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Post mission disposal design in the Laplace plane leveraging orbital perturbations

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Abstract

This paper focuses on post mission disposal of a spacecraft targeting an Earth reentry. The natural orbital perturbation is exploited and enhanced with impulsive manoeuvres which moves a spacecraft into a trajectory evolving naturally to Earth re-entry. The dynamics of the orbit of a spacecraft under the effects of the Earth oblateness and the gravitational attractions from the Moon and the Sun is averaged twice and then a third average over the right ascension of the node (RAAN) is applied which is known as the elimination of the node. The Laplace plane is proposed as the reference plane for the dynamics of the orbit instead of traditional equator, largely improved the accuracy of the triple averaged model. The proposed technique is applied to a test case and the results obtained are validated through a high-fidelity model.
Keywords: Post Mission Disposal, Space Debris, Manoeuvre Design, Laplace Plane

1. Introduction

The space object population has been increasing since the beginning of the space era and becoming much more rapidly in last decades due to the deployment of mega-constellations. A large number among all the space objects are debris which is threatening the safety of functioning spacecrafts and future missions. In response to this situation, the Inter-Agency Space Debris Coordination Committee (IADC) published space debris mitigation guidelines specifying mitigation measures, among which post mission disposal (PMD) is of importance, preventing prolonged stay in geostationary orbit (GEO) and limiting passage in low Earth orbit (LEO) [1]. Successful PMD make large contribution to debris mitigation and remediation. However, PMD implementation could be economically more expensive as extra propellant is consumed, which discourages spacecraft operators from implementing PMD strategies and meeting mitigation guidelines. This issue can be well mitigated if the natural orbit perturbations is exploited [2, 3]. On the other hand, one of the obstacles of PMD design is high computational cost of optimisation process as numerical orbit propagation of decades is involved. Using semianalytical models in the manoeuvre optimisation could well tackle this problem [2, 4].[**?**] This paper develops a triple averaged model for orbital perturbations, averaging disturbing functions over mean anomaly of a spacecraft, mean anomaly of a third body and the RAAN. The Laplace plane is proposed as the reference plane for the dynamics of the orbit instead of traditional equator [2, 5, 3, 6, 7]. The Laplace plane is a position where the effects of the Earth oblateness and third body perturbations from the Moon and the Sun are comparable [8, 9, 10]. In this fashion, the accuracy of the triple averaged model can be largely improved [11, 12].

The remaining part of the paper is organised as follows. Section 2 develops the triple averaged dynamics model for orbital perturbation in the Laplace plane and obtain the averaged Hamiltonian. Section 3 reports the PMD strategy design technique of a HEO spacecraft and then gives a case study of applying the proposed technique to a test case. Finally, Section 4 concludes the paper and summarizes the main results of the paper.

2. Semianalytical dynamics of a spacecraft relative to the classical Laplace plane

The dynamics of a spacecraft in a Highly Elliptical Orbit (HEO) with a high apogee is mainly affected by the Earth's oblateness and gravitational attractions of the Moon and the Sun. Such dynamics is typically described as a perturbed two-body problem.

$$\ddot{\boldsymbol{r}} = -\frac{\mu}{r^3}\boldsymbol{r} + \boldsymbol{f} \tag{1}$$

where $\boldsymbol{r}$ is the position vector of the spacecraft, $r = \|\boldsymbol{r}\|$ its magnitude, μ the gravitational parameter of the Earth, and $\boldsymbol{f}$ the perturbing acceleration due to various effects other than the central gravitational attraction.

If the sources of perturbations are all conservative, the perturbing acceleration can be rewritten as

$$\boldsymbol{f} = \frac{\partial \mathcal{R}}{\partial \boldsymbol{r}} \tag{2}$$

where $\mathcal{R}$ is a disturbing function of the corresponding perturbation.

The disturbing function due to the gravitational attraction of a third body of mass m_j is given by

$$\mathcal{R}_j = \mu_j \left(\frac{1}{\|\boldsymbol{r} - \boldsymbol{r}_j\|} - \frac{\boldsymbol{r} \cdot \boldsymbol{r}_j}{r_j^3} \right) \tag{3}$$

where μ_j is the gravitational parameter of the j-th perturbing body, $\boldsymbol{r}_j$ the corresponding position vector relative to the Earth centre, and r_j its magnitude. Given the assumption of $r \ll r_j$, only the lowest order term in the Legendre expansion is retained,

$$\mathcal{R}_j = \frac{\mu_j r^2}{r_j^3} P_2(\cos\theta_j) = \frac{\mu_j}{2r_j^5}\left[3\left(\boldsymbol{r}\cdot\boldsymbol{r}_j\right)^2 - r^2 r_j^2\right] \tag{4}$$

where $P_2(\cdot)$ is the second order Legendre polynomial, θ_j the angle between $\boldsymbol{r}$ and $\boldsymbol{r}_j$. Average the disturbing function over one period of the perturbing body, using

