section 5.4.

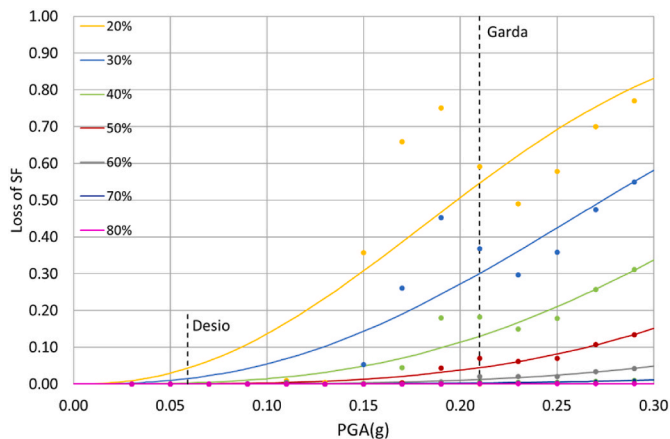


Fig. 12. Fragility curves built on the entire examined sample of Buildings in Desio town.

5.4. The vulnerability index

In this section, the consolidated approach proposed by Benedetti and Petrini (Benedetti and Petrini (1984) on the construction of the Vulnerability Index V.I. is used for comparison purposes. The previous data extracted from the CARTIS database are used also in this case. Following the V.I. procedure (Benedetti and Petrini, 1984), it was possible to construct the trilinear damage-PGA curves for each of the 10 buildings under consideration.

The trilinear curves correlating a possible level of damage in the considered range of PGA values show a rather homogeneous structural response for the examined masonry typology (Fig. 13). The result is closely linked to the V.I.s computed for these 10 buildings with an average value, 26.5, while only in one case the V.I. is noted to be particularly low, V.I. = 12.1 (Fig. 13a). In this case the building had undergone maintenance works which improved its seismic vulnerability and lowered the propensity for loss of performance.

Fig. 13b represents a single three linear plot created with a V.I. calculated as an average of each building V.I. index (see Fig. 13a). Furthermore, also the upper and lower bounds defined by the standard deviation are plotted (Fig. 13b).

From the fragility curves in Fig. 12 it can be noted that the examined masonry typology has sufficient requisites to guarantee a good response to seismic actions in low to moderate seismicity areas (0.05–0.15 PGA), while for areas with moderate to high seismicity (0.15–0.25 PGA), the probability to reach 50% of loss of performance is around 15%, which although still limited should be not neglected.

What is described by the fragility curves is only one aspect of the problem; in fact, even if most of the buildings belonging to a given typology will respond in a very similar way to the just observed behaviour, we should not disregard that each of them could present also other

important individual elements of seismic vulnerability, not known in this first general analysis based on CARTIS data. For a complete analysis of the structural response to horizontal forces it is necessary to combine the results of the fragility curves with the results of the expected damage starting from the value of the vulnerability index. Let's compare the results of Fig. 12 with the results of Fig. 13b for 2 levels of thresholds of loss of performance (20% and 50%).

Regarding the 50% of loss of performance: In Fig. 12 it can be observed that for the accelerations expected in Desio, the probability to reach that threshold is nearly null. This remains true also for higher values of PGA expected in the Lombardy region. In the trilinear curve of Fig. 13b, for the accelerations expected in Desio the expected damage is around 0.07. This value is still low, but higher than the value obtained with the fragility curves because this method considers the observed vulnerabilities of the buildings which are not included in the other method.

Regarding the 20% of loss of performance: in Fig. 12 for the acceleration characteristic of Desio, the probability to reach the threshold  $\bar{s}f \leq 20\%$  is estimated around 5%, instead for the Garda Lake area, is around 55%. In the trilinear curve of Fig. 13b, for a PGA of 0.06 of Desio, the expected damage is always 0.05, which although look similar compared to the previous value have a different significance. This is due to the fact that the trilinear curves are constructed based on the observed damage, while the CARTIS datasheet which we have used in our method, do not include damage observation. Both methods provide complementary information on the complete seismic response of the building object of study. Therefore, it emerges the necessity to correlate to the computations of the safety factor also with an analysis that takes into consideration the influence of local vulnerabilities in the final computed value of the SF.

