A numerical model for the unified analysis of soil sedimentation-consolidation phenomena

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Abstract

Soil sedimentation and consolidation, whether they are naturally or anthropically driven processes, are closely interconnected. While theoretical models are available to separately describe the behaviour of either 'fluid' suspensions or 'solid' soil, no well-established unified theory exists able to also describe the transition effectively between the two states.

In this work, a model aimed at capturing sedimentation, consolidation, and the transition between the two phenomena via a simplified 'interaction coefficient' [1], [2] was revised and applied to the simulation of relevant engineering problems using finite element software COMSOL Multiphysics. Finite kinematics to reproduce the evolution of large strains was modelled using a Lagrangian coordinate framework.

The numerical model was applied to the simulation of experimental results from the literature, including deposition tests involving uniform suspensions of both fine- and coarse-grained soil and backfilling sedimentation tests of a sand-water mixture. In particular, the simulation of backfilling tests required the definition of a bespoke numerical strategy to reproduce the injection of the sand-water mixture in Lagrangian coordinates.

The numerical modelling framework developed in this work may be efficiently used in the analysis of a number of engineering applications concerning sedimentation and consolidation, including industrial problems where a suspension must be separated into solid and fluid components, and offshore geotechnical problems involving submarine pipeline trench backfilling and land reclamation.

Keywords: sedimentation-consolidation, fluid-solid transition, land reclamation, pipeline backfilling, offshore engineering

Introduction

Sedimentation may be defined as the deposition of solid material from a fluid, typically air or water, from a state of suspension [3]. Consolidation is the gradual volume reduction in fully saturated soil due to pore fluid drainage (e.g. [4]). These interrelated processes, typically occurring successively, may last from minutes to decades, and are essential in various engineering applications. It is worth observing that while the sedimentation process is characterized by the absence of interparticle force chains, consolidation is defined by the formation of a sediment network structure (so-called solid skeleton) which can carry a part of its own weight [5].

In recent decades, increased dredging activities, coupled with environmental regulations, have driven more controlled use of dredged materials. Land reclamation is an efficient method, where slurry is deposited in containment facilities, undergoing sedimentation and consolidation. A unified sedimentation-consolidation theory is essential for designing such facilities, as storage capacity depends on the duration of these processes [6], [7], [8], [9].

A similar issue arises in oil sands mining, where separating water from fine tailings is key to controlling pond volumes [5].

Offshore pipeline engineering also relies on sedimentation and consolidation. Pipelines, often buried for protection, may experience flotation or sinking if discharge rates during backfilling exceed sedimentation rates, potentially leading to damage [10], [11], [12], [13].

This work simulates sedimentation and consolidation under large strain using COMSOL Multiphysics. The model was built gradually, by first implementing Gibson et al.'s [14] large strain consolidation theory and validating it against analytical solutions [15], [16]. A more general large strain model was then implemented, incorporating Kynch's [17] hindered settling theory, that was shown to align with Gibson et al.'s [14] model in stress-free conditions, as in soil suspension. This was carried out following Pane & Schiffman's [1] theory linking sedimentation and consolidation by introducing an 'interaction coefficient' to transition from zero effective stress in sedimentation to classical Terzaghi effective stress in consolidation. This model was implemented in COMSOL Multiphysics and validated against the solution presented by [2].

Finally, the model was used to replicate experimental data from Been [18] on suspension settling and Eikhout [19] on sand backfilling. A bespoke numerical strategy was adopted to simulate sand-water mixture sedimentation experiments in a Lagrangian coordinate system.

