Contents lists available at ScienceDirect

Journal of International Money and Finance

journal homepage: www.elsevier.com/locate/jimf

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ARTICLE INFO

Article history: Available online 3 June 2023

JEL classification: E31 E52 F63

Keywords: Price level targeting Monetary and fiscal policy interactions

ABSTRACT

The adoption of a "makeup" strategy has been one of the proposals in the review of the Fed's monetary policy framework. Another suggestion, to avoid the zero lower bound, has been a more active role for fiscal policy. We put together these ideas to study price level targeting under a fiscally-led regime. We find that following a deflationary demand shock: (i) the central bank should increase (rather than decrease) the policy rate; (ii) the central bank, thus, avoids the zero lower bound; (iii) price level targeting is welfare improving with respect to inflation targeting, unless one considers a nonstandard inflation targeting rule with a negative inflation coefficient and a high degree of smoothing.

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1. Introduction

One of the most discussed proposals about monetary policy strategies, especially in the U.S. and Canada, has been the adoption of a (temporary) price level targeting (PLT) framework (Bernanke, 2017; Bernanke, 2019). According to this proposal, whenever the economy is in the proximity of the ZLB, the central bank should commit to a lower-for-longer rate to fully offset any shortfalls of inflation from target.¹ At the Jackson Hole 2020 symposium, Jay Powell announced a change in the Fed's monetary procedure towards an average inflation target, which is akin to PLT with a finite time window for measuring inflation. Therefore, there will be periods when inflation will overshoot the stated inflation target to make up for its previous undershooting.²

In addition to the adoption of new monetary procedures, many observers suggested to use fiscal measures to stimulate aggregate demand in the presence of adverse shocks, even before the outbreak of the Covid-19 pandemic. Most of the

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https://doi.org/10.1016/j.jimonfin.2023.102876







^{*} The views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank. Ascari acknowledges generous support from the Ministry of the University, MIUR, PRIN 2020, Prot. 2020LKAZAH.

¹ See on this point even Williams (2017), Evans (2012) and Yellen (2018). Note that another often-cited option to avoid the zero lower bound(ZLB) is increasing the inflation target. However, Andrade et al. (2019) find that the case for increasing the inflation target is much reduced when the central bank adopts a price level targeting approach. PLT was adopted in the 1930s by the Sveriges Riksbank that, in the more recent past, has re-pondered that option, as the Bank of Canada did (see, for this past Swedish experience, Honkapohja and Mitra, 2020).

² Note that the average inflation target is a two-sided regime: whenever inflation overshoots the target, as it is currently happening, the Fed should lower inflation below the target for a while to make up for the ongoing high inflation.

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proposals are in the tradition of the standard "monetary regime", where fiscal policy is passive, in the sense that fiscal expansions are backed by higher taxes, at least in the long run.³ For example, Evans et al. (2008) claimed that monetary policy alone may not prevent deflationary spirals and should be complemented with aggressive (but passive) fiscal policy. Draghi (2019) invoked a more active role for (passive) fiscal policy since he imputed the slide into disinflation to an unbalanced policy mix, in favor of monetary policy. Lagarde (2019) called for euro area fiscal support claiming that monetary policy could not be the only game in town. In practical terms, most countries tried to mitigate the negative effects of the pandemic by adopting fiscal stimulus measures of unprecedented magnitude.

Especially during difficult times, the interaction between monetary and fiscal policy becomes crucial. Talking about PLT, Bernanke (2017) stated that central bank independence should be protected *without* "ruling out temporary periods of monetary-fiscal coordination that may be essential for achieving key policy goals". When interest rates (are about to) hit zero and the price level is falling, monetary policy can completely lose control on inflation and, according to Sims (2000), moving to an active fiscal policy (i.e., unbacked by taxes) could be beneficial. Two dynamics are at work during a deflation episode: an accelerationist dynamic that makes prices decrease further, and real balance effects that instead increase prices. "For real balance effects to rule out or prevent deflationary spirals, fiscal policy must be seen not to be committed to keeping the real value of primary surpluses in line with the current outstanding real value of government debt so that this will not be backed by increased future real taxation. Policy-makers should understand that under some circumstances budget balancing can become bad policy" (Sims, 2000).

However, the implications of adopting a PLT for different monetary-fiscal policy mixes have not been studied so far in the literature. Several authors have discussed the potential advantages of price-level stabilization over inflation stabilization in terms of improved welfare, lower inflation variability, and a more favorable inflation-output gap trade-off (see, among the others, Giannoni, 2014; Svensson, 1999; Vestin, 2006). Other studies theoretically justify a PLT approach to address the zero lower bound (ZLB) problem and to escape deflationary traps (see Evans, 2012; Eggertsson and Woodford, 2003; Billi, 2018). All these studies, however, either abstract from fiscal policy or consider passive fiscal policy configurations, while monetary policy is assumed to be active. On the other hand, a large literature analyses the effects of alternative monetary-fiscal mixes (both monetary- and fiscally-led regimes) to counteract recessions and deflationary traps in an inflation targeting (IT) context (see, among others, Davig and Leeper, 2011; Bianchi and Melosi, 2017; Ascari et al., 2020). There is no such analysis, however, for the PLT case, and we aim to fill this gap. To our knowledge, this paper is the first attempt to shed some light on PLT in the context of a fiscally-led regime, a regime where, as Eggertsson (2006) claims, the government uses fiscal policy to "commit to being irresponsible". In particular, we want to study how the adoption of a PLT rule modifies the results about equilibrium uniqueness under rational expectations in a simple New Keynesian model in a fiscally-led regime. In this respect, our paper could be considered as the counterpart, under PLT, of the analysis in Bhattarai et al. (2014) under IT.

Our main result is that in a fiscally-led regime a central bank with a PLT rule should raise (rather than cut) the policy rate after a negative demand shock. This leads the economy away from the deflationary trap and allows the central bank to avoid the zero bound. This result resonates with the neo-Fisherian recipe to increase the interest rates in order to spur inflation, as argued by Uribe (2018). However, here the logic is very different from the neo-Fisherian perspective. First, under PLT and active fiscal policy, determinacy requires the price level coefficient in the interest rate rule to be negative. Hence, the nominal interest rate increases if the price gap is negative. Second, after a demand shock, this "inverse" reaction of the policy rate to inflation amplifies the wealth effects due to the fiscal theory of the price level. Despite there is nothing normative in our PLT rule, we show that PLT is generally welfare-improving with respect to IT after a demand shock, under both a monetary and a fiscally-led regime, when the standard assumption of a positive coefficient in the IT Taylor rule is employed. Nonetheless, if we consider the awkward case of an IT rule with a negative inflation coefficient, PLT no longer outperforms IT after a demand shock in a fiscally-led regime. Rather, the two approaches return similar responses and both avoid the ZLB. We find that IT can outperform PLT in terms of welfare when the negative inflation coefficient is coupled with a high degree of interest rate smoothing.

The paper is organized as follows. Section 2 first introduces two of the main proposals – namely price level targeting and active fiscal policies – put forth to adjust the actual policy framework to face the current and future recessions and then proposes a simple DSGE model with a PLT rule and studies its determinacy properties. Since it is not possible to solve this model analytically, we also introduce a flexible-price version of the model to obtain useful analytical insights from its solutions. Section 3 shows the effects of a deflationary demand shock both in the monetary- and in the fiscally-led regimes, comparing the impulse response functions under a PLT rule to those under an IT rule. This section includes a welfare analysis of the two approaches and a description of the effects of the demand shock when the ZLB is taken into account. Furthermore, it compares the results under a fiscally-led regime employing a PLT rule to those one gets considering an IT rule with a negative inflation coefficient. Section 4 reports additional robustness checks, while Section 5 concludes.

³ According to Leeper (1991)'s taxonomy, a policy authority is active when it pursues its objective unconstrained by the actions of the other authority; otherwise it is passive. Two policy configurations ensure the existence of a unique stable equilibrium under rational expectations: the monetary regime (active monetary/passive fiscal mix) and the fiscal regime (passive monetary/active fiscal mix).

2. A simple New Keynesian model with PLT

Before the Covid-19 crisis, the Federal Reserve was discussing how to review its inflation-targeting framework. The reasons for this review were low inflation, which had been running below the 2% target for much of the 2009–2020 expansion, subdued inflation expectations, and low interest rates, which increased the risk of falling back into a ZLB episode. This eventually occurred when the U.S. was hit by the Covid-19 pandemic and arguably encouraged the Fed to adopt an average inflation target starting from August 2020. According to an IT rule, a central bank aims to reach 2% inflation every period and commits to correct the inflation deviations from the intended target. Instead, under a makeup strategy, like PLT or average inflation targeting, a central bank should reverse previous shocks to the price level, abandoning the "bygones are bygones" approach. Formally, under PLT the monetary authority defines a target path for the price level (p_i^* , in logs) and responds to deviations of the price level (p_i) from this path. The corresponding log-linear monetary policy rule has the form:

$$\widehat{R}_t = \phi_p(p_t - p_t^*) + \theta_t, \tag{1}$$

where \hat{R}_t is the nominal interest rate (in log deviations from the steady state) and θ_t is a monetary policy shock.

The main, traditional argument in favour of PLT is the long-run predictability of the price level, while the major disadvantage is the increased short-run volatility of both inflation and output, with respect to the IT benchmark (see Lebow et al., 1992; Haldane and Salmon, 1995). If credible, the history-dependence introduced by the PLT approach can achieve better economic outcomes thanks to the management of market expectations. After a deflationary shock in a traditional monetary-led regime, the central bank adopting a PLT approach commits to keep rates lower for longer to tackle deflationary expectations. This increases inflation expectations that, in turn, reduce real rates and scale down the risk of hitting the ZLB.⁴

The situation in the aftermath of the Covid-19 outbreak, characterized by low inflation, (e.g., Blanchard, 2020), huge fiscal stimulus and major central banks that accommodate the increased government spending, can be seen as a period of fiscal dominance. In such a context of active fiscal policy, what would happen if the proposal of a PLT approach were put into practice? At the moment there is no framework linking PLT and fiscally-led regimes.⁵

Our aim is to address this point by illustrating, more generally, the key properties of PLT allowing for monetary-fiscal interactions. To this end, we consider a simple three-equation New Keynesian model and extend it with either an IT or PLT rule and a fiscal policy block.⁶ The resulting log-linearized model is given by the following equations:⁷

$$\frac{1}{\overline{c}}\hat{y}_t = \frac{1}{\overline{c}}E_t\hat{y}_{t+1} - \left(\hat{R}_t - E_t\hat{\pi}_{t+1}\right) + (1 - \rho_\varepsilon)\varepsilon_t,\tag{2}$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{y}_t + v_t, \tag{3}$$

$$\hat{R}_t = \phi_\pi \hat{\pi}_t + \theta_t. \tag{4-IT}$$

$$\widehat{R}_t = \widehat{R}_{t-1} + \phi_p \widehat{\pi}_t + \Delta \theta_t, \tag{4-PLT}$$

$$\hat{b}_{t} = \frac{1}{\beta} \hat{b}_{t-1} - \frac{1}{\beta \bar{b}} \hat{\tau}_{t} - \frac{1}{\beta} \hat{\pi}_{t} + \hat{R}_{t},$$
(5)

$$\hat{\tau}_t = \gamma b_{t-1} + \psi_t. \tag{6}$$

where $\kappa = (1/\bar{c} + \xi)(1 - \alpha)(1 - \alpha\beta)/\alpha$. Eq. (2) is an Euler equation, where \hat{y}_t is output, $\hat{\pi}_t$ is inflation and ε_t is a demand shock.⁸ Eq. (3) is a New Keynesian Phillips curve, with a supply shock v_t . The monetary policy rule is represented by either Eq. (4-IT), which is a traditional IT rule, or Eq. (4-PLT), which is a PLT (or superinertial) rule that can be derived from (1) using the fact that $p_t^* = p_{t-1}^* + \pi^*$, where π^* is the inflation target. Finally, the last two equations represent the fiscal policy block, formed by the government budget constraint (5), where \hat{b}_t is real public debt, and a fiscal rule (6) that assumes that lump-sum taxes respond to lagged debt and to a fiscal policy shock ψ_t .⁹

⁴ Although we assume central bank credibility throughout the paper, we discuss the importance of commitment, for both monetary and fiscally-led regimes, at the end of Section 3.1.

⁵ The only exception is the so-called "going direct" approach (see Bartsch et al., 2019) that advocates a more explicit coordination between monetary and fiscal policy to be undertaken just in some defined circumstances, with an explicit exit strategy and with an explicit inflation objective for which both monetary and fiscal authorities are held accountable. The inflation target should be met through a monetary financing of a fiscal expansion (a fiscally-led regime) and the central bank should make up for past inflation shortfalls (a make-up strategy in line with PLT).

⁶ The model is log-linearized around a steady state where the interest rate is positive, inflation is equal to its target, and output is equal to its natural level – the so-called intended steady state. It is well-known that the three-equation New Keynesian model exhibits another possible deflationary steady state under IT. However, Ambler and Lam (2016) show that the latter does not exist under PLT. Hence, it seems natural to compare the two strategies using the only common steady state that exists in both cases.

⁷ Hatted variables indicate log-deviations from the steady state, while variables without a subscript indicate steady state values. The parameters of the model are described in Table 3, while Section A.1 contains a full derivation of the model.

⁸ Following Liu et al. (2009), ε_t can be modelled as a preference shock on the intertemporal discount rate.

⁹ All exogenous shocks evolve according to stationary AR(1) processes, with autoregressive parameters equal to ρ_i for $i = \varepsilon$, v, θ , ψ , r, where the shock r_t is introduced in Section 2.2.

Section 2.1 contains the analytical results about determinacy for this simple New Keynesian model. However, as the rational expectations solutions under sticky prices cannot be derived analytically, in Section 2.2 we show the solutions for a flexible-price version of the model.

