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Periodic Ground Resonance Analysis with Harmonic Decomposition

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Abstract. Ground resonance is a rotorcraft instability that involves an interaction between the rotor and its support, consisting of the fuselage and the landing apparatus. In cases where the rotor lacks symmetry, the equations of motion are periodic. This paper compares the classic time-periodic Floquet approach to investigate stability with the truncated linear time-invariant harmonic decomposition. Then, various cases are presented dealing with variations in some basic rotor characteristics, such as the number of blades. These findings show the capacity of the decomposition to represent time-periodic modes and present a criterion for the optimal application of this decomposition, which is extremely advantageous because it leads to a linear model for further analyses.

Introduction

Ground resonance is an instability caused by the coupling between structural vibrations and the periodic inertia forces originating from the lead-lag degrees of freedom of helicopter blades. This phenomenon is avoided through dampers connecting each blade with the rotor mast (or with another blade for interblade configurations). This problem was first studied by Coleman and Feingold [1], then the analysis was generalised by Hammond [2] for cases where the formulation cannot be reduced to a linear time-invariant form, typically if nonsymmetric behaviour is analysed, i.e. the failure of a single lag damper, take-off or landing scenarios involving wind or uneven surfaces. As periodic coefficients appear in the equations, modal analysis is more complex, especially in the context of preliminary design or nonlinear stability.

The following sections explain briefly the classic Floquet method and the novel harmonic decomposition. After that, a succinct overview of the rotor model is provided, with a focus on multiblade coordinates. Subsequently, results are shown for a four-bladed rotor with one inoperative damper, followed by a discussion of rotors with more failed dampers, different damper attachment and a changed number of blades. Conclusively, the relationship between the number of rotor blades, multiblade coordinates and modal participation is highlighted to delineate a reasonable criterion to apply the harmonic decomposition.

Modal analysis of periodic systems and linear time-invariant formulation

Floquet analysis for linear time-periodic systems. The Floquet approach, as formulated by Peters et al. [3], enables the stability analysis of Linear Time-Periodic (LTP) systems. Let a LTP system be defined as $d\{x(t)\}/dt = [A(t)]\{x(t)\}$, with $[A(t)] = [A(t+T)]$, i.e. periodic of period T. If the transition matrix $[\Phi(t)]$ is defined as $\{x(t)\} = [\Phi(t)]\{x(0\}$, then the stability of the system may be discerned from the eigenvalues of $[\Phi(T)]$, the *characteristic multipliers* Λ, or from the *characteristic exponents* $\eta = 1/T\, log(\Lambda) = \lambda + i\omega$. ω is not unique (the complex logarithm is multi-valued), as any integer multiple of $\Omega = 2\pi/T$ may be added without loss of meaning of the solution. Nonetheless, LTP stability is based on λ or $|\Lambda|$, which are uniquely defined. In particular, the system is unstable if at least one $\lambda > 0$.

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LTI Harmonic decomposition. Lopez and Prasad [4] propose a linear time-invariant *Harmonic Decomposition* (HD) of an LTP system (where $\psi = \Omega t$), with the state vector expanded as:

$$\{x\} = \{x^0\} + \sum_{n=1}^{N_H} \quad \{x^{nc}\} \, cos(n\psi) + \{x^{ns}\} \, sin(n\psi) \tag{1}$$

N_H is arbitrary and must be chosen properly, since $N_H \to \infty$ causes the HD modes to tend exactly to the LTP modes, whereas truncation to a finite number of harmonics leads to approximation.

A possible criterion for minimum N_H is based on the modal participation of the different harmonics to states and modes.

After differentiating in time Eq. (1) and substituting it in the system equations, integration is performed over one revolution for the average term x^0; whereas the system equations are multiplied by $cos(j\psi)$ or $sin(j\psi)$ with $j = 1, \dots, N_H$ and integrated over one rev to obtain the equations for x^{jc} and x^{js}. This formulation is computationally efficient, as the integration of the matrix terms may be substituted with the Fourier series expansion of $[A]$. The eigenvalues of the system are *equivalent to the Floquet exponents of the LTP system* ([4]).

Modal participation. After presenting the HD, modal participation [3,4] is introduced to provide an in-depth assessment of the harmonic content of modes. The modal participation of the n-th harmonic to the j-th state and k-th mode is defined as:

$$\phi_{j,k,n} = |c_{j,k,n}| / \sum_l \quad |c_{j,k,l}|, \quad -N_H \leq l \leq N_H \tag{2}$$

where $c_{j,k,n}$ is the n-th harmonic coefficient of the Fourier series expansion of $V_{j,k}(t)$, the j-th state of the k-th LTP eigenvector.

Given the equivalence of the LTI HD formulation to the LTP problem, the k-th LTI HD eigenvector contains the Fourier coefficients directly and can be used alternatively to compute the modal participation. The addition of integer multiples of Ω to the imaginary part of η, or equivalently the choice of the modes associated with higher harmonics in the LTI HD, does not change the harmonic content of the modes, rather their numbering, i.e. the value of n is shifted.

Ground resonance model and multiblade coordinates

The rotor model is originally from [2]. The j-th blade is linked to a rigid arm with a hinge, which allows a lag motion ζ_j. All blades constitute the rotor, spinning with a constant angular speed Ω. The arm is connected to the rotor mast and the structure of the rotorcraft, referred to as the “hub”, whose dynamics is represented through two damped harmonic oscillators. This simple model encapsulates the dynamic interactions causing ground resonance. This system is unstable and requires additional dampers mounted between each blade and the rotor head (Hub-to-Blade or H2B) or between adjacent blades (Blade-to-Blade or B2B). The full derivation of all model equations, with both damper connections, is discussed in Bassi [5].

