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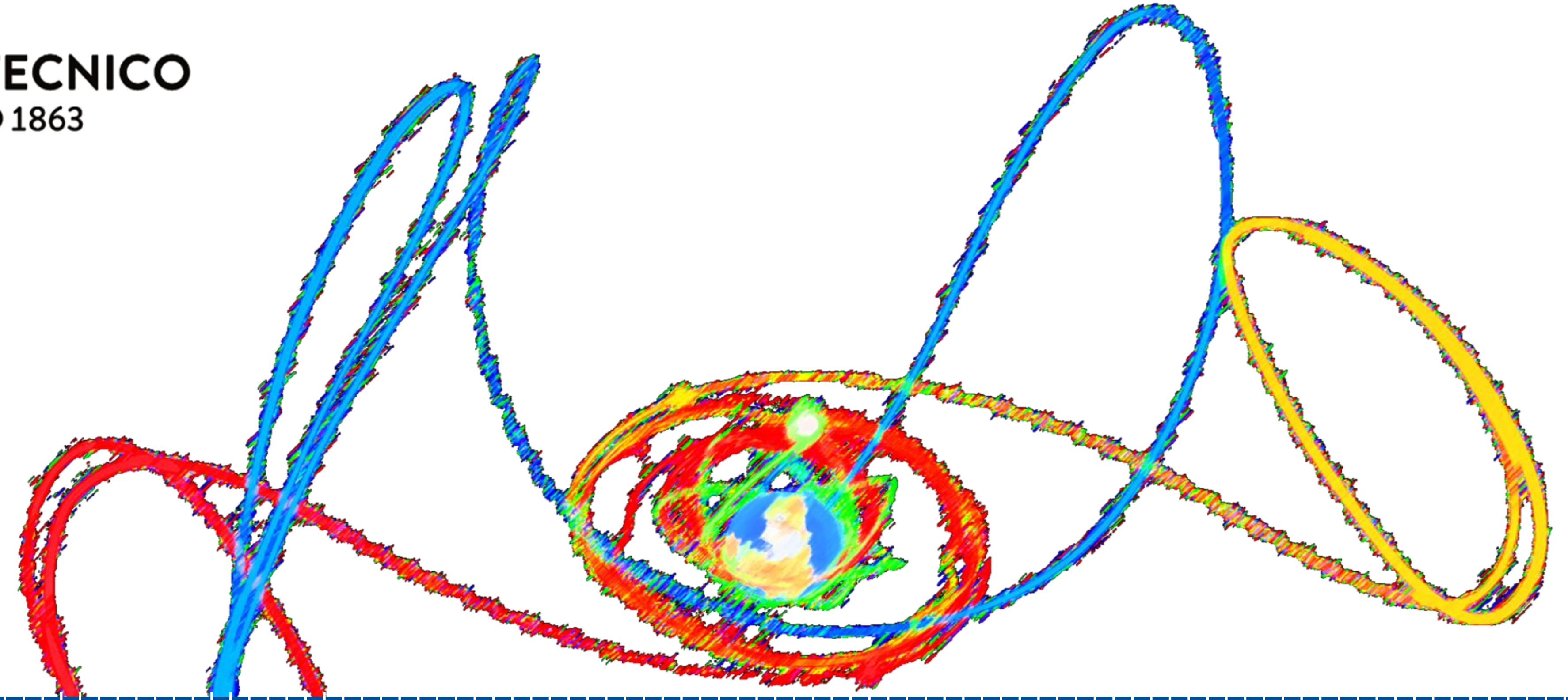


# Autonomous collision avoidance with operational constraints

Eduardo Maria Polli, Juan Luis Gonzalo, Camilla Colombo

International Conjunction Assessment Workshop

Toulouse, France, 18-20 June 2025

# ACTIVA

## Advanced Control Techniques for Increased on-board Autonomy [1]

### Conjunction assessment (GMV)

- Onboard collision avoidance detection
- Catalogue of neighbourhood objects
- Decision making
- Data uplink
- Inter-satellite link

**ACTIVA**
**Advanced Control Techniques for Increased on Board Autonomy**

### Collision avoidance manoeuvre (PoliMi)

- Semi-analytical fuel-optimal guidance for:
  - Impulsive manoeuvre
  - Low-thrust manoeuvre
- Computational efficiency
- Inclusion of operational constraints
  - Eclipses
  - Pointing
  - Maximum rotational velocity

[1] Oliveira, T., et al., Modular and Scalable Collision Avoidance System for Enhanced Satellite Autonomy (2025), in 9th European Conference on Space Debris, Bonn, Germany, 1–4 April

# Fuel-optimal manoeuvre

## Optimal impulsive manoeuvre

- Maximum deviation [2]: $\boldsymbol{\delta r}(t_{CA}) = \boldsymbol{T}\boldsymbol{\delta v}(t_{CAM})$

1. Define quadratic form: $\boldsymbol{\delta r}^2 = \boldsymbol{\delta v}^T \boldsymbol{T}^T \boldsymbol{T} \boldsymbol{\delta v}$
2. Solve eigenvalue problem of $\boldsymbol{T}^T \boldsymbol{T}$
3. Optimal $\boldsymbol{\delta v}$ is parallel to the eigenvector associated to the maximum eigenvalue

- Maximum deviation on the b-plane [3]: $\boldsymbol{\delta b}^2(t_{CA}) = \boldsymbol{\delta v}^T \boldsymbol{T}^T \boldsymbol{B}^T \boldsymbol{B}\, \boldsymbol{T} \boldsymbol{\delta v}$
  - $\boldsymbol{B}$ rotates to the b-frame

- Maximum squared Mahalanobis distance [4]: $\boldsymbol{\delta m}^2(t_{CA}) = \boldsymbol{\delta v}^T \boldsymbol{T}^T \boldsymbol{B}^T \boldsymbol{Q}^* \boldsymbol{B}\, \boldsymbol{T} \boldsymbol{\delta v}$
  - $\boldsymbol{Q}^*$ scales $\boldsymbol{\delta b}^*$ by the combined uncertainties

[2] Conway, B. A. (2001). Near-optimal deflection of earth-approaching asteroids. Journal of Guidance, Control, and Dynamics, 24(5), 1035-1037.
[3] Vasile, M., & Colombo, C. (2008). Optimal impact strategies for asteroid deflection. Journal of guidance, control, and dynamics, 31(4), 858-872.
[4] Bombardelli, C., & Hernando-Ayuso, J. (2015). Optimal impulsive collision avoidance in low earth orbit. Journal of Guidance, Control, and Dynamics, 38(2), 217-225.

# Operational constraints

## Overview

### Eclipse constraints

- Applicable only to electric propulsion low-thrust [3]

### Pointing constraints

- Maintain satellite operative during manoeuvre

### Thruster failure

- Keep algorithm flexible to account for possible failures

### Maximum rotational speed

- Limit attitude control to redirect thrust direction

### Objects catalogue

- Assess post-manoeuvre conjunctions with neighbourhood objects

[3] Polli, E. M., et al. (2025). Semi-Analytical Model for Autonomous Fuel-Optimal Low-Thrust Collision Avoidance with Eclipse Constraints. In 35th AAS/AIAA Space Flight Mechanics Meeting, 19–23 January

# Pointing constraint

## Allowed thrust directions

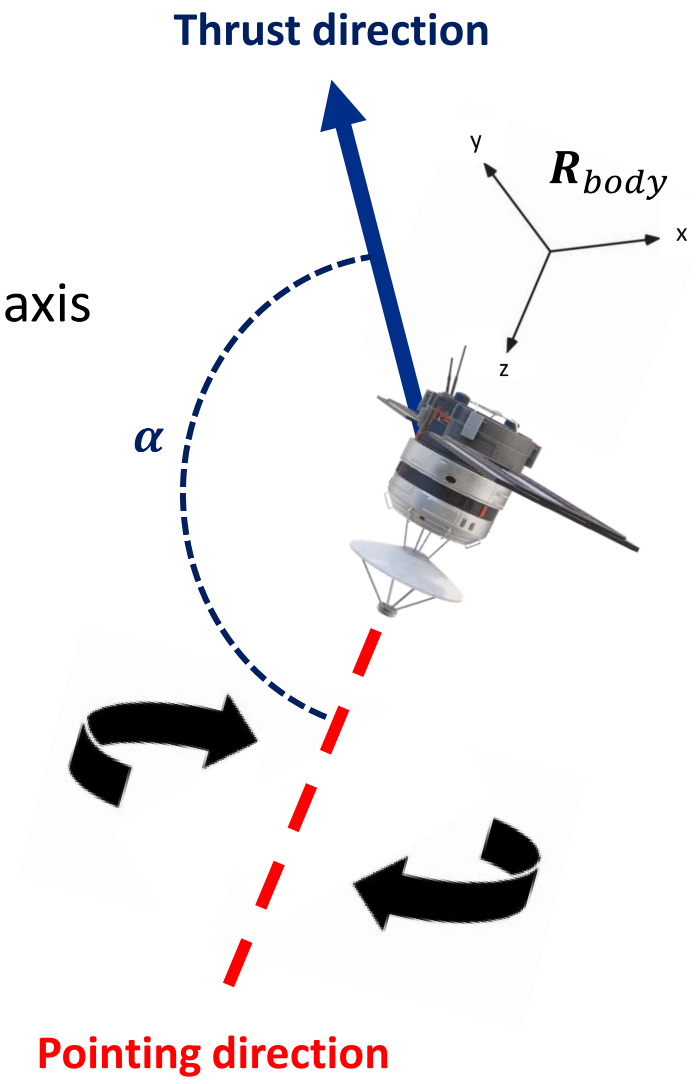
- Satellite can rotate around pointing direction

