On Data Reutilization for Historian Based Predictive Control

J. M. Maestre[1], Eva Masero[1], J. R. Salvador[2], D. R. Ramirez[1] and Q. Zhu[3]

Abstract— **This paper presents a robust finite-horizon control scheme based on data that produces feasible control sequences. The scheme makes use of a database that includes information from prior experiences of the same and others controllers handling similar systems. By the convex combination of feasible histories plus an auxiliary control law that deals with uncertainties, this scheme can be used as a robust historian-based predictive controller. Further application could include a cooperative learning-based strategy in which multiple controllers share their previous executions to gain collective benefits in terms of performance. The validity of the proposed controller is tested in a simulated case study.**

Index Terms— **Predictive control; Data-based control; Cloud-based control.**

I. INTRODUCTION

Many applications of model predictive control (MPC) in the industry, *e.g.*, [1]–[3], have stimulated the interest in approaches combining data-based and learning methods with predictive control schemes [4]. We say *renewed* because there was already some learning flavor in many control methods based on historical data, *e.g.*, those based on neural networks [5], adaptive models [6], scenarios [7], and the extension of terminal regions [8], to name a few examples. In the recent literature, we can find an MPC scheme for repetitive tasks that learns from previous executions to improve its performance, *e.g.*, [9], where the theoretical properties are derived from the use of safe sets, which can be defined as regions of the state space where there exists a control law that guarantees constraint satisfaction for all successive time steps. This topic is also addressed in [10], [11], where methods for expanding regions of safe states are proposed. Learning is also used to infer the plant model, as in [12], where experimental data on system inputs and outputs are used to feed a nonparametric machine learning method.

This paper focuses on reutilizing past historical information following the method presented in [13], where a predictive data-based approach that feeds the plant with a convex combination of previously applied trajectories is proposed for linear systems in the ideal case that successive executions of the system remain constant and disturbance-free. In the present study, we extend the original approach to outweigh lack of robustness for constraint satisfaction, *e.g.*, caused by parametric uncertainties and process noise. For that reason, the trajectories employed by the historian-based controller must fulfill a set of conditions to assure robust constraint satisfaction. To this end, we can apply a strategy analogous to traditional tube-based MPC [14], with the difference that here it is the data-based controller that provides *nominal* disturbance-free trajectories for the current system. Therefore, the newly proposed method can even deal with databases in which the information has been generated by different instances of the system being controlled (*e.g.*, consider a set of autonomous vehicles that share a cloud-based database).

The outline of the rest of the paper is as follows. Section II presents the problem settings and the control strategy. Sections III and IV detail the design and implementation of the nominal and auxiliary controllers. Section V illustrates simulation results, and Section VI shows concluding remarks.

II. PROBLEM FORMULATION

The system dynamics is considered to follow a discrete-time linear time-invariant (LTI) model with an unknown vector of parameters $w \in \mathbb{W} \subseteq \mathbb{R}^{n_w}$:

$$x^+ = A(w)x + B(w)u, \tag{1}$$

where $x,\ x^+ \in \mathbb{R}^{n_x}$ are the current and successor states, and $u \in \mathbb{R}^{n_u}$ is input of the system. Likewise, $A(w) \in \mathbb{R}^{n_x \times n_x}$ and $B(w) \in \mathbb{R}^{n_x \times n_u}$ are, respectively, the state-transition and the input-to-state matrices, which depend on the realization of parametric uncertainty w. We assume that:

1) All system realizations $(A(w), B(w))$ are controllable.
2) There exists a robust control law $u = Kx$ that can stabilize all system realizations $(A(w), B(w))$.
3) Two different realizations of w, say $w_i, w_j \in \mathbb{W}$, lead to two different system realizations (A_i, B_i) and (A_j, B_j), with

$$A_i - A_j \in \Delta A_w,\ \ B_i - B_j \in \Delta B_w, \tag{2}$$

i.e., differences between system realizations are bounded. Here, ΔA_w and ΔB_w are polytopic sets accounting for the possible parameter variations.

