Fast SHEPWM Solution Method for Wind Power Converter Based on State Equations

Ning Li, Shiqian Zhang, Xiaokang Liu, Yan Zhang, and Lin Jiang

***Abstract*—Selective harmonic elimination pulse width modulation (SHEPWM) is a modulation strategy widely used for three-level wind power grid-connected converters. Its purpose is to eliminate specified sub-low frequency harmonics by controlling switching angle. Furthermore, it can reduce fluctuation of the microgrid system and improve system stability. Intelligent algorithms have been applied to the SHEPWM solution process to mitigate calculation complexity associated with the algebraic method, as well as the need to set the initial value. However, disorder of the optimization result causes difficulty in satisfying incremental constraint of the three-level NPC switching angles, and affects the success rate of the algorithm. To overcome this limitation, this paper proposes a fast SHEPWM strategy to optimize the result obtained by the intelligent algorithm. The SHEPWM can be realized by solving switching angles through a state equations-based mathematical model, which is constructed by using the initial variables randomly generated by the intelligent algorithm as the disturbance. This mathematical model improves the success rate of calculation by simplifying constraint representation of switching angles and solving the disorder problem of the optimization result. At the same time, a method based on the circle equation and the trigonometric function is applied to the initial variable assignment of the state equation, which further improves the speed and accuracy of the solution, realizes a more thorough filtering effect, and further reduces the impact of sub-low frequency harmonics on a wind power integrated system. Finally, simulation and experiment results have been used to prove the effectiveness of the proposed SHEPWM strategy when combined with intelligent algorithms.**

***Index Terms*—Wind power converter, adaptive genetic algorithm, selective harmonic elimination pulse-width modulation (SHEPWM), state equation, success rate.**

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I. INTRODUCTION

IN the field of wind power generation, sub-low frequency harmonics are very harmful to stability of the microgrid system [1]. In the application of high-voltage and high-power wind power grid-connected converters, compared to a two-level inverter, the multi-level inverter has lower voltage stress, AC output voltage $\mathrm{d}v/\mathrm{d}t$, as well as total harmonic distortion (THD) of the output signal [2]–[4], which will directly reduce fluctuation of the microgrid system.

The pulse width modulation (PWM) strategy, which is critical for the three-level neutral-point-clamped (NPC) inverter (see Fig. 1), is addressed in this paper. Among available PWM strategies, sinusoidal PWM (SPWM) and space vector PWM (SVPWM) are commonly used at high switching frequencies. However, as switching frequency decreases, SPWM and SVPWM become inefficient at suppressing harmonics. This drawback is particularly notable in high-voltage and high-power applications, where a lower switching frequency is the key to equipment loss reduction. When compared to traditional modulation methods, the selective harmonic elimination PWM (SHEPWM) can achieve harmonic suppression by calculating the precise switching angles, yet maintaining a low switching frequency and thus low system loss. Owing to these advantages, the SHEPWM strategy has become a research hotspot for three-level NPC inverters since proposed.

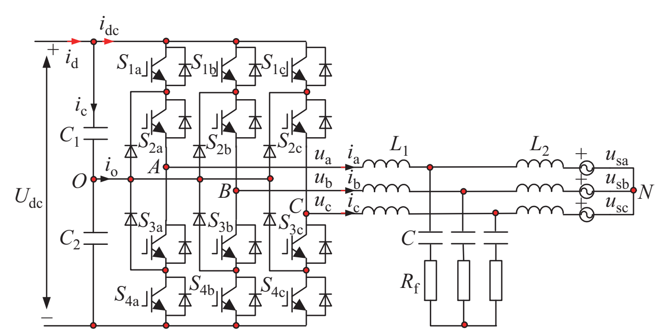

Fig. 1. Simplified circuit topology of the three-level NPC system.

The core problem of the SHEPWM method is the solution of the nonlinear transcendental equations to obtain switching angles, and existing methods are divided into numerical methods and intelligent algorithms. Numerical methods, such as gradient optimization and Newton iterative method, need to set initial values during the solution process. These initial values are of paramount importance since they determine the success rate, convergence speed, and calculation accuracy of the algorithm. However, choosing proper initial values is complicated in practice. The method resorting to direct use of the empirical formula is limited by topology condition and number of switching angles, and use of the cg-superposition SPWM method [5] and the SPWM method further increases solution complexity [6], [7]. Reference [8] uses the Chudnovsky algorithm to transform the equations to be solved into an algebraic equation set. Although the number of unknown variables is reduced, the order of unknown variables and complexity of the formula are increased. In order to reduce complexity of the mathematical calculation process in the numerical algorithm, the Walsh function transformation is used in [9] to simplify the transcendental equations into a piecewise linear system, but it still does not eliminate dependency of the accuracy on given initial values. The technique in [10] improves result accuracy and shortens calculation time by combining the Newton-Raphson iterative operation with the least square linearization, but it requires a more stringent initial value range. Based on the principle of polynomial interpolation, an algorithm combining evaluation of numerical interpolation points and linear equations is proposed in [11] to improve calculation speed of numerical solutions. However, this algebraic framework uses the extended n-dimensional Björck-Pereyra algorithm, which requires a large computational amount and complex implementation process, thus increasing cost of hardware equipment.

As a major advantage, solving nonlinear equations by means of intelligent algorithms eliminates dependency on initial values [12]. In practice, algorithms proposed earlier, such as genetic algorithm (GA), particle swarm optimization (PSO), and simulated annealing (SA), are more mature and have been widely used. However, these traditional intelligent algorithms are prone to local optima and excessive iteration times, which cause a heavy burden of intermediate calculations. To avoid this situation, several improvements have been proposed in literature to traditional intelligent algorithms. Reference [13] adopts the colonial competition algorithm (CCA) and proves its superiority by comparison versus GA and PSO algorithms. However, the number of verified switching angles is insufficient to demonstrate its ability to solve more complex equations. Reference [14] uses the chaotic ant colony algorithm (ACA) to obtain more accurate calculation results, but high calculation complexity limits its practical applications. Indeed, it is difficult for these adjusted algorithms to achieve the online solution requirement imposed by closed-loop control.

In order to solve the aforementioned problems of traditional intelligent algorithms, more advanced intelligent algorithms have been introduced, or multiple algorithms are combined [15], [16]. Despite advancement of the searching method, intelligence algorithms alone cannot overcome the limitation of the switching angle constraint in nonlinear transcendental equations, since they can hardly achieve sorting of the optimized result. In a three-level topology, phase voltage expression is determined by the number of levels and manifests as a set of transcendental equations with inconsistent positive and negative unknowns. Since switching angles obtained under the three-level framework should satisfy the incremental constraint, direct use of intelligent algorithms to solve the optimization problem reduces calculation efficiency. Conversely, a circuit topology with more levels enables listing the phase voltage expressions with the same sign, which is favored by many scholars, [17]–[20].