- Rotation matrix $\boldsymbol{R}_{body}$ from TNH [5] frame to body-frame with 3rd axis along pointing direction:

$$\delta m^2 = \boldsymbol{\delta v}^T \boldsymbol{R}_{body}^T \boldsymbol{T}^T \boldsymbol{B}^T \boldsymbol{Q}^* \boldsymbol{B} \boldsymbol{T} \boldsymbol{R}_{body} \boldsymbol{\delta v}$$
$$= \boldsymbol{\delta v}^T \boldsymbol{A} \boldsymbol{\delta v}$$

- Cone constraint applied by imposing that:

$$\delta v = \begin{bmatrix} \delta v_x \\ \delta v_y \\ \cos\alpha \end{bmatrix} \quad \text{with} \quad |\delta v| = 1$$

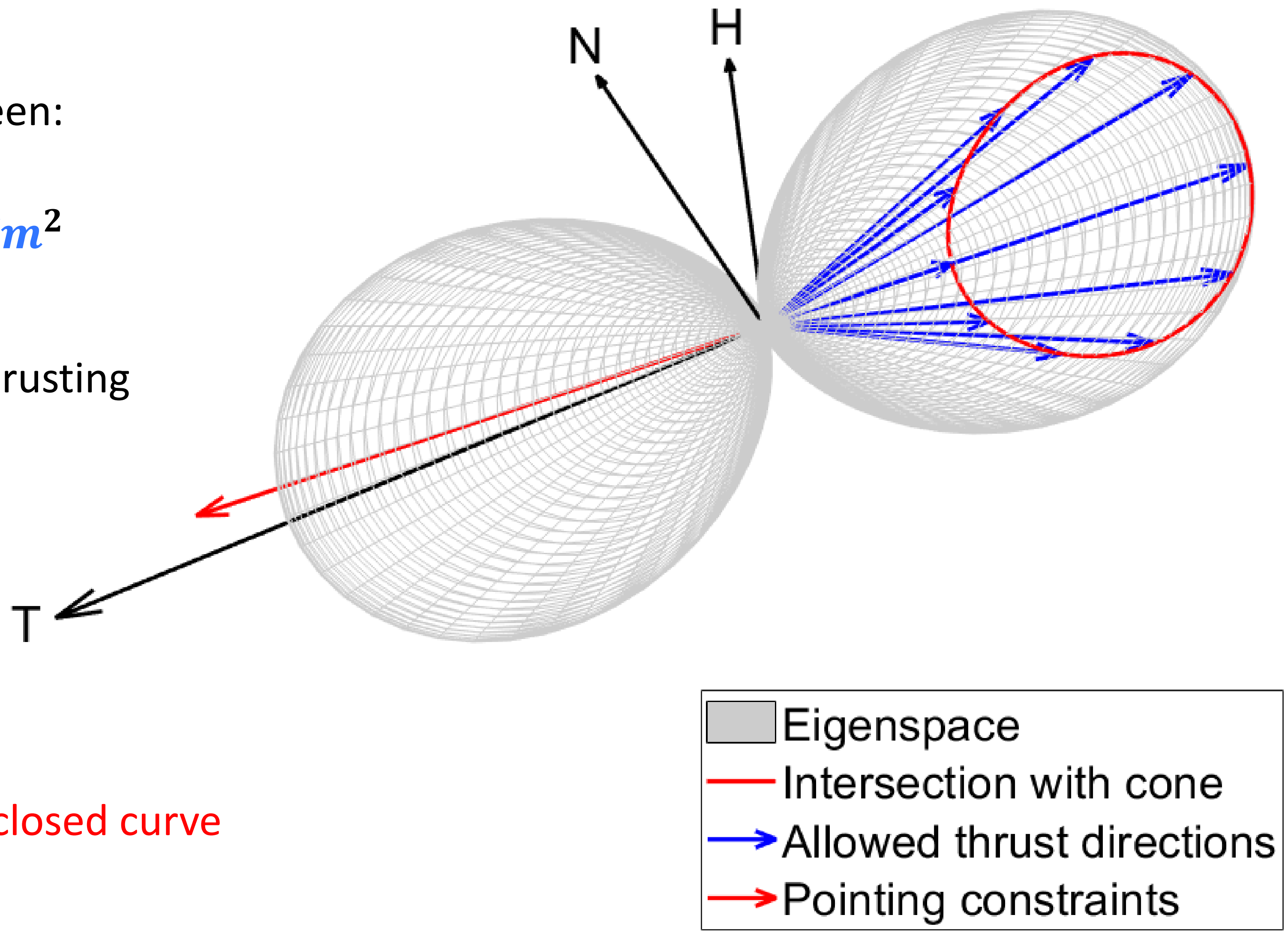



[5] Gonzalo, J. L., Colombo, C., & Di Lizia, P. (2021). Analytical framework for space debris collision avoidance maneuver design. Journal of Guidance, Control, and Dynamics, 44(3), 469-487.

# Pointing constraints

## Rayleigh quotient space

- The STM provides a linear mapping between:
  - The $\boldsymbol{\delta v}$ in the TNH frame
  - The squared Mahalanobis distance $\boldsymbol{\delta m^2}$
- The surface expresses the optimality of thrusting in a specific direction
- Closed curve as intersection between:
  - Eigenspace
  - Thrust direction cone
- Find the maximum effect that lies on the closed curve



# Cone constrained eigenvalue problem

## Lagrangian formulation

- The Lagrangian $\mathfrak{L}$ is defined as:

$$\mathfrak{L} = \boldsymbol{\delta v}^T A \boldsymbol{\delta v} + \lambda(1 - \boldsymbol{\delta v}^T \boldsymbol{\delta v}) + \nu(\boldsymbol{c} - K\boldsymbol{\delta v})$$

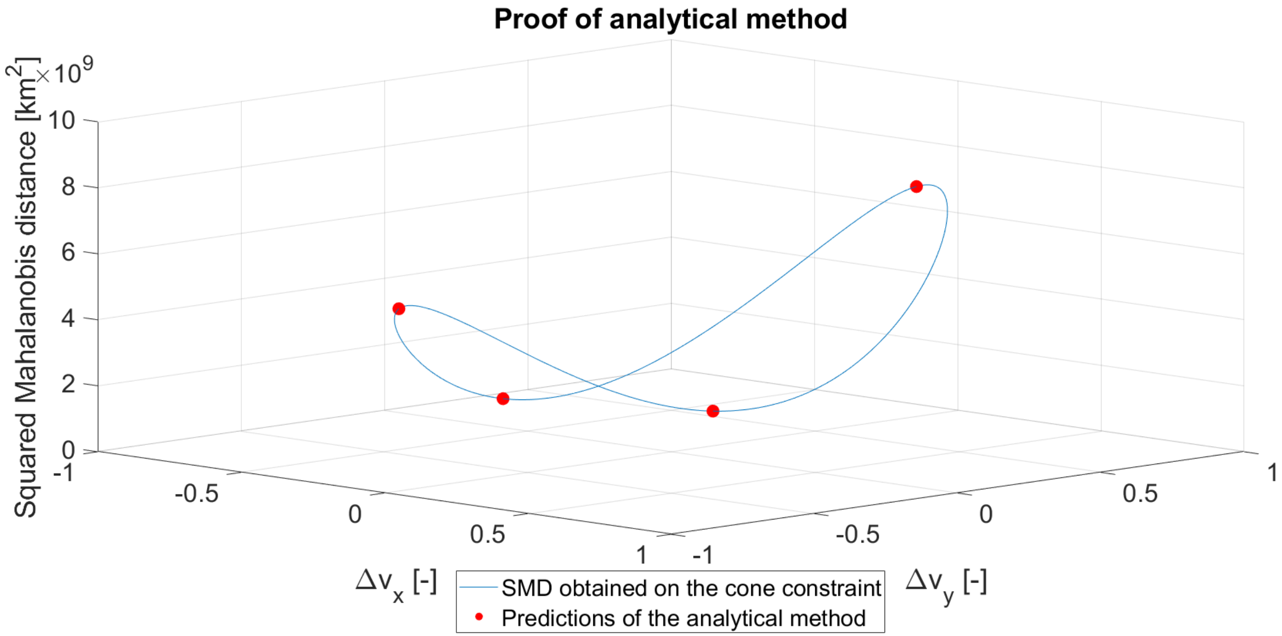
$$K = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{c} = \begin{bmatrix} 0 \\ 0 \\ \cos\alpha \end{bmatrix}$$

- The solution of the system is of the form:

$$\boldsymbol{\delta v} = \begin{bmatrix} \dfrac{A_{13}\lambda + A_{12}A_{23} - A_{22}A_{13}}{\lambda^2 - (A_{11} + A_{22})\lambda + A_{11}A_{22} - A_{12}^2} \\ \dfrac{A_{23}\lambda + A_{12}A_{13} - A_{11}A_{23}}{\lambda^2 - (A_{11} + A_{22})\lambda + A_{11}A_{22} - A_{12}^2} \\ 1 \end{bmatrix} \cos\alpha$$