6. Conclusions

The research is developed in the context of the Italian urban areas located in regions of moderate seismic hazard. Focus is posed on historical masonry residential buildings, which require continues maintenance and hence also evaluations of their seismic structural safety.

To carry out reliable, inexpensive checks that can guide protective actions, agile tools are needed, which can use data that are easily available from the municipal administrative bodies (or from the CARTIS database organized by the National Department of Civil Protection). In this research, a probabilistic method was developed such that can respond to this purpose.

The method uses a set of data computed by approaches already tested in the literature dealing with the seismic assessment of masonry buildings, which is based also on the requirements of the Italian Code NTC 2018. A deterministic safety factor SF is computed as the minimum of static assessment and global horizontal actions assessment. Assuming a range of accelerations, based on the local seismic hazard it is possible to compute the associated safety factor SF. Using the computed

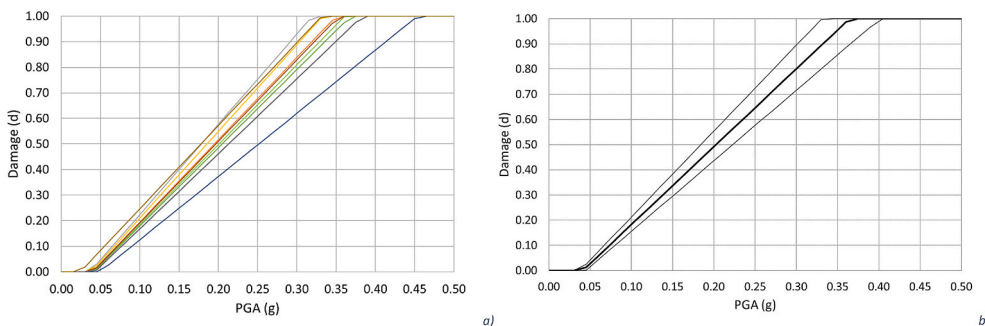


Fig. 13. Damage – PGA ratio for the masonry typology characteristic of 10the buildings examined in Desio (MB): a) V.I. calculated for each studied building; b) average V.I. for the typology (Average V.I. = 26.50 plus and minus the standard deviation 6.26).

set of data, the methodology was extended here in a probabilistic framework with the concept of fragility curves. This helps to formulate hypotheses on the possible loss of structural performance, measured as a decrease in the safety factor SF, as the intensity of shaking in PGA increases. The procedure was programmed through VBA (Visual Basic for Applications) and Excel in order to automate the here described steps into a simple tool.

The first application in the case study of Campi Alto di Norcia, where the response to a series of earthquakes was known in advance, helps to obtain a first validation of the proposed methodology. Accordingly, the results demonstrate that the predicted loss of performance matches well with the observed experimental response. The building, after the last reinforcement intervention, has displayed a new unexpected damage mechanism, which is now very difficult to repair, as it has consistently been demonstrated in the past after an earthquake, and now risks final demolition. Unfortunately, the observed damage and the prediction presented here confirmed how the use of modern (non-compatible) structural interventions does not help to increase the performance of a historical stone masonry building as simple as the one analysed here.

A second application was considered to demonstrate the ease of applying the method on a large scale. A portion of the historic centre of Desio town (MB) was analysed, based on CARTIS forms that were compiled for at least ten buildings belonging to the same typology.

The masonry typology studied here in the second case is a widespread and a rather common typology throughout the Lombardy Region. Therefore, the analysis carried out for an area of medium-low seismicity shows a moderate performance, can be generalised also for the same building typology located in an area of medium-high seismicity.

This is the first attempt to apply this procedure, and in both cases, it was demonstrated how it is possible to estimate in probabilistic terms the expected structural damage in the considered range of accelerations.

Further studies are required to validate the proposal. Therefore, additional work will be focused in two directions: a) to deepen the relationship between the damage predicted through the fragility curves and the observed damage, as well as the suitability of the chosen damage index; b) contribute to the active application of tools and methods developed in this research by integrating them in online platforms through API (Open Application Programming Interfaces).

#### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

#### Data availability

Data will be made available on request.

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