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Radar observation and recontruction of Cosmos 1408 fragmentation

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A B S T R A C T

The population of objects in space has increased dramatically over recent decades. Space debris now represents the majority of objects in space resulting from inactive satellites, breakups, collisions and fragmentations. It has become a concern for institutions all over the world and, as such, it has led to the fostering of several programmes to counter the issues. Among these, the use of ground-based sensors for Space Surveillance Tracking (SST) activities and services and tools for analysing fragmentations play a crucial role.

This work presents the activities carried out by Politecnico di Milano, Italian Space Agency and Italian National Institute of Astrophysics in this framework, using data from SST networks and the observation measurements from Bistatic Radar for LEo Survey (BIRALES), an Italian bistatic radar belonging to the EUropean Space Surveillance and Tracking (EUSST), which contributed most to the monitoring of the cloud of fragments. Exploiting Two-Line Elements (TLEs) of observed fragments, a reverse engineering approach is used to reconstruct a fragmentation in orbit through the use of the software suite PUZZLE developed at Politecnico di Milano. The analyses focus on studying the fragmentation of the Cosmos 1408 satellite, which occurred on November 15th 2021 following an Anti-SATellite (ASAT) missile test. More than 1000 trackable pieces and millions of smaller debris (estimated from numerical analysis) were produced by this event, increasing the population of inactive objects around the Earth, and threatening nearby orbiting objects.

First, the processing method adopted from BIRALES in observing Cosmos debris is presented and discussed and a critical analysis about the derivable information is conducted. Then, these data and those from SST network observations are used to identify the epoch and the location of the fragmentation. In this procedure, the software toolkit PUZZLE, developed by Politecnico di Milano within a project funded by the Italian Space Agency and extended through the European Research Council, is used.

1. Introduction

Over the last decades, human-made satellites have become valuable assets for different purposes, from science and research, to observation, telecommunication, navigation, as well as for military purposes. The number of launches has steadily increased in the last few years and hence the number of orbiting objects. Currently, almost 9000 satellites are still in space, but the overall population of Resident Space Objects (RSO) consists of more than 31,000 objects greater than 10 cm [1]. This is because the majority of RSO is made of space debris, and it is estimated that the number exceeds 130 million for pieces smaller than 1 cm [1]. The increasing threat of potential collisions that would damage active satellites and create new debris, as well as the risk of uncontrolled re-entries that could damage ground infrastructure, has thus led to the development of space surveillance networks and Space Surveillance & Tracking (SST) initiatives to mitigate these risks, catalogue new objects, and provide information and services to stakeholders.

Most of the debris were generated from over 600 recorded fragmentation events, the most notable ones being the 2007 Fengyun-

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1C Anti-Satellite Test (ASAT), the Iridium 33/Cosmos 2251 collision in 2009, and the recent Cosmos 1408 ASAT, occurred on November 15th 2021 at around 02:47 UTC [2], which resulted in more than 1000 new pieces of debris [2,3]. This paper focuses on the observations of the Cosmos 1408 breakup performed by Politecnico di Milano, the Italian National Institute of Astrophysics (INAF) and the Italian Space Agency ASI and details the use of the PUZZLE software for the characterisation of the fragmentation.

The paper is organised as follows: Section 2 describes the BIRALES sensor architecture, Section 3 describes the PUZZLE model and its updated versions, Section 4 presents the analyses carried out with BIRALES and PUZZLE on the Cosmos 1408 fragmentation, and Section 5 summarises the main results.

2. BIRALES sensor

BIstastic RAdar for LEo Survey, or simply BIRALES, is one of the Italian radar sensors belonging to the EUropean Space Surveillance and Tracking (EUSST) framework [4]. The transmitter, Radio Frequency Transmitter (TRF), is a 7 m wide parabolic antenna located at the Italian Joint Test Range of Salto di Quirra in Sardinia. The receiver is part of the Northern Cross radiotelescope, situated at the Medicina Radio Astronomical station (close to Bologna) where it is operated by INAF. In this case, the antenna consists of an array of 8 parallel cylinders (each 23.5 m long and 7.5 m wide) arranged along the North-South directions, with a total collecting area of about 1400 square meters, and this allows to detect small objects with a size of 10 cm at 1000 km. Each cylinder contains 4 receiver elements along its focal line, hence a total of 32 receivers are available, as shown in Fig. 1. The cylinders can be moved only in elevation and their pointing remains unchanged throughout the observation. Both the transmitter and receiver of BIRALES are currently undergoing an upgrade process, which will improve the sensor sensitivity in the future [5,6].