- Critical points are found by imposing:

$$|\boldsymbol{\delta v}(\lambda)| = 1$$

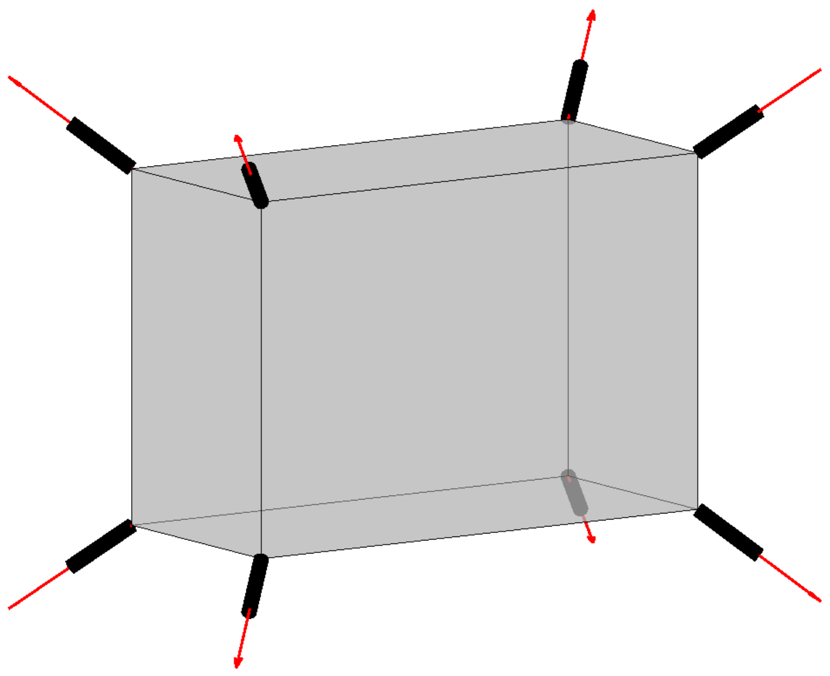


# Reaction control system (RCT)

## Thrust authority polytope

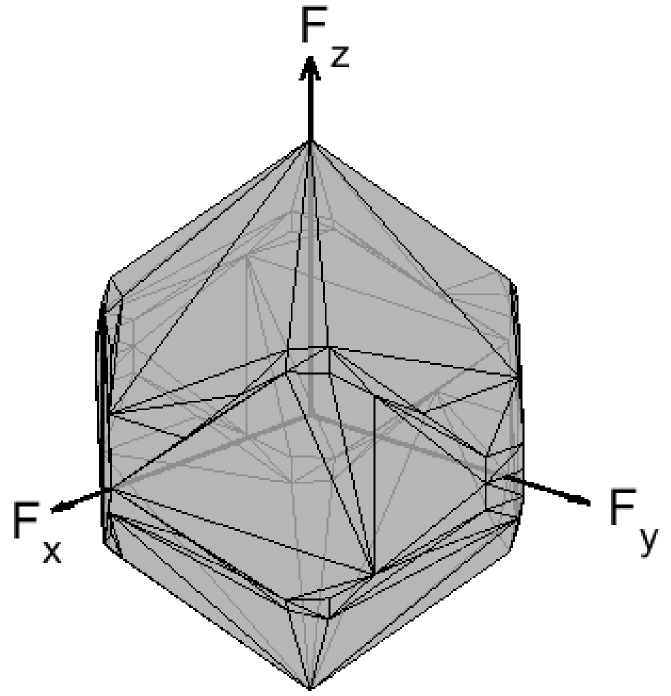
### RCT case

- All thrusting direction are available
- Zero torque is generally imposed



### Polytope approach

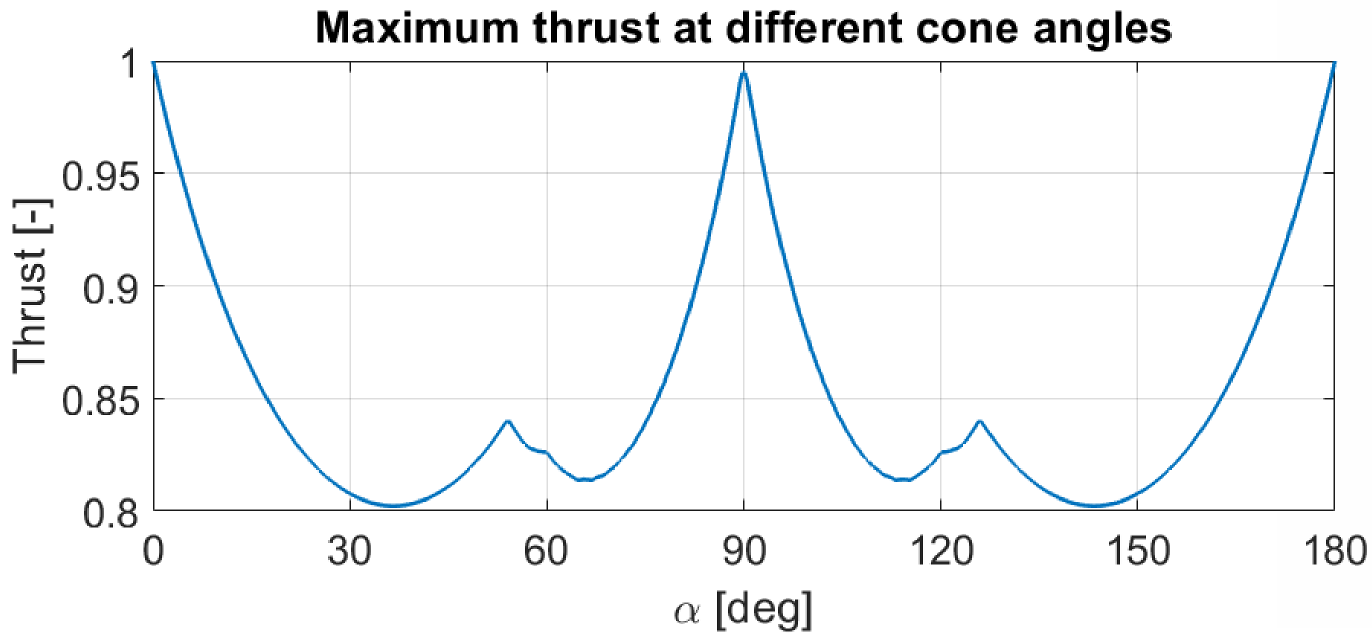
- Maximum acceleration along all directions
- STM does not work because not constant

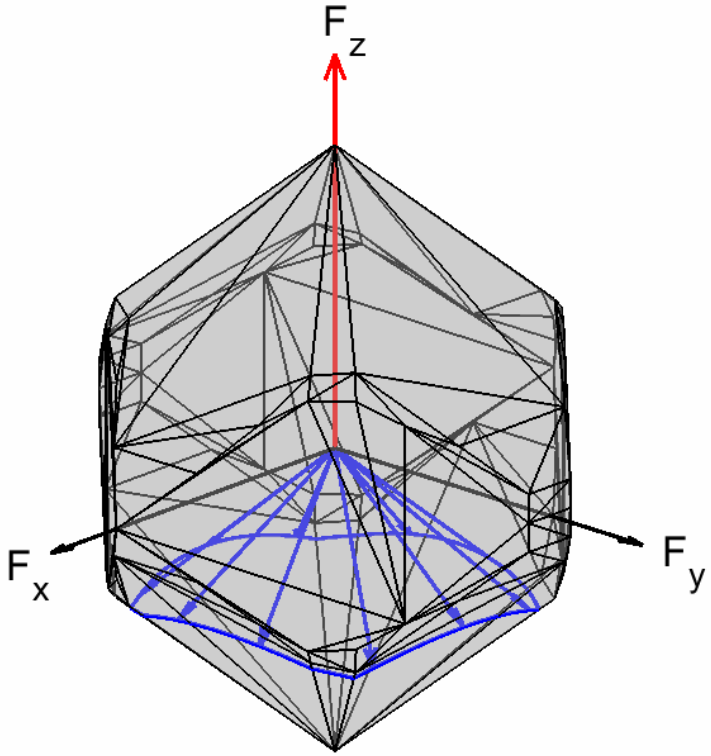


# Pointing constraints with RCT

## Methodology

- Satellite free to rotate around pointing direction
- For fixed $\alpha$ consider maximum thrust on the intersection
- Cone constraint approach applied for $\alpha \in [0,180]$ deg
- Reduce dimensionality from 3D to 2D