2.1. Determinacy analysis

The determinacy properties of the model depend on whether fiscal policy is passive ($\gamma > \overline{b}(1 - \beta)$) or active ($\gamma < \overline{b}(1 - \beta)$), and this is true for both inflation and price level targeting.

In the IT case, the New Keynesian model returns the usual determinacy conditions: under passive fiscal policy, determinacy requires to satisfy the Taylor principle, i.e. active monetary policy ($\phi_{\pi} > 1$), while under active fiscal policy determinacy is achieved if monetary policy is passive ($\phi_{\pi} < 1$). Similarly, in the PLT case, under passive fiscal policy, determinacy requires ϕ_p to be positive. This parametric region defines the monetary regime (regime M or active monetary/passive fiscal mix), with Ricardian agents and no wealth effects, so active monetary policy corresponds to $\phi_p > 0$. Conversely, under active fiscal policy determinacy active fiscal mix) where agents are non-Ricardian and wealth effects kick in, so passive monetary policy corresponds to $\phi_p < 0$ (see A.3). Recall that a PM/AF regime couples a low response of interest rates to inflation with a low response of taxes to public debt. This is a fiscal dominance regime where price changes ensure the solvency of the government and inflation is determined by the needs of fiscal policy. With an active fiscal policy, taxes do not adjust strongly enough to satisfy the intertemporal government budget constraint at prevailing prices. In this case, an increase in the value of government debt (that can be due to an increase in interest rates and/or a tax decrease) leads to a positive wealth effect on households, that hold public debt. In turn, this leads to higher spending and higher inflation.¹⁰

Therefore, the model has a unique solution in two distinct parametric regions, as summarized in Table 1, which divides the parameter space into four regions depending on whether monetary and fiscal policies are active or passive. In addition, similar to the analysis of Leeper (1991) who only considered the IT case, we also find that the system returns indeterminacy when both authorities behave passively, while it has no stable solutions when both policies are active.

Giannoni (2014) develops a similar analysis considering, however, just the case of PLT coupled with a passive fiscal policy. We confirm his finding that under PLT the determinacy condition for monetary policy is less restrictive than the Taylor principle: the reaction of interest rate to an increase in inflation must be positive but it can be even less than one-for-one in case of price-level stabilization. The new result of our analysis pertains to the case of active fiscal policy. There is determinacy whenever the nominal interest rate moves in opposite direction with respect to inflation ($\phi_p < 0$): if inflation decreases, the central bank must increase the nominal interest rate, so that the real interest rate increases unambiguously. This finding is akin to the so-called neo-Fisherian effect advocated by Uribe (2018): to exit a liquidity trap, a central bank should raise nominal interest rates to produce an increase in inflation.¹¹ Interestingly, the same would happen by adopting a PLT rule combined with a fiscal regime: whenever inflation decreases, the central bank should increase the interest rate. As explained below, however, the economic mechanism underlying our result is very different and has to do with the wealth effect induced by the fiscal theory of the price level. Similar effects could arise in the fiscal regime if we consider the barely explored case of a central bank that adopts an IT approach but reacts with a negative coefficient to inflation. From the bottom-right quadrant of Table 1, we see that determinacy would still be preserved. Section 3.4 will study the similarities between this specification and the PLT case.

2.2. Analytical solutions under flexible prices

We now resort to a flexible-price model to analyse the properties of equilibria produced by monetary and fiscal policy rules, depending on whether policy authorities behave actively or passively. The main reason to consider this simple setup is that PLT introduces an extra endogenous state variable and makes it impossible to get analytical expressions for the solutions in the sticky-price version of the model. Even in this simplified setup, in the fiscal regime with a PLT rule, there are two endogenous state variables to consider, namely public debt and the nominal interest rate. Despite this, we are able to obtain the analytical solution and gain some insights that carry over to the sticky-price version of the model.

The flexible-price model is composed of the previous monetary policy rules (4-IT) and (4-PLT) and of the following linearized equations:

$$\hat{R}_{t} = E_{t}\hat{\pi}_{t+1} + r_{t}, \tag{7}$$

$$\hat{b}_t = \frac{1}{\beta} \left(1 - \frac{\gamma}{\bar{b}} \right) \hat{b}_{t-1} + \hat{R}_t - \frac{1}{\beta} \hat{\pi}_t - \frac{1}{\beta \bar{b}} \psi_t.$$
(8)

¹⁰ Note that these wealth effects arise even in the absence of public expenditure and in an environment with lump-sum taxes.

¹¹ The well-known Fisher identity delivers a one-to-one long-run relationship between the nominal interest rates and inflation given that the real interest rate is determined only by non-monetary factors. As a consequence, an increase in the nominal rate, perceived to be permanent, will increase inflation.

Table 1

Determinacy regions for different combinations of monetary and fiscal policy.

| | Passive fiscal policy: $\gamma > \overline{b}(1 - \beta)$ | Active fiscal policy: $\gamma < \overline{b}(1 - \beta)$ |
|---|--|---|
| Active monetary policy: $\begin{cases} \phi_{\pi} > 1 & (\text{IT}) \\ \phi_{p} > 0 & (\text{PLT}) \end{cases}$ | Monetary regime: determinate, Ricardian, no wealth effects | Explosive |
| Passive monetary policy: $\begin{cases} \phi_\pi < 1 & (\text{IT}) \\ \phi_p < 0 & (\text{PLT}) \end{cases}$ | Indeterminate | Fiscal regime: determinate, non-Ricardian, wealth effects |

Table 2

Rational expectation solutions for inflation.

| Rational expectation solutions for mi | | |
|---------------------------------------|-----|---|
| Monetary regime | PLT | $\hat{\pi}_t = -\frac{1}{\phi_p} \widehat{R}_{t-1} - \frac{1}{1+\phi_p - \rho_g} \theta_t + \frac{1}{\phi_p} \theta_{t-1} + \frac{1}{1+\phi_p - \rho_r} r_t$ |
| | IT | $\hat{\pi}_t = -rac{1}{\phi_\pi - ho_	heta} 	heta_t + rac{1}{\phi_\pi - ho_r} r_t$ |
| Fiscal regime | PLT | $\hat{\pi}_t = \tfrac{K}{J} \hat{b}_{t-1} - \tfrac{(a-1)\beta}{J} \widehat{R}_{t-1} - \tfrac{(a-1)\beta}{J} \left(\tfrac{\frac{g}{2}-1}{\frac{g}{p}-\rho_s} \theta_t - \theta_{t-1} \right) + \tfrac{1}{\frac{g}{p}-\rho_r} r_t - \tfrac{K}{J\beta b} \tfrac{1}{\frac{g}{p}-\rho_\phi} \psi_t$ |
| | IT | $\hat{\pi}_t = \frac{\beta\phi_{\pi}-a}{\beta\phi_{\pi}-1}\hat{b}_{t-1} + \frac{1-a}{\beta\phi_{\pi}-1}\frac{1}{g}\frac{1}{\rho-\phi_0}\theta_t + \frac{1}{g}\frac{1}{\rho-\rho_r}r_t + \frac{a-\beta\phi_{\pi}}{\beta\phi_{\pi}-1}\frac{1}{\beta b}\frac{1}{g}\frac{1}{\rho-\phi_{\pi}}\psi_t$ |
| with coefficients: | | $a = 1 - rac{\gamma}{b}$ $J = a(\beta\phi_p - 1) + \beta$ $K = a(\beta(1 + \phi_p) - a)$ |

Eq. (7) is a Fisher equation, where r_t is the exogenous stochastic process followed by the real interest rate (i.e., a demand shock) while Eq. (8) is obtained by substituting the fiscal rule inside the government budget constraint.

We use the method by Bhattarai et al. (2014) to find the rational expectation solution for inflation (derivations in A.3). Table 2 reports the solutions under PLT and IT in the two regimes.¹²

As under IT, the solution for inflation under PLT depends just on the monetary and the demand shock in the regime M, and in regime F it depends on government debt and fiscal shocks too. Differently from the IT case, however, a super-inertial interest rate rule implies that the nominal interest rate is an endogenous state variable. Therefore, under PLT, inflation depends on the lagged interest rate (\hat{R}_{t-1}) under both regimes. In the M regime we have that $\phi_p > 0$, so the coefficient relating past interest rate to current inflation $(-1/\phi_p)$ is negative. In the F regime, since $\phi_p < 0$, the coefficient relating current inflation to past interest rate, though more involved $((1 - a)\beta/J)$, turns out to be positive.¹³ These coefficients explain how under PLT the monetary authority "makes-up" for a possible shortfall in inflation. The intuition goes as follows. Imagine that $\hat{\pi}_{t-1}$ decreases. Since PLT introduces history dependence in monetary policy, the policymaker should compensate the negative shock to inflation that occurred in the past by increasing current inflation ("bygones are not bygones"). However, given (1) and the determinacy conditions, the central bank increases the interest rate \hat{R}_{t-1} in an F regime (it decreases \hat{R}_{t-1} under an M regime). As a consequence, it must be that the increase in \hat{R}_{t-1} in the F regime (the decrease in regime M) increases $\hat{\pi}_t$, that is, there must be a positive relationship between \hat{R}_{t-1} and $\hat{\pi}_t$ in regime F (and a negative one in regime M). As a consequence, in the F regime $\hat{\pi}_t$ will be positively affected (or negatively affected in the M regime) by the variation in \hat{R}_{t-1} .

The next section will investigate the effects of a negative demand shock in the New Keynesian model. Our simple flexibleprice model, however, is of help in understanding the mechanisms behind those effects. As one can see from Table 2, the coefficient relating $\hat{\pi}_t$ to r_t (i.e., a demand shock) is always positive, in both regimes and under both IT and PLT: that is, a negative demand shock brings inflation down. However, plugging the inflation solution in the monetary rule we can gauge the response of the interest rate to this shock. We obtain that, under both IT and PLT in the M regime and under IT only in the F regime, after a reduction in inflation the central bank should answer by decreasing the interest rate. Conversely, if one adopts a PLT approach in the F regime, remembering that in that case $\phi_p < 0$, the recipe to go out from a deflationary trap is to increase interest rates. Higher interest rates raise bondholders' interest receipts while taxes do not rise, so that households' wealth increases, spurring aggregate demand and inflation.¹⁴

3. A negative demand shock

Section 3.1 reconsiders the New Keynesian model to present the simulated impulse responses to a demand shock under both the M and the F regime, to highlight the behavior of the two types of monetary policy rules. Section 3.2 compares the

¹² The solution under IT turns out to be very similar to the well-known solutions derived by Leeper (1991).

 $^{^{13}\,}$ In this flexible price model this is true for all the active fiscal policies with a $\gamma <$ 0.

¹⁴ For example, by exploiting the negative dependence of $\hat{\pi}_t$ to ψ_t in the F regime under both IT and PLT (see A.3), one can provide a similar discussion for fiscal shocks.

Table 2

| Parameter | Value | Description |
|----------------------|-------|---|
| α | 0.66 | Calvo probability of not readjusting prices |
| β | 0.995 | Intertemporal discount factor |
| ξ | 2 | Inverse Frisch elasticity of labor supply |
| b | 1.96 | Debt-taxes ratio at steady state |
| Ē | 0.8 | Consumption-output ratio at steady state |
| ρ_{ε} | 0.9 | Autoregressive parameter of the demand shock |
| γ | 0.2 | Tax coefficient in the fiscal rule (passive fiscal policy) |
| | 0 | Tax coefficient in the fiscal rule (active fiscal policy) |
| ϕ_{π} | 1.2 | Inflation coefficient in the IT rule (active monetary policy) |
| | 0.9 | Inflation coefficient in the IT rule (passive monetary policy) |
| ϕ_n | 1.2 | Price-level coefficient in the PLT rule (active monetary policy) |
| · r | -0.1 | Price-level coefficient in the PLT rule (passive monetary policy) |

welfare losses under IT and PLT following the same shock, while Section 3.3 includes impulse responses that explicitly take into account the presence of a zero bound for nominal rates. In Section 3.4 we reassess these exercises by comparing the PLT rule to a nonstandard IT rule with a negative inflation coefficient and a high degree of interest rate smoothing.

Unless otherwise stated, the simulations of this section are based on the benchmark calibration reported in Table 3.

3.1. Impulse responses

We now want to study the dynamics of economic variables when hit by a negative demand shock (see Fig. 1). We analyse both the M regime and the F regime and we compare the impulse response functions under IT (Eq. (4-IT)) to those under PLT (Eq. (4-PLT)). While the literature on PLT is limited to the study of active monetary/passive fiscal configurations, this paper extends and enriches the analysis to consider the dynamic of the model in a fiscally-led regime.

The right column of Fig. 1 exhibits the well-known results valid in an M regime: a negative demand shock reduces inflation, output, and the nominal interest rate, and by a greater extent under IT. Therefore, in a monetary regime, there are advantages from the adoption of a PLT approach.

The left panel turns to the F regime: a negative demand shock leads to a smaller response of inflation under PLT than under IT, while output declines more markedly on impact, though it exhibits a less persistent dynamic and returns to equilibrium after five periods. To have an intuition on the underlying mechanism, we can solve forward the budget constraint (5) to get the government debt valuation equation:

$$\hat{b}_{t-1} - \hat{\pi}_t = E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{1}{\bar{b}} \hat{\tau}_{t+j} - \beta \left(\hat{R}_{t+j} - \hat{\pi}_{t+j+1} \right) \right].$$
(9)

This equation shows that the real value of government debt (left-hand side, LHS) must be backed by the present value of future primary surpluses (right-hand side, RHS). Active fiscal policy implies that unbacked fiscal expansions – for example, an increase in government deficit – induces wealth effects: agents will substitute out their debt holdings and raise their demand for consumption goods, causing an increase in current prices to a level that restores balance between the LHS and the RHS. Under IT, output decreases and so does inflation, creating a negative inflation tax on government's nominal liabilities that increases real government debt (the LHS increases). Since fiscal policy does not adjust surpluses, the LHS is larger than the RHS so wealth effects kick in and these make future inflation increase (inflation reversal), and output too. The inflation increase keeps the government budget constraint satisfied.