The radar couples two different systems: the TRF can transmit a Frequency Modulated Continuous Wave (FMCW) chirp centred at 412.5 MHz and with 4 MHz bandwidth, and an unmodulated Continuous Wave (CW) signal at 410.085 MHz. The former system is used to measure the range of the targets, while the latter provides Doppler measurements. A total maximum power of 5 kW can be supplied, and it can be split as needed between the two signals according to the observations needs (i.e., to increase the sensitivity of one of the two systems). Range processing is performed by combining the received data from 3 cylinders (12 elements) into a single analog beam; instead, Doppler shift measurements are obtained exploiting multiple digital beams formed combining all 32 elements. In this way it is possible to cover the complete receiver Field of View (FoV) that spans about 6 in both North-South and East-West directions.

However, due to the sensor arrangement, grating lobes are present in any receiver beam and so it is difficult to determine which lobe is illuminated (which is linked to the angular position of the object). Currently, the static multibeam architecture is also exploited to derive the angular track in the receiver FoV. However, the entire BIRALES back-end is undergoing an upgrade to exploit an adaptive beamforming technique to derive this measurement. This will allow to reach high-resolution angular estimation. A more detailed description of BIRALES architecture and the planned upgrade to derive the angular track is provided in [7].

3. PUZZLE

PUZZLE is a software toolkit developed at Politecnico di Milano [8], initially under a contract with ASI, and later improved, under an European Research Council (ERC) project, including a long-term analysis [9] and uncertainty propagation [10]. The objective is to identify which objects, included in a set from a catalogue, originated via a breakup and, whenever a fragmentation is identified, to characterise it in terms of position and epoch of the event. Moreover, the software has a routine to model the fragmentation to identify the orbital regions at risk of possible collisions in the future.

The model investigates the evolution of the orbits of a set of unclassified objects in the form of TLE (retrieved from daily catalogues) by propagating them backwards (using the SGP4 propagator [11,12]) and searching for possible clustering of objects, that could represent a fragmentation. The propagation is coupled with pruning and clustering algorithms (e.g., geometrical filters exploiting the Minimum Orbital Intersection Distance (MOID) [13] or the hierarchical clustering method [14]) used to identify close approaches between the analysed objects and to remove the undesired ones. The close encounters are then used to find epochs characterised by concentration of fragments, which are considered as fragmentation. In case the latter is detected, the information on

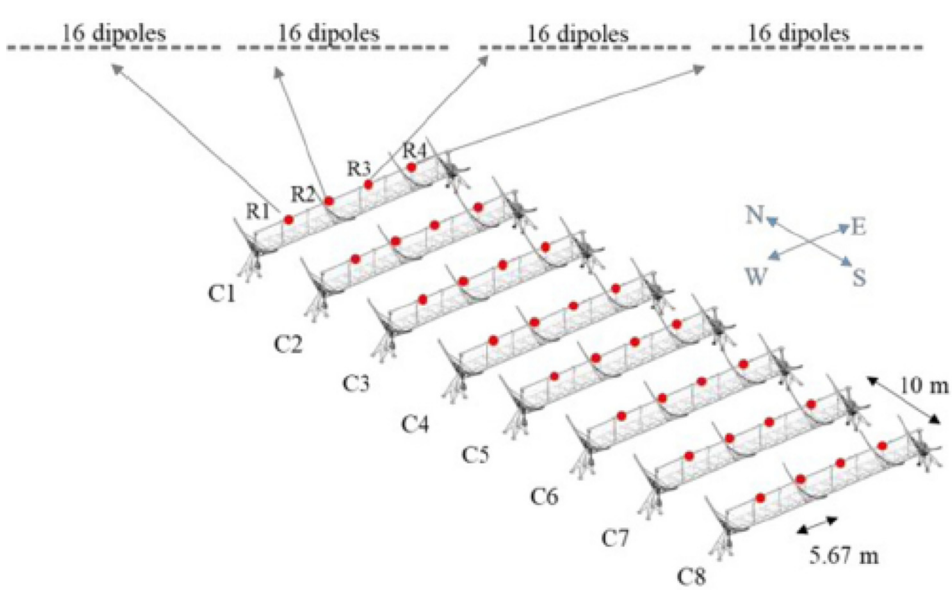

Fig. 1. BIRALES receiver configuration.