Model formulation

Governing equation

The core equation of the model is the one proposed by [1], providing a 1D finite strain formulation of both sedimentation and consolidation:

$$\frac{\partial e}{\partial t} = \mp\left(\frac{\gamma_s}{\gamma_f}-1\right)\frac{d}{de}\left(\frac{k}{1+e}\right)\frac{\partial e}{\partial z} + \frac{\partial}{\partial z}\left[-\frac{k}{\gamma_f(1+e)}\beta(e)\frac{d\sigma'}{de}\frac{\partial e}{\partial z}\right] + \frac{\partial}{\partial z}\left[-\frac{k}{\gamma_f(1+e)}\frac{d\beta(e)}{de}\sigma'\frac{\partial e}{\partial z}\right]$$

In the above, e is void's ratio, γ_s and γ_f the solids and fluid unit weights, k the hydraulic conductivity, σ' the effective stress and $\beta(e) \in [0,1]$ a so-called 'interaction coefficient'. The Lagrangian coordinate z, defining the thickness of an element of solid material, is related to Eulerian coordinate x as [16]:

$$z(x) = \int_0^x \frac{1}{1+e}\,dx$$

It is observed that when $\beta(e) = 0$ the above governing equation becomes Kynch's [17] equation describing hindered settling, while at $\beta(e) = 1$ Gibson et al.'s [14] finite strain consolidation equation is recovered.

Numerical Model

A 1D domain was defined, with the standard x-coordinate used in COMSOL Multiphysics referring to the Lagrangian 'solid thickness' coordinate defined above as z.

A *Stabilized Convection-Diffusion Equation* interface was introduced to simulate all considered problems of sedimentation or coupled sedimentation-consolidation. This interface allowed to reduce numerical instabilities by the *Streamline diffusion* and *Crosswind diffusion* strategies, which are automatically implemented in the interface. The reference convection-diffusion equation transforms into a simple convection equation when dealing with sedimentation problems, for which the diffusion coefficient is set to zero.

Initial and boundary conditions

As regards initial conditions, an initially homogeneous layer is considered, i.e. with constant void ratio throughout z.

A Dirichlet boundary condition was imposed at the top boundary, with void ratio equal to the initial one. For the bottom boundary, when considering it as impervious, the corresponding (Neumann) boundary condition in terms of void ratio was derived, starting from the traditional soil mechanics assumption in terms of excess pore pressure $\partial u/\partial z(0,t) = 0$, as [20]:

$$\frac{\partial e}{\partial z}(0,t) = \frac{\partial e}{\partial \sigma'}\left(\gamma_s - \gamma_f\right)$$

Numerical strategy to simulate mass inflow

A standard Kynch's hindered settling (sedimentation) problem, involving a constant solid mass within the domain, is easily solved in the Lagrangian coordinate z representing the thickness of the solids in the considered suspension layer, automatically satisfying the solids continuity. On the other hand, the problem of underwater sand backfilling of a pipeline (experimentally reproduced by Eikhout [19]), required a special numerical strategy to be solved in the Lagrangian reference system. In fact, this process involves the gradual discharge of a sand-water mixture within a trench, at a rate that is larger than the sedimentation rate of sand particles, leading to formation of a suspension in the domain surrounding the pipe. This implies a mass inflow into the domain, starting from clear water at zero solid concentration (i.e. at $e \to \infty$) and reaching a maximum concentration at the end of discharge. To solve the problem in the Lagrangian reference system, the mass discharge is simulated introducing a 'dummy reservoir' above the problem domain (Figure 1).

The volume of the reservoir, which is represented by a line of given length in 1D problems, is evaluated such that the total amount of solid mass in the domain corresponds to the mass discharged in the backfilling experiment. Two different hydraulic conductivities are defined for the two strata. The hydraulic conductivity of the reservoir is calibrated to reproduce the experimental discharge rate, which is larger than the sedimentation rate of sand particles in the domain.

Interaction coefficient function

The above-described interaction coefficient β was introduced to describe the evolution of effective stress during deposition. When $\beta = 0$ the particles (or aggregates of particles) are so distant that their static interaction in terms of force chain development is negligible, and the mixture behaves as a dispersion. When the soil is completely formed, there is full particle-to-particle contact and the interaction coefficient is equal to unity, i.e. the conventional effective stress principle holds and the mixture behaves as a soil [1]. At intermediate concentrations, the soil-water mixture is no longer a suspension, but it is not a soil yet. Pore fluid velocities are still high, and thus drag forces are comparable with effective stresses which start to develop. However, from a macroscopic point of view, the above presented governing equation is still valid in this intermediate range.