Output and inflation decrease on impact under PLT too. However, as a response to the inflation reduction, under PLT the central bank raises the nominal interest rate and this, in turn, raises real interest rates, $\hat{R}_t - \hat{\pi}_{t+1}$. This has two consequences. First, there is a much larger reduction in output on impact. Second, the rise in real rates widens the difference between the LHS (which increases on impact) and the RHS, because real rates enter with a negative sign in the RHS. Hence, wealth effects are now larger, inducing a faster rebound of both inflation and output.

After a decrease in inflation, the central bank raises the interest rate and this, in turn, creates wealth effects that spur inflation. As previously stressed, here the recipe to go out from a deflationary episode is similar to the one proposed exploiting the neo-Fisherian effect: increase interest rates to stimulate inflation. However, here the logic is very different from the neo-Fisherian perspective, and it is based on the wealth effects due to the fiscal theory of the price level. Furthermore, contrary to what happens under IT, if the central bank is adopting a PLT approach, a negative demand shock does not make the nominal interest rate decrease. In this case, PLT also serves the purpose of preventing the economy from hitting the ZLB.

Note that, throughout the paper, we assume a central bank fully committed to PLT. However, the lack of credibility could seriously impair its effectiveness, both under a traditional monetary regime and under a fiscal regime. In the M regime, a one-sided PLT approach that, after a negative demand shock, commits to keep rates lower for longer to tackle deflationary expectations, needs a credible commitment to increase inflation expectations that reduce real rates. This is even true for a



Fig. 1. Impulse response function to a negative demand shock. Notes: Solid blue lines correspond to inflation targeting, dashed red lines correspond to price level targeting. Parametrization in Table 3. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

two-sided PLT, as the one we consider, where the central bank reacts to an inflationary shock too, committing to keep rates higher to decrease inflation expectations. However, if compared to a two-sided PLT approach, a one-sided approach, which reacts just to deflationary shocks, is usually more credible: it is easier, indeed, to believe in a prolonged period of low rates, despite improvements in unemployment, than in a long period of high rates (to make-up for higher inflation) that should be maintained despite an ongoing recession. In the F regime, after a deflationary (inflationary) shock, the central bank adopting a PLT approach commits to keep rates higher (lower) for longer to tackle deflationary (inflationary) expectations. Even here, full commitment is crucial but, in this case, the central bank, to be credible, should commit to adopt a passive monetary policy or, in other words, not to respond too aggressively to inflation in order to create the positive (negative) wealth effects that spur (decrease) inflation.

3.2. Welfare analysis

Following Gorodnichenko and Shapiro (2007), we undertake a welfare analysis to evaluate the performance of IT and PLT rules following a demand shock. To do so, we employ the following loss function:



Fig. 2. Loss function after a demand shock.

$$\mathscr{L} = \sum_{t=0}^{\infty} \beta^t \left(\omega_\pi \hat{\pi}_t^2 + \omega_y \hat{y}_t^2 + \omega_R \hat{R}_t \right) = \omega_\pi \mathscr{L}_\pi + \omega_y \mathscr{L}_y + \omega_R \mathscr{L}_R, \tag{10}$$

where ω_{π} , ω_{y} and ω_{R} are the weights on inflation, the output gap and the deviation of the interest rate from its target path.¹⁵ As in Gorodnichenko and Shapiro (2007), we bias the findings against PLT by not including a possible term on the price level gap and by associating a relatively large weight to the output gap ($\omega_{y} = 1$), since PLT usually increases the volatility of output with respect to IT.

Fig. 2 shows how the loss function is affected as the inflation (or price level) coefficient in the monetary rule changes in the M and F regimes. For the analysis, we fix the fiscal policy coefficient and let the monetary policy coefficients vary: in the F regime, we set γ to zero and focus on passive monetary policy coefficients ($\phi_{\pi} < 1$ under IT and $\phi_{p} < 0$ under PLT); in the M regime, we set γ to 0.2 and focus on active monetary policy coefficients ($\phi_{\pi} > 1$ under IT and $\phi_{p} > 0$ under PLT). The yellow and red lines in the figure describe the components of the loss function when the weights ω_{π} and ω_{y} are each set equal to one (while the other weights are set to zero) and the blue line corresponds to the total loss, when $\omega_{\pi} = \omega_{y} = \omega_{R} = 1$ contemporaneously.¹⁶

¹⁵ Following Giannoni (2014), we include in the loss the (third) term reflecting the central bank's will to minimize the variability of interest rates. According to Woodford (1999), one of the reasons to do so is the central bank's will to try to avoid the zero nominal interest-rate floor.

¹⁶ The loss function is computed by simulating for 1000 periods the third-order solution of the model around the intended steady state. The nonlinear version of the model is presented in A.1. For the sake of clarity, in Fig. 2 we omit the loss function when $\omega_R = 1$ and the others are set to zero but, for completeness, we report the total loss function that includes even the interest rate component.

In the M regime, we obtain the well-known result, e.g., Schmitt-Grohe and Uribe (2007), that monetary policy can completely stabilize the output gap and inflation facing a demand shock by making the inflation coefficient (or the price level one) tend to infinity. This would be the preferred policy configuration to face a demand shock. Note that both the output gap and the inflation gap components decrease with ϕ_{π} (and ϕ_{n}).

Our focus, however, is on the F regime. In the IT case, the overall loss function (blue line) increases as ϕ_{π} approaches the limit value of 1, so a very passive monetary policy response would be preferred. As monetary policy becomes more active, the output gap component (red line) decreases, while the inflation gap component (yellow line) increases sharply. The latter effect dominates in determining the overall loss. The opposite happens in the PLT case. The overall loss is decreasing in ϕ_p , so that a less passive policy is preferred.

As expected, a PLT rule determines a larger output gap volatility and a smaller inflation gap volatility than an IT rule. However, the overall welfare loss function is much smaller under PLT, so that, conditional on a demand shock, PLT generally outperforms IT.



Fig. 3. Impulse response function to a negative demand shock accounting for the ZLB. Notes: Solid blue lines correspond to inflation targeting, dashed red lines correspond to price level targeting. Parametrization in Table 3. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

3.3. The zero lower bound case

Following Eggertsson and Woodford (2003), we now account for the presence of a ZLB constraint that prevents the central bank from providing the appropriate monetary stimulus following a negative demand shock that hits the economy and depresses output and inflation. Even in this case, we consider both types of monetary policy rules under both regimes. We report the impulse response functions in Fig. 3. Differently from the previous exercises, the impulse responses are here based on the piecewise linear solution method of Guerrieri and Iacoviello (2015), to take into account the presence of an occasionally binding constraint. In both regimes, under IT the nominal interest rate hits the zero bound immediately and remains constrained for five periods, while a PLT rule always allows the central bank to steer the economy clear of a liquidity trap. In the M regime (right panel), inflation and output decrease by a greater extent under IT, while they hardly move under PLT. Even the path of debt is more favorable under PLT. In the F regime (left panel), output reacts more strongly under PLT than IT, while inflation remains closer to its steady state value under PLT than under IT, in accordance with the findings of Fig. 1. Debt displays a smaller increase under IT.



Fig. 4. Impulse response function to a negative demand shock for $\phi_{\pi} < 0$ in the F regime. Notes: Solid blue lines correspond to inflation targeting, dashed red lines correspond to price level targeting. Parametrization: $\phi_{\pi} = -0.5$ and $\rho_{R} = 0.9$, $\phi_{p} = -0.1$, $\gamma = 0$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

3.4. IT under fiscal dominance: the case of a negative inflation reaction coefficient

As we saw in Section 2, the determinacy analysis in case of fiscal dominance returns a price-level coefficient ϕ_p lower than zero for the PLT rule, and an inflation coefficient ϕ_{π} lower than one for the IT rule. Since we are allowing ϕ_p to be negative under PLT, we may wonder how a negative value of ϕ_{π} would modify the results in the IT case. We want to focus on an IT rule in an F regime and, for the sake of comparison, we want this rule to be very similar to its counterpart under PLT. To this end, a potential candidate is the following:

$$\widehat{R}_t = \phi_\pi \widehat{\pi}_t + \rho_R \widehat{R}_{t-1} + \theta_t, \tag{11}$$

with $\phi_{\pi} < 0$ and a high degree of interest rate smoothing. We assess the performance of this rule along several dimensions. Unless otherwise stated, throughout this section we set $\phi_{\pi} = -0.5$ and $\rho_{R} = 0.9$, while for fiscal policy we set $\gamma = 0$.

Impulse responses. With the adopted specification, the response of inflation, output, and debt after a negative demand shock is very similar under PLT and IT (see Fig. 4). To contrast the fall of inflation, the central bank raises the nominal interest rate under both IT and PLT. The increase in the nominal interest rate is lower under IT but, due to the smoothing term, it is more persistent. The rise of the interest rate now causes strong wealth effects even under IT and these induce the same quick rebound of both inflation and output observed under PLT.

Welfare. Fig. 5 shows how the loss function in the F regime varies when we let the inflation coefficient in the monetary rule change assuming even negative values. Comparing this figure with the left panel of Fig. 2, one can see how this new specification for the IT rule alters the results for welfare. Now the loss functions are nearly constant, much flatter than under PLT. At the zero cut-off the welfare loss assumes the same minimum value under both IT and PLT but, generally, the overall welfare loss is smaller under IT. Therefore, a rule with a negative ϕ_{π} and a high level of smoothing seems to outperform PLT from the welfare point of view, at least conditional on a demand shock.

Zero lower bound. As already shown in Fig. 4, with a negative reaction to inflation in the IT rule, the nominal interest rate actually increases in response to a negative demand shock under fiscal dominance. The same happens if the central bank adopts a PLT rule. Therefore, both approaches are adequate to avoid the ZLB. The pattern of output, inflation, and debt is roughly the same under both types of monetary policy rules. For completeness, we report the impulse responses in Fig. 6.

A.4 shows the case with $\phi_{\pi} < 0$ without any interest rate smoothing. Impulse responses turn out to be similar to the ones in this section (again using $\phi_{\pi} = -0.5$), while the total welfare loss is similar to Fig. 5 for negative ϕ_{π} , but then increases steeply for positive ϕ_{π} , since there it coincides with the benchmark results of Fig. 2. Therefore, we can conclude that the smoothing term, rather than the mere negative ϕ_{π} , is the main driver of the flatter total loss under IT obtained in Fig. 5. Furthermore, without smoothing, it is still true that both IT and PLT allow avoiding the ZLB, but with IT output and inflation are higher than with PLT and the path of debt becomes more favorable.



Fig. 5. Loss function after a demand shock for $\phi_{\pi} < 0$ in the F regime. Notes: On the left the case under IT with $\phi_{\pi} < 0$ and $\rho_{R} = 0.9$, on the right the case under PLT already shown in Fig. 2.



Fig. 6. Impulse response function to a negative demand shock in the F regime, accounting for the ZLB, with $\phi_{\pi} < 0$ and smoothing in the IT rule. Notes: Solid blue lines correspond to inflation targeting, dashed red lines correspond to price level targeting. Parametrization: $\phi_{\pi} = -0.5$ and $\rho_{R} = 0.9$ (IT), $\phi_{p} = -0.1$ (PLT), $\gamma = 0$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

4. Robustness

In this section we want to extend our analysis to consider the effect of different shocks and parametrization. In Section 4.1 we focus on a fiscal shock and a supply shock.¹⁷ In Section 4.2 we assess how the previous results for a demand shock change if we use alternative parametrizations for the monetary coefficients ϕ_{π} and ϕ_{p} , for the fiscal coefficient γ , and for the loss function weights.

¹⁷ Results for a monetary shock, which do not reveal large differences between IT and PLT, are omitted for brevity but are available from the authors upon request.

4.1. The effects of other shocks

4.1.1. Fiscal shock

Thanks to the Ricardian equivalence, a shock to lump-sum taxes in the M regime turns out to be ineffective under both IT and PLT. For this reason, we focus our attention on the effects of a fiscal shock in the F regime only. In the following simulations we set to zero the persistence of the fiscal shock.

Impulse responses. Fig. 7 shows the effects of a reduction in lump-sum taxes in the F regime. Under IT, the tax cut decreases the present discounted value of surpluses (the RHS of the government debt valuation, Eq. (9)). The government debt owned by households exceeds the present discounted value of surpluses (LHS > RHS) and this represents a positive wealth effect: agents anticipate that surpluses will not be covered by future fiscal adjustments and thus they convert bonds into current consumption goods. As a consequence, spending and inflation surge. The increase in inflation is accommodated by the central bank, which raises nominal rates but less than one for one with inflation. This stimulates output and makes the real debt decrease.



Fig. 7. Impulse response function to an expansionary fiscal shock in the F regime. Notes: Solid blue lines correspond to inflation targeting, dashed red lines correspond to price level targeting. Parametrization in Table 3 (F regime only), with $\rho_{\psi} = 0$.



Fig. 8. Loss function after a fiscal shock in the F regime.

The dynamic is different under PLT. As before, a tax cut brings the present discounted value of surpluses below the value of government bonds owned by household (LHS > RHS), engineering a positive wealth effect that boosts spending and inflation. However, following the increase in the price level, the central bank now decreases the nominal rate, causing the real interest rate to fall by a greater extent than under IT. The more real rates decrease, the more they offset the reduction in the budget surplus. Therefore, the wedge between the LHS and the RHS will be smaller, and so will the wealth effect, resulting in a milder increase in inflation and in the real debt. Furthermore, the largest fall in real rates stimulates output.

