

Economic NMPC for Multiple Buildings Connected to a Heat Pump and Thermal and Electrical Storages

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Abstract: This paper studies the impact of different types of energy storage integrated with a heat pump to improve energy efficiency in multiple radiant-floor buildings. In particular, the buildings and the heating generation system are decoupled through a 3-element mixing valve, which enforces a fixed flow rate but a variable temperature in the inlet water entering the building pipelines. The paper presents an optimal control formulation based on an Economic Nonlinear MPC scheme, in order to find the best compromise among different goals: make the heat pump work when it is more efficient, store electrical energy when it is cheap, store thermal energy in the tank when the heat pump is more effective, modulate the inlet water temperature to satisfy the user's comfort constraints, exploit the buildings thermal inertia. The nonlinearity of the system stems from the variable flow rate into the hot water tank due to the variable action of the mixing valve. The model is also time-varying due to the fact that the heat pump efficiency depends on external conditions. The simulation results show that the proposed optimal control algorithm is able to economically distribute energy among all storages in order to insure cost benefits (almost 20% electricity cost saving) and comfort satisfaction with the feasible computational effort.

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Keywords: Building energy automation, Heat pump, thermal and electrical storages, Economic MPC.

1. INTRODUCTION

Heat pump systems are usually described, for sizing and control purposes, through their coefficient of performance (COP), which is a function of several variables, including external inputs, such as outside air temperature and humidity, and load-dependent ones such as inlet water temperature coming from the load Moran et al. (2010). As pointed out in Verhelst et al. (2012), the lower the inlet water temperature is, the higher the heat pump performance is. As a result, since radiant-floor use low-temperature emission heating systems, they are well suited for the combination with heat pump systems Olesen et al. (2002). In Verhelst et al. (2012), the effect of different COP models on the optimal control performance for the application of a radiant-floor building was studied. It was established that there is up to 5% energy cost saving compared to classic curve heating techniques. Heat pump systems can also be beneficial for demand-response applications, where the main focus is to shift demands from on-peak to off-peak periods, as discussed in Arteconi et al. (2017). In this case, heat pumps are usually connected to a buffer hot water tank (HWT), both to increase the COP and to decouple

the (thermal) load and the (electrical) heat generation (see Arteconi et al. (2013), Arteconi et al. (2017)). The space heating control coupled with heat pump is well studied in the literature, e.g. in Fischer and Madani (2017), Rastegarpour et al. (2020). There are also numerous control strategies to improve the energy consumption and comfort conditions in buildings (Awadelrahman et al. (2017), del Mar Castilla et al. (2013), Mantovani and Ferrarini (2014)). However, the heat pump performance control is still a challenging issue due to the variability of the performance of the heat pump itself due to operating and environmental conditions. In particular, when dealing with renewable energy sources, storage is increasingly necessary. In that context, the combination of heat pumps with different types of energy storages - electrical and thermal - is an attractive solution to better shape the electrical load in a demand-response scenario and to shift the heat pump work when it is more efficient and when the price of electrical energy is lower. The scenario addressed in this paper extends the classic building heated by a heat pump, and consists of a set of different buildings with different desired comfort levels and different levels of flexibility.