State and inputs of the system are subject to convex polytopic constraints with the origin in their interior, *i.e.*,

$$x \in \mathbb{X} \subseteq \mathbb{R}^{n_x},\ \ u \in \mathbb{U} \subseteq \mathbb{R}^{n_u}. \tag{3}$$

A direct consequence of (2) and (3) is that it is possible to recast the uncertainty of the system parameters as an unknown disturbance. For example, the viewpoints corresponding to the system realizations i and j can be related as

$$\begin{aligned} x^+ &= A_i x + B_i u \\ &\in (A_j \oplus \Delta A_w)x + (B_j \oplus \Delta B_w)u \\ &= A_j x + B_j u + \eta, \end{aligned} \tag{4}$$

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with $\eta \in \Omega \triangleq \Delta A_w \mathbb{X} \oplus \Delta B_w \mathbb{U}$ being a polytopic convex set that contains the origin in its interior. The symbol $\oplus$ denotes the Minkowski sum: $\mathbb{X} \oplus \mathbb{Y} \triangleq \{x + y : x \in \mathbb{X},\ y \in \mathbb{Y}\}$. Finally, note that in case of need, set Ω can be enlarged to deal with additional uncertainty sources.

A. Database and control goal

Any system realization has access to a database that contains a set $\mathcal{T} \triangleq \{1, \cdots, T\}$ of feasible trajectories for different realizations of (1). Here, note that we do not make any particular assumption about how the trajectories are generated. In particular, a trajectory $t \in \mathcal{T}$ is composed of a sequence of samples $\big(x_r(k_r), u_r(k_r)\big)$ for $k_r = \{1, \ldots, k^t_{\text{end}}\}$ that satisfy (1) and (3) for the particular system realization, with k^t_{end} being the time instant where the sequence t reaches the origin. Table I shows an example of the considered database, where each trajectory also provides us with meta-information about the trajectory identifier and the system realization in which that data was generated. Each row of the table can be considered as a partial trajectory with information about the time instant, the state, and the control input.

TABLE I
EXAMPLE OF DATABASE STRUCTURE

Traj. ID	System ID	k_r	$x_r(k_r)$	$u_r(k_r)$
1	1	1	$x_r(1)$	$u_r(1)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
1	1	k^1_{end}	$x_r(k^1_{\text{end}})$	$u_r(k^1_{\text{end}})$
2	1	1	$x_r(1)$	$u_r(1)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
2	1	k^2_{end}	$x_r(k^2_{\text{end}})$	$u_r(k^2_{\text{end}})$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

The control objective is to regulate the system to the origin while minimizing the following global infinite-horizon cost.

$$J^{\infty} = \sum_{k=0}^{\infty} \ell\big(x(k), u(k)\big), \tag{5}$$

where $\ell(\cdot,\cdot)$ is the stage cost, which is defined by the positive definite weighting matrices $Q \in \mathbb{R}^{n_x \times n_x}$ and $R \in \mathbb{R}^{n_u \times n_u}$:

$$\ell\big(x(k), u(k)\big) = x(k+1)^\top Q x(k+1) + u(k)^\top R u(k). \tag{6}$$

For the sake of implementability, it is desirable to consider a finite horizon N, so that the minimization of (5) is replaced by that of

$$J^{N} = \sum_{k=0}^{N-1} \ell\big(x(k), u(k)\big) + f\big(x(k+N)\big), \tag{7}$$

where $f(x(k+N)) = x^\top P x$ is a terminal cost function, with $P \succ 0$.

B. Control strategy

We control the system realization (A_i, B_i) using a data-based controller that combines the trajectories contained in $\mathcal{T}$. A robust feedback controller is also employed to deal with the differences η due to the uncertain system realization (recall Eq. (4)). That is, the control law becomes

$$u(x) = v(x) + v^e(x, \bar{x}), \tag{8}$$

where $v(x)$ corresponds to the first element of a sequence of control actions generated by the data-based law (obtained, *e.g.*, following [13]), and $v^e(x, \bar{x}) = K(x - \bar{x})$ rejects the differences between the system state x and the corresponding nominal value $\bar{x}$, which is the predicted state in k computed at instant $k-1$, *i.e.*, $\bar{x}(k) = x(k|k-1)$. Since there is a lack of information to compute $x(1|0)$ at instant $k = 0$, we consider $\bar{x}(1) = x(1|0) = x(1)$ at $k = 1$. A block diagram of the proposed dual control law is shown in Fig. 1.