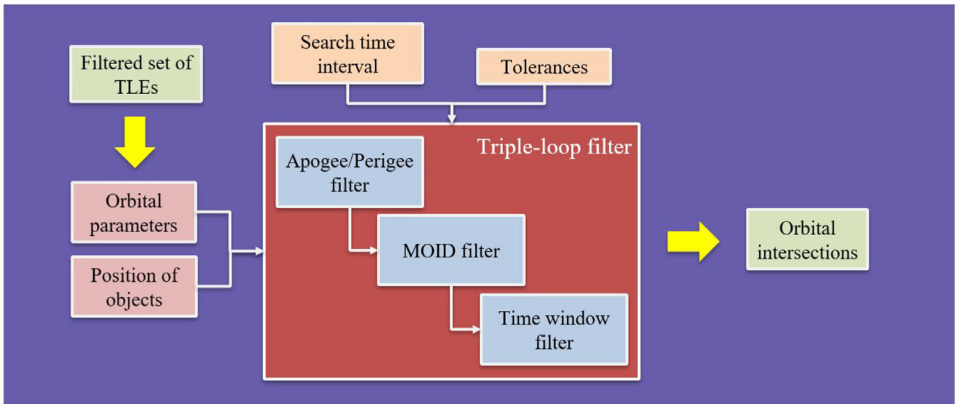

Fig. 2. Block diagram showing the filtering steps (from Romano et al. [8]).

the location and epoch of the event are used to scan a set of active or inactive objects (both spacecraft and rocket bodies) to associate the parent(s) that generated the found fragments. In the end, the NASA standard breakup model [15] is used to estimate the area-to-mass and relative velocity distributions of the generated fragments for that event. It is important to state that the entire procedure is carried out without a-priori assuming any recent breakup event. In addition, the initial set of TLEs is scanned through a filter to remove possible outliers from the process.

PUZZLE was further improved including uncertainty propagation inside the routine [10]. A hybrid non-linear uncertainty propagator was developed, coupling gaussian mixture model and unscented transformation algorithms. Starting from a value of uncertainty α, a normal distribution is generated for each TLE (or for each initial state associated to each considered object). The distribution is then decomposed into a sum of N Gaussian distributions through a gaussian mixture model. The obtained kernels are identified considering the mean of the distribution, which is used, along with the UT model, to generate K modified TLEs (for each object) to be added to the initial set.

3.1. Filtering steps

This section briefly introduces the main filtering steps (Fig. 2) performed by PUZZLE (a more complete description can be found in [8,9]). The filter consists of three main steps, and is inspired by that of Hoots et al. [16]. The first filter compares the heights of the apogee and perigee of the orbits of two objects to investigate if their relative geometry allows close approaches between the two objects. Thus, the maximum perigee radius $q = \max(r_{P_1}, r_{P_2})$ and the minimum apogee radius $Q = \min(r_{A_1}, r_{A_2})$ are compared against a predefined threshold as

$$q - Q \leq \Delta \qquad (1)$$

The objects that survive the first step, are compared against each other checking the MOID between the orbits. The computation is performed using the algebraic method proposed by Gronchi [13]. Also in this case, a threshold is defined to discard undesired objects.

The last filter checks whether it is possible for the two objects to be in proximity of the MOID at the same time within a selected time period. This is performed by defining angular windows around the positions of the previously identified MOID along the two orbits, by converting those windows to time windows using Kepler's equations, and by adding multiples of the periods of both orbits to the endpoints of each window. In this way, a sequence of time windows is defined throughout the searching interval of the fragmentation. The windows are then investigated to determine possible overlaps: if at least two intervals along the orbits overlap, the two objects are able to experience a close encounter within the specified time frame. If all the filtering steps are satisfied, a close encounter is possible and hence the two objects are further investigated to determine the actual encounter distance in order to identify the closest approach within the research window. This process is repeated for all the objects in the set, comparing two objects at a time. Then, in order to detect the fragmentation epoch and location, the evolution of all the close encounters between all the survived objects is investigated considering only those encounters that are below a defined threshold, which is the close encounter distance margin. In this way, clusters of objects can be identified, and a possible fragmentation detected.

