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Event-triggered robust control for multi-player nonzero-sum games with input constraints and mismatched uncertainties

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Abstract

In this article, an event-triggered robust control (ETRC) method is investigated for multi-player nonzero-sum games of continuous-time input constrained nonlinear systems with mismatched uncertainties. By constructing an auxiliary system and designing an appropriate value function, the robust control problem of input constrained nonlinear systems is transformed into an optimal regulation problem. Then, a critic neural network (NN) is adopted to approximate the value function of each player for solving the event-triggered coupled Hamilton–Jacobi equation and obtaining control laws. Based on a designed event-triggering condition, control laws are updated when events occur only. Thus, both computational burden and communication bandwidth are reduced. We prove that the weight approximation errors of critic NNs and the closed-loop uncertain multi-player system states are all uniformly ultimately bounded thanks to the Lyapunov's direct method. Finally, two examples are provided to demonstrate the effectiveness of the developed ETRC method.

KEYWORDS

adaptive dynamic programming, event-triggered robust control, input constraints, neural networks, nonzero-sum games, uncertain multi-player nonlinear systems

1 | INTRODUCTION

Due to dynamic uncertainties in practical systems, the controller has to be robust to avoid the control performance deterioration of the closed-loop system; in turn, this implies that the control system is effective whenever the actual system slightly deviates from its nominal conditions. During the past few decades, many robust control (RC) methods have been developed for nonlinear systems in the control community.[1-8] Lin et al.[2] transformed the RC problem into an optimal control (OC) problem by introducing a modified performance index function. To remove the requirement of knowing the system dynamics, Wang et al.[3] extended this strategy to deal with uncertain nonlinear systems by employing model parameters learned on the input-output data. In the OC problem, it is necessary to solve the Hamilton–Jacobi–Bellman (HJB) equation whose analytic solution is intractable for nonlinear systems.[9,10] Fortunately, adaptive dynamic programming (ADP) and reinforcement learning (RL) are suitable techniques proposed to overcome this difficulty by computing forward-in-time.[11-24]

Many ADP or RL-based methods have been reported to solve OC problems for continuous-time (CT) nonlinear systems with input constraints,[16] external disturbances and uncertainties,[25-31] failures[32,33] and so forth. For example, Liu et al.[28] proposed a decentralized control method for large-scale nonlinear (LSN) systems with matched interconnections with an online learning OC method. Zhao et al.[29] extended this method to LSN systems with unknown mismatched interconnections by establishing a set of neural network (NN)-based decentralized observers. Wang et al.[30] proposed a RC method for uncertain nonlinear systems based on online policy iteration. And then, for nonlinear systems with uncertainties,[31] the robust tracking control was transformed into an OC by constructing an augmented nominal system and a modified cost function. However, the aforementioned works were developed by using the ADP-based time-triggered control (TTC) mechanism, which may increase the computational burden and not well managing the communication resource.

As a well-recognized effective technique for solving the above problems, the event-triggered strategy has attracted extensive attention in the ADP community. Under the event-triggered strategy, the control input is updated when the designed triggering condition is violated.[34-41] Zhang et al.[36] developed an ADP-based event-triggered control (ETC) scheme for solving H_∞ control in the perspective of zero-sum game problems. The control law and the disturbance were updated by event-triggered and time-triggered mechanisms, respectively. To design a robust controller for nonlinear systems with uncertainties, many ADP-based event-triggered RC (ETRC) methods have been developed.[39-41] Wang et al.[40] proposed an ADP-based ETRC method for a class of uncertain nonlinear systems with input constraints. Yang et al.[41] developed an online integral RL-based ETC method to address robust constrained control problems for nonlinear systems with external disturbances. In these aforementioned works, the basic idea lies in that the ETRC problem is transformed into an event-triggered OC (ETOC) problem by a modified cost function. However, these methods only considered matched uncertainties, and thus were not applicable to systems with mismatched uncertainties. Zhang et al.[42] decomposed the mismatched uncertainty into a matched and a mismatched components, and constructed a modified value function to transform the ETRC problem into an ETOC problem.

Although RC schemes have been extensively reported, they considered the single controller only. In practice, many complex systems are controlled by multiple controllers, which can be regarded as multiple players trying to minimize their individual value functions, such as microgrid systems, traffic systems and so forth.[43,44] For nonzero-sum games (NZSGs), multiple players work neither in fully cooperation nor in fully competition. Zhang et al.[44] developed an ADP-based ETC method for multi-player NZSGs of unknown nonlinear systems. However, related research of multi-player nonlinear (MPN) systems with uncertainties is still in its infancy. Meanwhile, control inputs are always constrained by limit bounds given the physical characteristics of actuators. To deal with input constraints, nonquadratic utility functions are employed to design the ADP-based controllers.[27,38,41] However, for MPN systems, input constraints are rarely considered. Therefore, it is urgent to develop an ADP-based ETRC scheme for NZSGs of MPN systems with input constraints and mismatched uncertainties.

Motivated by the aforementioned works, this article focuses on solving the NZSG problem of MPN systems with input constraints and mismatched uncertainties. The contributions and novelties of this work are summarized in the following three aspects.

1. Different from existing works[45,46] on developing the OC laws for multi-player systems without uncertainties, this article develops an ADP-based ETRC method for MPN systems with uncertainties.
2. In contrast to existing time-triggered RC schemes[3,25,47,48] for nonlinear systems with uncertainties, this article investigates an ADP-based ETRC method for NZSGs of input-constrained MPN systems with mismatched uncertainties.

3. Unlike existing works[38,42] which developed ETRC methods for systems with single controller only, this article extends ADP-based ETRC method to tackle NZSGs of uncertain MPN systems. Different from References 39-41 which focused on developing ETRC for nonlinear systems with matched uncertainties, this article considers mismatched uncertainties in the input-constrained MPN systems. Furthermore, both the OC laws and the auxiliary OC laws are updated when events are triggered to reduce the computational and communication burdens.
4. Since a critic-only strategy is adopted to obtain the control laws, the computational burden is further reduced comparing to the popular methods which employed the critic-actor structure.[12,18]

The article is organized as follows. In Section 2, the problem statement is introduced. In Section 3, an ADP-based ETRC method is developed, and stability analysis is provided. In Section 4, two simulation examples are provided to validate the proposed theoretical results. Section 5 briefly concludes this article.

2 | PROBLEM STATEMENT

Consider a class of CT uncertain MPN systems described by

$$\dot{\mathcal{X}}(t) = \mathcal{F}(\mathcal{X}(t)) + \sum_{a=1}^{\mathcal{N}_u} \left(\mathcal{G}_a(\mathcal{X}(t))\, u_a(t) + \mathcal{H}_a(\mathcal{X}(t))\, \xi_a(\mathcal{X}(t))\right), \tag{1}$$

where $\mathcal{X}(t) \in \mathbb{R}^n$ is the system state with $\mathcal{X}(0) = \mathcal{X}_0$, u_a is the ath control input whose qth element is denoted as u_{aq}, and bounded as $|u_{aq}| \leq \bar{u}_a, q = 1, 2, \ldots, m_a$, $\bar{u}_a \in \mathbb{R}$ is the upper-bound of u_{aq}, and $\mathcal{N}_u$ is the number of players. $\mathcal{F}(\cdot) \in \mathbb{R}^n$, $\mathcal{G}_a(\cdot) \in \mathbb{R}^{n\times m_a}$ and $\mathcal{H}_a(\cdot) \in \mathbb{R}^{n\times s_a}$ are continuously differentiable matrix functions with $\mathcal{H}_a(\mathcal{X}) \neq \mathcal{G}_a(\mathcal{X})$ and $\mathcal{F}(0) = 0$. $\xi_a(\mathcal{X})$ is the unknown norm-bounded nonlinear perturbation, that is, there exists a known upper function such that $\|\xi_a(\mathcal{X})\| \leq \Xi_a(\mathcal{X})$ with $\xi_a(0) = 0$ and $\Xi_a(0) = 0$.

Assumption 1. The function describing system dynamics $\mathcal{F}(\mathcal{X}) + \sum_{a=1}^{\mathcal{N}_u} (\mathcal{G}_a(\mathcal{X})u_a(t) + \mathcal{H}_a(\mathcal{X})\xi_a(\mathcal{X}))$ is Lipschitz continuous on a compact set $\Omega \in \mathbb{R}^n$, and the system (1) is controllable.

Assumption 2. There exists a nonnegative function $\Lambda_a(\mathcal{X})$ such that

$$\|\mathcal{G}_a^{+}(\mathcal{X})\mathcal{H}_a(\mathcal{X})\xi_a(\mathcal{X})\|^2 \leq \Lambda_a^2(\mathcal{X}),$$

where $\mathcal{G}_a^{+}(\mathcal{X})$ denotes the Moore–Penrose pseudo-inverse of function $\mathcal{G}_a(\mathcal{X})$.[38,42,47]

To address the RC problem, the uncertain term $\mathcal{H}_a(\mathcal{X})\xi_a(\mathcal{X})$ can be decomposed into two components

$$\mathcal{H}_a(\mathcal{X})\xi_a(\mathcal{X}) = \underbrace{\mathcal{G}_a(\mathcal{X})\mathcal{G}_a^{+}(\mathcal{X})\mathcal{H}_a(\mathcal{X})\xi_a(\mathcal{X})}_{\text{Term I}} + \underbrace{(I_n - \mathcal{G}_a(\mathcal{X})\mathcal{G}_a^{+}(\mathcal{X}))\mathcal{H}_a(\mathcal{X})\xi_a(\mathcal{X})}_{\text{Term II}},$$

where Terms I and II are the matched and the mismatched components, respectively. The RC problem is converted into an OC problem by constructing the auxiliary system

$$\dot{\mathcal{X}}(t) = \mathcal{F}(\mathcal{X}) + \sum_{a=1}^{\mathcal{N}_u} \left(\mathcal{G}_a(\mathcal{X})u_a + \hbar_a(\mathcal{X})w_a\right), \tag{2}$$

where $\hbar_a(\mathcal{X}) = (I_n - \mathcal{G}_a(\mathcal{X})\mathcal{G}_a^{+}(\mathcal{X}))\mathcal{H}_a(\mathcal{X})$, and $w_a(\mathcal{X}) \in \mathbb{R}^{s_a}$ is the auxiliary control law.