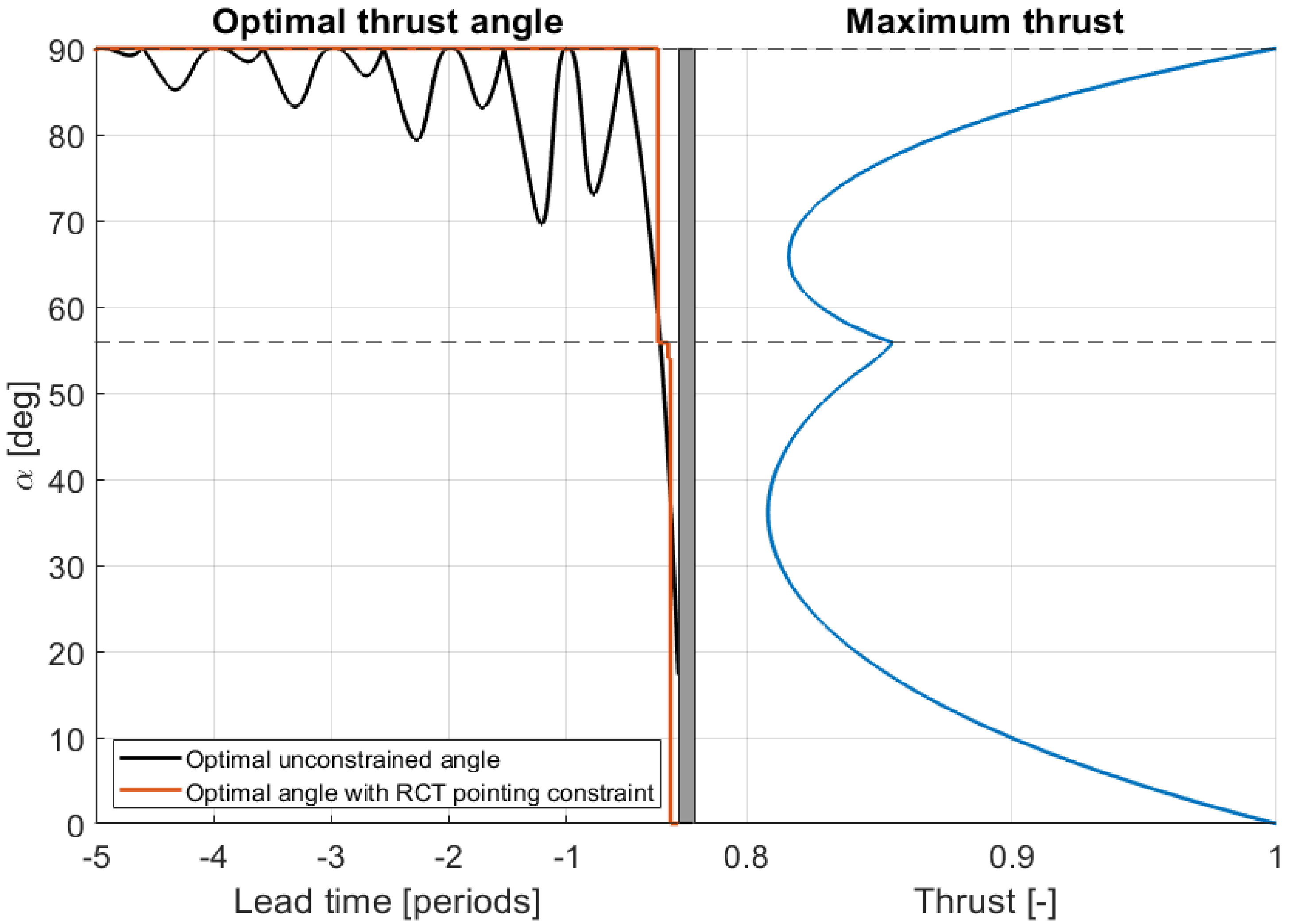

Failure constraints embedded with pre-computed polytopes

# Pointing constraints with RCT

## Sentinel testcase

- Positions at TCA:

$$r_1 = \begin{bmatrix} -432 \\ 1076 \\ -6987 \end{bmatrix} \quad r_2 = \begin{bmatrix} -454 \\ 1068 \\ -6988 \end{bmatrix}$$



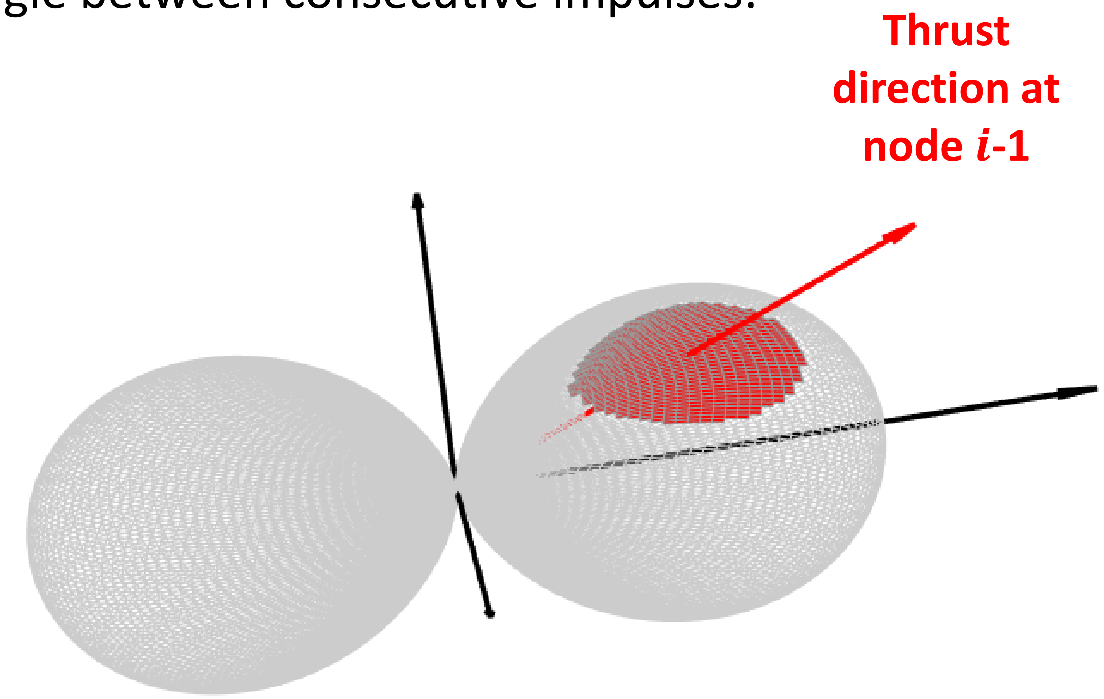
# Maximum rotational speed

## Low-thrust

- Low-thrust profile obtained with Sims-Flanagan transcription [6] with time step $\delta t$
- Maximum rotational speed $\omega_{max}$ imposed as maximum angle between consecutive impulses:

$$\theta_{max} = \frac{\omega_{max}}{\delta t}$$

- In the **red surface** $\theta_i \leq \theta_{max}$
- Constrained thrust direction as intersection between:
  - Boundary of locus of feasible directions
  - Plane defined by $\delta v_{i-1}$ and $\delta v_i$




Rayleigh space at node i

[6] Sims, J. A., & Flanagan, S. N. (1997). Preliminary design of low-thrust interplanetary missions.

# Results

## Impulsive manoeuvre

$$r_1 = \begin{bmatrix} -432 \\ 1076 \\ -6987 \end{bmatrix} \text{km} \qquad v_1 = \begin{bmatrix} 7.40 \\ -0.98 \\ -0.61 \end{bmatrix} \text{km/s} \qquad r_2 = \begin{bmatrix} -454 \\ 1068 \\ -6988 \end{bmatrix} \text{km} \qquad v_2 = \begin{bmatrix} 4.69 \\ 5.81 \\ 0.58 \end{bmatrix} \text{km/s}$$

$\Delta t = 3$ periods $\Delta$v = 2 m/s Target PoC = $10^{-8}$

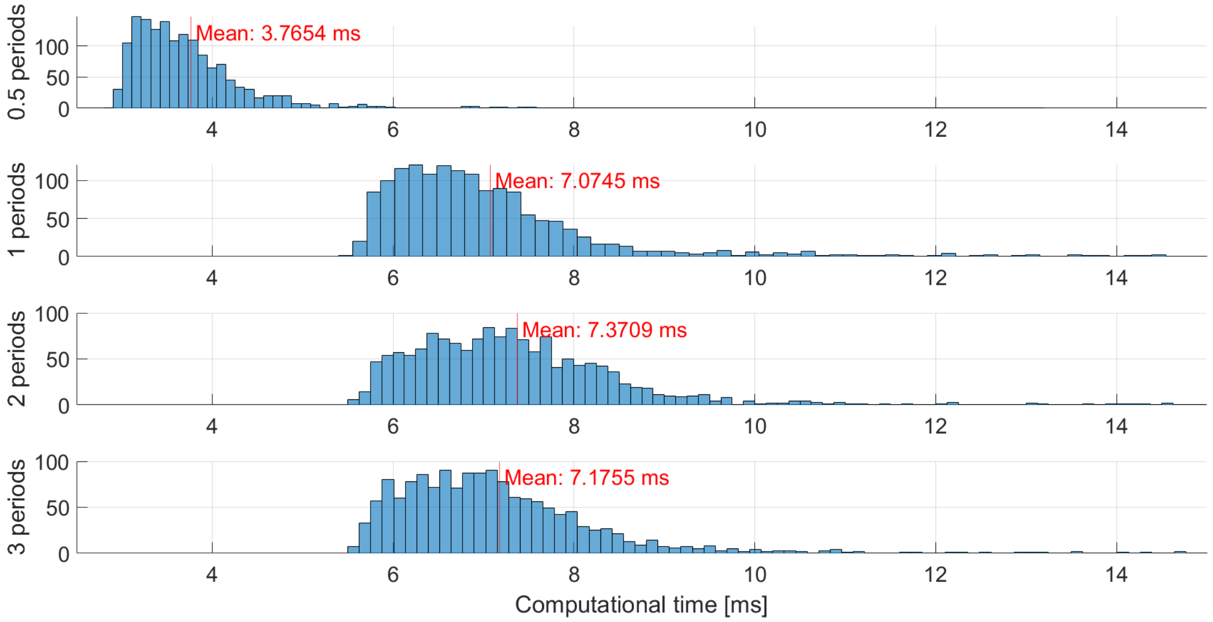
| Case | Thrust in TNH [m/s] | Time before TCA [periods] | Total $\Delta v$ [m/s] | Collision probability [-] |
|---|---|---|---|---|
| **Nominal** | $\begin{bmatrix} -0.9999 \\ -0.0053 \\ 0.0021 \end{bmatrix}$ | 2.5504 | 1.4337 | $1.1111 \times 10^{-8}$ |
| $\alpha = \frac{2}{3}\pi$ | $\begin{bmatrix} -0.8660 \\ -0.5001 \\ 0.0001 \end{bmatrix}$ | 2.5022 | 1.6341 | $1.1175 \times 10^{-8}$ |
| **RCT** | $\begin{bmatrix} -0.9999 \\ 0.0009 \\ 0.0021 \end{bmatrix}$ | 2.5504 | 1.4337 | $1.1111 \times 10^{-8}$ |