Pane & Schiffman [1] did not provide details on the numerical implementation of their model, and presented a discontinuous 'step' function for $\beta(e)$ that was bound to bring about convergence issues in the numerical code. In this work, the interaction coefficient was modelled by a Step function in COMSOL Multiphysics with continuous first and second derivatives [20]. This function describes a smooth transition between 1 and 0, once the extreme void ratios are encountered, thus increasing numerical stability of the analysis.

Simulation Results

The above-described model was first tested to separately reproduce a sedimentation and a large strain consolidation process [20], showing very good agreement with the corresponding analytical solutions, respectively provided by [17] and [15], [16].

Next, the full capabilities of the model were tested by successfully reproducing the numerical solution of Jeeravipoolvarn et al. [2]. In Figure 2 the results of these simulations are shown in terms of void ratio isochrones, representing a process of reduction of void ratio by sedimentation and consolidation of a clayey material. It can be observed that over time, a clear water–suspension interface settles and the suspension–soil interface builds from bottom up.

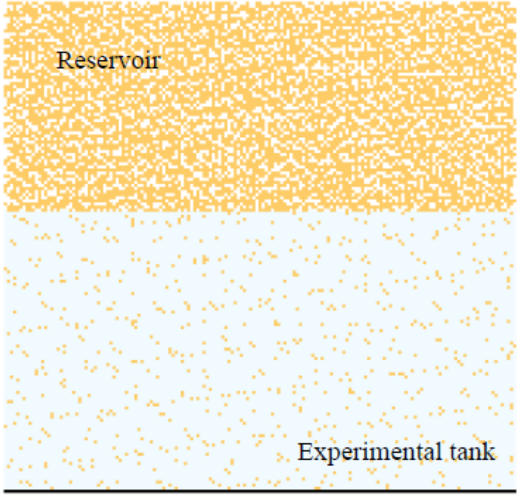

Figure 1. Schematic representation of 1D sedimentation of sand from a 'dummy' reservoir to the layer representing the experimental tank [19] initially full of water and of sand at very low concentration.

Finally, the model was validated by reproducing available experimental data. Two different experimental datasets with diverse engineering applications were selected to this aim.

Been's [18] experiments involving the settlement and consolidation of a natural mud with 30% clay content, with potential applications in the fields of dredging and land reclamation, were first reproduced. Figure 3 and Figure 4 respectively show the void ratio isochrones and the fall of water-slurry interface curve together with iso-void ratio lines for Been's experiment #6, involving both sedimentation, transition and consolidation phases. Comparison between experimental data and simulations in Figure 3 proves good capability of the model to capture all involved processes.

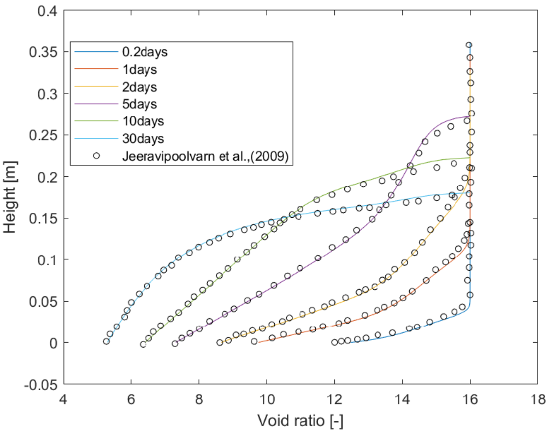

Figure 2. Void ratio isochrones for a sedimentation-consolidation process involving a clayey soil. Solid lines represent results from COMSOL simulation, dotted lines results by [2].

Further model validation was provided by reproducing Eikhout's [19] backfilling test in a pipe-sand interaction scaled physical model, with potential application in the offshore pipeline industry. These simulations, due to the mass inflow at the upper boundary, required the definition of a 'dummy reservoir' as detailed above. Figure 5 shows a comparison between simulated and experimentally measured concentration profiles of the sand-water suspension along depth at different times. Also in this case, the model is able to adequately capture the experimental trends. It should be observed that in this specific case, dealing with highly permeable sandy soil, the most significant part of the analysis involves sedimentation while the consolidation process is deemed negligible due to its short timeframe of development.