The value function of each player corresponding to (2) is defined as

$$\mathcal{V}_a(\mathcal{X}) = \int_t^{\infty} \left(\delta \sum_{b=1}^{\mathcal{N}_u} \Lambda_b^2(\mathcal{X}(\tau)) + \vartheta \sum_{b=1}^{\mathcal{N}_u} \Xi_b^2(\mathcal{X}(\tau)) + \mathcal{Z}_a(\mathcal{X}(\tau), \mathcal{U}_u(\tau), \mathcal{D}_w(\tau))\right) d\tau, \tag{3}$$

where $\mathcal{Z}_a(\mathcal{X}, \mathcal{U}_u, \mathcal{D}_w) = \mathcal{X}^\mathsf{T} Q_a \mathcal{X} + \mathcal{R}_a(\mathcal{U}_u) + \gamma^2 \sum_{b=1}^{\mathcal{N}_u} w_b^\mathsf{T} w_b$, $\mathcal{D}_w = [w_1, w_2, \dots, w_{\mathcal{N}_u}]$, $\mathcal{U}_u = [u_1, u_2, \dots, u_{\mathcal{N}_u}]$, $Q_a \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix, $\mathcal{R}_a(\mathcal{U}_u)$ is a nonnegative utility function, and δ, ϑ, and γ are positive constants. To solve the constrained control problem, $\mathcal{R}_a(\mathcal{U}_u)$[40,49] is chosen as

$$\mathcal{R}_a(\mathcal{U}_u) = 2\sum_{b=1}^{\mathcal{N}_u} \int_0^{u_b} \bar{u}_b \tan \mathrm{h}^{-1}(\nu/\bar{u}_b)^\mathsf{T} d\nu,$$

where $\tanh^{-1}(\cdot)$ is the inverse of the hyperbolic tangent function $\tanh(\cdot)$. Taking the time derivative of (3), we have

$$0 = \delta \sum_{b=1}^{\mathcal{N}_u} \Lambda_b^2(\mathcal{X}) + \vartheta \sum_{b=1}^{\mathcal{N}_u} \Xi_b^2(\mathcal{X}) + \mathcal{Z}_a(\mathcal{X}, \mathcal{U}_u, \mathcal{D}_w) + \nabla \mathcal{V}_a^\mathsf{T}(\mathcal{X}) \left(\mathcal{F}(\mathcal{X}) + \sum_{b=1}^{\mathcal{N}_u} (\mathcal{G}_b(\mathcal{X}) u_b + \hbar_b(\mathcal{X}) w_b) \right)$$

with $\mathcal{V}_a(0) = 0$, where $\nabla \mathcal{V}_a(\mathcal{X}) = \partial \mathcal{V}_a(\mathcal{X}) / \partial \mathcal{X}$. The Hamiltonian of system (2) is given by

$$\begin{aligned} \mathcal{H}_a(\nabla \mathcal{V}_a(\mathcal{X}), \mathcal{X}, \mathcal{U}_u, \mathcal{D}_w) &= \delta \sum_{b=1}^{\mathcal{N}_u} \Lambda_b^2(\mathcal{X}) + \vartheta \sum_{b=1}^{\mathcal{N}_u} \Xi_b^2(\mathcal{X}) + \mathcal{Z}_a(\mathcal{X}, \mathcal{U}_u, \mathcal{D}_w) \\ &\quad + \nabla \mathcal{V}_a^\mathsf{T}(\mathcal{X}) \left(\mathcal{F}(\mathcal{X}) + \sum_{b=1}^{\mathcal{N}_u} (\mathcal{G}_b(\mathcal{X}) u_b + \hbar_b(\mathcal{X}) w_b) \right). \end{aligned} \tag{4}$$

Define $\mathcal{A}(\Omega)$ as a set of admissible control pairs. According to Reference 9, the optimal value function $\mathcal{V}_a^*(\mathcal{X})$ is obtained by solving the HJ equation as

$$\min_{u_a, w_a \in \mathcal{A}(\Omega)} \mathcal{H}_a(\nabla \mathcal{V}_a^*(\mathcal{X}), \mathcal{X}, \mathcal{U}_u, \mathcal{D}_w) = 0 \tag{5}$$

with $\mathcal{V}_a^*(0) = 0$, where $\nabla \mathcal{V}_a^*(\mathcal{X}) = \partial \mathcal{V}_a^*(\mathcal{X}) / \partial \mathcal{X}$. Therefore, the OC law $u_a^*(\mathcal{X})$ and the auxiliary OC law $w_a^*(\mathcal{X})$ are derived as

$$u_a^*(\mathcal{X}) = -\bar{u}_a \tanh\left(\frac{1}{2\bar{u}_a} \mathcal{G}_a^\mathsf{T}(\mathcal{X}) \nabla \mathcal{V}_a^*(\mathcal{X}) \right), \tag{6}$$

$$w_a^*(\mathcal{X}) = -\frac{1}{2\gamma^2} \hbar_a(\mathcal{X})^\mathsf{T} \nabla \mathcal{V}_a^*(\mathcal{X}). \tag{7}$$

Substituting (6) and (7) into (5), we have

$$\begin{aligned} 0 &= \mathcal{X}^\mathsf{T} Q_a \mathcal{X} + \mathcal{R}_a(\mathcal{U}_u^*(\mathcal{X})) + \delta \sum_{b=1}^{\mathcal{N}_u} \Lambda_b^2(\mathcal{X}) + \vartheta \sum_{b=1}^{\mathcal{N}_u} \Xi_b^2(\mathcal{X}) + \nabla \mathcal{V}_a^{*\mathsf{T}}(\mathcal{X}) \left(\mathcal{F}(\mathcal{X}) + \sum_{b=1}^{\mathcal{N}_u} \left(\mathcal{G}_b(\mathcal{X}) u_b^* + \hbar_b(\mathcal{X}) w_b^* \right) \right) \\ &\quad + \gamma^2 \sum_{b=1}^{\mathcal{N}_u} w_b^{*\mathsf{T}} w_b^*, \end{aligned} \tag{8}$$

where $\mathcal{U}_u^*(\mathcal{X}) = [u_1^*, u_2^*, \dots, u_{\mathcal{N}_u}^*]$. As it is well known, (8) is a time-triggered HJ equation. Although ADP-based TTC methods[12,15] can solve the HJ equation, they often involve heavy computational burden and the resource waste. To address these problems, we propose an ADP-based ETRC method in the following section.

Remark 1. In multi-agent systems, the controller of each agent is constructed by collecting the states of its neighbors only in accordance with the communication topology. Then, all independent controllers construct the multiple controllers for multi-agent systems. Different from multi-agent systems, the multi-player system considered in this article is driven by multiple controllers with shared global system states.

Remark 2. It is worth mentioning that the traditional quadratic function is generally suitable to tackle control problems of systems without input constraints. In order to eliminate the influence of the constrained control input, a modified nonquadratic function which adopts the $\tanh(\cdot)$ function is constructed in the value function. Then, the constrained OC law $u_a^*(\mathcal{X}) = -\bar{u}_a \tanh\left(\frac{1}{2\bar{u}_a}\mathcal{G}_a^{\mathsf{T}}(\mathcal{X})\nabla\mathcal{V}_a^*(\mathcal{X})\right)$ is obtained, where $\tanh(\cdot) \in (-1, 1)$ can guarantee that the control input varies within the constraints.

Remark 3. It is noticed that the auxiliary system (2) is derived on the basis of the uncertain system (1), which implies their dynamics are different. In References 38 and 42, a RC method was developed by transforming the RC problem into an OC problem with a modified value function, and the theoretical analysis in Theorem 1 shows that the problem transformation is reasonable. Inspired by these works, in this article, the RC problem of uncertain system (1) is transformed into an OC problem of the auxiliary system (2) by introducing a modified value function, which reflects the upper bound of uncertainties. It implies that the OC laws are designed considering the bias between the mathematical model and the actual system. That is to say, the designed OC laws can guarantee the closed-loop uncertain system (1) to be stable. According to the Lyapunov stability theorem, the OC laws of the auxiliary system (2) can guarantee the uncertain system (1) to be asymptotically stable.

Remark 4. For multi-player nonzero-games, the value function of each player is usually defined as $\mathcal{V}_a(\mathcal{X}) = \int_t^\infty \left(\mathcal{X}^{\mathsf{T}}(\tau)Q_a\mathcal{X}(\tau) + \sum_{b=1}^{\mathcal{N}_u} u_b^{\mathsf{T}}(\tau)R_b u_b(\tau)\right) d\tau$, where Q_a and R_b are symmetric positive definite matrices. However, control inputs are always constrained due to the physical characteristics. To deal with this difficulty, inspired by Wang et al.,[40] Zhu et al.,[49] the nonquadratic utility function $\mathcal{R}_a(\mathcal{U}_u)$ is established to limit the amplitude of the control input. Moreover, to reduce the influence of uncertainties, the utility function $\gamma^2\sum_{b=1}^{\mathcal{N}_u} w_b^{\mathsf{T}} w_b$ of the auxiliary OC input w_a, the upper-bounded functions $\delta\sum_{b=1}^{\mathcal{N}_u}\Lambda_b^2(\mathcal{X})$ and $\vartheta\sum_{b=1}^{\mathcal{N}_u}\Xi_b^2(\mathcal{X})$ are integrated in the value function. In other words, the influence of uncertainties and control input constraints are considered in designing the value function.

3 | EVENT-TRIGGERED ROBUST CONTROLLER DESIGN

3.1 | ADP-based event-triggered robust control

Define a monotonic sequence of triggering instants $\{\varrho_p\}_{p=0}^\infty$ with $\varrho_p < \varrho_{p+1}$, $p \in \mathbb{N}$, where ϱ_p is the pth sampling instant. The sampled state can be written as

$$\hat{\mathcal{X}}_p(t) = \mathcal{X}(\varrho_p), \ \ \forall t \in [\varrho_p, \varrho_{p+1}).$$

The event-triggered error function $e_p(t)$ is defined as

$$e_p(t) = \hat{\mathcal{X}}_p(t) - \mathcal{X}(t), \forall t \in [\varrho_p, \varrho_{p+1}).$$

Then, the ETC law and the auxiliary ETC law are formulated as

$$u_a(\hat{\mathcal{X}}_p) = u_a\left(e_p(t) + \mathcal{X}(t)\right), \tag{9}$$

$$w_a(\hat{\mathcal{X}}_p) = w_a\left(e_p(t) + \mathcal{X}(t)\right). \tag{10}$$

Furthermore, the auxiliary system (2) becomes

$$\dot{\mathcal{X}}(t) = \mathcal{F}(\mathcal{X}) + \sum_{a=1}^{\mathcal{N}_u}\left(\mathcal{G}_a(\mathcal{X})u_a(\hat{\mathcal{X}}_p) + \hbar_a(\mathcal{X})w_a(\hat{\mathcal{X}}_p)\right).$$

Thus, the ETOC law (6) and the auxiliary ETOC law (7) can be derived as

$$u_a^*(\hat{\mathcal{X}}_p) = -\bar{u}_a \tanh\left(\frac{1}{2\bar{u}_a}\mathcal{G}_a^{\mathsf{T}}(\hat{\mathcal{X}}_p)\nabla\mathcal{V}_a^*(\hat{\mathcal{X}}_p)\right), \tag{11}$$

$$w_a^*(\hat{\mathcal{X}}_p) = -\frac{1}{2\gamma^2}\hbar_a(\hat{\mathcal{X}}_p)^{\mathsf{T}}\nabla\mathcal{V}_a^*(\hat{\mathcal{X}}_p), \tag{12}$$

where $\nabla\mathcal{V}_a^*(\hat{\mathcal{X}}_p) = \partial\mathcal{V}_a^*(\hat{\mathcal{X}}_p)/\partial\hat{\mathcal{X}}_p$. Therefore, the event-triggered HJ equation can be expressed as

$$\begin{aligned}\mathcal{H}_a\left(\nabla\mathcal{V}_a^*(\mathcal{X}), \mathcal{X}, \overline{\mathcal{U}}_u^*(\hat{\mathcal{X}}_p), \overline{\mathcal{D}}_w^*(\hat{\mathcal{X}}_p)\right) &= \nabla\mathcal{V}_a^{*\mathsf{T}}(\mathcal{X})\left(\mathcal{F}(\mathcal{X}) + \sum_{b=1}^{\mathcal{N}_u}\left(\mathcal{G}_b(\mathcal{X})u_b^*(\hat{\mathcal{X}}_p) + \hbar_b(\mathcal{X})w_b^*(\hat{\mathcal{X}}_p)\right)\right) + \mathcal{X}^{\mathsf{T}}Q_a\mathcal{X} \\ &\quad + \mathcal{R}_a\left(\overline{\mathcal{U}}_u^*(\hat{\mathcal{X}}_p)\right) + \delta\sum_{b=1}^{\mathcal{N}_u}\Lambda_b^2(\mathcal{X}) + \vartheta\sum_{b=1}^{\mathcal{N}_u}\Xi_b^2(\mathcal{X}) + \gamma^2\sum_{b=1}^{\mathcal{N}_u}w_b^{*\mathsf{T}}(\hat{\mathcal{X}}_p)w_b^*(\hat{\mathcal{X}}_p),\end{aligned}$$

where $\overline{\mathcal{U}}_u^*(\hat{\mathcal{X}}_p) = [u_1^*(\hat{\mathcal{X}}_p), u_2^*(\hat{\mathcal{X}}_p), \dots, u_{\mathcal{N}_u}^*(\hat{\mathcal{X}}_p)]$ and $\overline{\mathcal{D}}_w^*(\hat{\mathcal{X}}_p) = [w_1^*(\hat{\mathcal{X}}_p), w_2^*(\hat{\mathcal{X}}_p), \dots, w_{\mathcal{N}_u}^*(\hat{\mathcal{X}}_p)]$.