III. DATA-BASED CONTROL LAW

Let us consider a particular system realization (A_i, B_i), which starts in state x_i. In the most restrictive case, there is no previous information in the database $\mathcal{T}$ on previous executions of this system, *i.e.*, we need to rely on information from other system realizations.

First, we consider that the information is obtained from a single system realization (A_j, B_j), with $j \neq i$. Since the sequences of system realization j might not be feasible for (A_i, B_i), we need to *check* their robustness. If we recall the dynamics of the system from the viewpoint of system realization j (Eq. (4)) and the double control law (Eq. (8)), it yields

$$x(k+1) \quad = \quad A_j x(k) + B_j\big(v(k) + Ke(k)\big), \tag{9}$$

where $e(k) = x(k) - \bar{x}(k) \in \Omega$ captures the difference between real and nominal values of the state. Hence, any candidate trajectory $\big(x_r(k_r), u_r(k_r)\big)$, with $k_r = \{1, \ldots, k^t_{\text{end}}\}$, must possess enough margin with the state constraints to allow the uncertainty given by the closed-loop dynamics of the errors due to model discrepancies. One way to do this is to check:

$$x_r(k) \oplus \mathcal{R} \in \mathbb{X},\ k_r = \{1, \ldots, k^t_{\text{end}}\}, \tag{10}$$

where $\mathcal{R}$ is a robust positively invariant (RPI) set $\mathcal{R}$, which is assumed to exist, *i.e.*,

$$\begin{aligned} &(A_j + B_j K)\mathcal{R} \oplus \Omega \subseteq \mathcal{R} \subseteq \mathbb{X}, \\ &K\mathcal{R} \subseteq \mathbb{U}. \end{aligned} \tag{11}$$

Therefore, we need to check the following conditions:

$$\begin{array}{lll} x_i & \in & x_r(1) \oplus \mathcal{R}, \\ x_r(k_r) & \in & \mathbb{X} \ominus \mathcal{R}, \\ u_r(k_r) & \in & \mathbb{U} \ominus K\mathcal{R}, \end{array} \tag{12}$$

with $k_r = \{1, \ldots, k^t_{\text{end}}\}$, where the symbol $\ominus$ denotes the Pontryagin difference defined as $\mathbb{X} \ominus \mathbb{Y} \triangleq \{x \in \mathbb{X} : x + y \in \mathbb{X},\ \forall y \in \mathbb{Y}\}$. In this way, we can determine which of the available sequences of the database are valid by considering a robust control problem from the viewpoint of subsystem (A_j, B_j). Details on the calculation of K and $\mathcal{R}$ are given in Section IV.

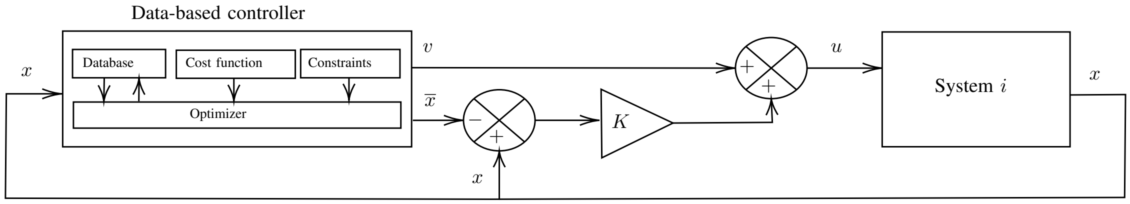

Fig. 1. Block diagram of the combined data-based and feedback controller.

By checking (12), we can find the set of robust feasible trajectories for each system j in the database, say $\mathcal{T}_j$.[1] Here, let $\mathcal{F}$ denote the set of system realizations providing at least one feasible trajectory, which is assumed to be nonempty. For a given N-length partial sequence $p_j \subseteq t \in \mathcal{T}_j$, $j \in \mathcal{F}$, obtained taking N consecutive states and control actions from a trajectory t, that is

$$p_j \quad \begin{cases} \mathbf{x_{p_j}} = \{x_{p_j}(0), x_{p_j}(1), \ldots, x_{p_j}(N-1)\} \\ \mathbf{u_{p_j}} = \{u_{p_j}(0), u_{p_j}(1), \ldots, u_{p_j}(N-1)\} \end{cases}.$$