# Results

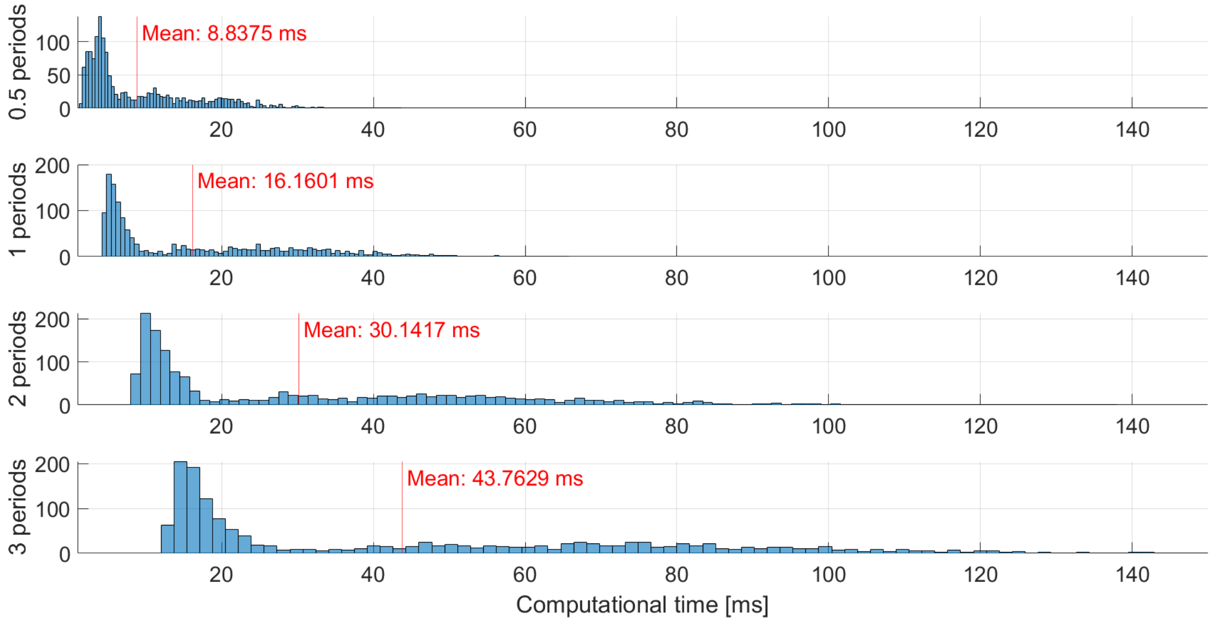
## Computational time

- Dataset of 1438 conjunctions
- Fuel optimal manoeuvre with target collision probability of $10^{-7}$

### Time optimal Impulsive




### Low-thrust



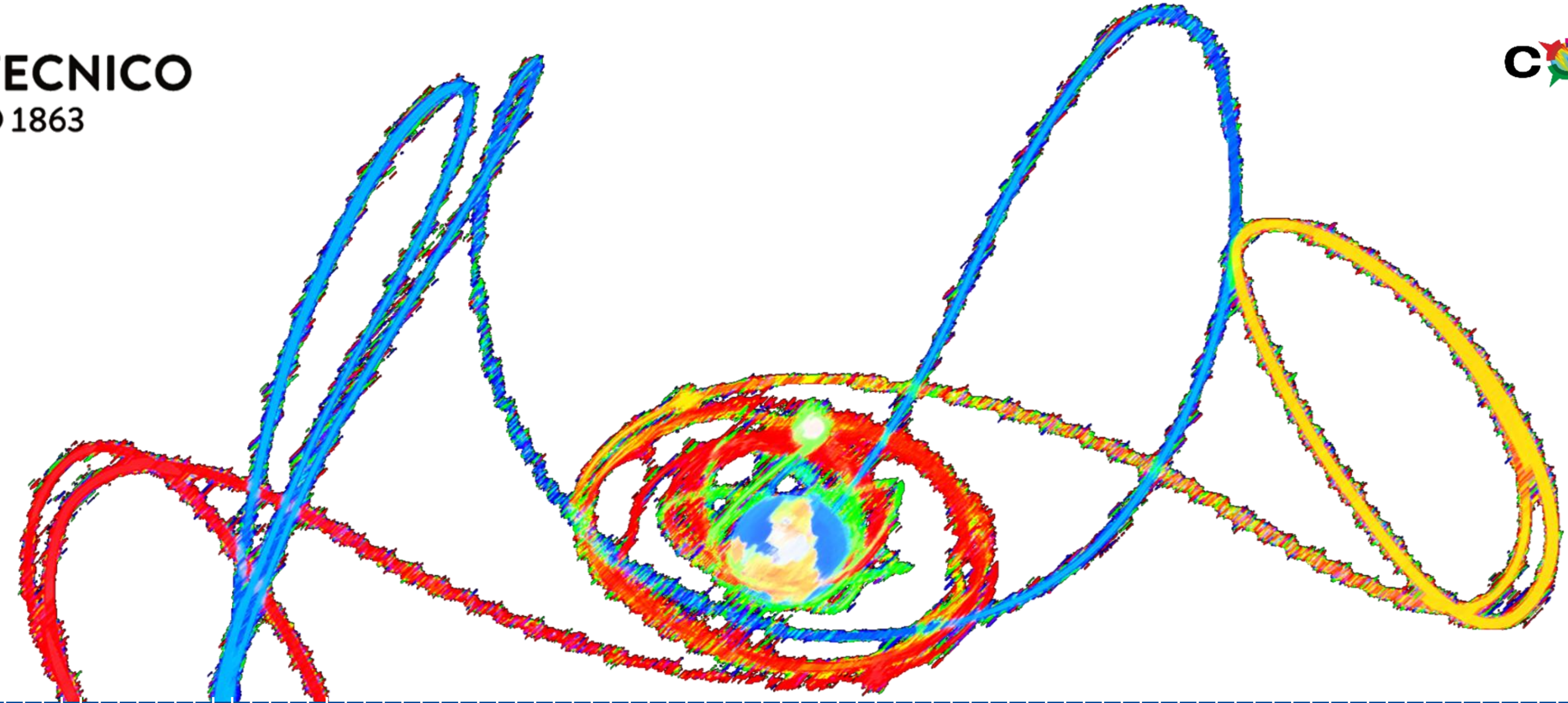
# Conclusions

## Summary

- Analytical CAM:
  - Linear relative dynamics with STM
  - Fuel-optimal with target collision probability
- Operational constraints:
  - Pointing constraints
  - Thruster failure
  - Maximum rotational speed
- Computational time:
  - Impulsive: max 7 ms
  - Low-thrust: 2 ms + 14 ms/period
- Convergence: 100%

## Future work

1. Conjunction:
   - Extend the model to non-Gaussian uncertainty constraint method for satellite equipped with RCT
2. Impulsive:
   - Develop method to improve initial guess for elliptical orbits
3. Low-thrust:
   - Improve computational time for low-thrust with cascade search
4. Sensitivity analysis:
   - Determine conditions that can lead to failure