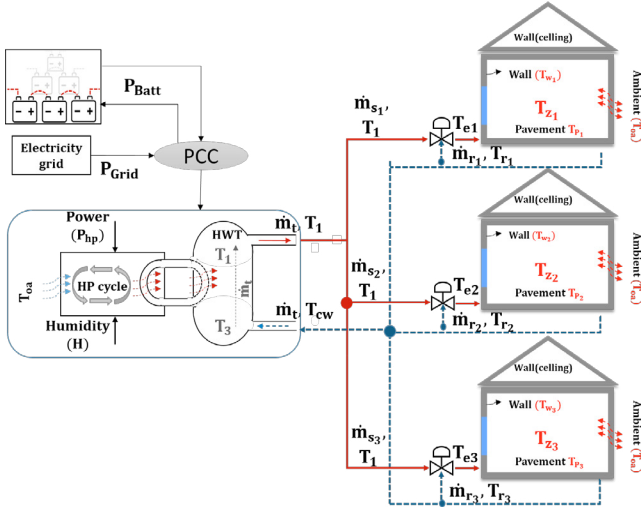


Fig. 1. Sketch of the system under study

The buildings are served by one hot water tank (HWT) connected to a modulating air-to-water heat pump. An electrical storage (battery) is also considered as an additional storage element, beside HWT. The two different types of storage enlarge the possibilities of finding good compromises between serving the load, storing electrical energy when it is cheap and running the heat pump when it is more efficient. Additionally, the load side and heating system are decoupled through a 3-element mixing valve which enforces a fixed flow rate and variable temperature for the inlet water into the building pipelines (see Fig. 1). Consequently, the HWT is subject to a nonlinear dynamic behavior due to variable water flow rate in the HWT.

The heat pump is modeled with its coefficient of performance (COP), which is a parameter that depends strongly on the weather and operating conditions. The heat pump power is mainly provided by the electricity grid, but the battery can shift the electrical load from on-peak periods to off-peak periods. The battery stores electrical energy before the heat pump transforms it into thermal energy, while the HWT stores thermal energy produced by the heat pump. Thus, the usage of these storage units are driven by different goals: maximizing economic convenience due to a variable electricity price (battery) and maximizing the heat pump performance (HWT). Moreover, the electrical energy stored in the battery can also be used for ancillary services to the grid. This paper combines different and possibly conflicting objectives in an optimal control problem formulation. In particular, in this paper a Nonlinear Model Predictive Control (NMPC) is adopted, which is an effective way to deal with the many physical constraints and nonlinear dynamics typical of the considered application case, as shown in Rastegarpour et al. (2020) and Maciejowski (2002). The simulation results show the effectiveness of the developed NMPC for the application of multiple residential buildings. The analysis and comparison of the NMPC performance is made for two scenarios: first, for a constant electricity price profile, where the NMPC decision is mainly affected by the COP variation. Second, for a day-night electricity tariff, where the NMPC needs to make a compromise between COP variation and electricity tariff. The modeling framework can seamlessly host real-time pricing scenarios.

2. MODEL DESCRIPTION

2.1 Building specification

The case study of the present paper consists of three residential radiant-floor buildings, each served by a three element mixing valve that guarantees a constant water flow rate inside of the building pipelines (see Fig. 1). Each building is modeled using the techniques developed in Rastegarpour et al. (2020) and Ferrarini et al. (2017), where the building dynamics is defined by a set of ordinary differential equations based on the energy balance equation for the building air, wall, pavement and pipes temperature. It reads as follows for $i = 1, 2, 3$:

$$\begin{aligned} C_w \frac{dT_{w_i}}{dt} &= k_{w,oa} (T_{oa} - T_{w_i}) + k_{w,z} (T_{z_i} - T_{w_i}) \\ C_z \frac{dT_{z_i}}{dt} &= k_{w,z} (T_{w_i} - T_{z_i}) + k_{p,z} (T_{p_i} - T_{z_i}) \\ C_p \frac{dT_{p_i}}{dt} &= k_{p,z} (T_{z_i} - T_{p_i}) + k_b (T_{r_i} - T_{p_i}) \\ C_r \frac{dT_{r_i}}{dt} &= k_b (T_{p_i} - T_{r_i}) + w C_{wp} (T_{e_i} - T_{r_i}) \end{aligned} \quad (1)$$

The state dynamics of the system corresponds to a single-zone model with four states, namely wall temperature $T_{w_i} [^{\circ}C]$, zone temperature $T_{z_i} [^{\circ}C]$, pavement temperature $T_{p_i} [^{\circ}C]$ and water pipeline temperature $T_{r_i} [^{\circ}C]$, while C_w , C_z , C_p and $C_r [J/K]$ are the respective heat capacities. The parameters $k_{w,oa}$, $k_{w,z}$, $k_{p,z}$, $k_b [W/K]$ are the overall heat transfer coefficients between respectively T_{oa} and T_w , T_z and T_w , T_{p_i} and T_{z_i} , T_{r_i} and T_{p_i} . T_{e_i} represents the inlet water temperature in the pipelines. The constant parameter $w [kg/s]$ denotes the water mass flow rate in the building pipelines and $C_{wp} [J/kgK]$ is the specific heat capacity of the water. Table 1 shows the value of the parameters used in the building model, which is the same in all three buildings. The inlet water temperature of