4. Analyses for reconstructing an ASAT event

The aim of the following analyses is to reconstruct the fragmentation of the Cosmos 1408 satellite which occurred on November 15, 2021. Section 4.1 describes the analysis performed using the TLEs from a catalogue to see if PUZZLE can associate the TLEs to the fragmentation. Then, Sections 4.2 and 4.3 present the analyses on the observations conducted using BIRALES and the reverse engineering study to reconstruct the fragmentation with PUZZLE using the TLEs produced with those observations.

4.1. Fragmentation detection using TLEs from a catalogue

The first analysis presented in this section is performed including in the initial set the TLEs associated to the Cosmos 1408 fragmentation, only. Then, a second test is presented in which additional TLEs were added to those associated to the Cosmos 1408 fragmentation.

4.1.1. Cosmos 1408 TLEs

The first analysis focuses on a set of objects comprising only the TLEs of the Cosmos 1408 debris, released by SpaceTrack [17] on December 1st 2021 (first available data), that is 16 days after the event. The catalogue includes 185 objects associated to the fragmentation. The objectives are to verify that the software can identify the fragmentation and to count how many fragments can be included in the final set. The analysis is carried out playing with one of the input parameters, that is the close encounters distance margin. This parameter states whether the close encounter between two analysed objects can be considered for the investigation of the event location (i.e., if the objects can be considered close enough) or not. The selection of small values is typically required to avoid the count of many close encounters which do not represent a real cluster of objects. However, being the propagation

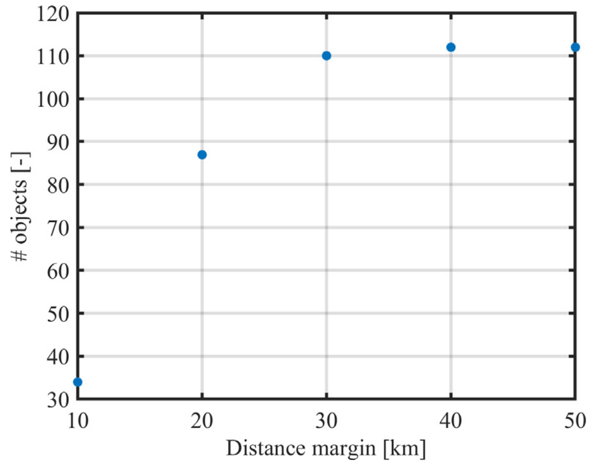

Fig. 3. Number of objects identified as function of the close encounters distance margin.

time near the limit in terms of accuracy for the SGP4 propagator and knowing a priori that the fragments belong to the fragmentation, it is possible to enlarge the distance margin for the test. Fig. 3 shows the results of the analysis. Choosing a lower distance margin implies a lower number of identified fragments (34 of 185 with 10 km). However, by enlarging this value, it is possible to include up to 112 fragments (i.e., about 60 %). It is important to mention that selecting wider margins for all the filtering parameters (not investigated in this work) could lead to an increase in the number of included objects. Finally, all the simulations estimated the epoch of the event on November 15th 2021 at 02:47 UTC 1.67 min.

4.1.2. Cosmos 1408 and additional TLEs

A test was then performed considering a set of objects including the Cosmos 1408 and 24 of its debris along with TLEs of known objects available on SpaceTrack on December 1, 2021. The 24 fragments belonging to the fragmentation have been randomly selected from the set of almost 90 fragments that in the Section 4.1.1 were identified when considering a distance margin of 20 km for the identification of the close encounters. The fragmentation is searched for within the 17 days prior to the reference epoch of the TLEs. The analysis will be evaluated in terms of the correct detection of the fragmentation, the correct identification of the fragments among the objects in the TLEs set and of the parent objects, and the computational time required. The initial TLEs set is composed of 1588 TLEs referring to 745 unique objects from different orbital regions. Table 1 shows the main parameters used for the analysis and the main results.

617 TLEs passed the filtering steps (more details about them in [8]), including all those related to the fragmentation. Then, setting the close encounters distance margin to 20 km, the software correctly detected the fragmentation (estimation of the event epoch in Table 1). Fig. 4 shows the close approaches below 20 km, between

Table 1
Main parameters used to detect the Cosmos 1408 breakup and main results.