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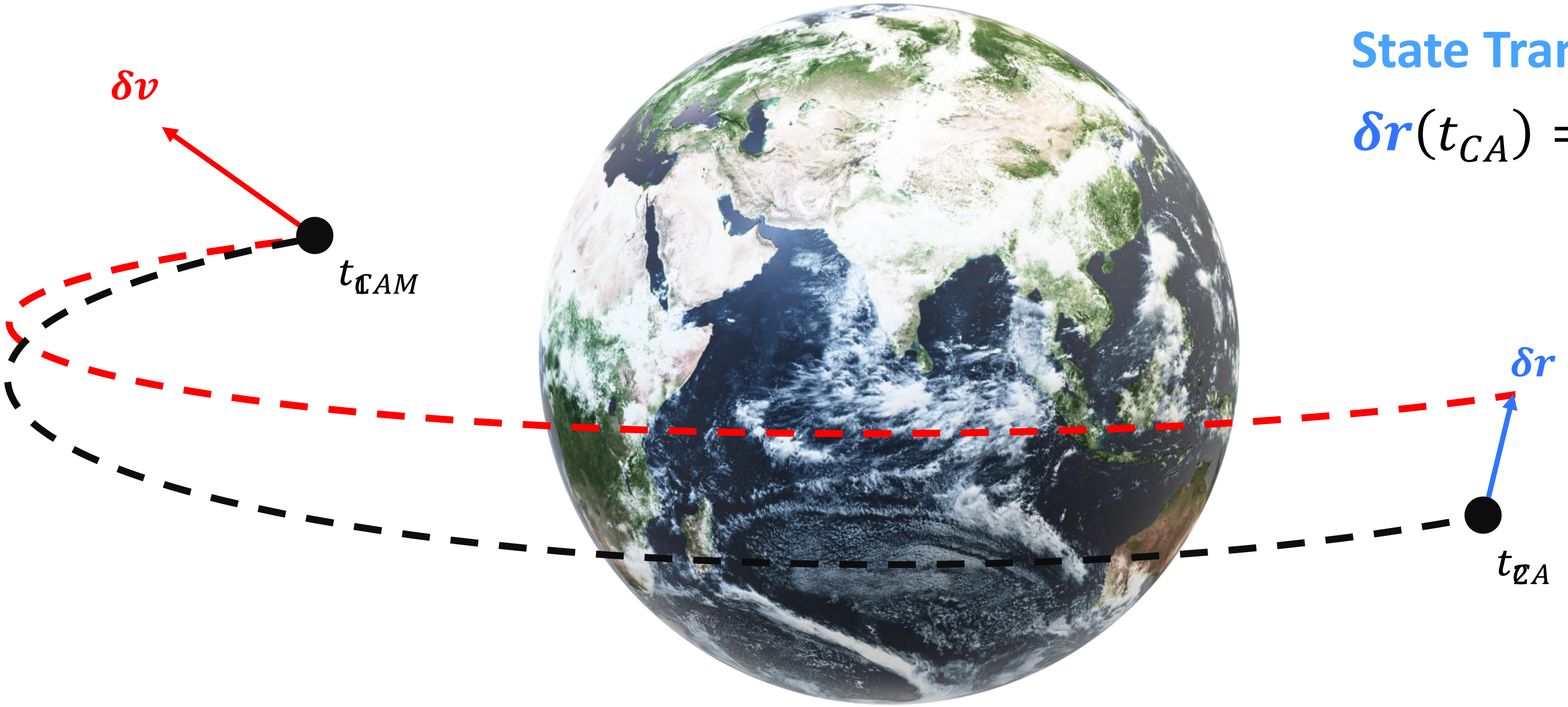
# BACKUP

# Fuel-optimal manoeuvre

## State Transition Matrix




**State Transition Matrix (STM)** [3]

$$\boldsymbol{\delta r}(t_{CA}) = \boldsymbol{T}\, \boldsymbol{\delta v}(t_{CAM})$$


[3] Gonzalo, J. L., Colombo, C., & Di Lizia, P. (2021). Analytical framework for space debris collision avoidance maneuver design. *Journal of Guidance, Control, and Dynamics*, *44*(3), 469-487.

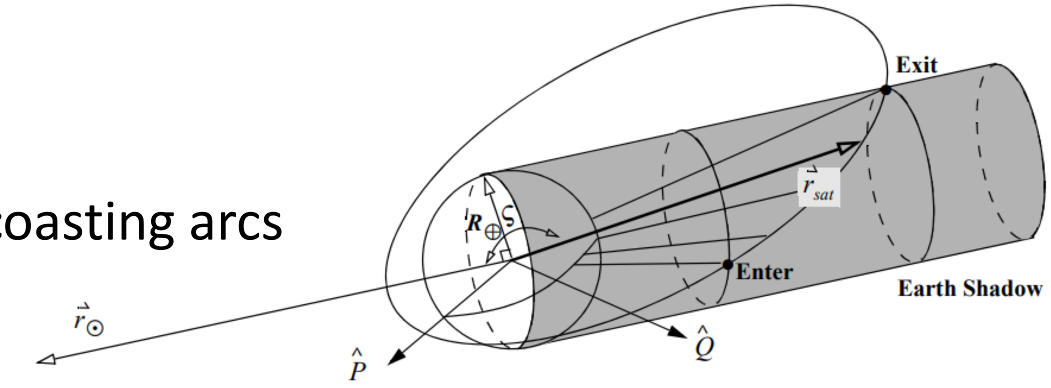
# Fuel-optimal manoeuvre

## Eclipse & transcription

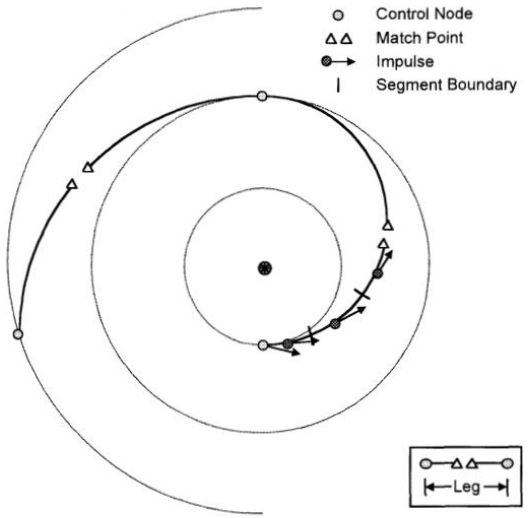
**Low thrust hypothesis:** Infinitesimal differences between nominal and perturbed trajectories

### Eclipse constraints:

- Analytical computation of shadow region boundaries [1]
- The nominal trajectory is subdivided into thrusting and coasting arcs




[1] Vallado, D. A. (2001). *Fundamentals of astrodynamics and applications*




### Sims-Flanagan transcription [2]:

- Thrusting arcs are discretised into smaller sub-arcs
- Low-thrust approximated by impulsive manoeuvre at mid-point

[2] Sims, J. A., & Flanagan, S. N. (1997). Preliminary design of low-thrust interplanetary missions.

# Fuel-optimal manoeuvre

## Algorithm

- **Maximum deviation on b-plane:**

# Fuel-optimal manoeuvre

## Algorithm

- **Maximum deviation on b-plane:**
  1. Divide the trajectory between thrusting and coasting arcs

# Fuel-optimal manoeuvre

## Algorithm

- **Maximum deviation on b-plane:**
  1. Divide the trajectory between thrusting and coasting arcs
  2. Divide the thrusting arcs into sub-arcs

# Fuel-optimal manoeuvre

## Algorithm

- **Maximum deviation on b-plane:**
  1. Divide the trajectory between thrusting and coasting arcs
  2. Divide the thrusting arcs into sub-arcs
  3. Compute the STM $\boldsymbol{T}$ for each sub-arc, assuming the mid-point as manoeuvre point

# Fuel-optimal manoeuvre

## Algorithm

- **Maximum deviation on b-plane:**
  1. Divide the trajectory between thrusting and coasting arcs
  2. Divide the thrusting arcs into sub-arcs
  3. Compute the STM $\boldsymbol{T}$ for each sub-arc, assuming the mid-point as manoeuvre point
  4. Solve the eigenproblem to find the optimal thrust directions $\boldsymbol{\delta v_i}$ for every sub-arc

# Fuel-optimal manoeuvre

## Algorithm

- **Maximum deviation on b-plane:**
  1. Divide the trajectory between thrusting and eclipse arcs
  2. Divide the thrusting arcs into sub-arcs
  3. Compute the STM $\boldsymbol{T}$ for each sub-arc, assuming the mid-point as manoeuvre point
  4. Solve the eigenproblem to find the optimal thrust directions $\boldsymbol{\delta v_i}$ for every sub-arc
  5. Compute the relative change at $t_{CA}$ as the sum of every sub-arc contribution $\boldsymbol{\delta b_i = B\,T_i \cdot \delta v_i}$

# Fuel-optimal manoeuvre

Data

- **Object #1:**
- $r_1 = [-5113.92199644821 \quad 27.0979570988554 \quad 5545.04274223079\,]$
- $v_1 = [-5.34166163081364 \quad -0.359926334888767 \quad -4.92355340569194\,]$