Table 1. Building parameters

C_w	$42 \times 10^5 [J/K]$	$k_{w,oa}$	$86 [W/K]$	w	$0.124 [kg/s]$
C_z	$6 \times 10^6 [J/K]$	$k_{w,z}$	$86 [W/K]$	C_{wp}	$4180 [J/kgK]$
C_p	$8 \times 10^5 [J/K]$	$k_{p,z}$	$594 [W/K]$		
C_r	$1.7 \times 10^6 [J/K]$	k_b	$506 [W/K]$		

the buildings pipelines (T_{e_i} , $i = 1, 2, 3$), can be evaluated through the energy balance equation on each valve as follows:

$$\begin{aligned} w &= \dot{m}_{s_i}(k) + \dot{m}_{r_i}(k) \\ w T_{e_i}(k) &= \dot{m}_{s_i}(k) T_1(k) + \dot{m}_{r_i}(k) T_{r_i}(k) \end{aligned} \quad (2)$$

where, $\dot{m}_{s_i} [kg/s]$ and $\dot{m}_{r_i} [kg/s]$ show respectively the water mass flow rate of the supply and return inputs of the i^{th} valve, see Fig. 1. $T_1(k)$ is also the water temperature of the top layer of the HWT, which is the same in the supply inputs of all valves. Subsequently, at each time instant k , the valve position $x_{v_i}(k)$ on the supply direction can be defined as follows:

$$x_{v_i}(k) = \frac{\dot{m}_{s_i}(k)}{w}, \quad i = 1, 2, 3 \quad (3)$$

2.2 Heat pump hot water tank

A typical modulating air-to-water heat pump for the residential buildings is considered in this study Daikin Europe (2012). In order to form the energy cost predictions of the heat pump, a model is required to report the ratio of the delivered thermal power (Q_{hp}) with respect to the electrical power consumed by the heat pump compressor (P_{hp}). Hence, in this paper, the heat pump is modeled by its COP function, which is assumed to be known over the prediction horizon and is computed at each time instant k by the following ratio:

$$COP(k) = \frac{Q_{hp}(k)}{P_{hp}(k)} \quad (4)$$

The HWT connected to the heat pump condenser has a linear relation with the heat pump due to the thermal power $Q_{hp}(k)$ transferred to the middle layer of the tank. The load side of the HWT is nonlinear when a variable circulated water flow rate is applied to the tank. The nonlinearity stems from the variable action of the mixing valves, which provide a variable water flow rate $\dot{m}_t(k)$ into the HWT as follows:

$$\dot{m}_t(k) = \sum_{i=1}^3 \dot{m}_{s_i} \quad (5)$$

where i denotes the building number. The control-oriented model of the HWT is based on a multi-node model defining the temperature dynamics of the fluid within the storage tank. In this approach, similar to the one used in Nash et al. (2017), the tank is discretized vertically into three nodes, where the first node is the top one. The final model is a three-state model, as described in Rastegarpour et al. (2018), where $T_1(k)$, $T_2(k)$, and $T_3(k)$ are the first, second and third layer, respectively.

2.3 Battery model

The battery model can be expressed as an integrator with a fixed charging and discharging efficiency ξ_B . Considering the power exchange of the battery $P_{Batt}(k)$ as an input of the model, the energy model of the battery can be defined as follows:

$$E_{Batt}(k+1) = E_{Batt}(k) - \xi_B P_{Batt}(k) \quad (6)$$

where the battery efficiency ξ_B is equal to 0.9. According to the battery model (6), a negative $P_{Batt}(k)$ corresponds to the charging mode of the battery.