Focusing on the constraint of switching angles in the equation system, a fast SHEPWM strategy, with the ability to quickly solve switching angles, is proposed in this paper to optimize the intelligent algorithm result based on the use of state equations. The major contributions are:

- Traditional SHEPWM mathematical model ignores the incremental relationship among the switching angles, and forms multiple sets of angle constraints that limit the success rate of the algorithm solution. Conversely, by resorting to state equations, this work connects initial variables randomly generated by an intelligent algorithm with the switching angles, solves the disorder issue of the optimization result, and simplifies the angle constraint to improve the success rate of the algorithm calculation.
- On the basis of the proposed framework, the switching angle has a decreasing trend w.r.t. the modulation index. Accordingly, this work improves the assignment method of the initial switching angle, which is an important variable in the proposed state equations model, by using circular equation- and trigonometric function-based expressions. This reduces influence of the initial variable on the calculation result, and enhances accuracy and speed of the algorithm.

The rest of this paper is organized as follows. Section II introduces the conventional SHEPWM mathematical model as well as its solution techniques. Section III proposes a fast SHEPWM strategy based on state equations, to optimize the result of the intelligent algorithm. This strategy is further improved by adopting two different assignment methods for the initial switching angle. Section IV presents simulation and experimental results when the proposed method is combined with the adaptive genetic algorithm (AGA) to solve the SHEPWM equations. Finally, Section V draws conclusions.

II. Traditional SHEPWM Model and Its Solutions

The traditional SHEPWM mathematical model carries out Fourier decomposition of the phase voltage, constructs a transcendental equation set of the switching angles, and solves these angles by the use of a suitable method.

A. Traditional SHEPWM Mathematical Model

The SHEPWM method for three-level inverters is designed, e.g., in [21]–[23], to eliminate unnecessary harmonics and control fundamental component amplitude by computing quarter symmetric pulse patterns. To this end, the Fourier expansion of the phase voltage is expressed as

$$U_x=\sum_{n=1}^{\infty}\left[a_n\cos(n\beta_n)+b_n\sin\beta_n\right] \tag{1}$$

where U_x represents the voltage of phase x, $x=a,b,c$. a_n and b_n are the Fourier coefficients, and β_n is the switching angle.

Due to the quarter wave symmetry of output voltage, even harmonics are absent. The obtained harmonic of the ith order is expressed by the first quadrant switching angles as

$$\begin{cases} a_n = 0 \\ b_n = \frac{4U_{\rm dc}}{n\pi}\left[\left(\sum_{i=1}^{\rm N} i_k \cdot \cos(n\beta_i)\right)\right],\ n = 1,5,7,11,\ldots \end{cases} \quad (2)$$

where $U_{\rm dc}$ is the DC side voltage of the capacitor, N is the number of switching angles, and

$$i_k = \begin{cases} 1 & \forall \text{ rising edge} \\ -1 & \forall \text{ falling edge} \end{cases}$$

When solving the transcendental equations, incremental constraint of switching angles should be considered. This yields

$$0 < \beta_1 < \beta_2 < \beta_3 < \cdots < \beta_{\rm N} < \frac{\pi}{2} \quad (3)$$

Line-to-line voltage of the three-phase system can inherently eliminate triplen harmonics of the frequency. To eliminate selected harmonics and obtain the fundamental wave, N optimal switching angles should be found based on non-linear equations (2)–(3). The solution of these angles requires an objective function that measures the elimination effectiveness of selected harmonics while controlling the fundamental component, yielding

$$F(\beta_1, \beta_2, \ldots \beta_{\rm N}) = |b_1 - E| + |b_5 - 0| + \cdots + |b_n - 0| \quad (4)$$

where E is the amplitude of the fundamental wave output by the inverter.

A modulation index can be obtained as the ratio between the fundamental wave amplitude and the DC voltage, i.e.,

$$M = \frac{E}{U_{\rm dc}} \quad (5)$$

B. Traditional Solution Method

When solving SHEPWM equations by means of an intelligent algorithm, a traditional GA that searches for the optimal solution by simulating a natural evolution process is widely used, [24], [25]. This algorithm has large search range coverage, and can evaluate multiple solutions in the search space, making it conducive to performing global optimization. However, due to fixed crossover and mutation probabilities, traditional GA is prone to local optimum when individual fitness values of a certain generation of the population are close. This causes a low success rate when solving the SHEPWM equations, and reduces accuracy of the convergence result.

C. Improved Solution Method

To overcome the shortcomings of traditional GA, this paper changes the crossover and mutation probabilities according to the concentration degree of the population fitness for each generation. An AGA is used to avoid slow convergence of the algorithm and the problem of local optimum. In the AGA, an index can be constructed to evaluate the convergence degree of a certain population, yielding [26]:

$$F_{\max} - \overline{F} \quad (6)$$

where, $F_{\max}$ and $\overline{F}$ represent the maximum and average fitness values of the population after each iteration, respectively.

The crossover probability P_c and mutation probability P_m are written as

$$P_{\rm c} = \begin{cases} k_1(F_{\max} - F')/(F_{\max} - \overline{F}), & F' \geq \overline{F} \\ k_3, & F' < \overline{F} \end{cases} \quad (7)$$

$$P_{\rm m} = \begin{cases} k_2(F_{\max} - F)/(F_{\max} - \overline{F}), & F' \geq \overline{F} \\ k_4, & F' < \overline{F} \end{cases} \quad (8)$$

where, k_1, k_2, k_3, and k_4 are defined transformation probabilities. F' and F are individual fitness values in the crossover stage and mutation stage, respectively.

Within the framework of AGA, if population convergence tends to be the same, the result of (6) becomes smaller, resulting in an increase in the crossover probability P_c and the mutation probability P_m. This increases diversity of population changes and facilitates exit of the local optimal region. On the other hand, in a certain generation of a population, P_c and P_m corresponding to an individual with better fitness will become smaller, so the best individual is retained. Since crossover and mutation probabilities increase with population fitness, the AGA algorithm reaches closer to the objective function value, while traditional GA and SA optimization methods are limited by fixed parameters, and their convergence speed will be lower.