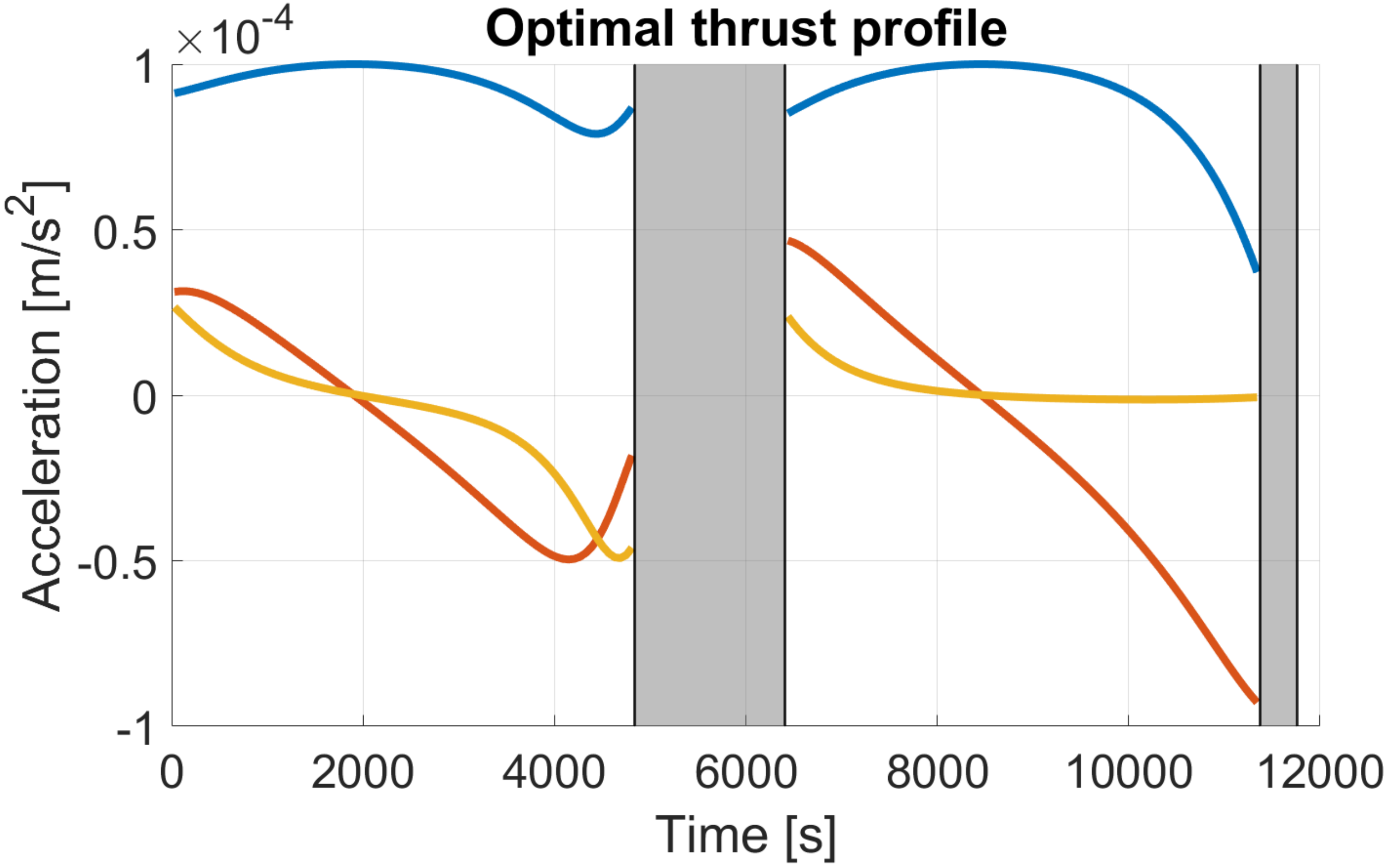
**Manoeuvrable object**
**Max acceleration: $10^{-1}$ mm/s**

- **Object #2:**
- $r_2 = [-5113.92258619383 \quad 27.0498590331835 \quad 5545.04444784058\,]$
- $v_2 = [5.39817661547525 \quad -0.143571778589103 \quad 4.88941890985276\,]$

**Nominal miss distance at $t_{CA}$:**
**$48.1$ m**

# Fuel-optimal manoeuvre

## Maximum miss distance on the b-plane



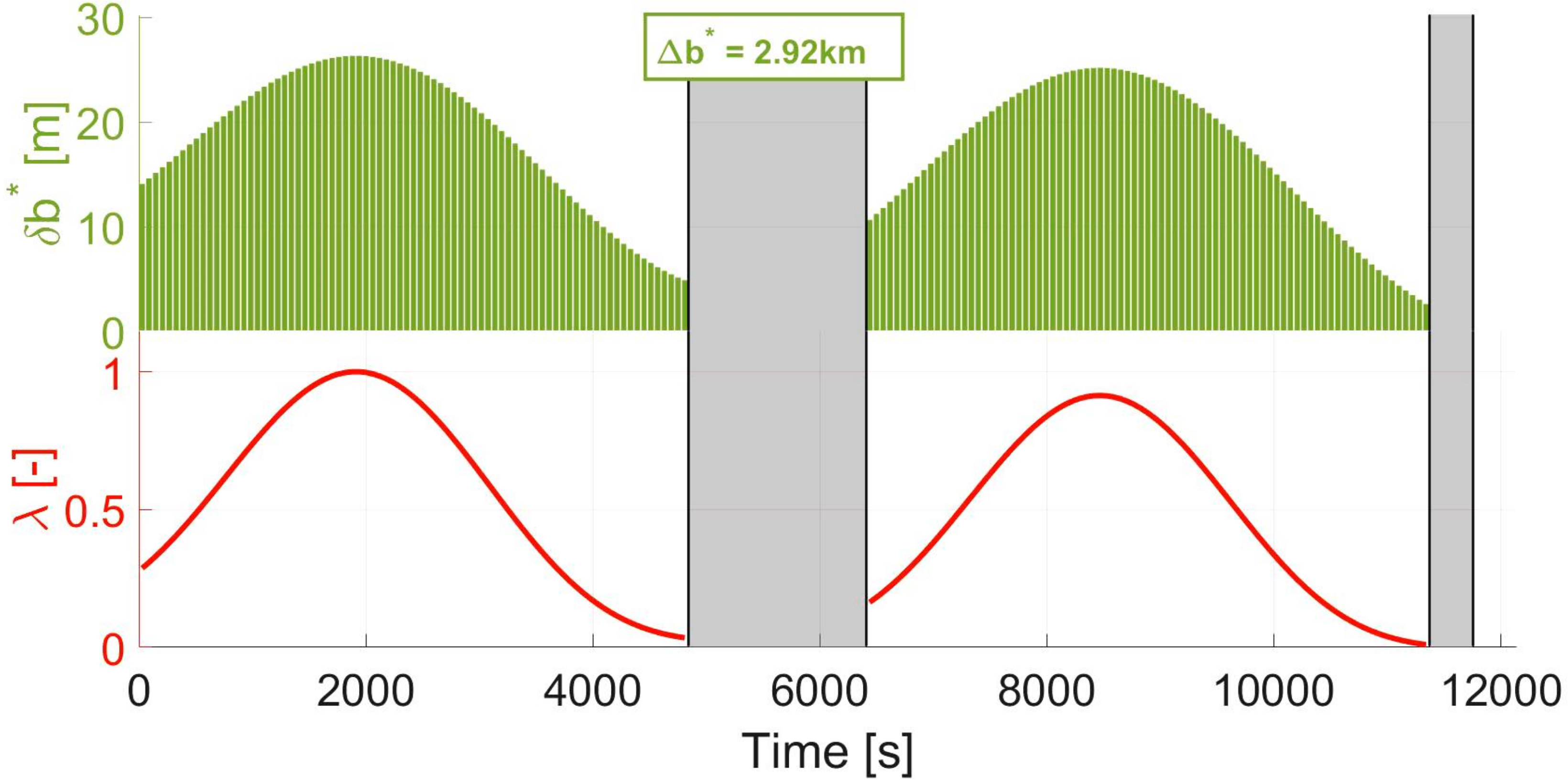

Optimal low-thrust profile in TNH frame for maximum deviation on the b-plane

**Numerical propagation**
$\Delta b^* = 2.89$ km

# Fuel-optimal manoeuvre

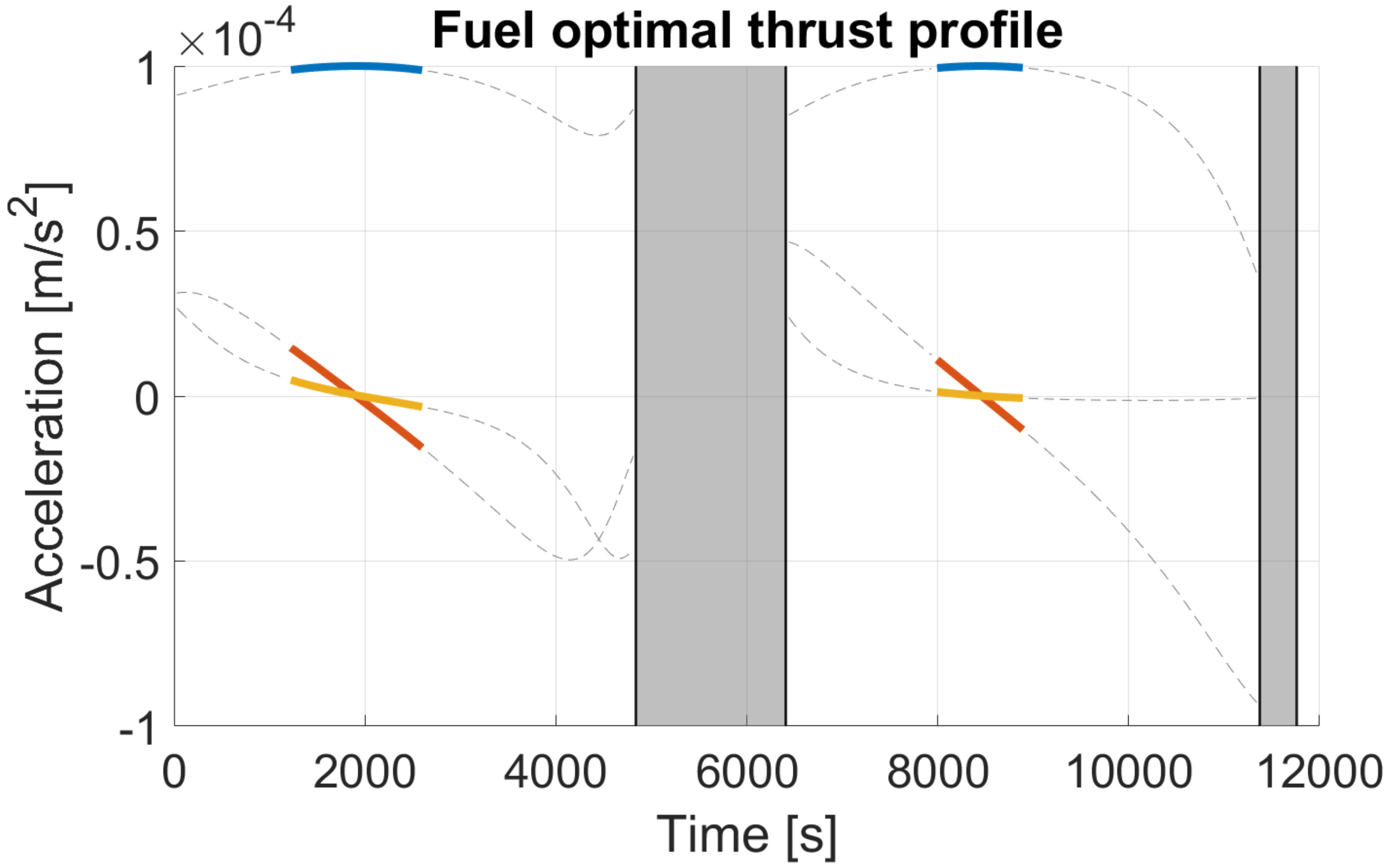
## Fuel optimality

- **Minimum fuel** consumption to achieved $\mathbf{\Delta b^* = 1\ km}$
- Pruning via discrete **Newton method** on the **eigenvalues** $\lambda$