Welfare. As shown in Fig. 8, the loss functions under IT and PLT follow a pattern similar to the benchmark of Fig. 2. In the IT case, the overall loss function increases as ϕ_{π} approaches unity, so a very passive monetary policy is preferred. The opposite is true in the PLT case: the overall loss decreases with ϕ_p , so a less passive policy is preferred. Even after a fiscal shock, a PLT rule is associated with a larger output volatility and a smaller inflation volatility than an IT rule. However, for very passive monetary policy, the overall welfare loss is smaller under IT while, for mildly passive monetary policies, PLT largely outperforms IT.

4.1.2. Supply shock

Impulse responses. We now turn to the effects of a supply shock, with persistence equal to 0.9. As Fig. 9 shows, this shock reduces output, increases inflation and, in our model where the monetary rule reacts just to inflation, increases the nominal interest rate in all specifications that we consider. In the M regime, while output reacts in the same way under both IT and PLT, the adoption of a PLT approach avoids a large spike in inflation, if compared to the IT case, so that the central bank does not respond with a large increase in the nominal interest rate. In the F regime, the advantages of adopting a PLT approach are less evident: while the decline in output is milder in the first two quarters, in the following ten quarters there is a lower output loss under IT and then the two approaches return similar results. On the other hand, the rise of inflation is lower under PLT and the central bank reacts by slightly decreasing the nominal rate instead of increasing it, as it does under IT.

Welfare. Differently from previous shocks, the main source of welfare loss is due to output volatility, while the loss due to inflation seems generally lower, as shown in Fig. 10. In the M regime, we find that monetary policy can completely stabilize inflation after a supply shock by making ϕ_{π} (or ϕ_{p}) tend to infinity. However, differently from what happens with a demand shock, by doing so the central bank generates a higher output loss and the overall welfare loss tends to increase under PLT, while it still decreases under IT. Except for monetary coefficients close to the cut-off value for determinacy, where PLT largely outperforms IT, for more active policies, IT slightly outperforms PLT.

In the F regime, the overall loss function for the IT case tends to increase as ϕ_{π} approaches the cut-off value of unity, so a very passive monetary policy would be preferred. The opposite happens in the PLT case, where the overall loss is decreasing in ϕ_p , so a less passive policy is preferred. If we compare the two rules, PLT largely outperforms IT when ϕ_{π} and ϕ_p are close to their cut-off value for determinacy, while for more passive policies the total loss can be lower under IT than under PLT. This result is driven by output gap volatility, which grows larger under PLT as monetary policy becomes more passive, while inflation volatility is always lower under PLT.



Fig. 9. Impulse response function to a supply shock. Notes: Solid blue lines correspond to inflation targeting, dashed red lines correspond to price level targeting. Parametrization in Table 3. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

4.2. Other admissible parametrizations

4.2.1. Monetary strategy

We now compare once more the impulse response functions following a deflationary demand shock under IT to those under PLT, to check if our previous results depend on a particular parametrization for the monetary policy parameters ϕ_{π} and ϕ_p . We still assume the benchmark calibration for passive and active fiscal policy but now we consider a range of values for the monetary policy parameters (i.e., $\phi_{\pi} \in [1.05, 3]$ or $\phi_p \in [0.05, 3]$ in the M regime and $\phi_{\pi} \in [-1, 0.95]$ or $\phi_p \in [-2, -0.05]$ in the F regime).¹⁸ The resulting impulse responses are shown in Fig. 11.

The results of Fig. 1 are generally confirmed. Except for the case of ϕ_p close to zero in the M regime, which causes an initial large drop in output and inflation and a big rise in debt, we find that in both regimes inflation, output, and the nominal in-

¹⁸ We chose these ranges so to include the values employed by Billi and Walsh (2022) ($\phi_{\pi} = 2$ in the M regime and $\phi_{\pi} = 0.8$ or $\phi_{\pi} = 0$ in the F regime), Davig and Leeper (2011) ($\phi_{\pi} = 1.29$ in the M regime and $\phi_{\pi} = 0.53$ in the F regime), and Giannoni (2014) ($\phi_{p} = 0.6$ in the M regime)



Fig. 10. Loss function after a supply shock.

terest rate decrease by a greater extent under IT. Therefore, we tend to confirm the advantages of adopting a PLT approach, and this is generally true for both regimes and across different parametrizations. If we focus our attention on the PLT rule in the F regime, there is again a much larger reduction in output on impact but the large wealth effects induce a much quicker rebound of both inflation and output. Under a PLT rule, when the central bank reacts with a large negative ϕ_p to the subsequent inflation rebound (after period 3), the nominal rate can happen to become negative but, for the majority of values of ϕ_p , a demand shock does not make the nominal interest rate decrease, preventing the economy from hitting the ZLB.

A policy rate path. Fig. 2 showed that, in a regime of fiscal dominance, both under IT and PLT, the welfare loss is minimized when $\phi_{\pi} = \phi_p = 0$. This is akin to pegging the interest rate. While interest rate pegging would generate indeterminacy if adopted under the traditional regime of monetary dominance, we stress that it would return determinacy and minimize the loss function in a fiscally-led regime.

4.2.2. Fiscal strategy

Fig. 12 shows what happens, following a negative demand shock, when the monetary authority adopts an IT or a PLT rule, when we consider different values of the fiscal rule coefficient (i.e., $\gamma \in [0.025, 0.5]$ in the M regime and $\gamma \in [-0.05 - 0.025]$ in the F regime).¹⁹

¹⁹ Again, these ranges include the values employed in Billi and Walsh (2022) ($\gamma = 0.2$ in the M regime and $\gamma = 0$ in the F regime) and in Davig and Leeper (2011) ($\gamma = 0.071$ in the M regime and $\gamma = -0.025$ in the F regime).



Fig. 11. Impulse response function to a negative demand shock for different values of the monetary policy parameters. Notes: Blue bands correspond to inflation targeting, red bands correspond to price level targeting. Parametrization of the F regime: $\phi_{\pi} \in [-1, 0.95]$ for IT, $\phi_{p} \in [-2, -0.05]$ for PLT, $\gamma = 0$ for both. Parametrization of the M regime: $\phi_{\pi} \in [1.05, 3]$ for IT, $\phi_{p} \in [0.05, 3]$ for PLT, $\gamma = 0.2$ for both. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 12 confirms the main findings of Fig. 1. As it is well known, in an M regime the path of lump-sum taxes (and of debt) does not affect inflation, output, and interest rates, thanks to the Ricardian equivalence. Therefore, the impulse responses of these variables are identical to those in Fig. 1, right column.

In an F regime, we consider the case of an irresponsible fiscal policy when the government cuts taxes as the level of debt rises ($\gamma < 0$). In the first few quarters, it is still true that a negative demand shock decreases inflation by a lesser extent under PLT, but now the more negative γ is, the earlier inflation under IT outpaces inflation under PLT. Results for output are qualitatively confirmed. Once again, we can use the government debt valuation Eq. (9) to better understand how results are affected by a change in γ . Under IT, the fall in inflation increases real government debt, so the LHS increases. Since, with an irresponsible fiscal policy, the government responds by cutting taxes, the RHS tends to fall. The resulting higher wedge between the LHS and the RHS induces huge wealth effects that make inflation and output decrease less, the more γ is negative. Therefore, the lower is γ , the less inflation (and output) will fall and the less the nominal rate will decrease – the lowest value of γ actually corresponds to the upper edge of the blue band in the left column of Fig. 12. Under PLT, as a response to the fall



Fig. 12. Impulse response function to a negative demand shock for different values of the fiscal policy parameter γ . Notes: Blue bands correspond to inflation targeting, red bands correspond to price level targeting. Parametrization of the M regime: $\phi_{\pi} = 1.2$ for IT, $\phi_{p} = 1.2$ for PLT, $\gamma \in [0.025, 0.5]$ for both. Parametrization of the F regime: $\phi_{\pi} = 0.9$ for IT, $\phi_{p} = -0.1$ for PLT, $\gamma \in [-0.05 - 0.025]$ for both. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

in inflation, the central bank raises the nominal interest rate and this increases the real interest rate, inducing an initial strong negative response of output. This effect widens the difference between the LHS (which increases on impact) and the RHS because the real rate appears with a negative sign on the RHS. In addition, the RHS decreases further since, as debt rises, taxes decrease. Hence, wealth effects are now very strong, inducing a rebound of both inflation and output that is quicker when γ is smaller. In this case, PLT also serves the purpose of preventing the economy from hitting the ZLB.

4.2.3. Alternative loss functions

The loss functions in the previous analysis were computed for benchmark fiscal rule parameters in Table 3. We tried different parametrizations for γ and found that the associated loss functions are very similar to the results of Fig. 2.

Furthermore, if we use the loss function weights previously employed by Woodford (1999) and Giannoni (2014) (i.e., $\omega_{\pi} = 1, \omega_{y} = 0.048$, and $\omega_{R} = 0.236$) we still confirm our previous findings.²⁰ Results are, again, roughly unchanged, following

²⁰ For the sake of brevity, these results are available from the authors upon request.



Fig. 13. Loss function after a demand shock without the interest rate variability term in the loss function.

a demand shock, even if we put to zero the weight associated with interest rates in the loss function, as shown in Fig. 13. The total loss associated with the PLT rule becomes lower and flatter in the F regime with respect to Fig. 2. The one in the M regime is almost identical to Fig. 2, signalling the low volatility of the nominal interest rate under PLT as shown by the impulse response functions. The total welfare loss for IT is much lower than in Fig. 2 and almost entirely given by the inflation component, as the value of ϕ_{π} gets closer to one. This reveals that values of ϕ_{π} close to one induce a large nominal interest rate volatility under both regimes M and F. If we compare the two monetary rules, PLT generally outperforms IT in minimizing the welfare loss, except in the F regime in the case of $\phi_{\pi} < 0$ – not shown in Fig. 2 – where total loss under IT is lower.

5. Conclusions

A negative demand shock has less severe consequences on the economy if the central bank is adopting a PLT rule: inflation, output, and the nominal interest rate decrease less than under an IT rule. While interest rates would be constrained by the ZLB under an IT framework, PLT allows the central bank to avoid it. We show that these results, well established for the traditional monetary regime, hold true even in the less studied fiscal regime. This case is of interest since it combines both the recent adoption of a makeup strategy and the need for more active fiscal policies to contrast deflationary risks.

Moreover, if the PLT rule is implemented during a regime of fiscal dominance, the central bank must *raise* the nominal interest rate following a deflationary demand shock and, by doing so, it avoids a prolonged period of low output and inflation. The nominal interest rate increases because, if fiscal policy is active, determinacy requires monetary policy to be passive. However, under PLT Leeper's determinacy condition is more restrictive: the inflation coefficient in the interest rate rule should be lower than zero, rather than one. Hence, the nominal interest rate increases if the deviation of the price level from

its target path (the price gap) is negative. Following a demand shock, this "inverse" reaction of the policy rate to inflation exacerbates wealth effects; while, if a shock hits the government surplus (see the tax shock above), it dampens them. Analysing the welfare loss function, we find that PLT generally dominates IT even from a social welfare point of view, unless one considers negative values of the response coefficients in the IT rule coupled with a high degree of inertia. Trivially, the PLT and IT rules become very similar under this configuration. Results are robust to alternative parameterizations for the monetary and fiscal coefficients.

Wealth effects are amplified even if one considers IT under fiscal dominance with a negative inflation coefficient in the reaction function coupled with a high degree of interest rate smoothing. In this case, we find that PLT is not superior to IT in the presence of a demand shock.

Bernanke (2017) proposal of a temporary price-level targeting entails a switch from IT to PLT whenever the economy is near the ZLB. According to our analysis, adopting a PLT approach could be beneficial, whatever the monetary/fiscal policy mix. Future work should address this topic by employing a Markov-switching approach to analyse both the switch from IT to PLT and that from a monetary to a fiscal regime. The Markov switching framework would, among other things, be able to account for the presence of expectations effects and, since expectations of future policy moves affect the present value of primary surpluses, this would shed more light on the role of wealth effects too.

Data availability

Data will be made available on request.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Sticky price model

A.1. Model setup

In the paper we consider a simple New Keynesian model with fiscal policy. The model is well known and a more detailed description can be found in the literature. For instance, <u>Bhattarai et al.</u> (2014) use a virtually identical version of the model. **Households.** The representative household maximises lifetime utility

$$E_0\sum_{t=0}^{\infty}\beta^t U(c_t,n_t,\varepsilon_t)$$

where $U(c_t, n_t, \varepsilon_t) = \varepsilon_t \left(\log c_t - \mu \frac{n_t^{1+\xi}}{1+\xi} \right)$, subject to the sequence of budget constraints

$$P_t c_t + R_t^{-1} B_t = P_t w_t n_t + B_{t-1} + \Pi_t - P_t \tau_t$$

Here, E_0 is the expectations operator conditional on information up to time t = 0, P_t is the price level, c_t is real consumption, n_t is labour hours, w_t is the level of real wages, Π_t are real profits remitted by intermediate firms, τ_t are real lump-sum taxes paid to the government, R_t is the gross nominal interest rate, B_t is the amount of nominal one-period government bonds that the consumer buys in t at the unit price R_t^{-1} . As for the parameters, β is the intertemporal discount factor, ζ is the inverse of the Frisch elasticity of labour supply, and μ is a scale parameter. The exogenous variable ε_t is an intertemporal preference shock.

From the first-order conditions, we can derive the Euler equation

$$\frac{1}{c_t} = \beta E_t \left[\frac{\varepsilon_{t+1}}{\varepsilon_t} \frac{1}{c_{t+1}} \frac{R_t}{\pi_{t+1}} \right],$$

where $\pi_t = P_t / P_{t-1}$, and the labour supply schedule

$$w_t = \mu n_t^{\xi} c_t.$$

Final good producers. In each period, a final good y_t is produced by perfectly competitive firms, using a continuum of intermediate inputs $y_{i,t}$ indexed by $i \in [0, 1]$ and a standard CES production function $y_t = (\int_0^1 y_{i,t}^{(\epsilon-1)/\epsilon} di)^{\epsilon/(\epsilon-1)}$, with $\epsilon > 1$. Final good producers' demand schedules for intermediate good quantities are $y_{i,t} = (P_{i,t}/P_t)^{-\epsilon}y_t$, where the aggregate price index is defined as $P_t = (\int_0^1 P_{i,t}^{1-\epsilon} di)^{1/(1-\epsilon)}$.