In the following example, we choose $k_1 = k_2 = 1.0$, $k_3 = 0.9$, and $k_4 = 0.4$. These values are properly selected based on simulation results, so that satisfactory iterative speed and calculation accuracy are obtained for the algorithm. Fig. 2 compares the solution processes of the traditional GA and AGA, when the expected result is set to be four light blue individuals. In the evolution process of the first two generations, differences between the populations are relatively large,

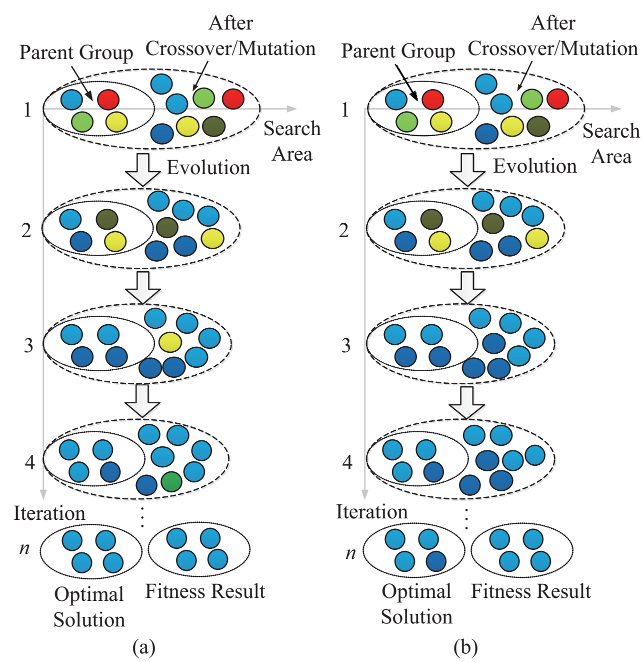

Fig. 2. Comparison of solution principles for different intelligent algorithms. (a) AGA, and (b) traditional GA.

and the iterative effects of the two intelligent algorithms are similar. Starting from the third generation, individuals of a population gradually get similar colors, indicating a smaller difference between populations. Traditional GA then encounters the problem of local optimum. Conversely, the AGA increases population diversity by increasing the probability of crossover and mutation, making it easier to find the desired individuals in a population.

The AGA can be implemented based on the flow chart shown in Fig. 3. Based on the different fitness values of each generation of the population, appropriate probabilities are independently selected for crossover and mutation operations, in order to solve the local optimum problem of traditional GA. Accordingly, when the AGA is used to solve the SHEPWM equations, the success rate of calculation is higher and the convergence result is more accurate. However, using traditional SHEPWM mathematical model, neither GA nor AGA can solve the sorting problem of the optimization result.

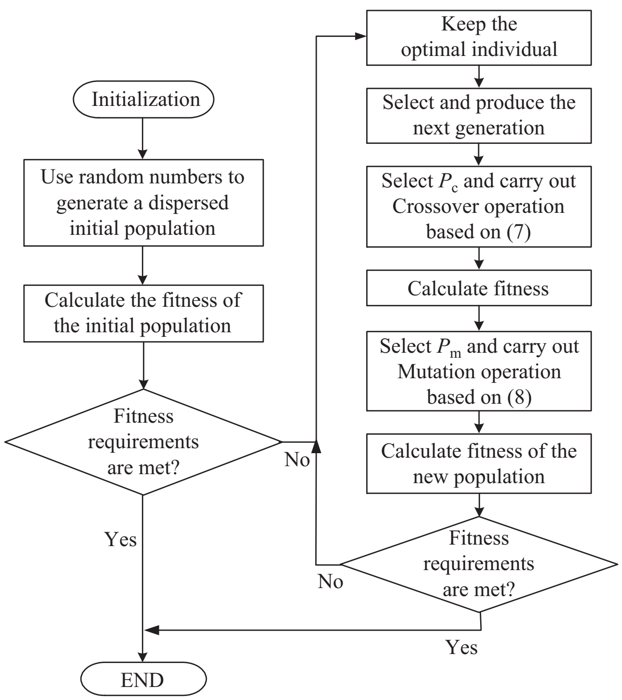

Fig. 3. Flow chart of the AGA for solving the SHEPWM problem.

III. State Equations-based Solution of SHEPWM Switching Angles

Solution of nonlinear transcendental equations has always been difficult during application of the SHEPWM strategy, especially for solving multiple switching angles of a three-level topology with wide modulation index range. Fig. 4 shows the phase voltage waveform of the NPC topology. Switching angles in the $1/4$ cycle occur alternately with rising and falling edges, resulting in cosine function coefficients with inconsistent positive and negative signs in (2). The result of intelligent algorithm optimization should strictly satisfy the multi-angle constraint defined by (3); however, if the intelligent algorithm based on traditional SHEPWM model is directly applied to the solution, the success rate can be low due to disorder of the calculation result.

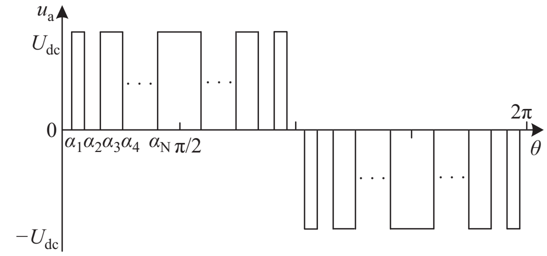

Fig. 4. Phase voltage waveform of the three-level NPC topology.

Focusing on this issue, a mathematical model based on state equations is developed in this paper to solve the SHEPWM with an improved success rate. Then, algorithm accuracy is enhanced by using improved assignment methods for the initial switching angle.

A. State Equations-based Solution of SHEPWM Model

In this paper, the concept of state equations in control science is adopted when solving the mathematical model of SHEPWM. During this process, the calculated switching angles are taken as state variables, and the initial variables randomly generated by the intelligent algorithm are taken as disturbance added each time. Then, state equations can be established based on the increasing order of switching angles, yielding

$$\begin{cases} \beta_1 = \alpha_1 \\ \beta_{i+1} = \beta_i + \left(1 - \beta_i \frac{2}{\pi}\right) \alpha_i \\ \alpha_{1,i,i+1} \in \left[0, \frac{\pi}{2}\right] \\ \max \beta_{i+1} \le \frac{\pi}{2}, \ i = 1, 2, \ldots, N-1 \end{cases} \tag{9}$$

where β_1 represents the first switching angle, which is also the initial state variable. β_i and β_{i+1}, as switching angles satisfying the relationship in (3), are the current state and next state variables, respectively. α_1 and α_i represent the initial disturbance, and the currently introduced disturbance, respectively. All disturbances belong to the initial variables randomly generated by the intelligent algorithm, and are within $[0, \pi/2]$.