Notably, periodic coefficients, linking the hub equations to the blade ones, emerge in the system matrices whenever asymmetry is present. Unless the hub is symmetric, and all blades and dampers are equal, they cannot be deleted. Moreover, as the equations of the blades are in a rotating frame, while the hub ones are in a fixed frame, they may all be written in a fixed frame through multiblade coordinates (MBC). The j-th lag angle ζ_j is expressed as the sum of collective and cyclic coordinates:

$$\zeta_j = Z_0 + Z_d(-1)^j + \sum_{n=1}^{N*} \quad Z_{nc} cos(n\psi_j) + Z_{ns} sin(n\psi_j) \tag{3}$$

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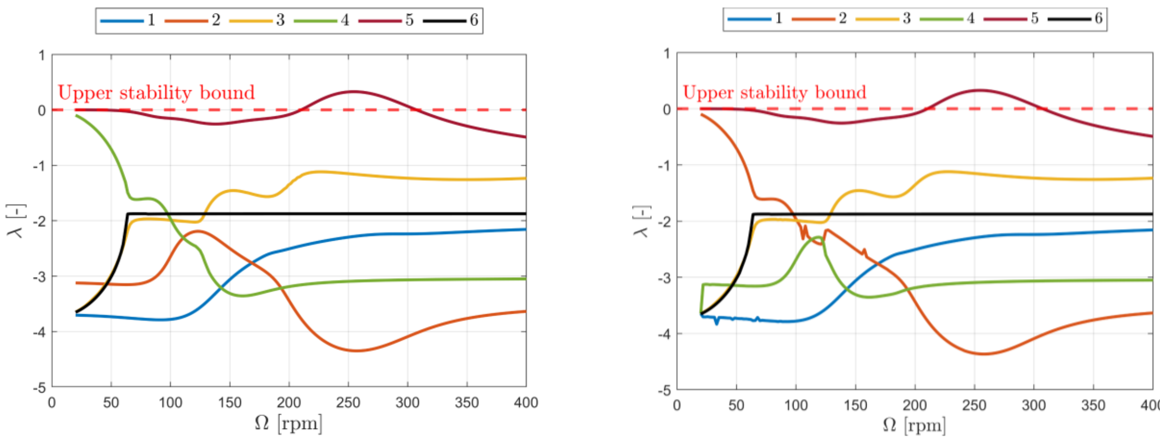
with $\psi_j = \Omega t + (2\pi)/N_b(j-1), j = 1,\ldots,N_b$, and $N^* = (N_b-1)/2$ for odd N_b (Z_d is absent), while $N^* = N_b/2 - 1$ for even N_b.

Stability of rotors and modal participation

Four-bladed rotor stability. The results of Fig. 1 deal with the hub-to-blade rotor with four blades, first presented in [2]. One of the blade dampers is inactive, so periodic coefficients appear. The time-periodic equations are harmonically decomposed up to the first harmonic, i..e $N_H = 1$ in Eq. (1). The system is correctly predicted to be unstable. The two approaches provide the same modes as in [2], except at low RPM, where the continuation algorithm tracks differently.



(a) HD modes *(b) Floquet modes*

Figure 1: Comparison of modes, 4 blades, H2B dampers

Modal participation in different rotors. Modal participation is computed in both configurations, then with a different number of inoperative dampers, and finally with different numbers of bladesN_b. For LTP modes, the principal logarithm is employed, whereas the modes in Fig. 1 are used in the HD case, thus achieving the same numbering for the harmonics.

The results for the 4-bladed rotor are also achieved with the two damper configurations, as well as when two adjacent or opposite dampers fail. Therefore, damper attachment and functionality do not affect modal participation. Regarding the variation of N_b, the harmonics with the highest modal participation in the unstable mode are reported in Table 1. Notably, all other modes never have harmonics higher than those in the table. To draw a link with the number of sinusoidal terms in the multiblade coordinates, the variable N^* from Eq. 2 is reported for each N_b, alongside the minimum N_H required to have a good approximation as in Fig. 1.

The conclusion is that the harmonic decomposition must contain at least as many harmonics as the cyclic terms in the MBC to approximate LTP modes.

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Table 1: Modal participation to unstable mode

N_b	3 or 4		5 or 6		7 or 8	
N^*	*1*		*2*		*3*	
min N_H	*1*		*2*		*3*	
States	Z_{1c}, Z_{1s}, x_H, y_H	Z_0, Z_d	Z_{2c}, Z_{2s}	Other	Z_{3c}, Z_{3s}	Other
Harm. with max $\phi_{j,k,n}$	*+1/-1*	0	*+2/-2*	As $N_b = 3{,}4$	*+3/-3*	As $N_b = 3{,}4{,}5{,}6$

Conclusion

The harmonic decomposition has been presented alongside its time-periodic counterpart, the Floquet method. The first provides an excellent approximation of modal damping if the proper number of harmonic terms has been employed, which in turn is determined from the number of cyclic coordinates in the MBC. In conclusion, the HD successfully and reliably approximates LTP modes, providing an ever-useful linear time-invariant representation.

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