Assumption 3. The OC law $u_a(\mathcal{X})$ is Lipschitz continuous such that

$$\|u_a^*(\mathcal{X}(t)) - u_a^*(\hat{\mathcal{X}}_p)\| \le \mathcal{L}_{ua}\|e_p(t)\|,$$

where $\mathcal{L}_{ua}$ is a positive constant.

Assumption 4. The system input gain function $\mathcal{G}_a(\mathcal{X})$ and $\hbar_a(\mathcal{X})$ are norm-bounded, that is, $\|\mathcal{G}_a(\mathcal{X})\| \le \bar{g}_a$ and $\|\hbar_a(\mathcal{X})\| \le \bar{\hbar}_a$, where $\bar{g}_a$ and $\bar{\hbar}_a$ are positive constants.[50,51]

Theorem 1. *Consider the uncertain MPN system (1), Assumptions 1–4, and the ETOC law $u_a^*(\hat{\mathcal{X}}_p)$ (11). There exists a matrix Q_a such that*

$$\varpi_1^2\lambda_{\min}(Q_a)\|\mathcal{X}\|^2 \ge \frac{3}{2}\|\nabla\mathcal{V}_a^*(\mathcal{X})\|^2, \tag{13}$$

and if the event-triggering condition satisfies

$$\|e_p(t)\|^2 \le \frac{2(1-\varpi_1^2)\sum_{a=1}^{\mathcal{N}_u}\lambda_{\min}(Q_a)\|\mathcal{X}\|^2}{\bar{g}^2\mathcal{N}_u^2\sum_{b=1}^{\mathcal{N}_u}\mathcal{L}_{ub}^2} = e_T^2, \tag{14}$$

where $\lambda_{\min}(Q_a)$ represents the minimal eigenvalue of Q_a, $\varpi_1 \in (0,1)$ is a design parameter, and e_T is the event-triggering threshold. Then, the closed-loop system (1) is asymptotically stable.

Proof. See Appendix A. ■

3.2 | NN implementation

The optimal value function $\mathcal{V}_a^*(\mathcal{X})$ can be approximated by a critic NN as

$$\mathcal{V}_a^*(\mathcal{X}) = \mathcal{W}_{ca}^{*\mathsf{T}}\sigma_{ca}(\mathcal{X}) + \xi_{ca}(\mathcal{X}), \tag{15}$$

where $\mathcal{W}_{ca}^* \in \mathbb{R}^{l_{ca}}$ is the ideal weight, $\sigma_{ca}(\mathcal{X}) \in \mathbb{R}^{l_{ca}}$ is the activation function, l_{ca} is the number of hidden neurons, and $\xi_{ca}(\mathcal{X}) \in \mathbb{R}$ is the approximation error. The partial derivative of (15) with respect to $\mathcal{X}$ can be expressed by

$$\nabla\mathcal{V}_a^*(\mathcal{X}) = \nabla\sigma_{ca}^{\mathsf{T}}(\mathcal{X})\mathcal{W}_{ca}^* + \nabla\xi_{ca}^{\mathsf{T}}(\mathcal{X}). \tag{16}$$

The approximate $\mathcal{V}_a^*(\mathcal{X})$ is formulated as

$$\hat{\mathcal{V}}_a(\mathcal{X}) = \hat{\mathcal{W}}_{ca}^{\mathsf{T}}\sigma_{ca}(\mathcal{X}),$$

where $\hat{\mathcal{W}}_{ca} \in \mathbb{R}^{l_{ca}}$ is the approximation of $\mathcal{W}^*_{ca}$. Similarly, we have

$$\nabla \hat{\mathcal{V}}_a(\mathcal{X}) = \nabla \sigma^{\mathsf{T}}_{ca}(\mathcal{X})\hat{\mathcal{W}}_{ca}.$$

According to (11), (12), and (16), the ETOC law (11) and the auxiliary ETOC law (12) are converted into

$$u^*_a(\hat{\mathcal{X}}_p) = -\bar{u}_a \tanh\left(\frac{1}{2\bar{u}_a}\mathcal{G}^{\mathsf{T}}_a(\hat{\mathcal{X}}_p)\left(\nabla\sigma^{\mathsf{T}}_{ca}(\hat{\mathcal{X}}_p)\mathcal{W}^*_{ca} + \nabla\xi^{\mathsf{T}}_{ca}(\hat{\mathcal{X}}_p)\right)\right), \tag{17}$$

$$w^*_a(\hat{\mathcal{X}}_p) = -\frac{1}{2\gamma^2}\hbar^{\mathsf{T}}_a(\hat{\mathcal{X}}_p)\left(\nabla\sigma^{\mathsf{T}}_{ca}(\hat{\mathcal{X}}_p)\mathcal{W}^{*\mathsf{T}}_{ca} + \nabla\xi^{\mathsf{T}}_{ca}(\hat{\mathcal{X}}_p)\right). \tag{18}$$

Then, the approximations of (17) and (18) are presented as

$$\hat{u}_a(\hat{\mathcal{X}}_p) = -\bar{u}_a \tanh\left(\frac{1}{2\bar{u}_a}\mathcal{G}^{\mathsf{T}}_a(\hat{\mathcal{X}}_p)\nabla\sigma^{\mathsf{T}}_{ca}(\hat{\mathcal{X}}_p)\hat{\mathcal{W}}_{ca}\right), \tag{19}$$

$$\hat{w}_a(\hat{\mathcal{X}}_p) = -\frac{1}{2\gamma^2}\hbar^{\mathsf{T}}_a(\hat{\mathcal{X}}_p)\nabla\sigma^{\mathsf{T}}_{ca}(\hat{\mathcal{X}}_p)\hat{\mathcal{W}}_{ca}. \tag{20}$$

By introducing the critic NN, the Hamiltonian (4) is approximated by

$$\begin{aligned}\hat{\mathcal{H}}_a\left(\hat{\mathcal{W}}_{ca}, \mathcal{X}, \hat{\mathcal{U}}_u(\hat{\mathcal{X}}_p), \hat{\mathcal{D}}_w(\hat{\mathcal{X}}_p)\right) &= \delta\sum_{b=1}^{\mathcal{N}_u}\Lambda^2_b(\mathcal{X}) + \vartheta\sum_{b=1}^{\mathcal{N}_u}\Xi^2_b(\mathcal{X}) + \mathcal{Z}_a\left(\mathcal{X}, \hat{\mathcal{U}}(\hat{\mathcal{X}}_p), \hat{\mathcal{D}}(\hat{\mathcal{X}}_p)\right)\\ &\quad + \hat{\mathcal{W}}^{\mathsf{T}}_{ca}\nabla\sigma_{ca}(\mathcal{X})\left(\mathcal{F}(\mathcal{X}) + \sum_{a=1}^{\mathcal{N}_u}\mathcal{G}_a(\mathcal{X})\hat{u}_a(\hat{\mathcal{X}}_p) + \sum_{a=1}^{\mathcal{N}_u}\mathcal{H}_a(\mathcal{X})\hat{w}_a(\hat{\mathcal{X}}_p)\right)\\ &\triangleq e_{ca},\end{aligned}$$

where $\hat{\mathcal{U}}_u(\hat{\mathcal{X}}_p) = [\hat{u}_1(\hat{\mathcal{X}}_p), \hat{u}_2(\hat{\mathcal{X}}_p), \ldots, \hat{u}_{\mathcal{N}_u}(\hat{\mathcal{X}}_p)]$ and $\hat{\mathcal{D}}_w(\hat{\mathcal{X}}_p) = [\hat{w}_1(\hat{\mathcal{X}}_p), \hat{w}_2(\hat{\mathcal{X}}_p), \ldots, \hat{w}_{\mathcal{N}_u}(\hat{\mathcal{X}}_p)]$. The critic NN is trained by minimizing the objective function $E_{ca} = (1/2)e^{\mathsf{T}}_{ca}e_{ca}$. Then, the critic NN weight $\hat{\mathcal{W}}_{ca}$ is updated by

$$\dot{\hat{\mathcal{W}}}_{ca} = -\alpha_c\frac{1}{(1+\Phi^{\mathsf{T}}_a\Phi_a)^2}\left(\frac{\partial E_{ca}}{\partial\hat{\mathcal{W}}_{ca}}\right) = -\alpha_c\left(\frac{e_{ca}}{(1+\Phi^{\mathsf{T}}_a\Phi_a)^2}\right)\Phi_a, \tag{21}$$

where $\alpha_c > 0$ is the learning rate, and $\Phi_a = \nabla\sigma_{ca}(\mathcal{X})\left(\mathcal{F}(\mathcal{X}) + \sum_{a=1}^{\mathcal{N}_u}\mathcal{G}_a(\mathcal{X})\hat{u}_a(\hat{\mathcal{X}}_p) + \sum_{a=1}^{\mathcal{N}_u}\hbar_a(\mathcal{X})\hat{w}_a(\hat{\mathcal{X}}_p)\right)$.

The training process of the critic NNs is shown in Figure 1. After training, the converged weight vectors of the critic NNs are employed to construct robust controllers which drive the uncertain system (1).

Lemma 1. *Considering the auxiliary system (2), the critic NN weight error dynamics is guaranteed to be uniformly ultimately bounded (UUB) with the updating law (21).*

Proof. The related proof is similar to that in References 33-38, so it is omitted here. ∎

Remark 5. The ideal weight vector $\mathcal{W}^*_{ca}$ which corresponds to the optimal value function $\mathcal{V}^*_a(\mathcal{X})$ is not unique. In this article, the goal is to find an approximate weight $\hat{\mathcal{W}}_{ca}$ to approximate the OC policy. In other words, the approximate weight vector $\hat{\mathcal{W}}_{ca}$ is required to converge to a small region of the ideal weight $\mathcal{W}^*_{ca}$. According to Lemma 1, the weight error dynamics of the critic NN is guaranteed to be UUB with updating law (21), that is, it converges to the optimum.

Remark 6. In this article, $E_{ca} = \frac{1}{2}e^{\mathsf{T}}_{ca}e_{ca}$ is defined as the objective function for training the critic NN. In existing ADP-based control methods, the gradient descent method is widely employed to train the critic NN weights through the updating rule (21). Moreover, many improved updating rules have been developed based on experience replay,[36] swarm intelligence,[7] and nested structure.[16] In this article, we focus on developing an ETRC scheme for multi-player NZSGs with input constraints and mismatched uncertainties. Thus, the typical updating rule is sufficient to train the critic NN weights of

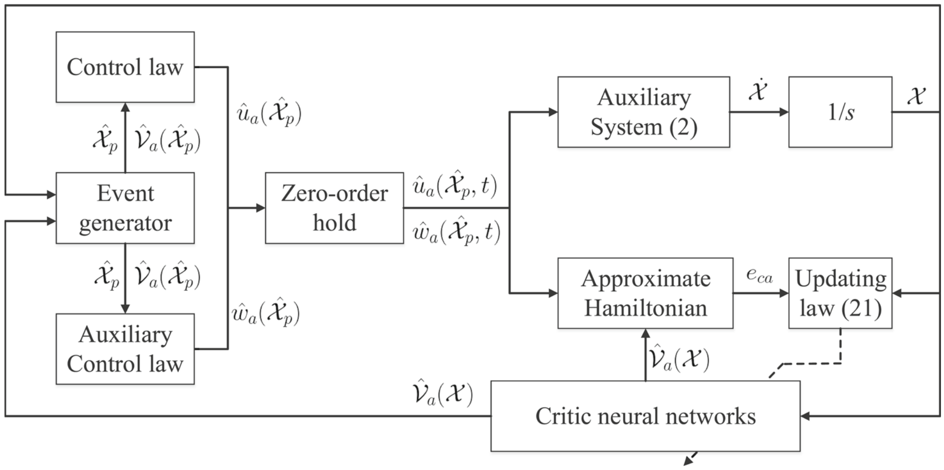

FIGURE 1 The training process of the critic NNs

each player, and we can also refer to aforementioned improved updating rules according to different control objectives. Furthermore, the simulation results demonstrate the rationale of the updating rule (21).