This set of state equations with increasing angle order solves the disorder problem of the optimization result. In addition, it simplifies the multiple constraints in (3) into a single one, making it faster to determine accuracy of the optimization result. Therefore, the intelligent algorithm result is improved both in terms of calculation speed and accuracy rate.

The proposed state equation model in (9) simplifies the original model described by nonlinear transcendental equations, and its effective solution makes use of the intelligent algorithm, followed by the conversion of intermediate variables (i.e., disturbances) into the sequential switching angles. The numerical iteration method, which relies on proper initial values that are tedious to obtain, is still not suitable for solving the proposed model.

B. Improved Assignment of Initial Switching Angle

Despite solution of the optimization result disorder, the fast SHEPWM strategy proposed here can have low accuracy if the initial switching angle β_1 is improperly assigned. This happens in the case of an excessively large initial disturbance α_1, which causes a larger value of the next state variable β_{i+1} and a resulting switching angle that approaches the constraint limit.

This limitation can be mitigated if the equivalence between β_1 and α_1 in (9) is modified. Indeed, several works in literature, e.g., [9], [25], [26], suggest the switching angle β has a non-linear trend of decreasing as the modulation index increases, as shown in Fig. 5. Accordingly, it is necessary to establish better assignment methods for β_1, to limit the value of the switching angle and improve accuracy of the algorithm. To this end, this paper proposes two alternatives to the constant assignment method in (9), as shown in the following.

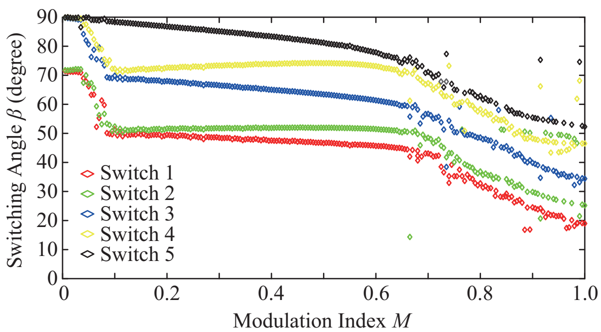

Fig. 5. Switching angle versus modulation index when $N = 5$.

1) Circle Equation Based Assignment

This method makes use of the relationship between abscissa and ordinate in the circle equation. According to (9), the disturbance α_1 and the initial switching angle β_1 both are in the range $[0, \pi/2]$. Besides, as a general trend, they should be positively correlated. Accordingly, one possibility that describes β_1 as a function of α_1 is through a proper arc, whose abscissa and ordinate correspond to α_1 and β_1, respectively. To this end, a circle with origin $(0, \pi/2)$ and radius $\pi/2$ is constructed on the (α_1, β_1) plane, and the lower arc in the first quadrant corresponding to the quarter circle is used as the assignment function. This gives

$$\beta_1 = \frac{\pi}{2} - \sqrt{\left(\frac{\pi}{2}\right)^2 - (\alpha_1)^2} \tag{10}$$

2) Trigonometric Function Based Assignment

Starting from the constant assignment function in (9), trigonometric functions and scale transformation are used to construct an assignment function description with the form of a concave curve and proper value range. This results in

$$\beta_1 = \frac{\pi}{2}[1 - \sin(\alpha_1)] \tag{11}$$

Figure 6 compares the three different assignment methods for β_1, where the proposed methods are represented by non-linear curves. With these improved methods, even if α_1 is close to the upper boundary, β_1 will be assigned a smaller value, thus avoiding the large switching angle that approaches the critical value and the resulting solution error. This conforms to the law between the modulation index and the switching angle described by [13], [14], [26], and improves algorithm accuracy.

C. SHEPWM Realization via Solution of State Equations-based Model

Application of the improved assignment methods to the fast SHEPWM strategy based on state equations is shown in Fig. 7. Algorithm flow to derive the switching angles can be

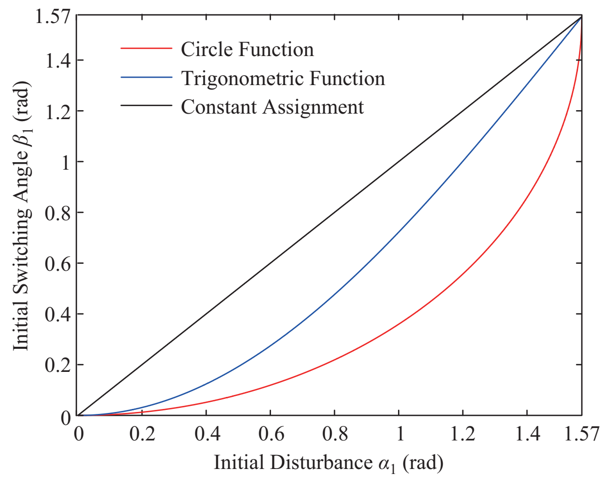

Fig. 6. Comparison of three assignment methods for β_1.

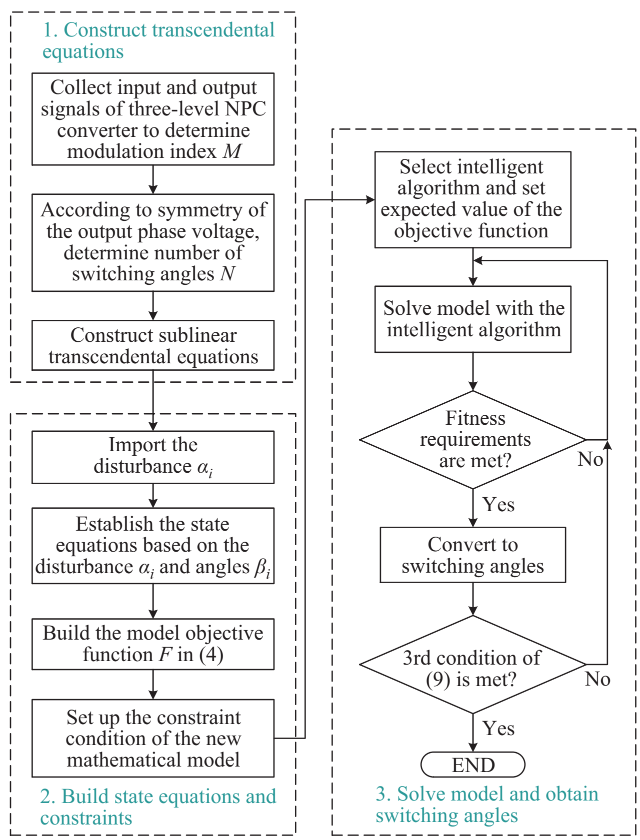

Fig. 7. Flow chart of switching angle solution.

summarized as follows.