Intermediate goods producers. A continuum of intermediate goods are produced by firms with production function $y_{i,t} = A_t n_{i,t}$. Intermediate producers compete monopolistically and set prices according to the usual Calvo mechanism. In ev-

ery period, each firm has a fixed probability $1 - \alpha$ to re-optimise its price $P_{i,t}^*$ in order to maximise the discounted stream of profits. With probability α , instead, the firm keeps its nominal price unchanged. Firms will discount profits *j* periods into the future by $Q_{t,t+j}\alpha^j$, where $Q_{t,t+j} = \beta^j \frac{U_{ct+k}}{U_{ct}}$ is the stochastic discount factor. Following the usual steps, the optimal relative price satisfies the first-order condition

$$p_{i,t}^* \equiv \frac{P_{i,t}^*}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{j=0}^{\infty} (\alpha \beta)^j U_{c,t+j} m c_{t+j} \left(\frac{P_{t+j}}{P_t}\right)^{\epsilon} y_{t+j}}{E_t \sum_{j=0}^{\infty} (\alpha \beta)^j U_{c,t+j} \left(\frac{P_{t+j}}{P_t}\right)^{\epsilon - 1} y_{t+j}},$$

or more compactly

$$p_t^* = \frac{\epsilon}{\epsilon - 1} \frac{x_{1,t}}{x_{2,t}},$$

with

$$\begin{split} x_{1,t} &= E_t \sum_{j=0}^{\infty} (\alpha \beta)^j U_{c,t+j} m c_{t+j} \Big(\frac{P_{t+j}}{P_t} \Big)^{\epsilon} y_{t+j} = U_{c,t} m c_t y_t + \alpha \beta E_t \big[\pi_{t+1}^{\epsilon} x_{1,t+1} \big], \\ x_{2,t} &= E_t \sum_{j=0}^{\infty} (\alpha \beta)^j U_{c,t+j} \Big(\frac{P_{t+j}}{P_t} \Big)^{\epsilon-1} y_{t+j} = U_{c,t} y_t + \alpha \beta E_t \big[\pi_{t+1}^{\epsilon-1} x_{2,t+1} \big]. \end{split}$$

Note that the real marginal cost $mc_t = w_t/A_t$ is identical for every firm, and so is the optimal price. Therefore, aggregate inflation evolves according to

$$1 = \alpha \pi_t^{\epsilon - 1} + (1 - \alpha) \left(p_{i,t}^* \right)^{1 - \epsilon}$$

Individual firms demand labour according to the relation $A_t n_{i,t} = (P_{i,t}/P_t)^{-\epsilon} y_t$. Aggregating this expression yields $A_t n_t = y_t s_t$, where $n_t \equiv \int_0^1 n_{i,t} di$ and $s_t \equiv \int_0^1 (P_{i,t}/P_t)^{-\epsilon} di$. The variable s_t measures the dispersion of relative prices across intermediate firms. s_t is bounded below by one and represents the resource costs (or inefficiency loss) due to relative price dispersion under the Calvo mechanism. s_t can be written recursively as

$$s_t = (1 - \alpha) \left(p_{i,t}^* \right)^{-\epsilon} + \alpha \pi_t^{\epsilon} s_{t-1}.$$

Fiscal and monetary policy. In real terms, the government budget constraint is given by

$$\frac{B_t}{R_t P_t} = \frac{B_{t-1}}{P_t} + g_t - \tau_t$$

For simplicity, we assume that real government spending is fixed at the steady state level $g_t = g$. After defining real debt as $b_t \equiv B_t/P_t$, we have

$$\frac{b_t}{R_t} = \frac{b_{t-1}}{\pi_t} + g - \tau_t.$$

The government sets the level of lump sum taxes according to the fiscal rule

$$\frac{\tau_t}{\tau} = \left(\frac{b_t}{b}\right)^{\phi_b} e^{\psi_t},$$

where ψ_t is a fiscal policy shock.

The central bank has an inflation target (π^*) and a price level target (P_t^*), and sets the nominal rate in response to the deviations from the targets:

$$\frac{R_t}{R} = \left(\frac{P_t}{P_t^*}\right)^{\phi_p} \left(\frac{\pi_t}{\pi^*}\right)^{\phi_\pi} e^{\theta_t}$$

where θ_t is a monetary policy shock. Note that the two targets are related by

$$P_t^* = P_{t-1}^* \left(\frac{P_{t-1}}{P_{t-1}^*}\right)^{1-\delta} \pi^*,$$

where $1 - \delta$ is the weight associated with past deviations from the price target when establishing the new price target for time *t*. When $\delta = 1$ the price target follows an exogenous path ($P_t^* = P_{t-1}^*\pi^*$), while for $\delta = 0$ the target is readjusted in each

period to be consistent with the inflation target ($P_t^* = P_{t-1}\pi^*$). In other words, bygone price deviations are bygone. To elaborate further, we can use the identity $P_t = P_{t-1}\pi_t$ to get

$$\frac{P_t}{P_t^*} = \frac{P_{t-1}\pi_t}{P_{t-1}^* \left(\frac{P_{t-1}}{P_{t-1}^*}\right)^{1-\delta}\pi^*} = \left(\frac{P_{t-1}}{P_{t-1}^*}\right)^{\delta} \frac{\pi_t}{\pi^*}.$$

Note that when $\delta = 0$ price deviations are identical to inflation deviations. We consider the following two versions of the monetary policy rule.

• Inflation targeting, with $\delta = 0$ and $\phi_p = 0$, so that the rule reduces to

$$\frac{R_t}{R} = \left(\frac{\pi_t}{\pi^*}\right)^{\phi_{\pi}} e^{\theta_t}.$$

• Price level targeting, with $\delta = 1$ and $\phi_{\pi} = 0$, so that:

$$\frac{R_t}{R} = \left(\frac{P_t}{P_t^*}\right)^{\phi_p} e^{\theta_t}.$$

If we divide the last expression by the same expression evaluated at time t - 1, we obtain

$$\frac{R_t}{R} = \frac{R_{t-1}}{R} \left(\frac{\pi_t}{\pi^*}\right)^{\phi_p} e^{\Delta \theta_t}.$$

Complete nonlinear model. The dynamics of aggregate variables are described by the following set of equations, here reproduced for convenience:

$$\begin{split} U_{c,t} &= \frac{\varepsilon_t}{c_t}, \\ 1 &= \beta E_t \left(\frac{\varepsilon_{t+1}}{\varepsilon_t} \frac{c_t}{c_{t+1}} \frac{R_t}{\pi_{t+1}} \right), \\ w_t &= \mu n_t^{\tilde{\varepsilon}} C_t, \\ 1 &= \alpha \pi_t^{\epsilon-1} + (1-\alpha) \left(p_t^* \right)^{1-\epsilon}, \\ p_t^* &= \frac{\epsilon}{\epsilon-1} \frac{x_{1,t}}{x_{2,t}}, \\ x_{1,t} &= \mathcal{E}_t \frac{y_{t}}{c_t} m C_t + \alpha \beta E_t \left[\pi_{t+1}^{\epsilon-1} x_{1,t+1} \right], \\ x_{2,t} &= \mathcal{E}_t \frac{y_{t}}{c_t} + \alpha \beta E_t \left[\pi_{t+1}^{\epsilon-1} x_{2,t+1} \right], \\ m C_t &= \frac{w_t}{A_t}, \\ s_t &= (1-\alpha) \left(p_t^* \right)^{-\epsilon} + \alpha \pi_t^{\epsilon} s_{t-1}, \\ A_t n_t &= s_t y_t, \\ y_t &= C_t + g, \\ \frac{b_t}{B_t} &= \frac{b_{t-1}}{\pi_t} + g - \tau_t, \\ \frac{\tau_t}{\tau} &= \left(\frac{b_{t-1}}{b} \right)^{\gamma} e^{\psi_t}, \\ \frac{R_t}{R} &= \left(\frac{p_t}{P_t^*} \right)^{\phi_p} \left(\frac{\pi_t}{\pi^*} \right)^{\phi_\pi} e^{\theta_t}, \\ \frac{p_t}{P_t} &= \left(\frac{p_{t-1}}{p_{t-1}^*} \right)^{\delta} \frac{\pi_t}{\pi^*}. \end{split}$$

Loglinearized model. The model can be log-linearized around the zero-inflation steady state to obtain the equations reported in Section 2. In particular, we have

$$\begin{split} &\frac{1}{\hat{c}}\hat{y}_t = \frac{1}{\hat{c}}E_t\hat{y}_{t+1} - \left(\hat{R}_t - E_t\hat{\pi}_{t+1}\right) + \hat{\varepsilon}_t - E_t\hat{\varepsilon}_{t+1}, \\ &\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa\hat{y}_t + \nu_t, \\ &\hat{b}_t = \hat{R}_t + \frac{1}{\beta}\hat{b}_{t-1} - \frac{1}{\beta}\hat{\pi}_t - \frac{1}{b\beta}\hat{\tau}_t, \\ &\hat{\tau}_t = \gamma\hat{b}_{t-1} + \psi_t, \end{split}$$

where we used the reparametrizations

$$\bar{c} = \frac{c}{y}, \qquad \bar{b} = \frac{b}{\tau}, \qquad \kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \left(\xi + \frac{1}{\bar{c}}\right),$$

and defined the supply-side shock v_t as

$$v_t = -rac{(1-lpha)(1-lphaeta)}{lpha}(\xi+1)\widehat{A}_t.$$

The model is closed by the monetary policy rule, in either its traditional IT version

$$\widehat{R}_t = \phi_\pi \widehat{\pi}_t + \theta_t,$$

or in the PLT version

$$R_t = R_{t-1} + \phi_p \hat{\pi}_t + \Delta \theta_t.$$

A.2. Determinacy analysis with PLT

Under PLT, the New Keynesian model of Section 2 is given by the following equations:

$$\begin{split} &\frac{1}{c} \hat{y}_{t} = \frac{1}{c} E_{t} \hat{y}_{t+1} - \left(\hat{R}_{t} - E_{t} \hat{\pi}_{t+1} \right) + (1 - \rho_{\varepsilon}) \varepsilon_{t}, \\ &\hat{\pi}_{t} = \beta E_{t} \hat{\pi}_{t+1} + \kappa \hat{y}_{t} + v_{t}, \\ &\hat{R}_{t+1} = \phi_{p} \hat{\pi}_{t+1} + \hat{R}_{t} + \Delta \theta_{t+1}, \\ &\hat{b}_{t+1} = \frac{1}{\beta} \left(1 - \frac{\gamma}{b} \right) \hat{b}_{t} + \hat{R}_{t+1} - \frac{1}{\beta} \hat{\pi}_{t+1} - \frac{1}{b\beta} \psi_{t+1}, \end{split}$$

which, disregarding expectations errors, can be written in matrix form as:

$$\begin{bmatrix} -\frac{1}{c} & -1 & 0 & 0\\ 0 & -\beta & 0 & 0\\ 0 & -\phi_p & 1 & 0\\ 0 & \frac{1}{\beta} & -1 & 1 \end{bmatrix} \begin{bmatrix} \hat{y}_{t+1} \\ \hat{\pi}_{t+1} \\ \hat{b}_{t+1} \end{bmatrix} = \begin{bmatrix} -\frac{1}{c} & 0 & -1 & 0\\ \kappa & -1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & \frac{1}{\beta} \left(1 - \frac{\gamma}{b}\right) \end{bmatrix} \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{b}_t \end{bmatrix} + \begin{bmatrix} 1 - \rho_\varepsilon & 0 & 0\\ 0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -\frac{1}{b\beta} \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ \Delta \theta_{t+1} \\ \psi_{t+1} \end{bmatrix}$$

Inverting the matrix on the left-hand side, we obtain

$$\begin{bmatrix} \hat{y}_{t+1} \\ \hat{\pi}_{t+1} \\ \hat{h}_{t+1} \\ \hat{b}_{t+1} \end{bmatrix} = \begin{bmatrix} \bar{c}\frac{\kappa}{\beta} + 1 & -\frac{c}{\beta} & \bar{c} & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 & 0 \\ -\frac{\kappa}{\beta} \phi_p & \frac{1}{\beta} \phi_p & 1 & 0 \\ -\frac{\kappa}{\beta^2} \left(\beta \phi_p - 1\right) & \frac{1}{\beta^2} \left(\beta \phi_p - 1\right) & 1 & \frac{1}{\beta} \left(1 - \frac{\gamma}{b}\right) \end{bmatrix} \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{b}_t \end{bmatrix} + \begin{bmatrix} \bar{c}(\rho_\varepsilon - 1) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{b\beta} \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ \Delta \theta_{t+1} \\ \psi_{t+1} \end{bmatrix}$$

or $Y_{t+1} = AY_t + B\epsilon_{t+1}$. One of the four eigenvalues of the matrix A is $\frac{1}{\beta} \left(1 - \frac{\gamma}{b}\right)$. To study determinacy we have to analyse the other three eigenvalues of the top-left submatrix:

$$\widetilde{A} = \begin{bmatrix} \overline{c} \frac{\kappa}{\beta} + 1 & -\frac{c}{\beta} & \overline{c} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ -\frac{\kappa}{\beta} \phi_p & \frac{1}{\beta} \phi_p & 1 \end{bmatrix}.$$

A.2.1. Monetary regime

The eigenvalue $\frac{1}{\beta} \left(1 - \frac{\gamma}{b}\right)$ is inside the unit circle when fiscal policy is passive, or

$$\bar{b}(1-\beta) < \gamma < \bar{b}(1+\beta).$$

The model contains two jump variables, so the Blanchard-Khan conditions for determinacy require \tilde{A} to have one eigenvalue inside and two eigenvalues outside the unit circle. Woodford (2003, appendix C) states these conditions in terms of the determinant, the trace and the sum of the principal minors of \tilde{A} , whose values are the following:

$$det\left(\widetilde{A}\right) = \frac{1}{\beta},$$

$$tr\left(\widetilde{A}\right) = 2 + \frac{1}{\beta} + \overline{c}\frac{\kappa}{\beta},$$

$$M\left(\widetilde{A}\right) = \frac{1}{\beta}\left(2 + \overline{c}\kappa + \beta + \overline{c}\kappa\phi_p\right).$$

In order to have two eigenvalues outside and one inside, one of the following three cases must hold.