Initial size of TLE set	$\sim$ 1600
Date of generation	1st December 2021
Time interval selected	17 days
Estimated epoch of the event	15th November 2021 02:47:59
Number of objects involved	24
Probable parent object(s)	Cosmos 1408 (ID 13552)
Computational time	8.3 min

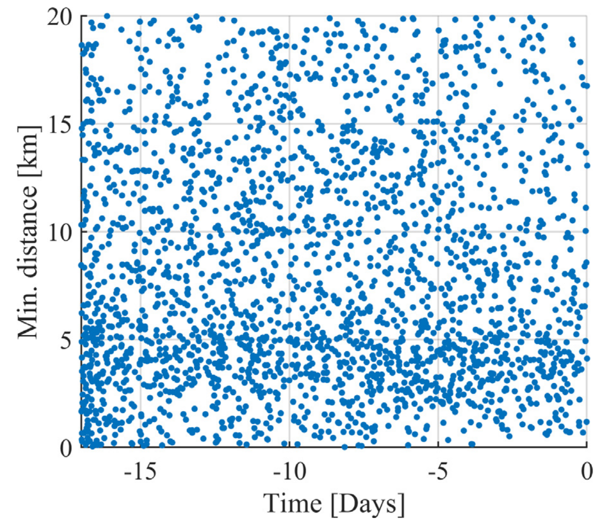

Fig. 4. Distribution in time and distance of the close encounters between the objects - Cosmos 1408 test using SpaceTrack TLEs.

the investigated objects, detected within the search time frame. On the x-axis, the number of days before the epoch of the investigated set of objects (i.e., December 1, 2021) is reported, where the 0 is associated with the epoch of the latest TLE.

4.2. BIRALES observation

The fragmentation of Cosmos 1408 was monitored by BIRALES in the days after the event, starting from November 16th 2021. The Tracking Data Messages (TDMs) collected by BIRALES were readily sent to the Italian Air Force, which then performed independent analysis.

The strategy adopted was to perform survey observations pointing to the inertial region of Cosmos 1408 at the epoch of fragmentation, where all the fragments were expected to fly through. It was also decided to transmit only the CW signal to optimize the Doppler shift and angular track quality.

Fig. 5 shows the November 16th detections which were compatible with the Cosmos 1408 fragmentation, each colour representing a different fragment detected. The compliance with the event was assessed by comparing the recorded Doppler shift measurements with the one which would have been expected by the parent transit (around −5030 Hz). It was supposed that much more fragments were in visibility during the acquisition, but most of them had a so small Radar Cross Section (RCS) that their Signal to Noise Ratio (SNR) did not overcome the detection threshold. For each TDM produced an orbit determination process was carried out. The procedure employed was a batch filter minimising the weighted residuals between the real measurements and the values projected from the computed orbit. TLE parameters were considered as optimisation variables and the parent TLE was used as first guess. A weighting strategy was run, to weight the Doppler shift measurement more than the angular track. In particular, the calibration campaign had determined 10 Hz Doppler shift accuracy, and this quantity was used to weight that measurement. Then, the angular track weight is taken as coherent with the beam resolution, that is in the order of 1e+00 deg. In addition, to guarantee the compatibility of the computed fragment orbit with the parent one, the objective function was modified with an a-priori residual that considers the relative position between Cosmos and the backward propagation of the computed fragment state of the event. This residual is weighted based on the time elapsed between the event and the orbit determination epoch. All the propagations were carried out