3. CONTROL OBJECTIVES

According to the dynamic model of the building, heat pump, HWT and battery the vector of decision variables defined as follows:

$$u(k) = \begin{bmatrix} P_{hp}(j|k) \\ P_{Batt}(j|k) \\ x_{v1}(j|k) \\ x_{v2}(j|k) \\ x_{v3}(j|k) \end{bmatrix}, j = k, \dots, N_{ch} + k - 1 \quad (7)$$

where N_{ch} denotes the control horizon which is considered equal to prediction horizon. Moreover, there are also some

external inputs such as outside air temperature $T_{oa}(k)$, COP(k) profile and electricity tariff, which are here considered as known time-varying profiles. The controller aims at making a compromise between the maximization of the COP, minimization of the overall electricity cost purchased from the main electricity grid, while minimizing the temperature set point tracking error, considering also the operational and actuators limitations. The cost function of the optimal control problem is then:

$$J(u, x) = \sum_{l=0}^{N_{ch}-1} \sum_{i=1}^3 \left(\Upsilon_P P_{Grid}(k+l|k) \Upsilon_{r,i} (T_{z,i}(k+l|k) - T_{sp,i})^2 \right) + \sum_{i=1}^3 \left(\Upsilon_\varepsilon \varepsilon_i^2 \right) + V^{term}, \quad (8)$$

where $P_{Grid}(j|k) = P_{hp}(j|k) - P_{Batt}(j|k)$ is the power exchange with the main utility grid penalized by normalized weighting coefficient Υ_P based on electricity price profile, and the terminal cost V^{term} approximates the infinite horizon cost:

$$V^{term} = x^T(k + N_{ch}|k) \Upsilon_v x(k + N_{ch}|k) \quad (9)$$

where Υ_v is a normalized weighting coefficient. Variables T_{sp1} , T_{sp2} and T_{sp3} are the set points of the buildings air temperatures for the first, second and third building, respectively, and Υ_{r1} , Υ_{r2} and Υ_{r3} are the respective normalized weighting coefficients. Moreover, the slack variable ε_1 , ε_2 and ε_3 with the normalized weighting coefficients $\Upsilon_{\varepsilon1}$, $\Upsilon_{\varepsilon2}$ and $\Upsilon_{\varepsilon3}$ are included in the problem formulation to guarantee feasibility at any time instant. We use the relaxed constraints:

$$T_{sp,i} - T_{flx,i} - \varepsilon_i \leq T_{z,i}(k) \leq T_{sp,i} + T_{flx,i} + \varepsilon_i \quad i = 1, 2, 3 \quad (10)$$

where $T_{flx,i}$ represents the building comfort flexibility defined by the occupants. A set of operational constraints ensures a safe operation of the electrical and thermal equipment. The HWT temperature in all layers is bounded as follows:

$$15^\circ C \leq T_i(k) \leq 60^\circ C \quad i = 1, 2, 3 \quad (11)$$

where $T_i(k)$ are the HWT temperature from top to bottom layer, respectively. The HWT has a lower bound of $15^\circ C$ to prevent any frost problem on the heat pump evaporator. Heat pump power input P_{hp} , the State of Charge (SOC) and battery power exchange $P_{Batt}(k)$ are bounded by:

$$\begin{aligned} 0kW &\leq P_{hp}(k) \leq 2kW \\ -1kW &\leq P_{Batt}(k+1) - P_{Batt}(k) \leq 1kW \\ 4kWh &\leq E_{Batt}(k) \leq 5kWh \end{aligned} \quad (12)$$

Additionally, as for the price profile, a day/night schedule is considered such that its tariff changes only once per day as follows:

$$\begin{aligned} P_e^{day} &= 0.35 \\ P_e^{night} &= 0.10 \\ P_e &= [P_e^{day} \ P_e^{night}] \end{aligned} \quad (13)$$

4. NONLINEAR MODEL PREDICTIVE CONTROL

As the dynamic model of the system described in Section 2 is nonlinear, the resulting optimal control problem holds nonlinear equality constraints. The sampling time required for the building heating system is sufficiently long to allow the computation of the online exact solution of the NMPC problem without any real-time issue. Considering the control objectives described in Section 3, together with the plant model described in Section 2, the main objective of the NMPC algorithm is to minimize the electricity cost purchased from the electricity grid, at each sampling time k for a given prediction (control) horizon N_p (N_{ch}). Hence, the vector of control policy $u(k)$ is defined by solving the following optimal control problem,

$$\begin{aligned} & \min J(u, x) \\ \text{s.t. } & \text{equations : (9) to (13)} \\ & \text{system dynamics} \end{aligned} \quad (14)$$

The sampling time T_s is chosen as 15 minutes. In this paper, the multiple shooting method is used Kirches et al. (2012). The system dynamics are discretized via an explicit Runge-Kutta4 scheme with a fixed step size $T_{RK4} = \frac{T_s}{4}$, resulting in four integration steps per shooting interval. The resulting nonlinear program is solved by a primal-dual interior point method, see Potra and Wright (2000), using the software IPOPT. Moreover, The differentiation of the discrete dynamics, required for the nonlinear solver, is performed via Algorithmic Differentiation using the CasADi software package. see Andersson et al. (2019).