3.3 | Stability analysis

Assumption 5. $\nabla\sigma_{ca}(\mathcal{X})$, $\nabla\xi_{ca}(\mathcal{X})$, $\tilde{\mathcal{W}}_{ca}$ and $\mathcal{W}^*_{ca}$ are norm-bounded, that is,

$$\|\nabla\sigma_{ca}(\mathcal{X})\| \le \nabla\overline{\sigma}_{ca}, \|\nabla\xi_{ca}(\mathcal{X})\| \le \nabla\overline{\xi}_{ca}, \max\{\|\tilde{\mathcal{W}}_{ca}\|, \|\mathcal{W}^*_{ca}\|\} \le \overline{W}_{ca},$$

where $\nabla\overline{\sigma}_{ca}$, $\nabla\overline{\xi}_{ca}$, and $\overline{W}_{ca}$ are positive constants.[37,41,51-55]

Assumption 6. The auxiliary OC law $w^*_a(\mathcal{X})$ is Lipschitz continuous, that is,

$$\|w^*_a(\mathcal{X}(t)) - w^*_a(\hat{\mathcal{X}}_p)\| \le \mathcal{L}_{wa}\|e_p(t)\|, \tag{22}$$

where $\mathcal{L}_{wa}$ is a positive constant.[44,50]

Theorem 2. *Considering the auxiliary system (2), the approximate ETOC law (19) and the approximate auxiliary ETOC law (20), the critic NN whose weights are tuned by (21), and Assumptions 1–6, the closed-loop auxiliary system is guaranteed to be UUB when the event-triggering condition*

$$\|e_p(t)\|^2 \le \frac{(1-\varpi_2^2)\sum_{a=1}^{\mathcal{N}_u}\lambda_{\min}(Q_a)\|\mathcal{X}\|^2}{\mathcal{L}_{uw}} = \hat{e}_T^2, \tag{23}$$

holds, where $\varpi_2 \in (0,1)$ and $\mathcal{L}_{uw} > 0$ are the design parameters, and $\hat{e}_T$ is the event-triggering threshold.

Proof. See Appendix A.2. ■

Remark 7. It is worth pointing out that the triggering condition (14) in Theorem 1 and the triggering condition (23) in Theorem 2 are different. Theorem 1 aims to guarantee the stability of the closed-loop system (1) theoretically. While, the triggering condition (23) is designed for the proposed ADP-based ETRC method which is actually implemented for the auxiliary system (2).

Remark 8. Theorem 2 proves that the system state $\mathcal{X}$ will converge to the compact set $\Omega_{\mathcal{X}}$ if the triggering condition (23) is satisfied. For Case 1, when the system state $\mathcal{X}$ lies outside the compact set $\Omega_{\mathcal{X}}$, and the designed event-triggering condition (23) holds, we have $\dot{L}_2(t) < 0$. For Case 2, the control input is updated when the event-triggering condition (23) is violated, and the event-triggering error e_p is reset to zero such that the event-triggering condition (23) is satisfied. Then, we have $\dot{L}_2(t) < 0$. Based on the analysis above, it concludes that the system state $\mathcal{X}$ will converge to the compact set $\Omega_{\mathcal{X}}$ as long as the event-triggering condition (23) is satisfied and $\mathcal{X}$ lies outside the compact set $\Omega_{\mathcal{X}}$. Thus, there is no conflict in these two conditions.

Remark 9. For Assumption 1, the Lipschitz condition is basic and popular for nonlinear systems in the control field.[6,23,36,41,44] In this article, we develop an ETRC method for uncertain MPN systems under this assumption. For Assumptions 2 and 4, since the controlled plant is controllable, the input gain functions $\mathcal{G}_a(\mathcal{X})$ and $\hbar_a(\mathcal{X})$ are reasonable to assume that they are norm-bounded by two positive constants.[37,41,44] Moreover, $G_a^+(\mathcal{X})$ is the Moore–Penrose pseudo-inverse of the input gain function $\mathcal{G}_a(\mathcal{X})$, which can be assumed to be norm-bounded. Meanwhile, it is reasonable to assume the uncertain term $\mathcal{H}_a(\mathcal{X})\xi_a(\mathcal{X})$ has an upper-bound function such that $\|G_a^+(\mathcal{X})\mathcal{H}_a(\mathcal{X})\xi_a(\mathcal{X})\|^2 \leq \Lambda_a^2(\mathcal{X})$ as in References 2,36,41, and 44. For Assumptions 3 and 6, the input gain functions $\mathcal{G}_a(\mathcal{X})$ and $\hbar_a(\mathcal{X})$ satisfy Lipschitz continuity according to Assumption 1. Meanwhile, $\nabla\mathcal{V}_a^*(\mathcal{X}) = \nabla\sigma_{ca}^{\mathsf{T}}(\mathcal{X})\mathcal{W}_{ca}^* + \nabla\xi_{ca}^{\mathsf{T}}(\mathcal{X})$ is assumed to satisfy the Lipschitz condition since the optimal value function $\mathcal{V}_a^*(\mathcal{X})$ is continuously differentiable. Furthermore, $\tanh(\cdot)$ is a hyperbolic tangent function and Lipschitz continuous. Thus, it is reasonable to assume that there exist two Lipschitz constants such that $\|u_a^*(\mathcal{X}(t)) - u_a^*(\mathcal{X}_p(t))\| \leq \mathcal{L}_{ua}\|e_p(t)\|$ and $\|w_a^*(\mathcal{X}(t)) - w_a^*(\hat{\mathcal{X}}_p)\| \leq \mathcal{L}_{wa}\|e_p(t)\|$.[32-42] For Assumption 5, the ideal weight vector $\mathcal{W}_{ca}^*$ is norm-bounded since it is a constant vector. According to Lemma 1, we can obtain that $\tilde{\mathcal{W}}_{ca}$ is norm-bounded, so it is reasonable to assume that $\max\{\|\tilde{\mathcal{W}}_{ca}\|, \|\mathcal{W}_{ca}^*\|\} \leq \overline{W}_{ca}$.[4,6,11,12,20-42]

Remark 10. The triggering instant ϱ_p can be calculated according to (14). Then, we can obtain the intersampling time $\Delta\varrho_p = \varrho_{p+1} - \varrho_p, p \in \mathbb{N}$. However, if the minimum intersampling time $\Delta\varrho_{\min} = \min\{\Delta\varrho_p\}, p \in \mathbb{N}$ is zero, the well-known Zeno behavior will occur.[56] Fortunately, related proves which demonstrated that the minimum intersampling time $\Delta\varrho_{\min} > 0$ have been given in detail in References 34-42. Thus, the proof of Zeno behavior exclusion is omitted here. Furthermore, simulation results in the following section are provided to show the minimum intersampling time $\Delta\varrho_{\min} > 0$.

4 | SIMULATION STUDIES

MPN systems have drawn much attention due to their wide practical applications.[43,44] In this section, we consider two general simulation examples to verify the effectiveness of the developed ADP-based ETRC method.

4.1 | Example 1

Consider the MPN system modified from Reference 50 with mismatched uncertainties as

$$\dot{\mathcal{X}} = \mathcal{F}(\mathcal{X}) + \mathcal{G}_1(\mathcal{X})u_1 + \mathcal{H}_1(\mathcal{X})\xi_1(\mathcal{X}) + \mathcal{G}_2(\mathcal{X})u_2 + \mathcal{H}_2(\mathcal{X})\xi_2(\mathcal{X}) \tag{24}$$

with

$$\mathcal{F}(\mathcal{X}) = \begin{bmatrix} \mathcal{X}_2 - 2\mathcal{X}_1 \\ X \end{bmatrix}, \mathcal{G}_1(\mathcal{X}) = \begin{bmatrix} 0 \\ \cos(2\mathcal{X}_1) + 2 \end{bmatrix}, \mathcal{G}_2(\mathcal{X}) = \begin{bmatrix} 0 \\ \sin(4\mathcal{X}_1^2) + 2 \end{bmatrix}, \mathcal{H}_1(\mathcal{X}) = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, \mathcal{H}_2(\mathcal{X}) = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix},$$
$$X = -0.5\mathcal{X}_1 - \mathcal{X}_2 - 0.25\mathcal{X}_2(\cos(2\mathcal{X}_1) + 2)^2 + 0.25\mathcal{X}_2(0.5\sin(4\mathcal{X}_1^2) + 2)^2),$$
$$\xi_a(\mathcal{X}) = \lambda_{a1}\mathcal{X}_1\cos(1/(\mathcal{X}_2 + \lambda_{a2})) + \lambda_{a3}\mathcal{X}_2\sin(\lambda_{a4}\mathcal{X}_1\mathcal{X}_2),$$

where $a = 1, 2$, $\mathcal{X} \in [\mathcal{X}_1, \mathcal{X}_2]^{\mathsf{T}} \in \mathbb{R}^2$ is the system state, λ_{a1}, λ_{a2}, λ_{a3}, and λ_{a4} are the unknown parameters randomly chosen as $\lambda_{a1} \in [-1, 1]$, $\lambda_{a2} \in [-100,100]$, $\lambda_{a3} \in [-0.2, 1]$, and $\lambda_{a4} \in [-100, 0]$, respectively.

After calculation, we can get

$$\mathcal{G}_1^+(\mathcal{X}) = [0, 1/(\cos(2\mathcal{X}_1) + 2)], (I - \mathcal{G}_1(\mathcal{X})\mathcal{G}_1^+(\mathcal{X}))\mathcal{H}_1(\mathcal{X}) = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix} \triangleq \hbar_1(\mathcal{X}),$$
$$\|\xi_1(\mathcal{X})\|^2 \leq \|\mathcal{X}\|^2 \triangleq \Xi_1^2(\mathcal{X}), \|\mathcal{G}_1^+(\mathcal{X})\mathcal{H}_1(\mathcal{X})\xi_1(\mathcal{X})\|^2 = 0 \triangleq \Lambda_1^2(\mathcal{X}),$$

$$\mathcal{G}_2^+(\mathcal{X}) = [0, 1/(\sin(4\mathcal{X}_1^2)+2)], (I - \mathcal{G}_2(\mathcal{X})\mathcal{G}_2^+(\mathcal{X}))\mathcal{H}_2(\mathcal{X}) = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix} \triangleq \hbar_2(\mathcal{X}),$$
$$\|\xi_2(\mathcal{X})\|^2 \le \|\mathcal{X}\|^2 \triangleq \Xi_2^2(\mathcal{X}), \|\mathcal{G}_2^+(\mathcal{X})\mathcal{H}_2(\mathcal{X})\xi_2(\mathcal{X})\|^2 = 0 \triangleq \Lambda_2^2(\mathcal{X}).$$

Then, the auxiliary system related to (24) is expressed as

$$\dot{\mathcal{X}} = \mathcal{F}(\mathcal{X}) + \mathcal{G}_1(\mathcal{X})u_1 + \hbar_1(\mathcal{X})w_1(\mathcal{X}) + \mathcal{G}_2(\mathcal{X})u_2 + \hbar_2(\mathcal{X})w_2(\mathcal{X}),$$

where $w_1, w_2 \in \mathbb{R}$ are the auxiliary control inputs. Based on (3), the value function is chosen as

$$\mathcal{V}_a(\mathcal{X}_0) = \int_0^\infty \left(\mathcal{X}^\top Q_a \mathcal{X} + \|\mathcal{X}\|^2 + \mathcal{R}_a(\mathcal{U}_u) + \sum_{b=1}^{\mathcal{N}_u} \|w_b\|^2 \right) dt,$$

where $Q_1 = 5I_2$, $Q_2 = 6I_2$ and

$$\mathcal{R}_a(\mathcal{U}_u) = 2\sum_{a=1}^{N=2} \int_0^{u_a} \bar{u}_a \tanh^{-1}(v/\bar{u}_a)^\top dv$$

with $\bar{u}_a = 0.5$. The learning rates of the critic NNs are set as $l_{c1} = l_{c2} = 0.8$. Since there is no guiding method to select activation functions, they are selected as $\sigma_{c1}(\mathcal{X}) = \sigma_{c2}(\mathcal{X}) = [\mathcal{X}_1^2, \mathcal{X}_1\mathcal{X}_2, \mathcal{X}_2^2]^\top$ based on repeated "trial and error". The weight vectors of the critic NNs are defined as $\hat{\mathcal{W}}_{ca} = [\hat{\mathcal{W}}_{ca}^1, \hat{\mathcal{W}}_{ca}^2, \hat{\mathcal{W}}_{ca}^3]^\top$. Let the initial system state be $x_0 = [1, -1]^\top$.