1) According to the input and output characteristics of the three-level NPC inverter, construct nonlinear transcendental equations of switching angles to be solved.

2) Select modulation index M and the number of switching angles N according to the actual situation.

3) Introduce disturbance α and build state equations related to switching angle β, and establish a new mathematical model.

4) Use a swarm intelligent algorithm to solve the model, and compare the operation result with the expected fitness. If it meets the requirement, proceed to the next step; otherwise, revise the model and solve it again.

5) Convert the operation result into the switching angle β_i to judge whether the third constraint in (9) is satisfied. If so, output the switching angles; otherwise, return to Step 4) and solve them again.

6) End.

As a specific example, the traditional mathematical model and the proposed SHEPWM method are compared in the case of five switching angles ($N = 5$). Accordingly, the harmonics to be eliminated are the 5th, 7th, 11th, and 13th in order. This gives

$$\begin{cases} b_n = \frac{4U_{\mathrm{dc}}}{n\pi}\left[\left(\sum_{i=1}^{5}(-1)^{i+1}\cos(n\beta_i)\right)\right], n = 1,5,7,11,13 \\ 0 < \beta_1 < \beta_2 < \beta_3 < \beta_4 < \beta_5 < \frac{\pi}{2} \\ F(\beta_1, \beta_2, \ldots, \beta_5) = |b_1 - E| + |b_5| + |b_7| + |b_{11}| + |b_{13}| \end{cases} \tag{12}$$

With the proposed SHEPWM strategy and the constant assignment method, the switching angles yield

$$\begin{cases} \beta_1 = \alpha_1 \\ \beta_2 = \beta_1 + \left(1 - \beta_1 \frac{2}{\pi}\right)\alpha_1 \\ \cdots \\ \beta_5 = \beta_4 + \left(1 - \beta_4 \frac{2}{\pi}\right)\alpha_4 \\ \beta_5 \leq \frac{\pi}{2} \end{cases} \tag{13}$$

For solution of such an illustrative case, different intelligent algorithms (GA, SA, and AGA) have been used. Specifically, M is 0.6 and population number is 100. The maximum number of iterations is fixed at 5000 to ensure a maximized success rate of convergence. The experiment is repeated 200 times. Finally, the calculation is successful if the third objective function in (12) converges to 0.5. Calculations were performed in the MATLAB 2018b environment, and on a standard laptop PC with an Intel (R) Core (TM) i5-5200U CPU running at 2.2 GHz and 8 GB of RAM.

Performances of different algorithms are compared in Table I. The AGA, owing to its adaptive crossover and mutation probabilities w.r.t. population fitness and advantage in solving the local optimum problem, provides faster iteration speed and higher accuracy compared with the other two intelligent algorithms (traditional GA and SA). This can be further proved by the convergence curves of three methods (not shown here for brevity). Besides, it has been shown the fast SHEPWM strategy proposed in this paper can increase the success rate of traditional intelligent algorithms to 100%. For better comparison, Table II summarizes performances of different algorithms used in [27]. In this case, the success rate of the solution can be increased to 100% by resorting to the exchange market algorithm (EMA), which corresponds to a more complex search method. Conversely, if traditional intelligent algorithms such as GA and PSO are used in this scenario with a traditional mathematical model, the success rates are relatively low. Therefore, the proposed SHEPWM strategy serves as a better alternative, since it not only achieves the same solution effectiveness as the EMA, but also allows for the use of traditional intelligent algorithms with low complexity.

TABLE I
COMPARISON OF DIFFERENT OPTIMIZATION TECHNIQUES TO SOLVE THE SHEPWM PROBLEM WITH FIVE SWITCHING ANGLES

Method	Algorithm	Iteration Time (p.u.)/(s)	Success Ratio (%)
Traditional	GA	1 / 4274.62	55
	AGA	0.1 / 419.08	70
	SA	1.2 / 5044.05	38
Proposed	GA	1 / 4271.55	100
	AGA	0.1 / 420.22	100
	SA	1.2 / 5042.65	100

TABLE II
SUCCESS RATIOS OF DIFFERENT METHODS COMPARED IN [27]

Method	Iteration Time (p.u.)	Success Ratio (%)
GA	10	41
PSO	0.5	54
EMA	1	100

Effectiveness of different assignment methods of the initial switching angle has been compared in the following two scenarios:

- In the first case, fitness function values using the AGA and the three assignment methods are calculated, and results are shown in Fig. 8. When M changes from 0.3 to 0.9, the proposed circle equation and trigonometric function based methods provide lower fitness values, indicating better objective function effectiveness.

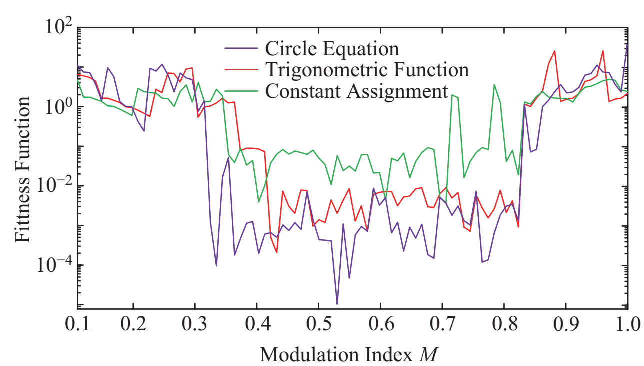

Fig. 8. Objective function curves using different β_1 assignment methods.

- In the second case, the cumulative distribution function (CDF) values of the solution success rate are compared for different assignment methods, as shown in Fig. 9. When fitness function value reaches 10^{-2}, the circle equation method leads to a cumulative success probability of 60%, the trigonometric function method leads to a

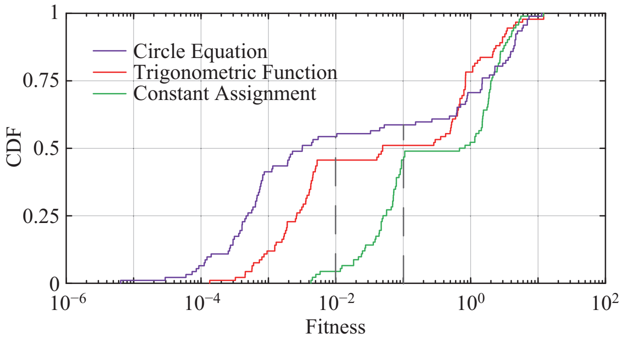

Fig. 9. CDFs of success rate for different β_1 assignment algorithms.

probability of 40%, and the constant assignment method causes a probability of less than 10%. The proposed assignment methods improve the calculation accuracy of the algorithm compared to the constant assignment method.