Let $J_{p_j}^N$ denote the performance of this sequence measured by (7). A feasible weight vector $\Lambda \in \mathbb{R}^m$ for the convex combination of control sequences when the current state measurement is $x_i(k)$ can be computed as

$$\begin{aligned} \min_{\Lambda \in \mathbb{R}^m} \quad & \sum_{j\in\mathcal{F}} \sum_{p_j \in \mathcal{T}_j} \lambda_{p_j} J_{p_j}^N, \\ \text{s.t.} \quad & \sum_{j\in\mathcal{F}} \sum_{p_j \in \mathcal{T}_j} \lambda_{p_j} x_{p_j}(0) = x_i(k), \\ & \sum_{p_j \in \mathcal{T}_j} \lambda_{p_j} = 1, \\ & \lambda_{p_j} \geq 0,\ p_j \subseteq t \in \mathcal{T}_j,\ j \in \mathcal{F}, \end{aligned} \tag{13}$$

where Λ is the set of weighs λ_{p_j}, with $p_j \subseteq t \in \mathcal{T}_j$ and $j \in \mathcal{F}$, which are the decision variables of (13).

Thus, let $\Lambda^* = \{\lambda_1^*, \lambda_2^*, \ldots, \lambda_m^*\}$ be the minimizer of (13), provided that it exists,[2] then

$$\mathbf{u_c} = \sum_{j\in\mathcal{F}} \sum_{p_j \in \mathcal{T}_j} \lambda_{p_j}^* \mathbf{u_{p_j}} \tag{14}$$

is a nominal solution for system (A_i, B_i) and the first element correspond to the control action $v(k) = \mathbf{u_c}(1)$. The auxiliary part of the control law becomes:

$$v^e(k) = K\big(x_i(k) - \sum_{j\in\mathcal{F}} \sum_{p_j \in \mathcal{T}_j} \lambda_{p_j}^{k-1} x_{p_j}(k|k-1)\big), \tag{15}$$

where $\lambda_{p_j}^{k-1}$ is the optimal weigh vector computed in $k-1$ and $x_{p_j}(k|k-1)$, which corresponds to the nominal state $\bar{x}$, is the predicted state in k computed at instant $k-1$.

As a consequence, the trajectory of the system realization i will lie within a tube around the trajectory generated by the data-based controller, which will be recursively feasible because the combined partial sequences reach the origin. At that moment, the state of the system will lie in a region $\mathcal{R}$ around the origin. Likewise, it is also possible to produce a new combination of partial sequences based on the most recent information available in the database following a receding-horizon strategy.

Algorithm 1 details the steps of the proposed robust historian-based controller. Note that we consider as a selection criterion that only the m feasible partial trajectories of the database can be combined to obtain the control input to relieve the computational burden.

Algorithm 1

Inputs: k, $x(k)$, $\mathcal{T}_j$, K, $\mathcal{R}$

1: Compute the distance between the current state measurement $x(k)$ and the state x_r of all rows in the database:

$$\mathrm{d}(x(k), x_r) = |x(k) - x_r|^2.$$

2: Select the m feasible partial sequences of the database with the lowest distance.
3: Evaluate the cost (7) of each candidate $p \in m$.
4: Solve (13) to optimize the weight vector Λ^* for the convex combination of the m control sequences.
5: Obtain $\mathbf{u_c}$ as (14) and then $v(k) = \mathbf{u_c}(1)$.
6: Calculate the auxiliary control input term $v^e(k)$ that penalizes the deviation between current and the corresponding nominal states using (15).
7: Apply $u(k) = v(k) + v^e(k)$ to the system.