The auxiliary system states are illustrated in Figure 2, where the states $\mathcal{X}_1$ and $\mathcal{X}_2$ all converge to a small region of zero (SRZ) after 5 s. From Figure 3, we can observe that $\hat{\mathcal{W}}_{c1}$ and $\hat{\mathcal{W}}_{c2}$ converge to $[0.1153, -0.4542, 0.6292]^\top$ and $[-0.5773, 0.2646, -0.3400]^\top$, respectively. Then, the converged critic NN weight vector $\hat{\mathcal{W}}_{ca}$ is applied to the proposed ETRC method (19) to drive the system (24).

Select $\lambda_{11} = 0.2$, $\lambda_{12} = 100$, $\lambda_{13} = 1$, $\lambda_{14} = -1$, $\lambda_{21} = -0.2$, $\lambda_{22} = 100$, $\lambda_{23} = -0.2$, and $\lambda_{24} = -100$. The state trajectories of the closed-loop system (24) and corresponding control signals are shown in Figure 4, which shows that the control signals can guarantee the closed-loop system (24) to approach a SRZ. From Figure 5, we can see that the minimum intersampling time $\Delta\varrho_{\min} > 0$, and the Zeno behavior is excluded. The event-triggering error and threshold are displayed in Figure 6A, we can find that the $\|e_p(t)\|$ and e_T both approach a SRZ after 5 s. Figure 6B describes that the required samples of time-triggered RC (TTRC) and ETRC methods are 300 samples and 101 samples, respectively. It means that 66% transmission has been reduced between the actuator and the controller. Thus, the proposed ADP-based ETRC method can reduce the computational burden, as well as save the communication resource.

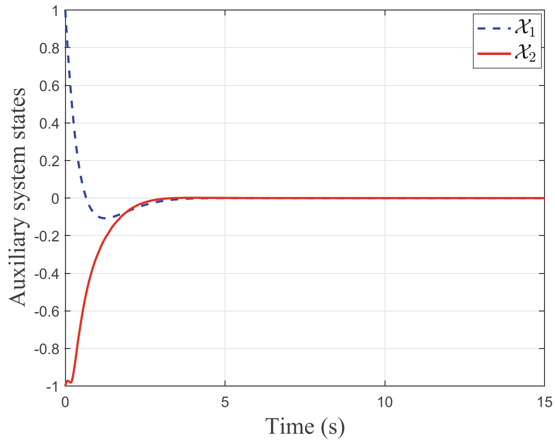

FIGURE 2 The auxiliary system states of Example 1.

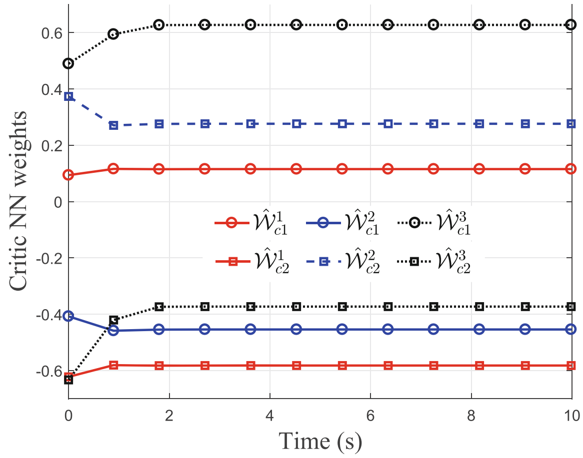

FIGURE 3 Convergence of critic NN weight vectors $\hat{\mathcal{W}}_{c1}$ and $\hat{\mathcal{W}}_{c2}$ of Example 1.

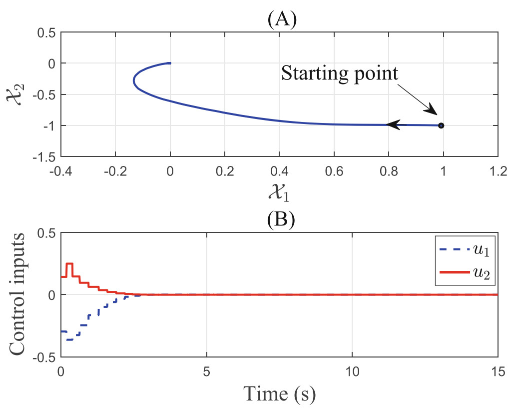

FIGURE 4 (A) State trajectories of closed-loop system of Example 1. (B) The ETC inputs of Example 1.

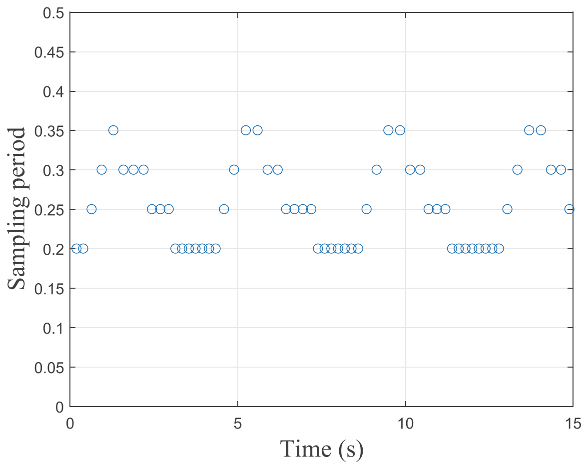

FIGURE 5 The sampling period of Example 1.

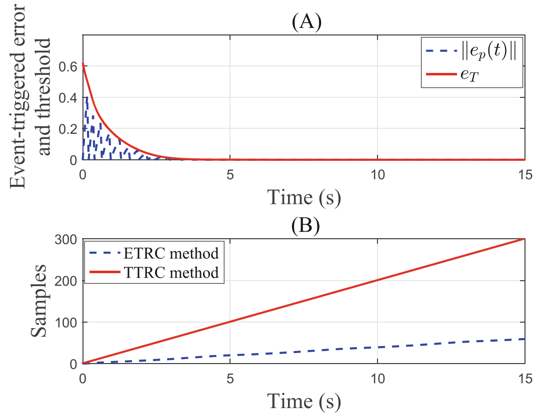

FIGURE 6 (A) Event-triggered error and threshold of Example 1. (B) Cumulative number of samples of Example 1.

Remark 11. From Assumptions 3 and 6, we know that there exist two Lipschitz constants $\mathcal{L}_{ua}$ and $\mathcal{L}_{wa}$ such that $\|u_a^*(\mathcal{X}(t)) - u_a^*(\hat{\mathcal{X}}_p)\| \leq \mathcal{L}_{ua}\|e_p(t)\|$ and $\|w_a^*(\mathcal{X}(t)) - w_a^*(\hat{\mathcal{X}}_p)\| \leq \mathcal{L}_{wa}\|e_p(t)\|$, respectively. Actually, they are selected by "trial and error" with repeated simulations. It is worth pointing out that the large Lipschitz constant may lead to more triggered events, and wasting more computational and communication resources, but small Lipschitz constant may cause system instability. Thus, appropriate Lipschitz constants are selected to tradeoff the triggering frequency and the control performance.

4.2 | Example 2

We consider the following MPN system with mismatched uncertainties as

$$\dot{\mathcal{X}} = \mathcal{F}(\mathcal{X}) + \mathcal{G}_1(\mathcal{X})u_1 + \mathcal{H}_1(\mathcal{X})\xi_1(\mathcal{X}) + \mathcal{G}_2(\mathcal{X})u_2 + \mathcal{H}_2(\mathcal{X})\xi_2(\mathcal{X}) + \mathcal{G}_3(\mathcal{X})u_3 + \mathcal{H}_3(\mathcal{X})\xi_3(\mathcal{X}) \tag{25}$$

with

$$\mathcal{F}(\mathcal{X}) = \begin{bmatrix} \mathcal{X}_2 \\ -2\sin\mathcal{X}_1 - 0.15\mathcal{X}_2 \end{bmatrix}, \mathcal{G}_1(\mathcal{X}) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathcal{H}_1(\mathcal{X}) = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix},$$
$$\mathcal{G}_2(\mathcal{X}) = \begin{bmatrix} 0 \\ 0.75 \end{bmatrix}, \mathcal{H}_2(\mathcal{X}) = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix}, \mathcal{G}_3(\mathcal{X}) = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \mathcal{H}_3(\mathcal{X}) = \begin{bmatrix} 0.25 \\ 0 \end{bmatrix},$$
$$\xi_a(\mathcal{X}) = \lambda_{a1}\mathcal{X}_1\cos(1/(\mathcal{X}_2 + \lambda_{a2})) + \lambda_{a3}\mathcal{X}_2\sin(\lambda_{a4}\mathcal{X}_1\mathcal{X}_2),$$

where $a = 1, 2, 3$, u_a is control input bounded by $|u_{aq}| \leq \bar{u}_a = 1.2$, λ_{a1}, λ_{a2}, λ_{a3}, and λ_{a4} are the unknown parameters with $\lambda_{a1} \in [-1, 1]$, $\lambda_{a2} \in [-100,100]$, $\lambda_{a3} \in [-0.2, 1]$, and $\lambda_{a4} \in [-100, 0]$.

Similar to Example 1, we have

$$\hbar_1(\mathcal{X}) = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, \hbar_2(\mathcal{X}) = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix}, \hbar_3(\mathcal{X}) = \begin{bmatrix} 0.25 \\ 0 \end{bmatrix}, \|\xi_a(\mathcal{X})\|^2 \leq \|\mathcal{X}\|^2 \triangleq \Xi_a^2(\mathcal{X}),$$
$$\|\mathcal{G}_a^+(\mathcal{X})\mathcal{H}_a(\mathcal{X})\xi_a(\mathcal{X})\|^2 = 0 \triangleq \Lambda_a^2(\mathcal{X}), a = 1, 2, 3.$$

Thus, the auxiliary system related to (25) can be expressed as

$$\dot{\mathcal{X}} = \mathcal{F}(\mathcal{X}) + \mathcal{G}_1(\mathcal{X})u_1 + \hbar_1(\mathcal{X})w_1(\mathcal{X}) + \mathcal{G}_2(\mathcal{X})u_2 + \hbar_2(\mathcal{X})w_2(\mathcal{X}) + \mathcal{G}_3(\mathcal{X})u_3 + \hbar_3(\mathcal{X})w_3(\mathcal{X}),$$

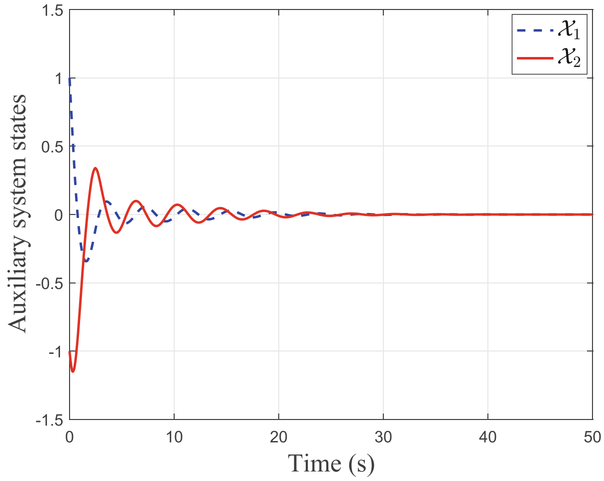

FIGURE 7 The auxiliary system states of Example 2.