Finally, Fig. 10 compares the overall level of harmonic content with the circle equation assignment method and a modulation index range of 0.3–0.9. It can be seen the THD levels in all cases have been effectively reduced to within 0.4% to achieve the aim of harmonic elimination.

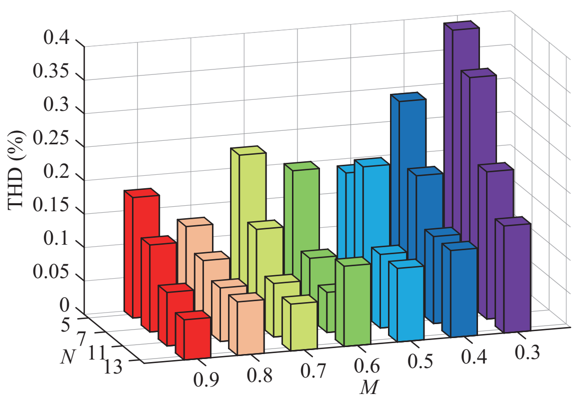

Fig. 10. THD levels with the circle equation assignment method for β_1.

IV. Simulation and Experimental Verification

A. Simulation Results and Analysis

This section presents selected simulation results to compare the SHEPWM methods. Simulation parameters of the three-level NPC topology are summarized in Table III.

TABLE III
Simulation Parameters

Type	Value	Comments
R	20 Ω	per-phase
L	5 mH	per-phase
U_{dc}	660 V	DC-link voltage
C_1, C_2	550 µF	DC-link capacitance
f	50 Hz	fundamental frequency

In the first simulation, N is set to 5, population size is set to 60, and number of iterations is set to 500. Termination and judgment conditions are (12)–(13). Under the condition that M is 0.6 and 0.8, respectively, more than 10 repeated experiments have been performed to determine optimal switching angles as shown in Tables IV and V. Fig. 11 compares the harmonic components of phase voltage with the proposed SHEPWM model and different solution methods. The AGA plus the circle equation assignment method achieves the best elimination effect of harmonics, limiting the levels of 5th, 7th, 11th, and 13th harmonics to within 0.1%.

TABLE IV
Switching Angles (in Degree) With $M = 0.6$

Method	$\alpha_1 \uparrow$	$\alpha_2 \downarrow$	$\alpha_3 \uparrow$	$\alpha_4 \downarrow$	$\alpha_5 \uparrow$
GA	45.682	51.647	61.782	73.654	78.960
AGA plus constant assignment	45.671	51.635	61.754	73.631	78.908
AGA plus trigonometric function assignment	45.573	51.578	61.553	73.488	78.564
AGA plus circle equation assignment	45.545	51.561	61.496	73.448	78.467

*↑ and ↓ represent rising edge and falling edge respectively.

TABLE V
Switching Angles (in Degree) With $M = 0.8$

Method	$\alpha_1 \uparrow$	$\alpha_2 \downarrow$	$\alpha_3 \uparrow$	$\alpha_4 \downarrow$	$\alpha_5 \uparrow$
GA	31.725	35.884	48.576	57.222	62.332
AGA plus constant assignment	31.553	35.747	48.444	57.004	62.126
AGA plus trigonometric function assignment	31.484	35.707	48.392	56.935	62.057
AGA plus circle equation assignment	31.375	35.638	48.312	56.820	61.954

*↑ and ↓ represent rising edge and falling edge respectively.

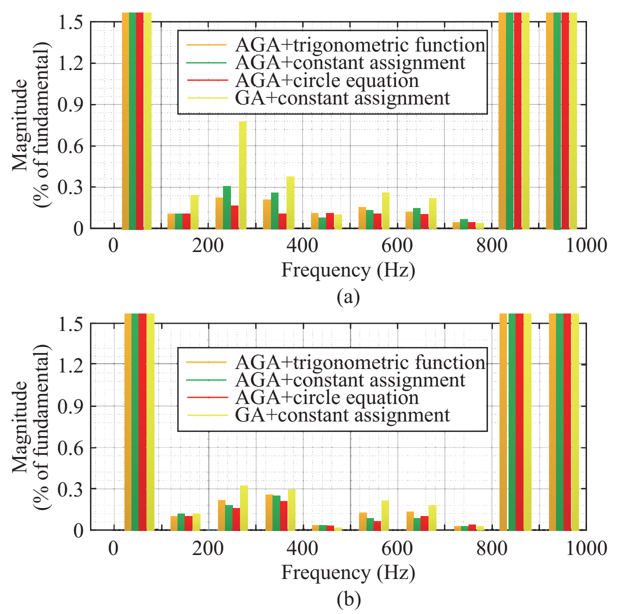

Fig. 11. Harmonic spectra with different algorithms when (a) $M = 0.6$, and (b) $M = 0.8$.

Next, N is reduced to 3, and the AGA plus circle equation assignment method is used to calculate the value of each switching angle as shown in Table VI when $M = 0.5$ and 0.9. Switching angles corresponding to different modulation indexes are brought into the simulation, and FFT analysis results of the output phase/line voltage, using the proposed optimization method and other methods in literature with different N and M, are compared in Table VII. To obtain the solution of the numerical method, appropriate initial values are selected by resorting to the cumbersome gravity center coincidence method. However, due to relatively large errors between the solved initial values and the final optimization result, calculation accuracy and the iteration speed are limited for the numerical method. Conversely, the proposed scheme integrated with an intelligent algorithm omits the complex solution process and reduces the amount of calculation especially for initial values, yet achieving a similar effect. Besides, compared to the SHEPWM method in [16] and the numerical method, when N is 5 and M is 0.6 or 0.8, line voltage using the proposed optimization method has a lower THD and has better reduction effect on 5th, 7th, 11th, and 13th harmonics, which are specified to be eliminated. When N is 3 and M is 0.9, phase voltage and line voltage THDs using the proposed method are smaller than that using the SHEPWM method in [28]. When M is 0.5, the phase voltage and line voltage THDs of the proposed algorithm are lower than of the SOPWM [22] and the SHEPWM [28]. In general, the AGA plus circle equation assignment method proposed in this paper is proved to exhibit superior effectiveness for elimination of specified sub-harmonics when N is 3 and 5, compared with existing methods in literature.