• Case 1. Two restrictions should be satisfied simultaneously:

$$1 - tr(\widetilde{A}) + M(\widetilde{A}) - \det(\widetilde{A}) < 0$$

$$-1 - tr(\widetilde{A}) - M(\widetilde{A}) - \det(\widetilde{A}) > 0$$

The first is verified for $\phi_p < 0$, but the second is never verified, so this case does not hold.

• Case 2. Three conditions are required:

$$1 - tr(\widetilde{A}) + M(\widetilde{A}) - \det(\widetilde{A}) > 0$$

-1 - tr(\tilde{A}) - M(\tilde{A}) - \det(\tilde{A}) < 0
$$\det(\tilde{A})^{2} - \det(\tilde{A})tr(\tilde{A}) + M(\tilde{A}) - 1 > 0$$

The first condition is satisfied for $\phi_p > 0$, the second is always true while the third holds for $\phi_p > \frac{1}{\beta} - 1$. • Case 3. Four conditions are required:

$$\begin{split} &1 - tr\big(\widetilde{A}\big) + M\big(\widetilde{A}\big) - \det\big(\widetilde{A}\big) > 0 \\ &-1 - tr\big(\widetilde{A}\big) - M\big(\widetilde{A}\big) - \det\big(\widetilde{A}\big) < 0 \\ &\det\big(\widetilde{A}\big)^2 - \det\big(\widetilde{A}\big)tr\big(\widetilde{A}\big) + M\big(\widetilde{A}\big) - 1 < 0 \\ &\left|tr\big(\widetilde{A}\big)\right| > 3 \end{split}$$

The first is satisfied for $\phi_p > 0$, the second and the fourth are always satisfied, the third holds true for $\phi_p < \frac{1}{\beta} - 1$.

In conclusion, from cases 2 and 3 we get the sole condition is $\phi_p > 0$. Therefore, when fiscal policy is passive, determinacy can be achieved for $\phi_p > 0$. If the Taylor rule satisfies this condition, monetary policy is said to be active.

A.2.2. Fiscal regime

The eigenvalue $\frac{1}{\beta} \left(1 - \frac{\gamma}{b}\right)$ is outside the unit circle when fiscal policy is active, i.e.

 $\gamma < \overline{b}(1-\beta)$ or $\gamma > \overline{b}(1+\beta)$.

To respect Blanchard-Khan conditions for determinacy, \tilde{A} should have two eigenvalues inside and one outside of the unit circle or, equivalently, we can require the inverse \tilde{A} to have two eigenvalues outside and one inside. The inverse matrix is

$$\widetilde{A}^{-1} = egin{bmatrix} 1 & ar{c}(1+\phi_p) & -ar{c} \ \kappa & ar{c}\kappa+eta+ar{c}\kappa\phi_p & -ar{c}\kappa \ 0 & -\phi_p & 1 \end{bmatrix}$$

For this matrix we have

$$det\left(\widetilde{A}^{-1}\right) = \beta$$
$$tr\left(\widetilde{A}^{-1}\right) = \bar{c}\kappa + \beta + \bar{c}\kappa\phi_p + 2$$
$$M\left(\widetilde{A}^{-1}\right) = 2\beta + 1 + \bar{c}\kappa.$$

As before, we should consider the three cases outlined above. We can exclude case 1 since the second condition is never verified. From case 2 we find that the first condition gives $\phi_p < 0$, the second is always true and the third is verified for $\phi_p < \frac{1}{\beta} - 1$ (that is a less stringent condition than $\phi_p < 0$). The third case can be excluded because the third condition gives rise to a contradiction. Therefore, when fiscal policy is active determinacy can be achieved for $\phi_p < 0$. If the Taylor rule satisfies this condition, monetary policy is said to be passive.

A.3. Rational expectations solutions in a flexible price model

We now solve our model to find the solution for inflation both in the monetary regime and in the fiscal regime. To find the solutions, we follow the procedure of Bhattarai et al. (2014), which is based on the spectral decomposition of matrix

 $A = VDV^{-1}$, where D and V are the matrices with the eigenvalues and the eigenvectors of A. The flexible-price model in Section 2.2 is the following:

$$\begin{split} \hat{R}_{t} &= E_{t} \hat{\pi}_{t+1} + r_{t} \\ \hat{R}_{t+1} &= \phi_{p} \hat{\pi}_{t+1} + \hat{R}_{t} + \Delta \theta_{t+1} \\ \hat{b}_{t+1} &= \frac{1}{\beta} \left(1 - \frac{\tau}{b} \gamma \right) \hat{b}_{t} + \hat{R}_{t+1} - \frac{1}{\beta} \hat{\pi}_{t+1} - \frac{1}{\beta} \frac{\tau}{b} \psi_{t+1}, \end{split}$$

which in matrix form becomes:

$$\begin{bmatrix} -1 & 0 & 0 \\ -\phi_p & 1 & 0 \\ \frac{1}{\beta} & -1 & 1 \end{bmatrix} \begin{bmatrix} \hat{\pi}_{t+1} \\ \hat{R}_{t+1} \\ \hat{b}_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\beta} (1 - \frac{\tau}{b} \gamma) \end{bmatrix} \begin{bmatrix} \hat{\pi}_t \\ \hat{R}_t \\ \hat{b}_t \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\beta} \frac{\tau}{b} \end{bmatrix} \begin{bmatrix} r_t \\ \Delta \theta_{t+1} \\ \psi_{t+1} \end{bmatrix}$$

Inverting the matrix on the left hand side, we obtain

$$\begin{bmatrix} \pi_{t+1} \\ R_{t+1} \\ b_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & \phi_p + 1 & 0 \\ 0 & 1 - \frac{1}{\beta} + \phi_p & \frac{1}{\beta} \left(1 - \frac{\tau}{b} \gamma \right) \end{bmatrix} \begin{bmatrix} \pi_t \\ R_t \\ b_t \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ -\phi_p & 1 & 0 \\ \frac{1}{\beta} - \phi_p & 1 & -\frac{1}{\beta} \frac{\tau}{b} \end{bmatrix} \begin{bmatrix} r_t \\ \Delta \theta_{t+1} \\ \psi_{t+1} \end{bmatrix},$$
(A1)

or $Y_{t+1} = AY_t + B\varepsilon_{t+1}$. To satisfy the Blanchard-Kahn conditions for determinacy, matrix A must have two eigenvalues inside and one outside the unit circle. The eigenvalues of A are 0, $(1 + \phi_p)$, and $\frac{1}{\beta}(1 - \frac{\tau}{b}\gamma)$.

Monetary regime. If $\frac{1}{\beta}(1 - \frac{\tau}{b}\gamma)$ is inside the unit circle, that is, in case of passive fiscal policy $(\gamma_{\tau} < \frac{b}{\tau}(1 + \beta))$, given the presence of one jump variable, to respect Blanchard-Khan conditions for determinacy, A' should have one eigenvalue inside and one outside of the unit circle which is the case if and only if $|tr(A')| > |1 + \det(A')|$ where $\det(A') = 0$ and $tr(A') = 1 + \phi_p$. This happens when $\phi_p > 0$.

Fiscal regimes. If $\frac{1}{\beta}(1 - \frac{\tau}{b}\gamma)$ is outside the unit circle, that is in case of active fiscal policy $(\gamma > \frac{b}{\tau}(1 + \beta))$, given the presence of one jump variable, to respect Blanchard-Khan conditions for determinacy, A' should have two eigenvalues inside the unit circle which is the case if and only if $|tr(A')| < |1 + \det(A')|$ and $|\det(A')| < 1$. This happens when $\phi_p < 0$.

In summary, determinacy requires either.

- $\phi_p > 0$ and $\left| \frac{1}{\beta} \left(\frac{\tau}{b} \gamma 1 \right) \right| < 1$, the AM/PF case, or
- $\phi_p < 0$ and $\left|\frac{1}{\beta} \left(\frac{\tau}{b} \gamma 1\right)\right| > 1$, the PM/AF case.

A.3.1. Solutions

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We now solve our flexible price model to find the solution for inflation both in the monetary and in the fiscal regime, both under PLT and under IT. We follow the procedure in Bhattarai et al. (2014).

PLT case. Under PLT the model reduces to

$$\begin{aligned} R_t &= E_t \hat{\pi}_{t+1} + r_t, \\ \hat{R}_t &= \phi_p \hat{\pi}_t + \hat{R}_{t-1} + \Delta \theta_t, \\ \hat{b}_t &= \frac{1}{\beta} \left(1 - \frac{\tau}{b} \gamma \right) \hat{b}_{t-1} + \hat{R}_t - \frac{1}{\beta} \hat{\pi}_t - \frac{1}{\beta} \frac{\tau}{b} \psi_t, \end{aligned}$$

which in matrix form becomes

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} E_t \hat{\pi}_{t+1} \\ \hat{R}_t \\ \hat{b}_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ \phi_p & 1 & 0 \\ -\frac{1}{\beta} & 0 & \frac{1}{\beta} (1 - \frac{\tau}{b} \gamma_{\tau}) \end{bmatrix} \begin{bmatrix} \hat{\pi}_t \\ \hat{R}_{t-1} \\ \hat{b}_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\beta} \frac{\tau}{b} \end{bmatrix} \begin{bmatrix} r_t \\ \Delta \theta_t \\ \psi_t \end{bmatrix}$$

Inverting the matrix on the left hand side, we obtain:

$$\begin{bmatrix} E_t \hat{\pi}_{t+1} \\ \hat{R}_t \\ \hat{b}_t \end{bmatrix} = \begin{bmatrix} \phi_p & 1 & 0 \\ \phi_p & 1 & 0 \\ \phi_p - \frac{1}{\beta} & 1 & \frac{1}{\beta} \left(1 - \frac{\tau}{b} \gamma \right) \end{bmatrix} \begin{bmatrix} \hat{\pi}_t \\ \hat{R}_{t-1} \\ \hat{b}_{t-1} \end{bmatrix} + \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -\frac{1}{\beta} \frac{\tau}{b} \end{bmatrix} \begin{bmatrix} r_t \\ \Delta \theta_t \\ \psi_t \end{bmatrix},$$
(A2)

or $Y_{t+1} = AY_t + B\varepsilon_t$. The procedure we follow to find the solutions is based on the spectral decomposition of matrix $A = VDV^{-1}$, where *D* and *V* are, respectively, the matrices whose elements are the eigenvalues and the eigenvectors of *A*:

$$A = \begin{bmatrix} v_{11} & v_{12} & 0 \\ v_{21} & v_{22} & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \phi_p + 1 & 0 \\ 0 & 0 & \frac{1}{\beta} \left(1 - \frac{\tau}{b} \gamma \right) \end{bmatrix} \begin{bmatrix} q_{11} & q_{12} & 0 \\ q_{21} & q_{22} & 0 \\ q_{31} & q_{32} & 1 \end{bmatrix}$$

For later use we note that:

$$\begin{split} D &= \begin{bmatrix} e_1 & 0 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & e_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \phi_p + 1 & 0 \\ 0 & 0 & \frac{1}{\beta} \left(1 - \frac{\tau}{b} \gamma \right) \end{bmatrix}, \\ V &= \begin{bmatrix} v_{11} & v_{12} & 0 \\ v_{21} & v_{22} & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 - \frac{\tau}{b} \gamma & \frac{\beta + \beta \phi_p + \frac{\tau}{b} \gamma - 1}{\beta + \beta \phi_p - 1} & 0 \\ -\phi_p \left(1 - \frac{\tau}{b} \gamma \right) & \frac{\beta + \beta \phi_p + \frac{\tau}{b} \gamma - 1}{\beta + \beta \phi_p - 1} & 0 \\ 1 & 1 & 1 \end{bmatrix}, \\ V^{-1} &= \begin{bmatrix} \frac{v_{22}}{v_{11} v_{22} - v_{12} v_{21}} & -\frac{v_{11}}{v_{11} v_{22} - v_{12} v_{21}} & 0 \\ -\frac{v_{21}}{v_{11} v_{22} - v_{12} v_{21}} & \frac{v_{11}}{v_{11} v_{22} - v_{12} v_{21}} & 0 \\ \frac{v_{21} - v_{22}}{v_{11} v_{22} - v_{12} v_{21}} & -\frac{v_{11} - v_{22}}{v_{11} v_{22} - v_{12} v_{21}} & 1 \end{bmatrix}. \end{split}$$

The system (A2) becomes:

$$\begin{bmatrix} E_t \hat{\pi}_{t+1} \\ \hat{R}_t \\ \hat{b}_t \end{bmatrix} = \begin{bmatrix} v_{11} & v_{12} & 0 \\ v_{21} & v_{22} & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \phi_p + 1 & 0 \\ 0 & 0 & \frac{1}{\beta} (1 - \frac{\tau}{b} \gamma) \end{bmatrix} \begin{bmatrix} q_{11} & q_{12} & 0 \\ q_{21} & q_{22} & 0 \\ q_{31} & q_{32} & 1 \end{bmatrix} \begin{bmatrix} \hat{\pi}_t \\ \hat{R}_{t-1} \\ \hat{b}_{t-1} \end{bmatrix} + \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -\frac{1}{\beta} \frac{\tau}{b} \end{bmatrix} \begin{bmatrix} r_t \\ \Delta \theta_t \\ \psi_t \end{bmatrix}.$$

We now define the vector of transformed variables $X_t = V^{-1}Y_t$, whose elements are

$$\begin{bmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \end{bmatrix} = \begin{bmatrix} q_{11} & q_{12} & 0 \\ q_{21} & q_{22} & 0 \\ q_{31} & q_{32} & 1 \end{bmatrix} \begin{bmatrix} \hat{\pi}_t \\ \hat{R}_{t-1} \\ \hat{b}_{t-1} \end{bmatrix} = \begin{bmatrix} q_{12}R_{t-1} + q_{11}\pi_t \\ q_{22}R_{t-1} + q_{21}\pi_t \\ b_{t-1} + q_{32}R_{t-1} + q_{31}\pi_t \end{bmatrix}$$

and rewrite the system as:

$$\begin{bmatrix} \mathbf{x}_{1,t+1} \\ \mathbf{x}_{2,t+1} \\ \mathbf{x}_{3,t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \phi_p + \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \frac{1}{\beta} \left(\mathbf{1} - \frac{\tau}{b} \gamma \right) \end{bmatrix} \begin{bmatrix} \mathbf{x}_{1,t} \\ \mathbf{x}_{2,t} \\ \mathbf{x}_{3,t} \end{bmatrix} + \begin{bmatrix} (q_{11} + q_{12})\Delta\theta_t - q_{11}r_t \\ (q_{21} + q_{22})\Delta\theta_t - q_{21}r_t \\ (q_{31} + q_{32} + 1)\Delta\theta_t - q_{31}r_t - \frac{1}{\beta}\frac{\tau}{b}\psi_t \end{bmatrix},$$
(A3)

,

or $X_{t+1} = DX_t + Z_t$.