5. SIMULATION RESULTS AND DISCUSSION

5.1 Simulation settings

In this section, the proposed algorithm is tested for the application of three $150m^2$ radiant-floor buildings with the same specifications as described in Table 1, but different desired comfort level. In this scenario, the building number 3 does not accept any flexibility, while the building one and two are more flexible:

$$\begin{aligned} T_{flx1} &= 0.5^\circ C, T_{sp1} = 18^\circ C \\ T_{flx2} &= 1^\circ C, T_{sp2} = 20^\circ C \\ T_{flx3} &= 0^\circ C, T_{sp3} = 18^\circ C \end{aligned} \quad (15)$$

The hot water required for the building heating is provided by a 200l HWT, which is connected to a heat pump with the maximum capacity of 2kW electrical power, which can deliver up to 12kW thermal power. In this study, as the main focus is on the energy resources management, a predefined COP profile is considered, see Fig. 2(a). The assumptions are also made of the perfect prediction of the external inputs, particularly the weather temperature. The ambient air temperature profile is shown in Fig. 2(c), which is a periodic signal representing a typical winter day in the Italy, with a daily mean temperature of $0^\circ C$. As already shown in (8), the discrepancies between the buildings air temperatures and their set-points are penalized by weighting coefficients $\Upsilon_{r,i}$, $i = 1, 2, 3$ to deliver a temperature as close as possible the one requested by the occupants. The slack variables are also penalized with a weighting factor, which is much larger than other coefficients to satisfy the desired comfort bound whenever the

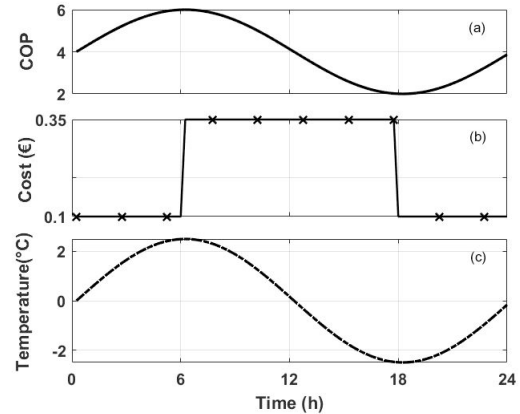


Fig. 2. External inputs initialization, (a): COP profile, (b): Electricity tariff, (c): Outside air temperature

problem is feasible. Table 2 shows all normalized weighting coefficient used in the NMPC formulation. The results are

Table 2. MPC parameters: normalized weighting coefficients

Υ_{r1}	1	Υ_{r3}	1	$\Upsilon_{\epsilon2}$	10
Υ_{r2}	1	$\Upsilon_{\epsilon1}$	10	$\Upsilon_{\epsilon3}$	10
Υ_p	$diag([P_e(k:k+N_p-1)]/Max(P_{Grid}))$				

discussed in two parts. First, the NMPC performance is studied for a constant electricity price scenario. Second, the analysis is performed for the day-night electricity tariff as shown in Fig. 2(b). For both scenarios, the results reveal that the NMPC can efficiently exploit the inertia of the HWT and of the buildings, and finds a compromise between the electricity tariff and the COP profile of the heat pump for charging or discharging of both the HWT and battery. As a result, it manages to economically shift the load from on-peak periods to the off-peak periods, while satisfying all desired comfort levels (15).