Conclusions

A model able to capture sedimentation, consolidation, and the transition between the two phenomena was presented in this work and applied to the simulation of relevant engineering problems using finite element software COMSOL Multiphysics, adopting a Lagrangian finite strain framework.

The numerical model was first tested by comparison with an existing numerical solution, then it was validated by reproducing two different experimental datasets: the first one [18] involving the settlement and consolidation of a clayey material with applications in the field of land reclamation, the second one [19] involving backfilling sedimentation tests of a sand-water mixture with applications in the field of offshore pipeline installation.

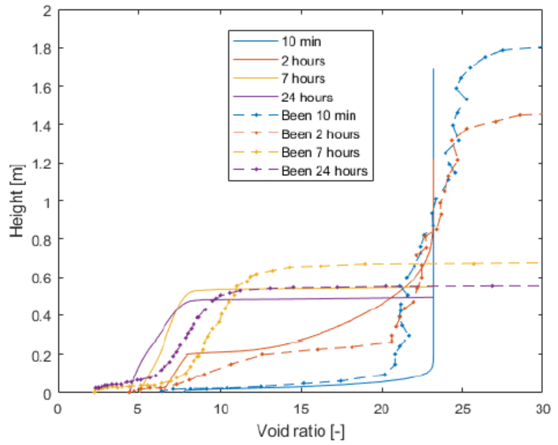

Figure 3. Void ratio isochrones. Solid lines represent results from COMSOL simulation, dotted lines experimental measurements by Been's [18] Experiment #6.

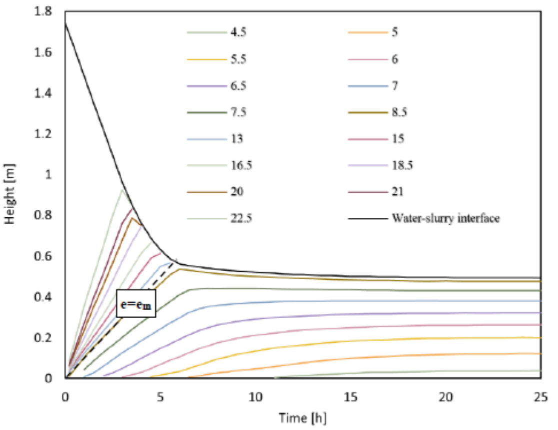

Figure 4. Fall of water-slurry interface curve and iso-void ratio lines at different heights and different times from simulations of Been's [18] Experiment #6.

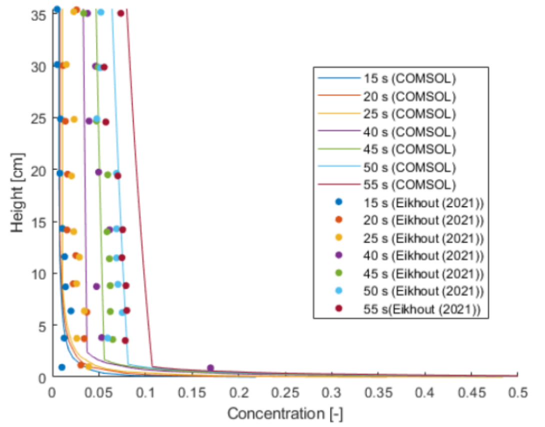

Figure 5. Concentration profiles along depth at different times for Eikhout's [19] Test 4. Continuous lines refer to simulation in COMSOL Multiphysics, dots to experimental results.

Comparison between simulations and experimental data shows satisfactory agreement in all considered cases, suggesting the flexibility and robustness of the proposed modelling approach.

Previous studies have provided limited detail on the numerical implementation of sedimentation-consolidation models. This work addresses this gap by demonstrating how to numerically implement such a model using COMSOL Multiphysics, a well-established and user-friendly software. This approach does not require advanced numerical expertise, making the model accessible to a broader range of users and facilitating its application in various scientific and engineering contexts, such as submarine pipeline trench backfilling and land reclamation.

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