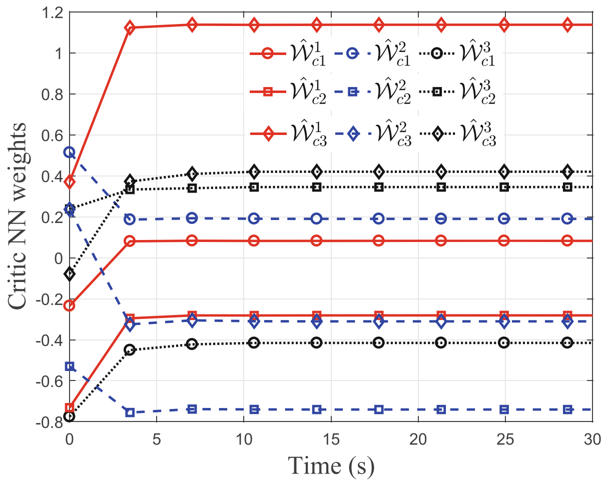

FIGURE 8 Convergence of critic NN weight vectors $\hat{\mathcal{W}}_{c1}$, $\hat{\mathcal{W}}_{c2}$, and $\hat{\mathcal{W}}_{c3}$ of Example 2.

where w_1, w_2, and $w_3 \in \mathbb{R}$ are the auxiliary control inputs. Let $Q_1 = 5$, $Q_2 = 6I_2$, and $Q_3 = 12I_2$. The value function is selected similarly to Example 1. Let the initial system state be $x_0 = [1, -1]^{\mathsf{T}}$.

Simulation results are shown in Figures 7–12. From Figure 7, we can find that the system states converge to a SRZ after 35 s. Figure 8 shows the critic NN weights $\hat{\mathcal{W}}_{c1}$, $\hat{\mathcal{W}}_{c2}$, and $\hat{\mathcal{W}}_{c3}$ converge to $[0.0825, 0.1898, -0.4153]^{\mathsf{T}}$, $[-0.2810, -0.7404, 0.3451]^{\mathsf{T}}$, and $[1.1371, -0.3102, 0.4200]^{\mathsf{T}}$, respectively. Then, we apply the converged critic NN weight vector $\hat{\mathcal{W}}_{ca}$ to construct the ETRC. The parameters of uncertainties $\xi_a(\mathcal{X}), a = 1, 2, 3$ are chosen as Table 1.

The state trajectories of closed-loop system (25) are shown in Figure 9, we can observe that the system states approach a SRZ. As displayed in Figure 10, the control inputs are piecewise signals which indicate that they are updated at the sampling time ϱ_p only and keep unchanged during the time interval $[\varrho_p, \varrho_{p+1})$. The minimum intersampling time $\Delta\varrho_{\min} > 0$ is shown in Figure 11, and it reveals the exclusion of the Zeno behavior. Figure 12A shows the event-triggering error $\|e_p(t)\|$ is beneath the threshold e_T, and both of them converge to SRZs as time increases. Figure 12B indicates that the ETRC method needs 106 samples, while the TTRC method needs 800 samples. It can be concluded that 86% transmission has been reduced between the actuator and the controller. Hence, the computational burden is reduced and the communication resource are saved.

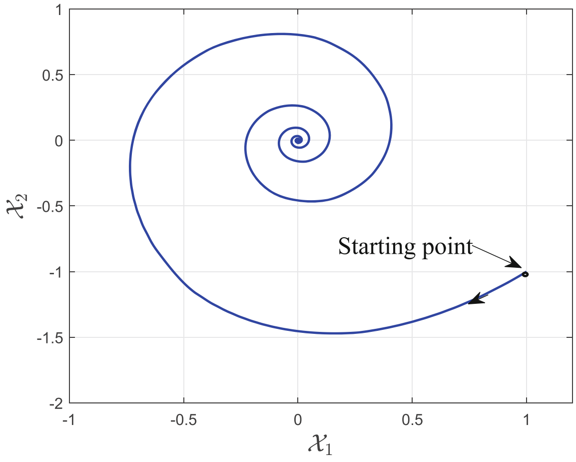

FIGURE 9 State trajectories of closed-loop system of Example 2.

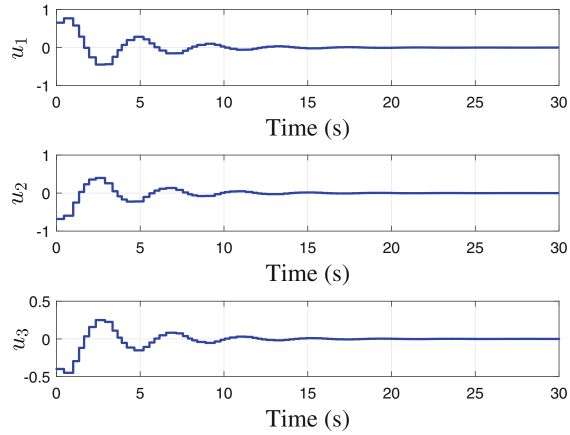

FIGURE 10 The ETC inputs of Example 2.

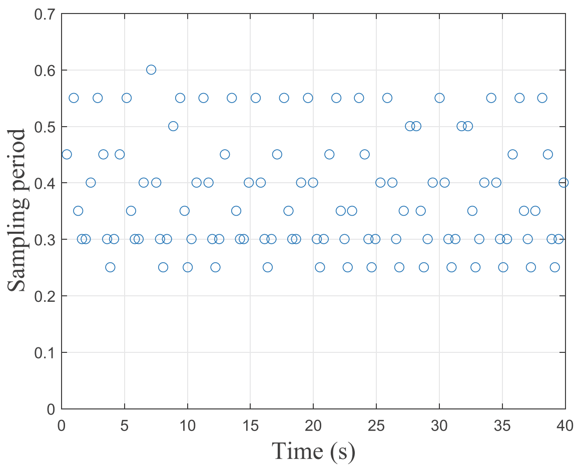

FIGURE 11 The sampling period of Example 2.

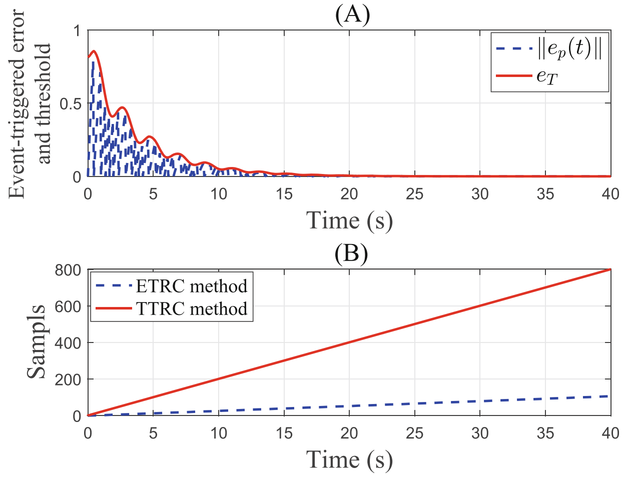

FIGURE 12 (A) Event-triggered error and threshold of Example 2. (B) Cumulative number of samples of Example 2.

TABLE 1 Parameters of uncertainties

	λ_{a1}	λ_{a2}	λ_{a3}	λ_{a4}
$a = 1$	0.2	100.0	1.0	−1.0
$a = 2$	−0.2	100.0	−0.2	−100.0
$a = 3$	−1.0	−100.0	0	−100.0

Remark 12. The initial critic NN weights are selected according to the initial admissible control input of each player. However, there is no guiding method to obtain the initial admissible control input, thus, we select the initial critic NN weights by repeated "trial and error".

5 | CONCLUSIONS

This article mainly focused on designing multiple event-triggered robust controllers for MPN systems with input constraints and mismatched uncertainties, and the computational burden and the communication resource are all saved. The RC problem is transformed into an OC problem by constructing an auxiliary system. To reduce the computational burden and save the communication resource, a novel triggering condition is presented to determine whether the control laws are updated or not. To derive the ETRC law of each player, a critic NN is employed to approximate its value function. Then, according to the Lyapunov stability theorem, both the critic NN weight error dynamics and the states of the closed-loop system are guaranteed to be UUB. Finally, two simulation examples are provided to verify the effectiveness of the proposed ETRC scheme.

In practical implementations, we notice that the independent players in MPN systems have their own transmission sequences, rather than taking the transmission actions synchronously. Therefore, it is worth considering how to relax the requirement of synchronous transmission actions in our future work.

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CONFLICT OF INTEREST

The authors declare that there is no conflict of interest regarding the publication of this article.

DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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APPENDIX A

A.1 Appendix

Select a Lyapunov function candidate as $L_1 = \sum_{a=1}^{\mathcal{N}_u} L_{1a}$, where $L_{1a} = \mathcal{V}_a^*(\mathcal{X})$. Its time derivative is expressed as

$$\begin{aligned}\dot{L}_{1a}(t) &= \nabla\mathcal{V}_a^{*\top}(\mathcal{X})\left(\mathcal{F}(\mathcal{X}) + \sum_{b=1}^{\mathcal{N}_u}\left(\mathcal{G}_b(\mathcal{X})u_b^*(\hat{\mathcal{X}}_p) + \mathcal{H}_b(\mathcal{X})\xi_b(\mathcal{X})\right)\right) \\ &= \nabla\mathcal{V}_a^{*\top}(\mathcal{X})\left(\mathcal{F}(\mathcal{X}) + \sum_{b=1}^{\mathcal{N}_u}\left(\mathcal{G}_b(\mathcal{X})u_b^*(\hat{\mathcal{X}}_p) + \mathcal{G}_b(\mathcal{X})\mathcal{G}_b^+(\mathcal{X})\mathcal{H}_b(\mathcal{X})\xi_b(\mathcal{X})\right)\right) + \nabla\mathcal{V}_a^{*\top}(\mathcal{X})\sum_{b=1}^{\mathcal{N}_u}\hbar_b(\mathcal{X})\xi_b(\mathcal{X}).\end{aligned} \tag{A1}$$

According to (8), we have

$$\begin{aligned}\nabla\mathcal{V}_a^{*\top}(\mathcal{X})\mathcal{F}(\mathcal{X}) = &-\mathcal{X}^\top Q_a\mathcal{X} - \mathcal{R}_a\left(\mathcal{U}_u^*(\mathcal{X})\right) - \gamma^2\sum_{b=1}^{\mathcal{N}_u} w_b^{*\top}w_b^* - \delta\sum_{b=1}^{\mathcal{N}_u}\Lambda_b^2(\mathcal{X}) - \nabla\mathcal{V}_a^{*\top}(\mathcal{X})\sum_{b=1}^{\mathcal{N}_u}\left(\mathcal{G}_b(\mathcal{X})u_b^* + \hbar_b(\mathcal{X})w_b^*\right) \\ &- \vartheta\sum_{b=1}^{\mathcal{N}_u}\Xi_b^2(\mathcal{X}).\end{aligned} \tag{A2}$$