5.2 Constant electricity price scenario

In the constant electricity cost scenario (see Fig. 3) the desired comfort level is always satisfied by the NMPC, and the available flexibility of building number 1 and building number 2 are used to shift the activation of the heat pump to more efficient time periods. More precisely, when the COP is high, i.e. from midnight to midday (labeled AM hereafter), the HWT is charged by the heat pump to save thermal energy in the HWT, which gets released during PM periods, when the heat pump COP is low. Conversely, during PM hours, the heat pump is mostly used to satisfy the load constraints, and is sometimes off. The thermal energy stored in the HWT is then exploited to serve the buildings. In these periods, the mixing valves are at their minimum value, so as to exploit as much as possible the buildings inertia. Clearly, using the 3-element mixing valve at the pipeline inlet of each building increases the degree of freedom of the NMPC scheme. Therefore, the NMPC scheme is able to provide different comfort levels in different buildings, as shown in Fig. 3(a,b,c). For example, as there is no flexibility in building number 3, valve number 3 is used to set the temperature of building number 3 at

the requested set point. There is an interaction between the valves, which explains the variation of valve number 3, see Fig. 3(f). Indeed, valve number 1 and valve number 2 act on the HWT temperature to exploit the flexibility of building number 1 and 2 (Fig. 3(a, b)). This causes rapid changes in the HWT temperature, which in turn is used by valve 3 to achieve a fixed air temperature in building number 3. Furthermore, the NMPC scheme tends to charge the battery during PM period, when the heat pump is almost off. Actually, with a fixed price profile, the NMPC shapes the battery power based on COP estimation only. In this time period, the heat pump is off due to the low COP value, and the electrical energy is stored in the battery for later use, in order to minimize the energy cost.

6. CONCLUSION

The present paper focuses on energy efficiency in smart buildings, enabling them as future crucial nodes of a smart microgrid, being able to change their own load profile. In this vision, two main factors have been considered here: the adoption of both thermal and electrical storage units and of suitable optimal control techniques to accommodate all the constraints, all the competing objectives, and the different dynamics of the components (buildings, pump, storages). The control algorithm is realized with a nonlinear time-varying MPC. The results discussed here are extremely encouraging. First, computational issues are solved, both from numerical point of view and computation burden: the 15-minute simulation step is executed in less than 2 seconds on a standard office personal computer equipped with Matlab toolbox. Then, the optimal control formulation proposed here proved to be consistent in many different scenarios, even if only two of them are described with the necessary detail here. According to the results obtained, it is possible to cut electricity cost by approximately 20% in the variable price scenario. Future directions include the development and integration of more complex models for the heat pump. Also the sensitivity to system parameters encourages the adoption of some adaptation and estimation technique, for the heat pump itself but also for the tank and for the building.

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5.3 Day/Night electricity price scenario

Fig. 4 illustrates the NMPC performance for the day-night electricity price scenario. As before, the desired comfort levels are fulfilled by the controller. However, unlike the fixed price scenario, here the NMPC scheme tends to increase the temperature of the building 1 and 2 to the maximum limit during off-peak periods to better exploit the inertia of the buildings during on-peak periods, see Fig. 4(a, b). In the fixed-price scenario, the buildings temperature was kept at a lower level to enable lower set points for the HWT. In this scenario, the NMPC scheme operates in four distinguishable phases. In the first quarter of each day, i.e. at off-peak time and when the COP is high, the NMPC scheme charges all available forms of storage: the battery, HWT and the building walls. The battery and HWT are charged to their higher limit and the valves are opened to bring the building at their maximum temperature. These effects can be observed in the first quarter of each day, see Fig. 4. In the last quarter of each day, in off-peak periods but when the COP is lower, the NMPC scheme tends to charge mainly the battery as electrical energy is cheap. The heat pump is used less as the COP is low. The battery is fully charged during off-peak periods regardless of the COP value, to be ready for discharging during on-peak periods, see Fig. 4(i). In the second quarter of each day, both the COP and electricity price are high, which is the most challenging period. The power purchased from the grid is then zero in this period as the priority is given to the electricity cost minimization. So, all the required electrical power is provided by the battery. As the COP is high, it is better for the heat pump to start heating the HWT and consequently heating the buildings. Therefore, the heat pump runs at almost full load, and the battery is discharged to satisfy the electric power demand and without purchasing electricity from the grid. In the meanwhile, all valves are mostly open to allow the heat to enter the building. Finally, in the third quarter of each day, the COP is low and the period is on-peak. The battery is therefore discharging. Moreover, the low COP value makes it uninteresting to run the heat pump. In this period, the HWT and building inertia are exploited the most, with the heat pump being switched off almost completely.