TABLE VI
SWITCHING ANGLES (IN DEGREE) OF AGA PLUS CIRCLE EQUATION ASSIGNMENT METHOD WHEN $N = 3$

M	$\alpha_1 \uparrow$	$\alpha_2 \downarrow$	$\alpha_3 \uparrow$
0.5	29.229	39.244	52.509
0.9	52.770	64.394	77.303

TABLE VII
SIMULATION COMPARISON BETWEEN DIFFERENT METHODS

Method	M/N	Phase voltage THD	Line voltage THD
SHE [16]	0.8/5	–	33.67%
Proposed Method**	0.8/5	**46.68%**	**28.82%**
Numerical Method	0.8/5	48.95%	30.77%
SHE [16]	0.6/5	–	43.41%
Proposed Method	0.6/5	**65.45%**	**39.03%**
Numerical Method	0.6/5	67.53%	40.44%
CBM [12]*	0.9/3	42.3%	29.1%
SOPWM [22]	0.9/3	47.0%	30.3%
SHE [28]	0.9/3	58.8%	57.1%
Proposed Method	0.9/3	**49.9%**	**37.16%**
CBM [12]	0.5/3	98.4%	60.5%
SOPWM [22]	0.5/3	112.0%	71.6%
SHE [28]	0.5/3	105.4%	81.6%
Proposed Method	0.5/3	**101.22%**	**78.96%**

*The results related to [12], [16] are taken from [28].
**Proposed Method is AGA plus circle equation assignment method.

B. Experimental Results and Analysis

In this work, an experimental platform based on DSP/TMS320F28335 (see Fig. 12) is used for verification. DC-link voltage is set to 660 V, the upper and bottom capacitors are 4700 mF, and dead time of switches is set to 2 µs. For real-time control, switching angle data using the AGA plus circle equation assignment method (see Tables IV and V) are stored in the microprocessor.

Figures 13 and 14 show measurement results of the phase voltage and line voltage when M is 0.6 and 0.8. From frequency spectra of the phase and line voltages (obtained by the embedded mathematical function of the oscilloscope), the specified 5th, 7th, 11th, and 13th harmonics have been effectively eliminated. At the same time, the triplen harmonics of the line voltage have been canceled due to symmetry of the three-phase topology. Though not shown here for brevity,

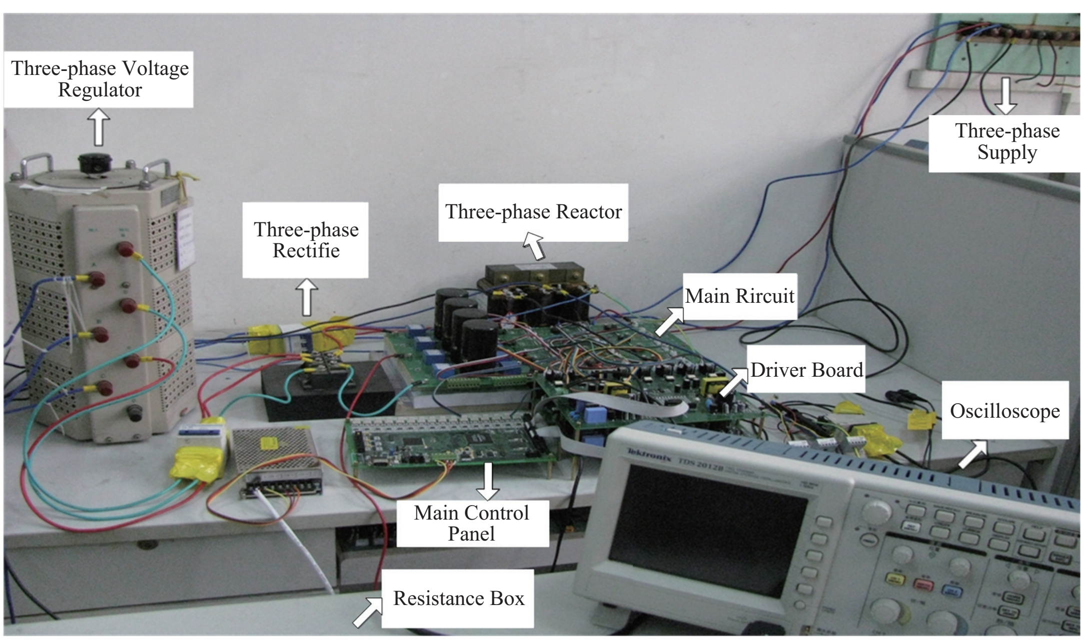

Fig. 12. Experiment platform of the three-level NPC converter.

results obtained by the numerical method are similar both in terms of time-domain waveforms and frequency-domain harmonics; however, a relevant solution process is much more complicated.

Finally, Table VIII and Fig. 15 compare performances of two SHEPWM methods, i.e., the proposed method and the method in [16], based on simulation and experimental results. From simulation results in Fig. 15(a), under both conditions (i.e., M equals 0.8 and 0.6), the proposed method has better suppression effect on the 7th, 11th, and 13th harmonics than the method in [16]. Measured harmonics are larger in experimental results due to the presence of power grid harmonics, as shown in Fig. 15(b). In this case, the proposed method maintains higher effectiveness than of [16].

V. Conclusion

This paper proposes a fast SHEPWM solution method for a wind power converter based on state equations. The initial variables of the intelligent algorithm and the required switching angles are regarded as introduced disturbances and state

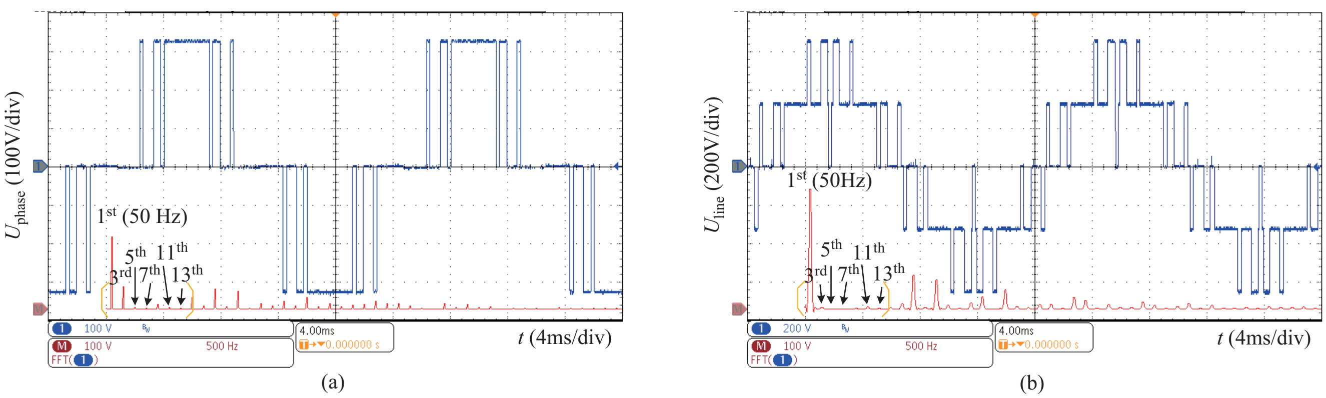

Fig. 13. Voltage measurement results and pertinent frequency spectrum ($M = 0.6$) for (a) phase voltage and (b) line voltage.