Substituting (A2) into (A1), and recalling Assumptions 2–4, we get

$$\begin{aligned}\dot{L}_{1a}(t) = &-\sum_{b=1}^{\mathcal{N}_u}\left(\delta\Lambda_b^2(\mathcal{X}) + \vartheta\Xi_b^2(\mathcal{X}) + \gamma^2 w_b^{*\top}(\mathcal{X})w_b^*(\mathcal{X})\right) - \mathcal{X}^\top Q_a\mathcal{X} - \mathcal{R}_a\left(\mathcal{U}_u^*(\mathcal{X})\right) + \nabla\mathcal{V}_a^{*\top}(\mathcal{X})\sum_{b=1}^{\mathcal{N}_u}\mathcal{G}_b(\mathcal{X})\mathcal{G}_b^+(\mathcal{X})\mathcal{H}_b(\mathcal{X})\xi_b(\mathcal{X}) \\ &+ \nabla\mathcal{V}_a^{*\top}(\mathcal{X})\sum_{b=1}^{\mathcal{N}_u}\mathcal{G}_b(\mathcal{X})\left(u_b^*(\hat{\mathcal{X}}_p) - u_b^*(\mathcal{X})\right) + \nabla\mathcal{V}_a^{*\top}(\mathcal{X})\sum_{b=1}^{\mathcal{N}_u}\hbar_b(\mathcal{X})\left(\xi_b(\mathcal{X}) - w_b^*(\mathcal{X})\right) \\ \leq &-\sum_{b=1}^{\mathcal{N}_u}\left(\delta\Lambda_b^2(\mathcal{X}) + \vartheta\Xi_b^2(\mathcal{X}) + \gamma^2 w_b^{*\top}(\mathcal{X})w_b^*(\mathcal{X})\right) - \mathcal{X}^\top Q_a\mathcal{X} - \mathcal{R}_a\left(\mathcal{U}_u^*(\mathcal{X})\right) + \frac{3}{2}\|\nabla\mathcal{V}_a^*(\mathcal{X})\|^2 \\ &+ \frac{1}{2}\left\|\sum_{b=1}^{\mathcal{N}_u}\mathcal{G}_b(\mathcal{X})\mathcal{G}_b^+(\mathcal{X})\mathcal{H}_b(\mathcal{X})\xi_b(\mathcal{X})\right\|^2 + \frac{1}{2}\left\|\sum_{b=1}^{\mathcal{N}_u}\mathcal{G}_b(\mathcal{X})\left(u_b^*(\hat{\mathcal{X}}_p) - u_b^*(\mathcal{X})\right)\right\|^2 + \frac{1}{2}\left\|\sum_{b=1}^{\mathcal{N}_u}\hbar_b(\xi_b(\mathcal{X}) - w_b^*(\mathcal{X}))\right\|^2 \\ \leq &-\sum_{b=1}^{\mathcal{N}_u}\left(\delta\Lambda_b^2(\mathcal{X}) + \vartheta\Xi_b^2(\mathcal{X}) + \gamma^2 w_b^{*\top}(\mathcal{X})w_b^*(\mathcal{X})\right) - \mathcal{X}^\top Q_a\mathcal{X} - \mathcal{R}_a\left(\mathcal{U}_u^*(\mathcal{X})\right) + \frac{3}{2}\|\nabla\mathcal{V}_a^*(\mathcal{X})\|^2 \\ &+ \frac{\mathcal{N}_u}{2}\sum_{b=1}^{\mathcal{N}_u}\bar{g}_b^2\Lambda_b^2(\mathcal{X}) + \frac{\mathcal{N}_u}{2}\sum_{b=1}^{\mathcal{N}_u}\mathcal{L}_{ub}^2\bar{g}_b^2\|e_p(t)\|^2 + \frac{\mathcal{N}_u}{2}\sum_{b=1}^{\mathcal{N}_u}\left\|\hbar_b\left(\xi_b(\mathcal{X}) - w_b^*(\mathcal{X})\right)\right\|^2 \\ \leq &-\mathcal{X}^\top Q_a\mathcal{X} - \mathcal{R}_a\left(\mathcal{U}_u^*(\mathcal{X})\right) + \frac{3}{2}\|\nabla\mathcal{V}_a^*(\mathcal{X})\|^2 + \frac{\mathcal{N}_u}{2}\sum_{b=1}^{\mathcal{N}_u}\bar{g}_b^2\mathcal{L}_{ub}^2\|e_p(t)\|^2 + \sum_{b=1}^{\mathcal{N}_u}\left(\frac{\mathcal{N}_u}{2}\bar{g}_b^2 - \delta\right)\Lambda_b^2(\mathcal{X}) \\ &+ \sum_{b=1}^{\mathcal{N}_u}\left(\frac{\mathcal{N}_u}{2}\bar{\hbar}_b^2 - \vartheta\right)\Xi_b^2(\mathcal{X}) + \sum_{b=1}^{\mathcal{N}_u}\left(\frac{\mathcal{N}_u}{2}\bar{\hbar}_b^2 - \gamma^2\right)w_b^{*\top}(\mathcal{X})w_b^*(\mathcal{X}).\end{aligned} \tag{A3}$$

Denoting

$$\Delta(\mathcal{X}) = \left(\frac{\mathcal{N}_u}{2}\overline{g}^2 - \delta\right)\sum_{b=1}^{\mathcal{N}_u}\Lambda_b^2(\mathcal{X}) + \left(\frac{\mathcal{N}_u}{2}\overline{\hbar}^2 - \vartheta\right)\sum_{b=1}^{\mathcal{N}_u}\Xi_b^2(\mathcal{X}) + \left(\frac{\mathcal{N}_u}{2}\overline{\hbar}^2 - \gamma^2\right)\sum_{b=1}^{\mathcal{N}_u} w_b^{*\mathsf{T}}(\mathcal{X})w_b^*(\mathcal{X}),$$

where $\overline{g} = \max\{\overline{g}_1, \overline{g}_2, \dots, \overline{g}_{\mathcal{N}_u}\}$ and $\overline{\hbar} = \max\{\overline{\hbar}_1, \overline{\hbar}_2, \dots, \overline{\hbar}_{\mathcal{N}_u}\}$, and based on (A3), we have

$$\begin{aligned}
\dot{L}_1(t) &= \sum_{a=1}^{\mathcal{N}_u}\dot{L}_{1a}(t) \\
&\leq -\sum_{a=1}^{\mathcal{N}_u}\mathcal{X}^\mathsf{T}Q_a\mathcal{X} - \sum_{a=1}^{\mathcal{N}_u}\mathcal{R}_a\left(\mathcal{U}_u^*(\mathcal{X})\right) + \frac{\mathcal{N}_u^2}{2}\overline{g}^2\sum_{b=1}^{\mathcal{N}_u}\mathcal{L}_{ub}^2\|e_p(t)\|^2 + \mathcal{N}_u\Delta(\mathcal{X}) + \frac{3}{2}\sum_{a=1}^{\mathcal{N}_u}\|\nabla\mathcal{V}_a^*(\mathcal{X})\|^2 \\
&\leq -\varpi_1^2\sum_{a=1}^{\mathcal{N}_u}\lambda_{\min}(Q_a)\|\mathcal{X}\|^2 + \frac{3}{2}\sum_{a=1}^{\mathcal{N}_u}\|\nabla\mathcal{V}_a^*(\mathcal{X})\|^2 + \mathcal{N}_u\Delta(\mathcal{X}) + (\varpi_1^2 - 1)\sum_{a=1}^{\mathcal{N}_u}\lambda_{\min}(Q_a)\|\mathcal{X}\|^2 + \frac{\mathcal{N}_u^2}{2}\overline{g}^2\sum_{b=1}^{\mathcal{N}_u}\mathcal{L}_{ub}^2\|e_p(t)\|^2.
\end{aligned}$$

By selecting the parameters as $\delta \geq \frac{\mathcal{N}_u}{2}\overline{g}^2$, $\vartheta \geq \frac{\mathcal{N}_u}{2}\overline{\hbar}^2$, and $\gamma^2 \geq \frac{\mathcal{N}_u}{2}\overline{\hbar}^2$, we have $\mathcal{N}_u\Delta(\mathcal{X}) \leq 0$, and $\dot{L}_1(t)$ becomes

$$\dot{L}_1(t) \leq -\varpi_1^2\sum_{a=1}^{\mathcal{N}_u}\lambda_{\min}(Q_a)\|\mathcal{X}\|^2 + \frac{3}{2}\sum_{a=1}^{\mathcal{N}_u}\|\nabla\mathcal{V}_a^*(\mathcal{X})\|^2 + (\varpi_1^2 - 1)\sum_{a=1}^{\mathcal{N}_u}\lambda_{\min}(Q_a)\|\mathcal{X}\|^2 + \frac{\mathcal{N}_u^2}{2}\overline{g}^2\sum_{b=1}^{\mathcal{N}_u}\mathcal{L}_{ub}^2\|e_p(t)\|^2.$$

Then, when conditions (13) and (14) hold, we have

$$\dot{L}_1(t) < 0, \ \forall\mathcal{X} \neq 0.$$

According to Lyapunov stability theorem,[57,58] for $L_1(t) > 0$ and $\dot{L}_1(t) < 0$, $\forall\mathcal{X} \neq 0$, we can conclude that the closed-loop system (1) is asymptotically stable.

A.2 Appendix

Choose the Lyapunov function candidate as

$$L_2(t) = L_{21}(t) + L_{22}(t), \tag{A4}$$

where $L_{21}(t) = \sum_{a=1}^{\mathcal{N}_u}\mathcal{V}_a^*(\mathcal{X})$ and $L_{22}(t) = \sum_{a=1}^{\mathcal{N}_u}\mathcal{V}_a^*(\hat{\mathcal{X}}_p)$.

Since the event triggering mechanism is introduced, the stability should be analyzed with the following two cases.

Case 1. Events are not triggered, that is, $\forall t \in [\varrho_p, \varrho_{p+1})$. It implies that $\dot{L}_{22}(t) = 0$. Taking the time derivative of $L_{21}(t)$, we can obtain

$$\dot{L}_{21}(t) = \sum_{a=1}^{\mathcal{N}_u}\left(\nabla\mathcal{V}^{*\mathsf{T}}_a(\mathcal{X})\left(\mathcal{F}(\mathcal{X}) + \sum_{b=1}^{\mathcal{N}_u}\left(\mathcal{G}_b(\mathcal{X})\hat{u}_b(\hat{\mathcal{X}}_p) + \hbar_b(\mathcal{X})\hat{w}_b(\hat{\mathcal{X}}_p)\right)\right)\right).$$

Then, based on (A2), we get

$$\begin{aligned}
\dot{L}_{21}(t) = &-\sum_{a=1}^{\mathcal{N}_u}\left(\mathcal{X}^\mathsf{T}Q_a\mathcal{X} + \mathcal{R}_a\left(\mathcal{U}_u^*(\mathcal{X})\right) + \delta\sum_{b=1}^{\mathcal{N}_u}\Lambda_b^2(\mathcal{X})\right) - \sum_{a=1}^{\mathcal{N}_u}\left(\vartheta\sum_{b=1}^{\mathcal{N}_u}\Xi_b^2(\mathcal{X}) + \gamma^2\sum_{b=1}^{\mathcal{N}_u}w_b^{*\mathsf{T}}(\mathcal{X})w_b^*(\mathcal{X})\right) \\
&+ \sum_{a=1}^{\mathcal{N}_u}\nabla\mathcal{V}_a^{*\mathsf{T}}(\mathcal{X})\sum_{b=1}^{\mathcal{N}_u}\mathcal{G}_b(\mathcal{X})\left(\hat{u}_b(\hat{\mathcal{X}}_p) - u_b^*(\mathcal{X})\right) + \sum_{a=1}^{\mathcal{N}_u}\nabla\mathcal{V}_a^{*\mathsf{T}}(\mathcal{X})\sum_{b=1}^{\mathcal{N}_u}\hbar_b(\mathcal{X})\left(\hat{w}_b(\hat{\mathcal{X}}_p) - w_b^*(\mathcal{X})\right)
\end{aligned}$$