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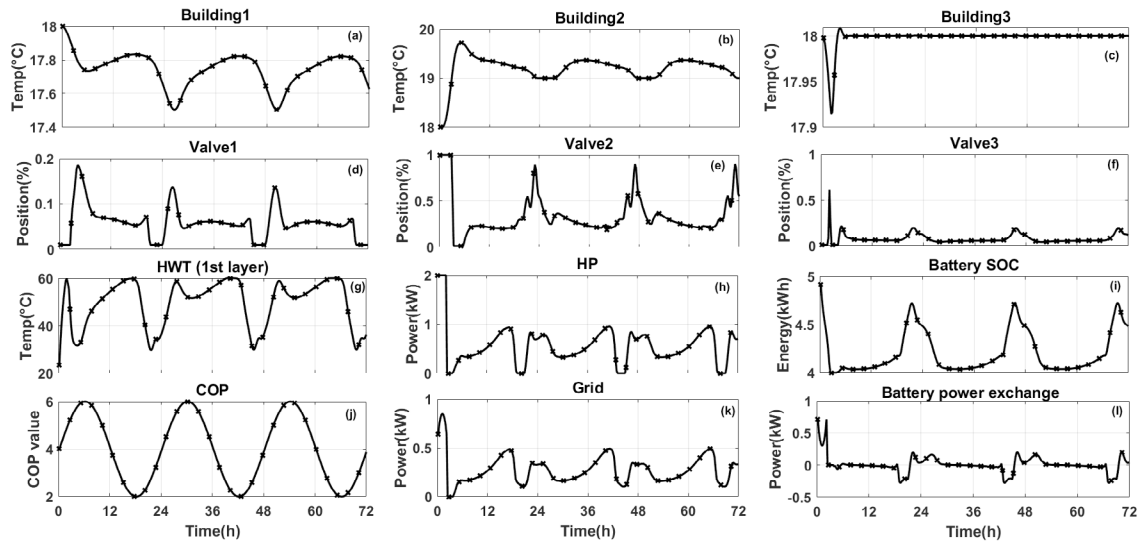


Fig. 3. NMPC performance analysis for the constant electricity price scenario

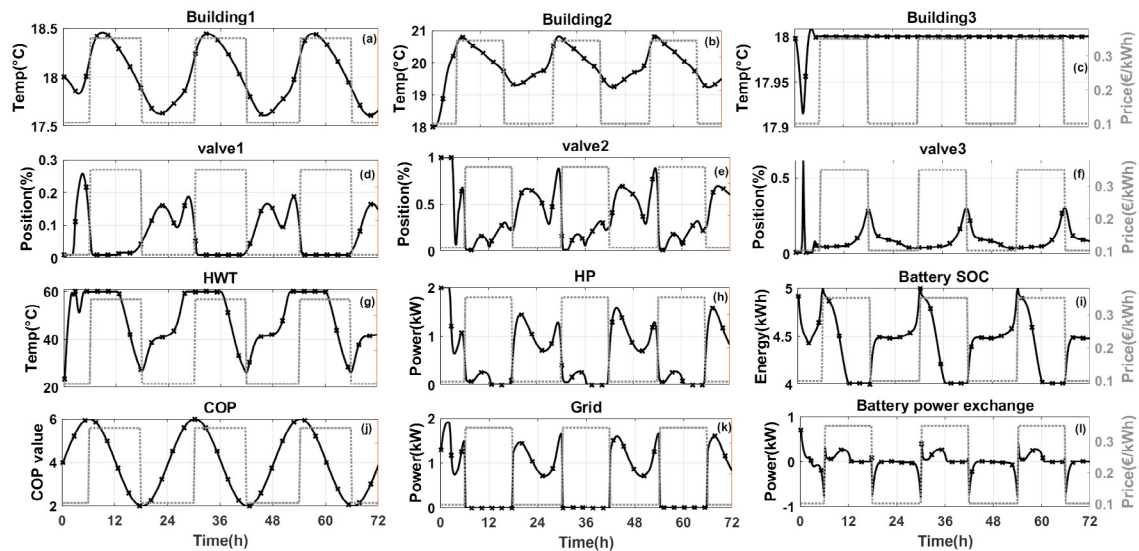


Fig. 4. NMPC performance analysis for the day-night electricity price scenario

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