IV. Auxiliary Control Law

The design of a robust feedback controller $u = Kx$ is based on standard linear matrix inequalities (LMIs) [15]. In particular, we assume that the uncertainty set $(A(w), B(w))$ of system realizations is polytopic with vertices denoted by set $\mathcal{V}$, leading to the dynamics:

$$x^+ = A_i x + B_i u, \quad i \in \mathcal{V}.$$

In this context, it is possible to find K by maximizing the trace of matrix W subject to the set of LMIs:

$$\begin{bmatrix} W & WA_i^\top + Y^\top B_i^\top \\ A_i W + B_i Y & W \end{bmatrix} \succ 0, \quad i \in \mathcal{V}, \tag{16}$$

where W and Y are the optimization variables. The stabilizing control law is $K = YW^{-1}$ and the corresponding Lyapunov function becomes $P = W^{-1}$. Then, given an ellipsoidal bound on the disturbance, it is straightforward to find an RPI for the system realizations by using standard LMIs methods. As for the polytopic case, one can employ methods such as the LP proposed in [16] to compute a common minimal RPI

[1] Feasible sequences can be extended with zeros so that their length becomes greater or at least equal to N, *i.e.*, $k_{\text{end}}^t \geq N$.

[2] One option to guarantee the existence of the minimizer is to use soft constraints in Problem (13).

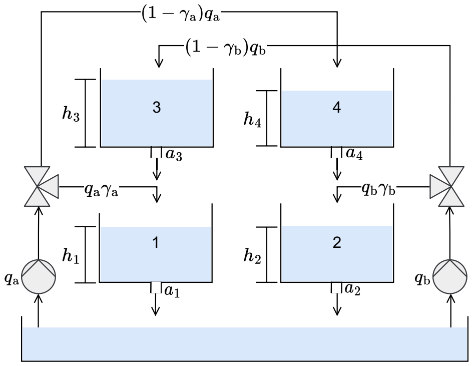

Fig. 2. Scheme of the quadruple-tank plant.

for the system realizations. Likewise, one can find a feedback gain and an RPI set for each vertex and then use the union of the RPI sets in the checking steps (12).

V. Case Study

We evaluate the proposed control scheme in the quadruple-tank plant presented in [17], which consists of four interconnected water tanks, as shown in Fig. 2.

The two upper tanks ($\#3$ and $\#4$) discharge flow rates to the lower ones ($\#1$ and $\#2$), and these, in turn, into a sinking tank. The plant is controlled by two pumps that keep the water circulation between tanks. There are also two three-way valves that divide the flow into two ways. Applying the mass balance and Bernoulli's law to the plant, we obtain the following:

$$\begin{aligned} S\frac{dh_1}{dt} &= -a_1\sqrt{2gh_1} + a_3\sqrt{2gh_3} + \gamma_{\mathrm{a}}\frac{q_{\mathrm{a}}}{3600}, \\ S\frac{dh_2}{dt} &= -a_2\sqrt{2gh_2} + a_4\sqrt{2gh_4} + \gamma_{\mathrm{b}}\frac{q_{\mathrm{b}}}{3600}, \\ S\frac{dh_3}{dt} &= -a_3\sqrt{2gh_3} + (1-\gamma_{\mathrm{b}})\frac{q_{\mathrm{b}}}{3600}, \\ S\frac{dh_4}{dt} &= -a_4\sqrt{2gh_4} + (1-\gamma_{\mathrm{a}})\frac{q_{\mathrm{b}}}{3600}, \end{aligned} \tag{17}$$

where h_n is the water level of tank $n \in \{1,2,3,4\}$, a_n is the cross section of the outlet pipe n, $S = 0.03\ \mathrm{m}^2$ is the cross section of all tanks, $\gamma_{\mathrm{a}}, \gamma_{\mathrm{b}} \in [0,1]$ are the opening of the three-way valves ($\gamma_{\mathrm{a}} = 0.3$,$\gamma_{\mathrm{b}} = 0.4$), $g = 9.81\ \mathrm{m/s}^2$ is the gravitational constant, and $q_{\mathrm{a}}, q_{\mathrm{b}}$ are the pump flow rates.