 $E_t Z_{2,t+}$

Monetary regime. Under the M regime, the eigenvalues e_1 and e_3 are inside the unit circle, e_2 is outside. We thus use the second row of (A3) to draw linear restrictions between the variables of the model. Remember that

$$x_{2,t} = q_{22}R_{t-1} + q_{21}\pi_t,$$

so the second line of the system (A3) can be rewritten as

$$x_{2,t+1} = e_2 x_{2,t} + (q_{22} + q_{21}) \Delta \theta_t - q_{21} r_t,$$

or

$$x_{2,t} = \frac{1}{e_2} x_{2,t+1} + \frac{1}{e_2} z_{2,t},$$

where $z_{2,t} = q_{21}r_t - (q_{22} + q_{21})\Delta\theta_t$. Substituting the future values of $x_{2,t}$ recursively, we obtain

$$x_{2,t} = \frac{1}{e_2} \sum_{k=0}^{\infty} \left(\frac{1}{e_2}\right)^k E_t z_{2,t+k}.$$
(A5)

Since $r_t = \rho_1 r_{t-1} + \varepsilon_{1,t}$ and $\theta_t = \rho_2 \theta_{t-1} + \varepsilon_{2,t}$, we have that

$$z_{2,t} = q_{21}r_t - (q_{22} + q_{21})\Delta\theta_t \text{ for } k = 0,$$

$$k = q_{21}\rho_1^k r_t - (q_{22} + q_{21})(\rho_2 - 1)\rho_2^{k-1}\theta_t \text{ for } k > 0,$$

which we can substitute into (A5) to obtain

$$x_{2,t} = \frac{q_{21}}{e_2 - \rho_r} r_t - \frac{q_{22} + q_{21}}{e_2} \left(\frac{e_2 - 1}{e_2 - \rho_\theta} \theta_t - \theta_{t-1} \right)$$
(A6)

(A4)

We now equate the right hand sides of (A4) and (A6) and use the definition of e_2 to get

$$q_{22}R_{t-1} + q_{21}\pi_t = \frac{q_{21}}{\phi_p + 1 - \rho_r}r_t - \frac{q_{22} + q_{21}}{\phi_p + 1}\left(\frac{\phi_p}{\phi_p + 1 - \rho_\theta}\theta_t - \theta_{t-1}\right).$$

Finally, solving for π_t and noting that $\frac{q_{22}}{q_{21}} = \frac{1}{\phi_n}$, we obtain the solution for inflation for the AM/PF case:

$$\pi_{t} = -\frac{1}{\phi_{p}}R_{t-1} + \frac{1}{1+\phi_{p}-\rho_{r}}r_{t} - \frac{1}{1+\phi_{p}-\rho_{\theta}}\theta_{t} + \frac{1}{\phi_{p}}\theta_{t-1}.$$

Fiscal regime. Under the F regime, the eigenvalue e_3 is outside the unit circle, while e_1 and e_2 are inside. We thus use the third row in (A3) to draw linear restrictions between model variables. Remember that

$$\mathbf{x}_{3,t} = \mathbf{b}_{t-1} + \mathbf{q}_{32}\mathbf{R}_{t-1} + \mathbf{q}_{31}\boldsymbol{\pi}_t,\tag{A7}$$

so the third line of the system (A3) can be rewritten as

$$x_{3,t+1} = e_3 x_{3,t} + (q_{31} + q_{32} + 1)\Delta\theta_t - q_{31}r_t - \frac{1}{\beta}\frac{\tau}{b}\psi_t$$

or

$$x_{3,t} = \frac{1}{e_3} x_{3,t+1} + \frac{1}{e_3} z_{3,t+1}$$

where $z_{3,t} = q_{31}r_t - (q_{31} + q_{32} + 1)\Delta\theta_t + \frac{1}{\beta}\frac{\tau}{b}\psi_t$. Substituting out the future values of $x_{3,t}$ recursively, we obtain

$$x_{3,t} = \frac{1}{e_3} \sum_{k=0}^{\infty} \left(\frac{1}{e_3}\right)^k E_t z_{3,t+k}.$$
(A8)

Since $r_t = \rho_1 r_{t-1} + \varepsilon_{1,t}$, $\theta_t = \rho_2 \theta_{t-1} + \varepsilon_{2,t}$, and $\psi_t = \rho_3 \psi_{t-1} + \varepsilon_{3,t}$, we have that

$$\begin{aligned} z_{3,t} &= q_{31}r_t - (q_{31} + q_{32} + 1)\Delta\theta_t + \frac{1}{\beta}\frac{\tau}{b}\psi_t \text{ for } k = 0, \\ E_t z_{3,t+k} &= q_{31}\rho_r^k r_t - (q_{31} + q_{32} + 1)(\rho_\theta - 1)\rho_\theta^{k-1}\theta_t + \frac{1}{\beta}\frac{\tau}{b}\rho_\psi^k\psi_t \text{ for } k > 0 \end{aligned}$$

which we can substitute into (A8) to obtain:

$$\begin{aligned} x_{3,t} &= \frac{q_{31}}{e_3 - \rho_r} r_t - \frac{q_{31} + q_{32} + 1}{e_3} \left(\frac{e_3 - 1}{e_3 - \rho_\theta} \theta_t - \theta_{t-1} \right) + \frac{\frac{1}{\beta} \frac{\tau}{b}}{e_3 - \rho_\psi} \psi_t(A9) \\ x_{3,t} &= \frac{q_{31}}{\frac{1}{\beta} (1 - \frac{\tau}{b} \gamma_\tau) - \rho_1} r_t - \frac{q_{31} + q_{32} + 1}{\frac{1}{\beta} (1 - \frac{\tau}{b} \gamma) - \rho_2} \Delta \theta_t + \frac{\frac{1}{\beta} \frac{\tau}{b}}{\frac{1}{\beta} (1 - \frac{\tau}{b} \gamma) - \rho_3} \psi_t \end{aligned}$$

We now equate the right hand sides of (A7) and (A9) and use the definition of e_3 to get

$$b_{t-1} + q_{32}R_{t-1} + q_{31}\pi_t = \frac{q_{31}}{\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma\right) - \rho_1}r_t - \frac{q_{31} + q_{32} + 1}{\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma\right)} \left(\frac{\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma\right) - 1}{\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma\right) - \rho_\theta}\theta_t - \theta_{t-1}\right) + \frac{\frac{1}{\beta}\frac{\tau}{b}}{\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma\right) - \rho_\psi}\psi_t$$

Finally, solving for π_t and noting that $v_{12} = v_{22}$ and $v_{21} = -v_{11}\phi_p$ we get, after substituting the values for q_{31} and q_{32} , the solution for inflation for the PM/AF case:

$$\pi_t = -\frac{(a-1)\beta}{J}R_{t-1} + \frac{K}{J}b_{t-1} + \frac{1}{\frac{a}{\beta} - \rho_1}r_t - \beta\frac{a-1}{J}\left(\frac{\frac{a}{\beta} - 1}{\frac{a}{\beta} - \rho_2}\theta_t - \theta_{t-1}\right) - \frac{K\frac{1}{\beta}\frac{t}{\beta}}{J\left(\frac{a}{\beta} - \rho_3\right)}\psi_t$$

with

$$a = 1 - \frac{\tau}{b}\gamma$$

$$J = \beta - a + a\beta\phi_p$$

$$K = a(\beta - a + \beta\phi_p)$$

and where

$$\frac{\partial \pi_t}{\partial R_{t-1}} > \mathbf{0}, \frac{\partial \pi_t}{\partial b_{t-1}} > \mathbf{0}, \frac{\partial \pi_t}{\partial \theta_t} > \mathbf{0}, \frac{\partial \pi_t}{\partial \theta_{t-1}} < \mathbf{0}, \frac{\partial \pi_t}{\partial r_t} > \mathbf{0}, \text{and} \frac{\partial \pi_t}{\partial \psi_t} < \mathbf{0}.$$

IT case. Under IT we have the following system:

$$\begin{aligned} R_t &= E_t \pi_{t+1} + r_t, \\ R_t &= \phi_\pi \pi_t + \theta_t, \\ b_t &= \frac{1}{\beta} \left(1 - \frac{\tau}{b} \gamma \right) b_{t-1} + R_t - \frac{1}{\beta} \pi_t - \frac{1}{\beta} \frac{\tau}{b} \psi_t. \end{aligned}$$

which in matrix form becomes:

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} E_t \pi_{t+1} \\ R_t \\ b_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ \phi_{\pi} & 0 & 0 \\ -\frac{1}{\beta} & 0 & \frac{1}{\beta} (1 - \frac{\tau}{b} \gamma) \end{bmatrix} \begin{bmatrix} \pi_t \\ R_{t-1} \\ b_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\beta} \frac{\tau}{b} \end{bmatrix} \begin{bmatrix} r_t \\ \theta_t \\ \psi_t \end{bmatrix}.$$

Inverting the matrix on the left-hand-side, we get:

$$\begin{bmatrix} E_t \pi_{t+1} \\ R_t \\ b_t \end{bmatrix} = \begin{bmatrix} \phi_{\pi} & 0 & 0 \\ \phi_{\pi} & 0 & 0 \\ \phi_{\pi} - \frac{1}{\beta} & 0 & \frac{a}{\beta} \end{bmatrix} \begin{bmatrix} \pi_t \\ R_{t-1} \\ b_{t-1} \end{bmatrix} + \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -\frac{1}{\beta} \frac{\tau}{b} \end{bmatrix} \begin{bmatrix} r_t \\ \theta_t \\ \psi_t \end{bmatrix},$$
(A10)

or $Y_t = AY_{t-1} + B\varepsilon_t$. The coefficient matrix *A* can be decomposed as $A = VDV^{-1}$, where *D* and *V* are, respectively, the matrices whose elements are the eigenvalues and the eigenvectors of *A*:

$$A = \begin{bmatrix} 0 & \nu_{12} & 0 \\ 1 & \nu_{22} & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \phi_{\pi} & 0 \\ 0 & 0 & \frac{a}{\beta} \end{bmatrix} \begin{bmatrix} q_{11} & 1 & 0 \\ q_{21} & 0 & 0 \\ q_{31} & 0 & 1 \end{bmatrix},$$

where $a = (1 - \frac{\tau}{b}\gamma)$ as defined above. The system (A10) becomes:

$$\begin{bmatrix} E\pi_{t+1} \\ R_t \\ b_t \end{bmatrix} = \begin{bmatrix} 0 & v_{12} & 0 \\ 1 & v_{22} & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \phi_{\pi} & 0 \\ 0 & 0 & \frac{a}{\beta} \end{bmatrix} \begin{bmatrix} q_{11} & 1 & 0 \\ q_{21} & 0 & 0 \\ q_{31} & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ R_{t-1} \\ b_{t-1} \end{bmatrix} + \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -\frac{1}{\beta} \frac{\tau}{\beta} \end{bmatrix} \begin{bmatrix} r_t \\ \theta_t \\ \psi_t \end{bmatrix}$$

For later use, note that:

$$D = \begin{bmatrix} e_1 & 0 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & e_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \phi_{\pi} & 0 \\ 0 & 0 & \frac{a}{\beta} \end{bmatrix},$$
$$V = \begin{bmatrix} 0 & \nu_{12} & 0 \\ 1 & \nu_{22} & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{\beta\phi_{\pi}-1}(a-\beta\phi_{\pi}) & 0 \\ 1 & -\frac{1}{\beta\phi_{\pi}-1}(a-\beta\phi_{\pi}) & 0 \\ 0 & 1 & 1 \end{bmatrix},$$
$$V^{-1} = \begin{bmatrix} -\frac{1}{\nu_{12}}\nu_{22} & 1 & 0 \\ \frac{1}{\nu_{12}} & 0 & 0 \\ -\frac{1}{\nu_{12}} & 0 & 1 \end{bmatrix}$$

Finally, letting $X_t = (x_{1,t}, x_{2,t}, x_{3,t})' = V^{-1}(\pi_t, R_t, b_t)'$, we rewrite the system as:

$$\begin{bmatrix} \mathbf{x}_{1,t+1} \\ \mathbf{x}_{2,t+1} \\ \mathbf{x}_{3,t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \phi_{\pi} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \frac{a}{\beta} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{1,t} \\ \mathbf{x}_{2,t} \\ \mathbf{x}_{3,t} \end{bmatrix} + \begin{bmatrix} (q_{11}+1)\theta_t - q_{11}r_t \\ q_{21}\theta_t - q_{21}r_t \\ (q_{31}+1)\theta_t - q_{31}r_t - \frac{1}{\beta}\frac{\tau}{b}\psi_t \end{bmatrix}$$

Each element of X_t is given by:

$$\begin{bmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \end{bmatrix} = \begin{bmatrix} R_{t-1} + q_{11}\pi_t \\ q_{21}\pi_t \\ b_{t-1} + q_{31}\pi_t \end{bmatrix}$$

and substituting it into (A3), we get:

$$\begin{bmatrix} x_{1,t+1} \\ x_{2,t+1} \\ x_{3,t+1} \end{bmatrix} = \begin{bmatrix} 0 \\ \phi_{\pi}q_{21}\pi_t \\ \frac{a}{\beta}b_{t-1} + \frac{a}{\beta}q_{31}\pi_t \end{bmatrix} + \begin{bmatrix} (q_{11}+1)\theta_t - q_{11}r_t \\ q_{21}\theta_t - q_{21}r_t \\ (q_{31}+1)\theta_t - q_{31}r_t - \frac{1}{\beta}\frac{\tau}{b}\psi_t \end{bmatrix},$$
(A11)

or $X_{t+1} = DX_t + Z_t$.