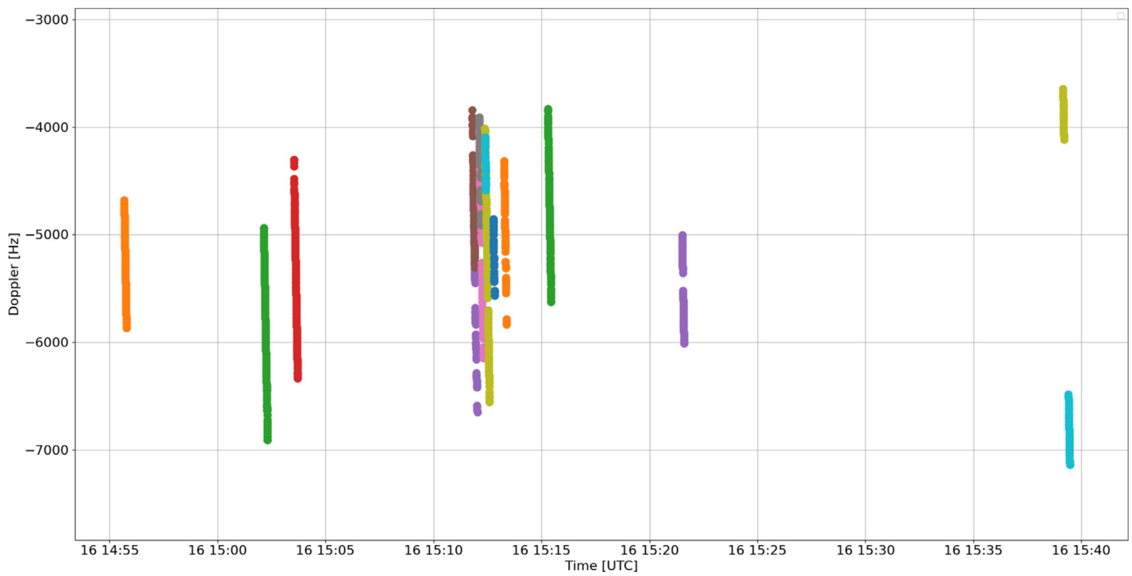

Fig. 5. BIRALES detections on November 16th 2021.

using the SGP4 model [11,12]. Through this strategy, completely unfeasible solutions are penalised but without enforcing an exact constraint to the minimisation problem. In addition, such an approach allows to take into account the uncertainty associated both to the fragmentation inertial position and the propagation model, by also improving the batch filter convergence. Overall, 14 TLEs were obtained which were then used in the following analysis. This is because the SGP4 backpropagation becomes more unreliable in the following days.

The Gabbard diagram of the corresponding debris is shown in Fig. 6. It is worth to notice that some fragments feature a remarkably small orbital perigee, and this can be linked to the orbit determination accuracy. For the sake of completeness, also these orbital states are considered in the PUZZLE analysis in Section 4.3.

4.3. Fragmentation detection using BIRALES data

This test includes now in the initial set the TLEs generated by the observation performed with BIRALES. The objective is to understand whether PUZZLE can identify the fragmentation using this type of data and assess the quality of the TLE data.

First, an analysis was carried out using the original version of the software, thus without the uncertainty propagation. The test considers just the TLEs associated to BIRALES, without adding additional TLEs from other catalogues. The test gave negative results, because the uncertainty associated to the solution was neglected.

Therefore, a second examination was carried out using the updated version of the software which includes the uncertainties as explained in Section 3. Even with the updated version, the software was initially unable to identify the fragmentation. Further investigations, using a graphical approach, showed that some of the TLEs generated had close encounters far from the event (in particular near the initial date), thus not allowing its proper identification. Fig. 7 shows the close encounters between the objects detected by BIRALES, in a time window of 2 days before the TLEs generation. From the figure it is possible to see that at 0, which is the initial epoch of the study (i.e., November 17, 2021), many close encounters are detected, leading to an estimate of the epoch of the event in 0. The latter is likely to be related to a very low quality of some generated TLEs. By removing those objects (5 of 14), the software can identify the fragmentation. It is worth to highlight that the low perigee fragments represented in Fig. 6 are included in the objects

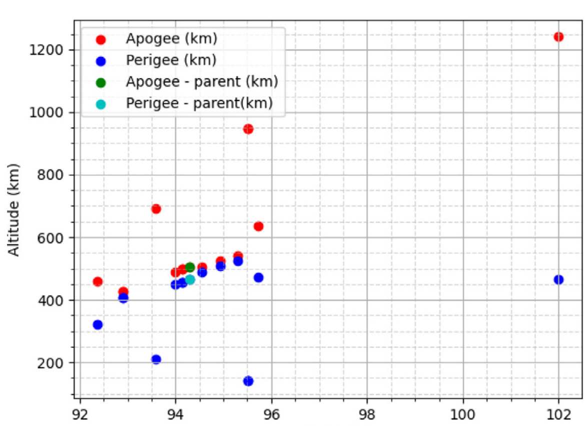

Fig. 6. Gabbard diagram of fragments identified by BIRALES.