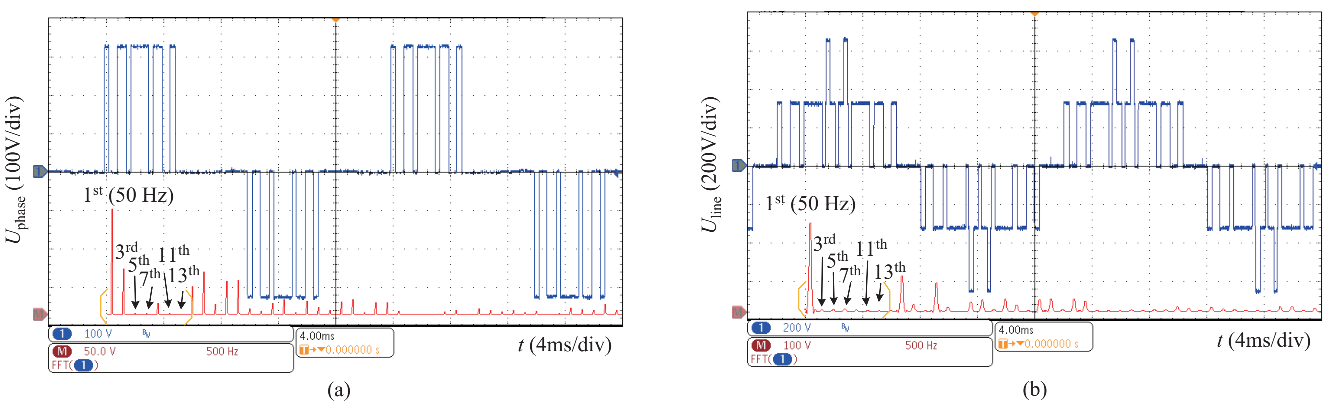

Fig. 14. Voltage measurement results and pertinent frequency spectrum ($M = 0.8$) for (a) phase voltage and (b) line voltage.

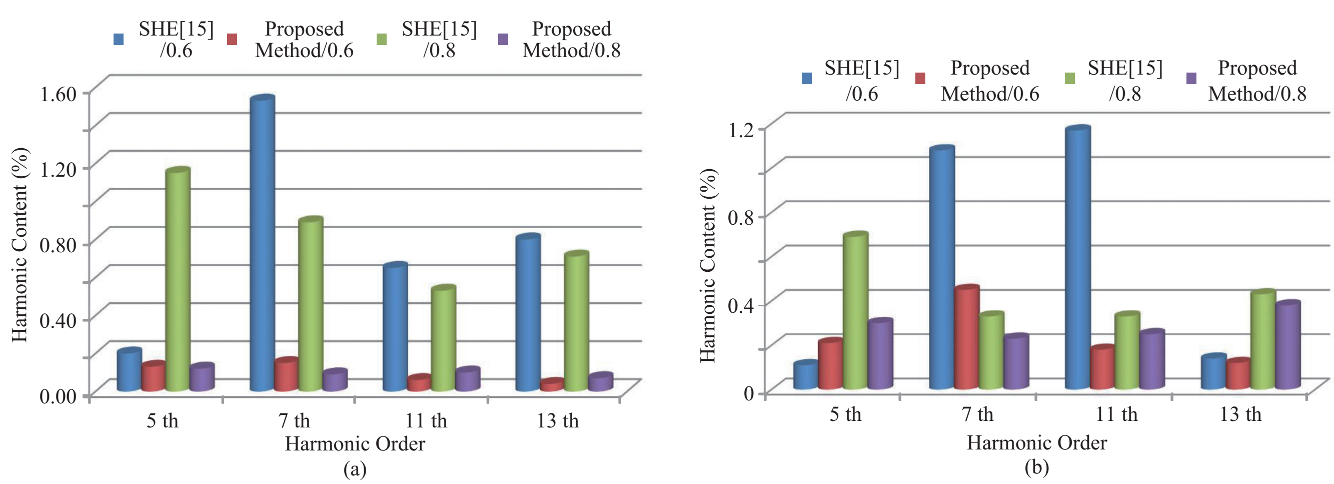

Fig. 15. Comparison of harmonic components using different SHEPWM solution techniques based on (a) simulation and (b) experimental results.

TABLE VIII
EXPERIMENTAL & SIMULATION RESULTS OF HARMONIC COMPONENTS ($N = 5$)

Method	M	5th (%)	7th (%)	11th (%)	13th (%)
Simulation					
SHE [16]	0.8	0.2	1.53	0.65	0.8
	0.6	1.15	0.89	0.53	0.71
Proposed Method	0.8	0.13	0.15	0.06	0.04
	0.6	0.12	0.09	0.10	0.07
Experimental					
SHE [16]	0.8	0.11	1.08	1.17	0.14
	0.6	0.69	0.33	0.33	0.43
Proposed Method	0.8	0.21	0.45	0.18	0.12
	0.6	0.30	0.23	0.25	0.38

variables at different time steps, respectively. Using the idea of the state equations, the success rate of the intelligent algorithm for solving SHEPWM nonlinear equations is improved to 100%. According to the negative correlation between M and switching angle, a circle equation and trigonometric function are used to improve the model, reducing dependency on the initial variables and improving calculation speed and accuracy, therefore further limiting impact of sub-low frequency harmonics on the system.

Simulation and experimental results prove the proposed mathematical model enables simple intelligent algorithms (GA, AGA) to achieve the same success rate of calculation as more complex intelligent algorithms (e.g., EMA [27]). When N equals 3 and 5, the improved model based on the circle equation assignment, combined with AGA can improve calculation accuracy of the traditional intelligent algorithm from 10^{-2} to 10^{-5}. Moreover, the proposed method leads to lower THDs of line and phase voltages and improved elimination effectiveness for specified harmonics, compared with many works in literature, e.g., [12], [16], [28].

Although traditional GA and AGA in wind power grid-connected converters have been verified in this paper as examples, the improved mathematical model universally applies to other intelligent algorithms. In general, the proposed strategy is characterized by simple and effective implementation, as well as enhanced calculation accuracy. These advantages facilitate online calculation of SHEPWM when a proper control scheme (e.g., the one proposed in [7]) is adopted.

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