Monetary regime. Under the M regime, the eigenvalues e_1 and e_3 are inside the unit circle, e_2 is outside. We thus use the second row of (A11) to draw linear restrictions between model variables:

$$x_{2,t} = q_{21}\pi_t.$$

From the second line of the system (A11):

$$x_{2,t+1} = \phi_{\pi} x_{2,t} + q_{21} \theta_t - q_{21} r_t,$$

or

$$x_{2,t} = \frac{1}{\phi_{\pi}} x_{2,t+1} - \frac{q_{21}}{\phi_{\pi}} \theta_t + \frac{q_{21}}{\phi_{\pi}} r_t.$$

Substituting out the future values of $x_{2,t}$ recursively, we obtain

$$x_{2,t} = rac{1}{e_2} \sum_{k=0}^{\infty} \left(rac{1}{e_2}
ight)^k E_t z_{2,t+k}^*,$$



Fig. 14. Impulse response function to a negative demand shock in the F regime with $\phi_{\pi} < 0$. Notes: Solid blue lines correspond to inflation targeting, dashed red lines correspond to price level targeting. Parametrization: $\phi_{\pi} = -0.5$, $\phi_{p} = -0.1$, $\gamma = 0$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

(A12)

(A13)

where

$$z_{2,t} = q_{21}\theta_t - q_{21}r_t.$$

Since $r_t = \rho_1 r_{t-1} + \varepsilon_{1,t}$ and $\theta_t = \rho_2 \theta_{t-1} + \varepsilon_{2,t}$, it implies:

$$E_t z_{2,t+k} = \rho_2^k q_{21} \theta_t - \rho_1^k q_{21} r_t$$

Plugging these equations into (A13), we obtain:

$$\mathbf{x}_{2,t} = -\frac{q_{21}}{\phi_{\pi} - \rho_2} \theta_t + \frac{q_{21}}{\phi_{\pi} - \rho_1} \mathbf{r}_t.$$
 (A14)

Equating the right hand sides of (A12) and (A14):

$$q_{21}\pi_t = -rac{q_{21}}{\phi_{\pi}-\rho_2} heta_t + rac{q_{21}}{\phi_{\pi}-\rho_1}r_t.$$

Solving for π_t we obtain the solution for inflation for the AM/PF case:

$$\pi_t = -\frac{1}{\phi_\pi - \rho_2}\theta_t + \frac{1}{\phi_\pi - \rho_1}r_t.$$

Fiscal regime. Under the F regime, the eigenvalue e_3 is outside the unit circle, e_1 is inside, e_2 must be inside too. We thus use the third row in (A11) to draw linear restrictions between model variables:

$$x_{3,t} = b_{t-1} + q_{31}\pi_t. \tag{A15}$$

From the third line of (A11) and remembering that $\theta_t = \rho_2 \theta_{t-1} + \varepsilon_{2,t}$, we get

$$\begin{aligned} \mathbf{x}_{3,t+1} &= \frac{\beta}{\beta} \mathbf{x}_{3,t} + (q_{31}+1)\theta_t - q_{31}r_t - \frac{1}{\beta}\frac{\tau}{b}\psi_t \\ \mathbf{x}_{3,t} &= \frac{\beta}{a} \mathbf{x}_{3,t+1} - \frac{\beta}{a} \Big[(q_{31}+1)\theta_t - q_{31}r_t - \frac{1}{\beta}\frac{\tau}{b}\psi_t \Big]. \end{aligned}$$

Substituting out the future values of $x_{3,t}$ recursively, we obtain

$$x_{3,t} = \frac{1}{e_3} \sum_{k=0}^{\infty} \left(\frac{1}{e_3}\right)^k E_t z_{3,t+k}^*$$
(A16)

where

$$\begin{aligned} E_t z_{3,t+k} &= -\rho_2^k (q_{31} + 1)\theta_t + q_{31}\rho_1^k r_t + \frac{1}{\beta} \frac{\tau}{b} \rho_3^k \psi_t \\ z_{3,t} &= -(q_{31} + 1)\theta_t + q_{31}r_t + \frac{1}{\beta} \frac{\tau}{b} \psi_t. \end{aligned}$$



Fig. 15. Loss function after a demand shock in the F regime. Notes: On the left the case under IT with $\phi_{\pi} < 0$, on the right the case under PLT.

Plugging these equations into (A16), we obtain:

$$x_{3,t} = -\frac{q_{31}+1}{\frac{a}{\beta}-\rho_2}\theta_t + \frac{q_{31}}{\frac{a}{\beta}-\rho_1}r_t + \frac{\frac{1}{\beta}\frac{\tau}{b}}{\frac{a}{\beta}-\rho_3}\psi_t.$$
(A17)

Equating the right hand sides of (A15) and (A17) and solving for π_t , we get:

$$\pi_t = \frac{\beta\phi_{\pi} - a}{\beta\phi_{\pi} - 1}b_{t-1} + \frac{1 - a}{(\beta\phi_{\pi} - 1)\left(\frac{a}{\beta} - \rho_2\right)}\theta_t + \frac{1}{\frac{a}{\beta} - \rho_1}r_t + \frac{\frac{1}{\beta}\frac{\tau}{b}(a - \beta\phi_{\pi})}{(\beta\phi_{\pi} - 1)\left(\frac{a}{\beta} - \rho_3\right)}\psi_t$$

where

$$\frac{\partial \pi_t}{\partial b_{t-1}} > 0, \quad \frac{\partial \pi_t}{\partial \theta_t} > 0, \quad \frac{\partial \pi_t}{\partial r_t} > 0, \quad \text{and} \quad \frac{\partial \pi_t}{\partial \psi_t} < 0.$$



Fig. 16. Impulse response function to a negative demand shock in the F regime, accounting for the ZLB, with $\phi_{\pi} < 0$ and no smoothing in the IT rule. Notes: Solid blue lines correspond to inflation targeting, dashed red lines correspond to price level targeting. Parametrization: $\phi_{\pi} = -0.5$, $\phi_{p} = -0.1$, $\gamma = 0$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

A.4. IT with $\phi_{\pi} < 0$ without smoothing

We now consider an IT rule like (11) with ϕ_{π} < 0 without considering, as instead we did in the main text, the presence of a smoothing term.

Impulse response function. With this specification we find, as in Section 3.4, that a negative demand shock decreases inflation and output and increases debt by roughly the same extent under PLT and IT (see Fig. 14). As a response to the reduction of inflation, the central bank increases the nominal interest rate under both IT and PLT. Even under IT there are now wealth effects that induce a much quicker rebound of both inflation and output that react very similarly under IT and PLT however now, without a smoothing term, the interest rate under IT increases more abruptly as a reaction to a decrease in inflation.

Welfare. Differently from Section 3.4, we here find welfare loss functions that are very similar to the benchmark case analyzed in the main text (compare Fig. 15 to Fig. 2).

Zero lower bound. In line with Section 3.4, after a negative demand shock in the presence of the ZLB, inflation and output decrease by roughly the same extent under PLT and IT while the nominal interest rate remains unconstrained under both approaches (see Fig. 16).

References

Ambler, S., Lam, J.-P., 2016. Inflation targeting, price-level targeting, the zero lower bound, and indeterminacy, mimeo.

Andrade, P., Galí, J., Bihan, H.L., Matheron, J., 2019. The optimal inflation target and the natural rate of interest, Brookings Papers on Economic Activity. Fall, pp. 173-230.

Ascari, G., Florio, A., Gobbi, A., 2020. Controlling inflation with timid monetary-fiscal regime changes. Int. Econ. Rev. 61, 1001–1024.

Bartsch, E., Boivin, J., Fischer, S., Hildebrand, P., 2019. Dealing with the next downturn: from unconventional monetary policy to unprecedented policy coordination, BlackRock Investment Institute, August. https://www.blackrock.com/us/individual/insights/blackrock-investment-institute/globalmacro-outlook

Bernanke, B.S., 2017. Temporary price-level targeting: An alternative framework for monetary policy, Ben Bernanke's Blog, October 12, Brookings.

Bernanke, B.S., 2019. Evaluating lower-for-longer policies: Temporary price-level targeting, Ben Bernanke's Blog, February 21, Brookings.

Bhattarai, S., Lee, I.W., Park, W.Y., 2014. Inflation dynamics: the role of public debt and policy regimes. J. Monet. Econ. 67, 93-108.

Bianchi, F., Melosi, L., 2017. Escaping the Great Recession. Am. Econ. Rev. 107, 1030-1058.

Billi, R.M., 2018. Price level targeting and risk management. Econ. Model. 73, 163-173.

Billi, R.M., Walsh, C.E., 2022. Seemingly Irresponsible but Welfare Improving Fiscal Policy at the Lower Bound, Working Paper Series 410, Sveriges Riksbank (Central Bank of Sweden)

Blanchard, O., 2020. Is there deflation or inflation in our future? https://voxeu.org/article/there-deflation-or-inflation-our-future, accessed: 2020-05-17. Davig, T., Leeper, E.M., 2011. Monetary-fiscal policy interactions and fiscal stimulus. Eur. Econ. Rev. 55, 211-227.

Draghi, M., 2019. Twenty Years of the ECB's monetary policy, Speech, June 18, ECB Forum on Central Banking, Sintra, 18 June 2019.

Eggertsson, G.B., 2006. The deflation bias and committing to being irresponsible. J. Money, Credit Bank. 38, 283–321.

Eggertsson, G.B., Woodford, M., 2003. The zero bound on interest rates and optimal monetary policy. Brookings Papers Econ. Activ. 34, 139-235.

Evans, C.L., 2012. Monetary policy in a low-inflation environment: developing a state-contingent price-level target. J. Money, Credit Bank. 44, 147–155. Evans, G.W., Guse, E., Honkapohja, S., 2008. Liquidity traps, learning and stagnation. Eur. Econ. Rev. 52, 1438–1463.

Giannoni, M.P., 2014. Optimal interest-rate rules and inflation stabilization versus price-level stabilization. J. Econ. Dynam. Control 41, 110-129.

Gorodnichenko, Y., Shapiro, M.D., 2007. Monetary policy when potential output is uncertain: Understanding the growth gamble of the 1990s. J. Monet. Econ. 54. 1132-1162.

Guerrieri, L., Iacoviello, M., 2015. OccBin: A toolkit for solving dynamic models with occasionally binding constraints easily. J. Monet. Econ. 70, 22-38. Haldane, A., Salmon, C., 1995. Three Issues on Inflation Targets, in Andrew Haldane (ed.), Targeting Inflation, Bank of England, pp. 170-201. Honkapohja, S., Mitra, K., 2020. Price level targeting with evolving credibility. J. Monet. Econ. 116, 88–103.

Lagarde, C., 2019. The future of the euro area economy, Speech, November 22, Frankfurt European Banking Congress, Frankfurt am Main, 22 November 2019. Lebow, D.E., Roberts, J., Stockton, D.J., 1992. Economic performance under price stability, Working Paper Series/ Economic Activity Section 125, Board of Governors of the Federal Reserve System (U.S.).

Leeper, E.M., 1991. Equilibria under 'active' and 'passive' monetary and fiscal policies. J. Monet. Econ. 27, 129-147.

Liu, Z., Waggoner, D., Zha, T., 2009. Asymmetric expectation effects of regime shifts in monetary policy. Rev. Econ. Dynam. 12, 284-303.

Schmitt-Grohe, S., Uribe, M., 2007. Optimal simple and implementable monetary and fiscal rules. J. Monet. Econ. 54, 1702-1725.

Sims, C.A., 2000. Comment on three lessons for monetary policy in a low-inflation era. J. Money, Credit Bank. 32, 967-972.

Svensson, L., 1999. Price-level targeting versus inflation targeting: a free lunch?. J. Money, Credit Bank. 31, 277–295.

Uribe, M., 2018. The Neo-Fisher Effect: Econometric Evidence from Empirical and Optimizing Models, NBER Working Papers 25089, National Bureau of Economic Research Inc.

Vestin, D., 2006. Price-level versus inflation targeting. J. Monet. Econ. 53, 1361-1376.

Williams, J.C., 2017. Preparing for the Next Storm: Reassessing Frameworks and Strategies in a Low R-star World, FRBSF Economic Letter.

Woodford, M., 1999. Optimal monetary policy inertia. Manchester School 67, 1-35.

Woodford, M., 2003. Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton University Press, Princeton, NI.

Yellen, J.L., 2018. Comments on monetary policy at the effective lower bound.