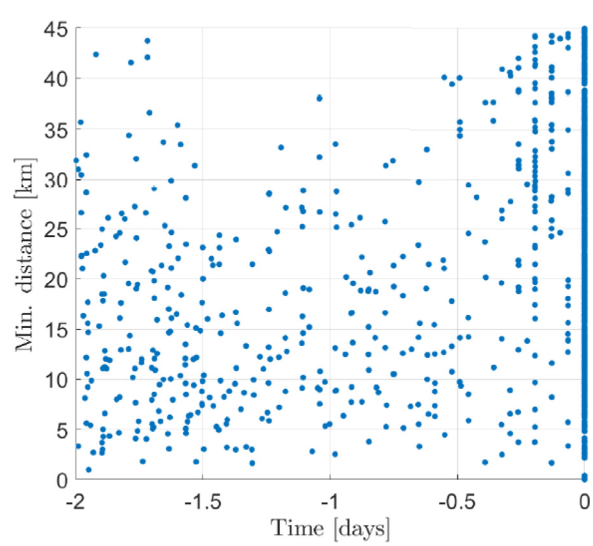

Fig. 7. Distribution in time and distance of the close encounters between the objects TLE generated using BIRALES observations.

Table 2
Input parameters - PUZZLE with uncertainty propagation.

Number of objects	9
Distance margin	30
Number of kernels	15
Uncertainty level	1%

filtered out at this step. The new test is performed considering the 9 remaining objects, and a distance margin of 30 km for the investigation of the close encounters. In addition, in order to account for the uncertainty associated to the TLEs generated by BIRALES, a level of 1% of uncertainty is added to each Keplerian orbital element, while 15 kernels are considered for the generation of the additional TLEs. All the input parameters are summarised in Table 2. Using those input parameters, the epoch of the event is estimated on November 15, 2021, at 02.46 UTC 1.67 min, including all the 9 objects.

The distance margin for the close approaches is wide due to the low accuracy of the data. However, selecting a lower value still allows the detection of the fragmentation, but with a greater error in estimating the epoch of the event.

5. Conclusions

The increasing levels of human activity in space (e.g., for science or military purposes), along with new mission architecture (e.g., large constellations), have led to an acceleration in the growth of space population around the Earth. This has caused, in parallel, the increase of the space debris population at an alarming rate. Indeed, this involves more breakup events, some of which are hardly predictable (e.g., collisions), while others even unpredictable (e.g., explosions). The early detection of these events becomes a key objective to keep the population of active objects safe, thus showing the importance of having Space Surveillance Tracking (SST) activities and services. This paper presented the joint work of Politecnico di Milano, ASI and INAF concerning the Cosmos 1408 fragmentation, showing the methodology adopted and the main results achieved.

The event was monitored by BIRALES, and 14 fragments TLEs were generated from the acquired measurements. Much more fragments were supposed to had crossed the sensor visibility region, but most of them were not detected, basically because of the too small RCS, which prevented their SNR to exceed the detection threshold. BIRALES is currently undergoing an improvement process, which regards both hardware (transmitting and receiving antennas) and software (back-end), and its sensitivity and quality are expected to improve in the future, allowing to detect smaller and more distant objects with a more accurate measurements generation. In particular, BIRALES measurements will be accurate enough to derive a fragment orbital state after a single sensor detection, with no need of a-priori catalogued information. This will allow to better track the fragments cloud, with a remarkable contribution both to the fragmentation and the reentry service [18,19].

Regarding the software PUZZLE, developed at Politecnico di Milano to reconstruct occurred fragmentations, this event represented an excellent opportunity to test its performance. Results showed that the software is able to identify most of the fragments included in the catalogue, even using TLEs dated 16 days after the event, that is near the limit of SGP4 propagator accuracy. Moreover, PUZZLE was able to correctly identify the fragmentation using a set of mixed data (i.e., data related to the fragmentation plus additional random objects). This was also a precious opportunity to test the updated version of the software which includes the uncertainty propagation. In addition, PUZZLE could correctly identify the fragmentation epoch starting from the orbital data determined from BIRALES measurements, as long as the fragments ephemerides uncertainties are considered in the process. However, the availability of more operational data produced nearer to the event could enhance the study process, and hence to show the real capability of the toolkit.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Andrea Muciaccia: Methodology, Software, Formal analysis, Writing – original draft, Writing – review & editing. **Luca Facchini:** Methodology, Software, Formal analysis, Writing – original draft. **Marco Felice Montaruli:** Methodology, Software, Writing – review & editing. **Giovanni Purpura:** Software. **Roberto Detomaso:** Methodology, Software. **Camilla Colombo:** Supervision, Funding acquisition. **Mauro Massari:** Supervision. **Pierluigi Di Lizia:** Supervision. **Alessandra Di Cecco:** Supervision. **Luca Salotti:** Supervision. **Germano Bianchi:** Supervision, Funding acquisition.

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