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Reply to “A rediscovery of stiff pentamodes. A comment on high bulk modulus pentamodes: The three-dimensional metal water"

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ABSTRACT

This letter responds to Milton's commentary [1], which questions the originality of the microstructure proposed by Brambilla *et al.* [2]. We trace the origins of this class of designs to Sigmund's 2000 work in 2D [3] and to the 3D extension introduced by Milton *et al.* in 2017 [4]. Finally, we highlight that our study makes several important contributions. One of these is the design of a material whose properties closely resemble those of water, making it ideal for acoustic applications.

The class of elastic materials with a bulk modulus much higher than the shear modulus is commonly known as Pentamode Materials (PMs), due to their five low-energy modes of deformation. PMs can be exploited to mimic the soft behaviour of liquids and gels, when they have high ratio between bulk and shear moduli. Their implementation through architected microstructures has proven useful to control mechanical waves (see, e.g., the references cited in our original letter and in the commentary by Milton [1]). The first geometries achieving these properties were independently proposed by Milton and Cherkaev [5] and by Sigmund [6] in 1995. They are made of links (typically having the shape of double cones) that approximate hinges in their thin extremities, making the shear modulus negligible. Most of the works in the literature build upon this concept, and address the design of two-dimensional bimode materials [7] and three-dimensional PMs [8] using hinge-like connections. To increase the bulk-to-shear ratio arbitrarily, it is crucial to replace this mechanism by sheaves of thin, parallel elements that act as a slider. For example, it is needed to reproduce the high bulk modulus of water in 3D, as we showed in 2025 [2]. This class of materials is referred to as *high-bulk modulus* or *stiff* PMs.

In our work, we claimed to be the first to propose such a replacement, but in his recent commentary, Milton states that he and his co-authors did so in 2017 [4]. Further insight is therefore needed regarding the specific contributions that led to the current state of the art. The scope of this letter is twofold. Firstly, we aim to clarify the key contributions of the existing literature on stiff pentamodes. Secondly, we acknowledge that some of these important works were omitted from our previous letter, we identify our wrong claims, and offer our apologies to the authors.

Two key works defined the class of stiff PMs prior to our own contribution. The first discovery was made by Sigmund in 2000 [3] through the use of topology optimisation. Among the various microstructures he proposed, the one shown in Fig. 1a is strikingly similar to the structure we put forward in 2025 (reported in Fig. 1b), with the main difference being that Sigmund's design was in 2D – hence, technically a bimode. This earlier contribution, although not always explicitly acknowledged in later discussions, anticipated several of the ideas that were subsequently brought to the fore.

The extension to 3D stiff PMs was later suggested by Milton *et al.* in 2017 [4]: the double cones links composing the classical 3D lattice were hypothetically replaced by a sheaf of fibres, in order to study the G-closure problem from a theoretical perspective and characterise the range of effective elasticity tensors achievable from given constituent

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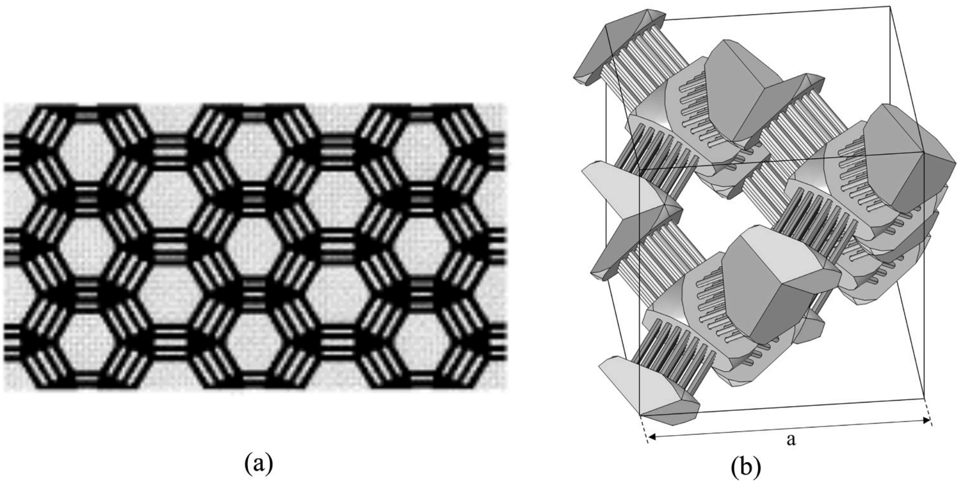

(a) (b)

Fig. 1. (a) two-dimensional bimode proposed by Sigmund 2000 [3], adapted from Figure 7 and reproduced with permission. (b) three-dimensional pentamode proposed by Brambilla *et al.* [2].

phases.

The use of thin, parallel elastic elements has long been employed in mechanics to create compliant devices for many applications. One of the oldest examples of this is the invention of ropes, which inspired our work. For this reason, we consider only these two works to be pertinent to this discussion.

While the works by Sigmund, Milton and co-authors are closely related to ours, they differ in scope and intent. The former aimed to characterise the range of effective static elasticity tensors that can be achieved from given constituent phases, with PMs analysed as a subcase. The latter details and analyses the unit cell geometry of a specific three-dimensional PM material. The difference between determining a material's theoretical limit and designing a practical, functional microstructure is clear.

For completeness, the main contributions of Brambilla *et al.* to the state of the art are summarized below:

- We presented the first complete design of a PM that closely approximates the acoustic properties of water. A parametric geometry is accurately described, optimised and analysed. We also showed how these parameters influence the emergence of a shear bandgap.
- The use of sheaves of thin fibres was only introduced after clearly demonstrating the infeasibility of the commonly adopted double-cone links in achieving PMs having bulk modulus with absolute value close to water. Moreover, the mechanics of sheaves was analysed in detail, revealing their ability to transmit bending moments between nodes–besides axial force and negligible shear.
- This design requires the mass to be concentrated at the nodes. This has a detrimental consequence visible only in the dynamic regime, that we first highlighted and discussed through the dispersion diagram of the lattice. These aspect is fundamental for acoustic applications, where the analysis of the pure static behaviour is not sufficient.

As a result, the consistency of our original letter can be appropriately restored by adding references to Sigmund [3,6] and Milton [4], along with minor revisions limited to the statements concerning the novelty of using 3D sheaves of beams.

Declaration of Competing Interest

None.

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