# Fuel-optimal manoeuvre

## Fuel optimal miss distance on the b-plane

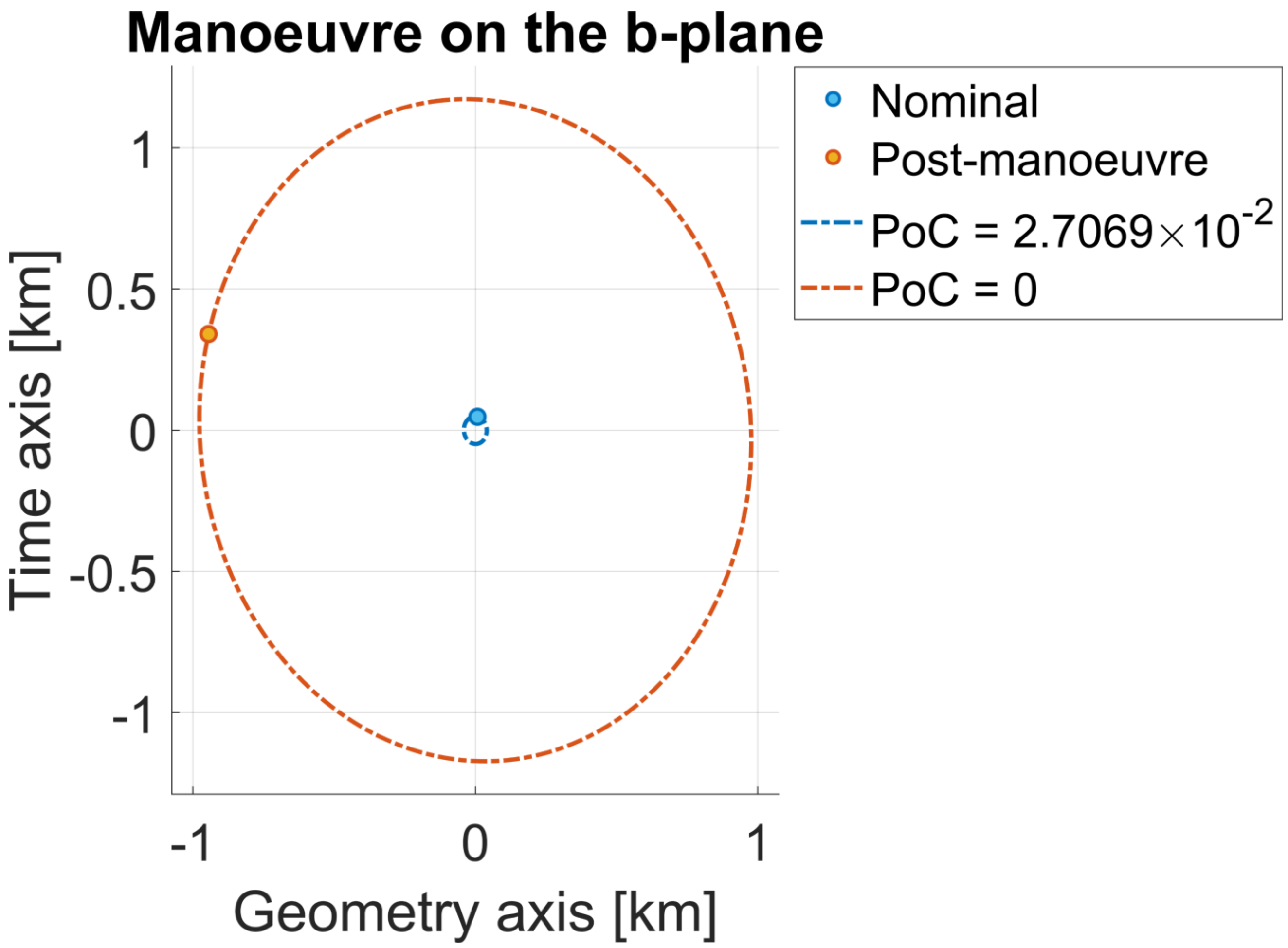



Fuel-optimal low-thrust profile in TNH frame to obtain a deviation on the b-plane of 1 km

**Numerical propagation**
$\Delta b^* = 1.01$ km

# Fuel-optimal manoeuvre

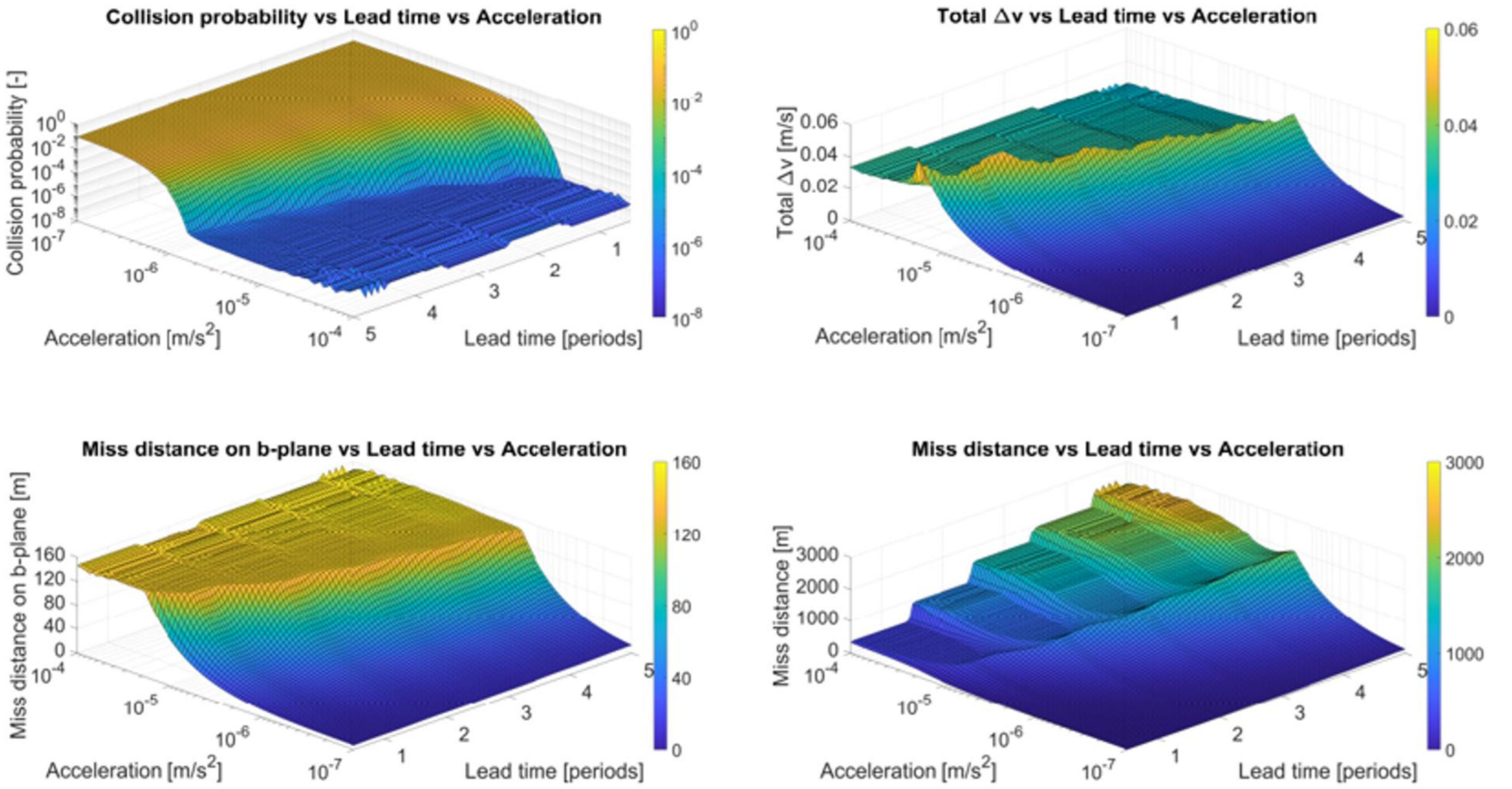
## Maneuver represented on the b-plane




Maneuver effects on the b-plane

# Fuel-optimal manoeuvre

## Sensitivity analysis




Sensitivity analysis on manoeuvre lead time and spacecraft acceleration to achieve a collision probability of $10^{-6}$

# Pointing constraints with RCT

## Non-perpendicular pointing constraint