The nonlinear system dynamics expressed by (17) can be linearized by its Taylor expansion. Given the operating point:

$$\begin{aligned} h^0 &= [h_1^0, h_2^0, h_3^0, h_4^0] = [0.5006, 0.4704, 0.5206, 0.4319]\ \mathrm{m} \\ q^0 &= [q_{\mathrm{a}}^0, q_{\mathrm{b}}^0] = [1.5355, 1.6794]\ \mathrm{m}^3/\mathrm{h}, \end{aligned}$$

the discrete linearized state-space dynamics are

$$\hat{x}^+ = A\hat{x} + B\hat{u}, \quad \hat{y} = C\hat{x}, \tag{18}$$

with $\hat{x} = [h_1 - h_1^0, \ldots, h_4 - h_4^0]^\top$ and $\hat{u} = [q_{\mathrm{a}} - q_{\mathrm{a}}^0,\ q_{\mathrm{b}} - q_{\mathrm{b}}^0]^\top$. The discrete matrices (A, B, C) of the nominal system using a sample time $t_{\mathrm{s}} = 30$ s are:

$$A = \begin{bmatrix} 0.6654 & 0 & 0.1919 & 0 \\ 0 & 0.5971 & 0 & 0.2250 \\ 0 & 0 & 0.7643 & 0 \\ 0 & 0 & 0 & 0.7077 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0684 & 0.0179 \\ 0.0254 & 0.0868 \\ 0 & 0.1461 \\ 0.1644 & 0 \end{bmatrix},$$

and $C = I$, which is the unit matrix of appropriate dimension.

Regarding constraints, the system is subject to the following operational state and input constraints:

$$\begin{aligned} 0.2\ \mathrm{m} \leq \{h_1, h_2, h_3, h_4\} \leq 1.2\ \mathrm{m}, \\ 0\ \mathrm{m/h}^3 \leq \{q_{\mathrm{a}}, q_{\mathrm{b}}\} \leq 3\ \mathrm{m/h}^3. \end{aligned}$$

A. Control parameters

We aim to achieve the reference levels of the two lower tanks (h_1^0, h_2^0) by minimizing an N-horizon cost function while satisfying some constraints. We consider a prediction horizon of $N = 5$, weighting matrices $Q = I$ and $R = 0.01 \cdot I$, and a simulation length of $N_{\mathrm{end}} = 2000$ s. The feedback gain K is calculated by solving (16):

$$K_{(A,B)} = \begin{bmatrix} -0.8572 & -0.6449 & 0.29360 & -3.1792 \\ -0.7400 & -1.0007 & -3.4503 & 0.07570 \end{bmatrix},$$

and the RPI set $\mathcal{R}$ is computed with the `MPT toolbox` of MATLAB taking into account K and the uncertainty polytopic set Ω for all system realizations.

B. Database and system realizations

The database is composed of a set $\mathcal{T}_o = \{1, \ldots, T_o\}$ with $T_o = 100$ trajectories, which have been obtained from the nominal plant (A, B) using PID controllers. These trajectories start at different points and steer the system to the operating point $\{h^0, q^0\}$, which becomes the origin in (18). We consider the most restrictive case where there is no information in the database about other system realizations. Each trajectory has 7000-second information of the plant operation. Hence, there are $T_o \cdot 7000/t_{\mathrm{s}}$ candidate trajectories at each time step, but only $m = 1000$ are considered to compute the control input.

C. Simulation results

We perform simulations for three system realizations: (A_1, B_1), (A_2, B_2), and (A_3, B_3), which are obtained by slightly changing the cross section of the outlet pipes of the nominal system (A, B), as detailed in Table II. Note that, for a practical application of the control method, the difference between systems should be small to apply robust arguments. As illustration, Figs. 3 and 4 display the results corresponding to the first and second system realizations and show trajectories of the tank levels (m) and the flow rates (m^3/h) considering four initial states: $(h_1^0, h_2^0) \pm (0.05, 0.05)$ with their corresponding h_3^0 and h_4^0, and the saturated flow rates $q_{\mathrm{a}}^{\mathrm{sat}}, q_{\mathrm{b}}^{\mathrm{sat}}$. As shown, our proposed robust data-based controller take advantages of information from the database, which only contains trajectories from the nominal system (A, B), to carry the tanks levels $\#1$ and $\#2$ to their operating points, *i.e.*, h_1^0 and h_2^0. Note that the levels h_1 and h_2 have steady-state errors because the database does not have offset-free trajectories. The mean relative error of these tank levels $n = \{1, 2\}$ computed as $E_{\mathrm{rel}}(\%) = (E_{\mathrm{rel}}^1 + E_{\mathrm{rel}}^2)/2$, with:

$$E_{\mathrm{rel}}^n = \frac{E_{\mathrm{abs}}^n}{h_n^0} \cdot 100, \quad E_{\mathrm{abs}}^n = \frac{\sum_{k=1}^{N_{\mathrm{end}}/t_{\mathrm{s}}} \left| h_n(k) - h_n^0 \right|}{N_{\mathrm{end}}/t_{\mathrm{s}}},$$

and the accumulated cost: $J_{\mathrm{acc}} = \sum_{k=1}^{N_{\mathrm{end}}/t_{\mathrm{s}}} \hat{h}(k)^\top\ Q\ \hat{h}(k) + \hat{q}(k)^\top\ R\ \hat{q}(k)$, with $\hat{h}(k) = [h_1(k), h_2(k)] - [h_1^0, h_2^0]$ and

TABLE II

NUMERICAL RESULTS OF THE ROBUST DATA-BASED CONTROLLER FOR THE DIFFERENT SYSTEM REALIZATIONS.

System Realizations	Cross section of outlet pipes (m)				Robust data-based controller	
	$a_1 \cdot 10^{-4}$	$a_2 \cdot 10^{-4}$	$a_3 \cdot 10^{-4}$	$a_4 \cdot 10^{-4}$	$E_{\text{rel}}(\%)$	J_{acc}
(A, B)	1.301	1.597	0.8758	1.026	2.1812	0.2465
(A_1, B_1)	1.341	1.627	0.8961	1.108	4.7457	0.2726
(A_2, B_2)	1.436	1.588	0.9769	0.9568	8.8432	3.4295
(A_3, B_3)	1.436	1.588	0.9769	1.226	8.9097	0.5190

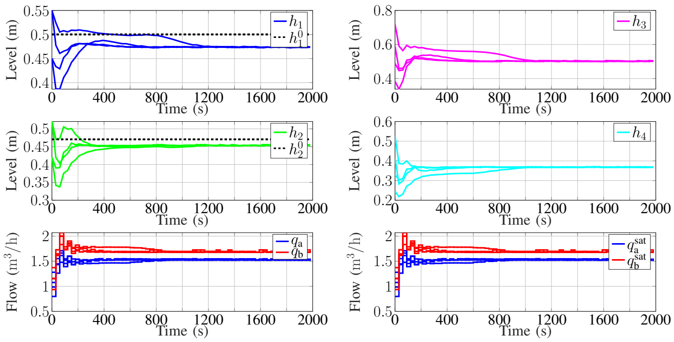

Fig. 3. Results for system realization (A_1, B_1) with the proposed approach.

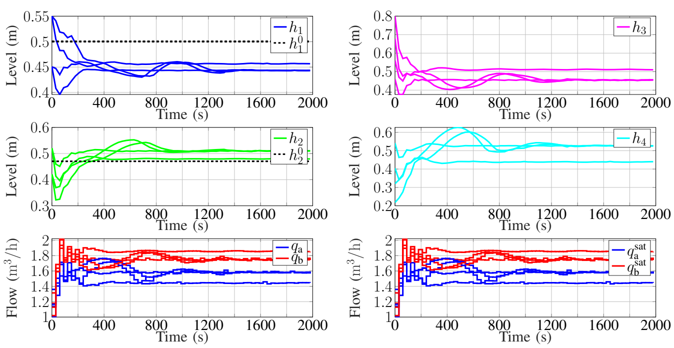

Fig. 4. Results for system realization (A_2, B_2) with the proposed approach.

$\hat{q}(k) = [q_{\text{a}}(k), q_{\text{b}}(k)] - [q_{\text{a}}^0, q_{\text{b}}^0]$ are shown in Table II. By combining the data-based and feedback controller, we can stabilize plant realizations with data from the nominal plant.

VI. CONCLUSIONS

A robust data-based predictive approach that allows a set of controllers to share information employing a common database has been proposed. An interesting application is cloud-based cooperative learning, where multiple controllers may cooperate and reuse their collective experiences. The proposed method is related to tube-based model predictive control that uses a bound on disturbance to drive the system by using a dual control law in which a data-based controller steers the nominal system, and the error is dealt with auxiliary feedback. Our results in the quadruple-tank plant show promising results and highlight the relevance of exchanging data between different system realizations in this context.

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