$$\leq -\sum_{a=1}^{\mathcal{N}_u}\left(\mathcal{X}^{\mathsf{T}}Q_a\mathcal{X}+\mathcal{R}_a\left(\mathcal{U}_u^*(\mathcal{X})\right)+\delta\sum_{b=1}^{\mathcal{N}_u}\Lambda_b^2(\mathcal{X})\right)-\sum_{a=1}^{\mathcal{N}_u}\left(\vartheta\sum_{b=1}^{\mathcal{N}_u}\Xi_b^2(\mathcal{X})+\gamma^2\sum_{b=1}^{\mathcal{N}_u}w_b^{*\mathsf{T}}(\mathcal{X})w_b^*(\mathcal{X})\right)$$
$$+\frac{1}{2}\left\|\sum_{b=1}^{\mathcal{N}_u}\mathcal{G}_b\left(\hat{u}_b(\hat{\mathcal{X}}_p)-u_b^*(\mathcal{X})\right)\right\|^2+\frac{1}{2}\left\|\sum_{b=1}^{\mathcal{N}_u}\hbar_b\left(\hat{w}_b(\hat{\mathcal{X}}_p)-w_b^*(\mathcal{X})\right)\right\|^2+\mathcal{N}_u\sum_{a=1}^{\mathcal{N}_u}\left\|\nabla\sigma_{ca}^{\mathsf{T}}(\mathcal{X})\mathcal{W}_{ca}^*+\nabla\xi_{ca}^{\mathsf{T}}(\mathcal{X})\right\|^2$$
$$\leq -\sum_{a=1}^{\mathcal{N}_u}\left(\mathcal{X}^{\mathsf{T}}Q_a\mathcal{X}+\mathcal{R}_a\left(\mathcal{U}_u^*(\mathcal{X})\right)+\delta\sum_{b=1}^{\mathcal{N}_u}\Lambda_b^2(\mathcal{X})\right)-\sum_{a=1}^{\mathcal{N}_u}\left(\vartheta\sum_{b=1}^{\mathcal{N}_u}\Xi_b^2(\mathcal{X})+\gamma^2\sum_{b=1}^{\mathcal{N}_u}w_b^{*\mathsf{T}}(\mathcal{X})w_b^*(\mathcal{X})\right)$$
$$+2\mathcal{N}_u\sum_{a=1}^{\mathcal{N}_u}(\nabla\overline{\sigma}_{ca}^2\overline{W}_{ca}^2+\nabla\overline{\xi}_{ca}^2)+\frac{\mathcal{N}_u}{2}\overline{g}^2\sum_{a=1}^{\mathcal{N}_u}\underbrace{\left\|\hat{u}_a(\hat{\mathcal{X}}_p)-u_a^*(\mathcal{X})\right\|^2}_{\Upsilon_1}+\frac{\mathcal{N}_u}{2}\overline{\hbar}^2\sum_{a=1}^{\mathcal{N}_u}\underbrace{\left\|\hat{w}_a(\hat{\mathcal{X}}_p)-w_a^*(\mathcal{X})\right\|^2}_{\Upsilon_2}. \tag{A5}$$

By applying Young's inequality $\|O+P\|^2 \leq 2\|O\|^2+2\|P\|^2$, and using (17)–(20), Υ_1 and Υ_2 become

$$\Upsilon_1=\left\|\left(\hat{u}_a(\hat{\mathcal{X}}_p)-u_a^*(\hat{\mathcal{X}}_p)\right)+\left(u_a^*(\hat{\mathcal{X}}_p)-u_a^*(\mathcal{X})\right)\right\|^2$$
$$\leq 2\left\|\hat{u}_a(\hat{\mathcal{X}}_p)-u_a^*(\hat{\mathcal{X}}_p)\right\|^2+2\left\|u_a^*(\hat{\mathcal{X}}_p)-u_a^*(\mathcal{X})\right\|^2$$
$$\leq 2\left\|-\bar{u}_a\tanh\left(\frac{1}{2\bar{u}_a}\mathcal{G}_a^{\mathsf{T}}(\hat{\mathcal{X}}_p)\nabla\sigma_{ca}^{\mathsf{T}}(\hat{\mathcal{X}}_p)\hat{\mathcal{W}}_{ca}\right)+\bar{u}_a\tanh\left(\frac{1}{2\bar{u}_a}\mathcal{G}_a^{\mathsf{T}}(\hat{\mathcal{X}}_p)\left(\nabla\sigma_{ca}^{\mathsf{T}}(\hat{\mathcal{X}}_p)\mathcal{W}_{ca}^*+\nabla\xi_{ca}^{\mathsf{T}}(\hat{\mathcal{X}}_p)\right)\right)\right\|^2+2\mathcal{L}_{ua}^2\|e_p(t)\|^2$$
$$\leq \frac{1}{2}\mathcal{L}_{\tanh}^2\|\mathcal{G}_a^{\mathsf{T}}(\hat{\mathcal{X}}_p)\|^2\left\|\nabla\xi_{ca}^{\mathsf{T}}(\hat{\mathcal{X}}_p)+\nabla\sigma_{ca}^{\mathsf{T}}(\hat{\mathcal{X}}_p)\tilde{\mathcal{W}}_{ca}\right\|^2+2\mathcal{L}_{ua}^2\|e_p(t)\|^2$$
$$\leq \frac{1}{2}\mathcal{L}_{\tanh}^2\overline{g}^2\left(\nabla\overline{\sigma}_c^2\overline{W}_{ca}^2+\nabla\overline{\xi}_{ca}^2\right)+2\mathcal{L}_{ua}^2\|e_p(t)\|^2, \tag{A6}$$

$$\Upsilon_2=\left\|\left(\hat{w}_a(\hat{\mathcal{X}}_p)-w_a^*(\hat{\mathcal{X}}_p)\right)+\left(w_a^*(\hat{\mathcal{X}}_p)-w_a^*(\mathcal{X})\right)\right\|^2$$
$$\leq 2\left\|\hat{w}_a(\hat{\mathcal{X}}_p)-w_a^*(\hat{\mathcal{X}}_p)\right\|^2+2\left\|w_a^*(\hat{\mathcal{X}}_p)-w_a^*(\mathcal{X})\right\|^2$$
$$\leq 2\left\|-\frac{1}{2\gamma^2}\hbar_a^{\mathsf{T}}(\hat{\mathcal{X}}_p)\nabla\sigma_{ca}^{\mathsf{T}}(\hat{\mathcal{X}}_p)\hat{\mathcal{W}}_{ca}+\frac{1}{2\gamma^2}\hbar_a^{\mathsf{T}}(\hat{\mathcal{X}}_p)\left(\nabla\sigma_{ca}^{\mathsf{T}}(\hat{\mathcal{X}}_p)\mathcal{W}_{ca}^{*\mathsf{T}}+\nabla\xi_{ca}^{\mathsf{T}}(\hat{\mathcal{X}}_p)\right)\right\|^2+2\mathcal{L}_{wa}^2\|e_p(t)\|^2$$
$$\leq \frac{1}{2\gamma^4}\|\hbar_a^{\mathsf{T}}(\hat{\mathcal{X}}_p)\|^2\left\|\nabla\xi_{ca}^{\mathsf{T}}(\hat{\mathcal{X}}_p)+\nabla\sigma_{ca}^{\mathsf{T}}(\hat{\mathcal{X}}_p)\tilde{\mathcal{W}}_{ca}\right\|^2+2\mathcal{L}_{wa}^2\|e_p(t)\|^2$$
$$\leq \frac{1}{2\gamma^4}\overline{\hbar}^2\left(\nabla\overline{\sigma}_c^2\overline{W}_{ca}^2+\nabla\overline{\xi}_{ca}^2\right)+2\mathcal{L}_{wa}^2\|e_p(t)\|^2, \tag{A7}$$

where $\mathcal{L}_{\tanh}$ is a Lipschitz constant of $\tanh(\cdot)$. Considering (A5) and (A6), $\dot{L}_{21}(t)$ becomes

$$\dot{L}_{21}(t)\leq -\sum_{a=1}^{\mathcal{N}_u}\mathcal{X}^{\mathsf{T}}Q_a\mathcal{X}+\mathcal{T}_0+\mathcal{L}_{uw}\|e_p(t)\|^2-\sum_{a=1}^{\mathcal{N}_u}\left(\mathcal{R}_a\left(\mathcal{U}_u^*(\mathcal{X})\right)+\delta\sum_{b=1}^{\mathcal{N}_u}\Lambda_b^2(\mathcal{X})+\vartheta\sum_{b=1}^{\mathcal{N}_u}\Xi_b^2(\mathcal{X})\right)-\mathcal{N}_u\gamma^2\sum_{b=1}^{\mathcal{N}_u}\|w_b^*(\mathcal{X})\|^2, \tag{A8}$$

where $\mathcal{T}_0=\frac{1}{4}(\mathcal{N}_u\mathcal{L}_{\tanh}^2\overline{g}^4+\frac{2}{\gamma^4}\mathcal{N}_u\overline{\hbar}^4+8\mathcal{N}_u)\sum_{a=1}^{\mathcal{N}_u}(\nabla\overline{\sigma}_{ca}^2\overline{W}_{ca}^2+\nabla\overline{\xi}_{ca}^2)$ and $\mathcal{L}_{uw}=\sum_{a=1}^{\mathcal{N}_u}(\mathcal{N}_u\overline{g}^2\mathcal{L}_{ua}^2+\mathcal{N}_u\overline{\hbar}^2\mathcal{L}_{wa}^2)$ are positive constants.

Considering $\dot{L}_{22}(t)=0$ and introducing (A8), the time derivative of $L_2(t)$ in (A4) is given by

$$\dot{L}_2(t)\leq -\varpi_2^2\sum_{a=1}^{\mathcal{N}_u}\lambda_{\min}(Q_a)\|\mathcal{X}\|^2+\mathcal{T}_0+\mathcal{L}_{uw}\|e_p(t)\|^2+(\varpi_2^2-1)\sum_{a=1}^{\mathcal{N}_u}\lambda_{\min}(Q_a)\|\mathcal{X}\|^2. \tag{A9}$$

Then, we have $\dot{L}_2(t)<0$ when conditions (23) satisfies and $\mathcal{X}(t)$ lies outside the compact set

$$\Omega_{\mathcal{X}}=\left\{\mathcal{X}:\|\mathcal{X}\|\leq\sqrt{\frac{\mathcal{T}_0}{\varpi_2^2\sum_{a=1}^{\mathcal{N}_u}\lambda_{\min}(Q_a)}}\right\}. \tag{A10}$$

Case 2. Events are triggered, that is, $\forall t = \varrho_{p+1}$. The difference of the Lyapunov function candidate $L_2(t)$ in (A4) is given as

$$\Delta L_2(t) = L_2(\hat{\mathcal{X}}_{p+1}) - L_2\left(\mathcal{X}(\varrho_{p+1}^-)\right)$$
$$= \Delta L_{21}(t) + \Delta L_{22}(t).$$

From Case 1, it shows $\dot{L}_2(t) < 0$ for all $t \in [\varrho_p, \varrho_{p+1})$. Considering the continuity of the auxiliary system state and the value function, we have

$$\Delta L_{21}(t) = \sum_{a=1}^{\mathcal{N}_u} \left(\mathcal{V}_a^*(\hat{\mathcal{X}}_{p+1}) - \mathcal{V}_a^*\left(\mathcal{X}(\varrho_{p+1}^-)\right)\right) \leq 0,$$
$$\Delta L_{22}(t) = \sum_{a=1}^{\mathcal{N}_u} \left(\mathcal{V}_a^*(\hat{\mathcal{X}}_{p+1}) - \mathcal{V}_a^*(\hat{\mathcal{X}}_p)\right) \leq -\nu\left(\|e_{p+1}(\varrho_p)\|\right),$$

where $\mathcal{X}(\varrho_{p+1}^-) = \lim_{\Delta t \to 0} \mathcal{X}(\varrho_{p+1} - \Delta t)$, $\nu(\cdot)$ is a class-$\mathcal{K}$ function and $e_{p+1}(\varrho_p) = \hat{\mathcal{X}}_{p+1} - \hat{\mathcal{X}}_p$. Therefore, $\Delta L_2(t) \leq -\nu(\|e_{p+1}(\varrho_p)\|)$, which implies that $L_2(t)$ decreases at $\forall t = \varrho_{p+1}$.

Based on the above analysis, it concludes that the closed-loop auxiliary system (2) is guaranteed to be UUB. It is noticed that $\hat{\mathcal{X}}_p$ is sampled from the system state $\mathcal{X}$. That is to say, $\hat{\mathcal{X}}_p$ is not an independent state, thus $\hat{\mathcal{X}}_p$ can also be guaranteed to be UUB if $\mathcal{X}$ has an ultimate bound.