$$\begin{aligned} \frac{1}{2\pi}\int_0^{2\pi}\frac{1}{r_j^3}\,\mathrm{d}M_3 &= \frac{1}{a_j^3(1-e_j^2)^{3/2}} \\ \frac{1}{2\pi}\int_0^{2\pi}\frac{1}{r_j^5}\boldsymbol{r}_j\boldsymbol{r}_j\,\mathrm{d}M_3 &= \frac{1}{2a_j^3(1-e_j^2)^{3/2}}(\mathbb{I}-\boldsymbol{w}_j\boldsymbol{w}_j) \end{aligned} \tag{5}$$

where a_j, e_j, M_j are semimajor axis, eccentricity, and mean anomaly of the orbit of the perturbing body, $\mathbb{I}$ the idemtensor, and $\boldsymbol{w}_j$ the normal unit vector of the orbital plane of the perturbing body. And the averaged disturbing function is

$$\overline{\mathcal{R}}_j = -\frac{\mu_j}{4a_j^3(1-e_j^2)^{3/2}}\left[3\left(\boldsymbol{r}\cdot\boldsymbol{w}_j\right)^2 - r^2\right] \tag{6}$$

which can be averaged again over one period of the spacecraft, using

$$\begin{aligned} \frac{1}{2\pi}\int_0^{2\pi}\frac{1}{r^3}\,\mathrm{d}M &= \frac{1}{a^3(1-e^2)^{3/2}} \\ \frac{1}{2\pi}\int_0^{2\pi}\boldsymbol{r}\boldsymbol{r}\,\mathrm{d}M &= \frac{1}{2}a^2[(1+4e^2)\boldsymbol{p}\boldsymbol{p}+(1-e^2)\boldsymbol{q}\boldsymbol{q}] \end{aligned} \tag{7}$$

where a, e, M are semimajor axis, eccentricity, and mean anomaly of the spacecraft orbit, $\boldsymbol{p}$ the unit vector in the orbital plane of the spacecraft pointing to the perigee, $\boldsymbol{w}$ the normal unit vector of the orbital plane of the spacecraft, and $\boldsymbol{q} = \boldsymbol{w}\times\boldsymbol{p}$. The double averaged disturbing function is

$$\begin{aligned} \overline{\overline{\mathcal{R}}}_j = &-\frac{3\mu_j a^2}{4a_j^3(1-e_j^2)^{3/2}}\left[\frac{1}{2}(1-e^2)\left(\boldsymbol{w}\cdot\boldsymbol{w}_j\right)^2\right. \\ &\left.+e^2\left(1-\frac{5}{2}\left(\boldsymbol{p}\cdot\boldsymbol{w}_j\right)^2\right)\right] \end{aligned} \tag{8}$$

The disturbing function due to the Earth's oblateness is given by

$$\mathcal{R}_{J_2} = -\frac{\mu J_2 R_\oplus^2}{2r^5}\left[3(\boldsymbol{r}\cdot\boldsymbol{k})^2 - r^2\right] \tag{9}$$

Averaging over one period of the spacecraft, we have

$$\overline{\mathcal{R}}_{J_2} = \frac{\mu J_2 R_\oplus^2}{4a^3(1-e^2)^{3/2}}\left[3(\boldsymbol{w}\cdot\boldsymbol{k})^2 - 1\right] \tag{10}$$

The Hamiltonian formulation of dynamic systems can give us insights on dynamical behaviours and qualitative results of long-term evolution of the system. The Hamiltonian formulation of a perturbed two-body problem is given by

$$\mathcal{H} = -\frac{\mu}{2a} - \mathcal{R}_{J_2} - \mathcal{R}_m - \mathcal{R}_s \tag{11}$$

including the effects of the Earth's oblateness and the gravitational attraction of the Moon and the Sun. After averaging over the fast angles twice, the Hamiltonian becomes

$$\overline{\overline{\mathcal{H}}} = -\frac{\mu}{2a} - \overline{\mathcal{R}}_{J_2} - \overline{\overline{\mathcal{R}}}_m - \overline{\overline{\mathcal{R}}}_s \tag{12}$$

We describe the equations of motion in terms of the angular momentum and eccentricity vectors, i.e., the Milankovitch elements, to avoid the involving of any reference frame. Define

$$\boldsymbol{e} = e\boldsymbol{p}, \quad \boldsymbol{h} = \sqrt{1-e^2}\boldsymbol{w}, \tag{13}$$

and we have

$$\boldsymbol{e}\cdot\boldsymbol{h} = 0, \quad \|\boldsymbol{e}\|^2 + \|\boldsymbol{h}\|^2 = 1. \tag{14}$$

The double averaged secular equations of motion under the Earth's oblateness and lunisolar perturbations are hence given by

$$\begin{aligned} \dot{\boldsymbol{h}} &= -\frac{\omega_{J_2}}{h^5}(\boldsymbol{k}\cdot\boldsymbol{h})\mathbb{K}\cdot\boldsymbol{h} - \sum_{j=1,2}\omega_j\boldsymbol{w}_j\cdot(5\boldsymbol{e}\boldsymbol{e}-\boldsymbol{h}\boldsymbol{h})\cdot\mathbb{W}_j \\ \dot{\boldsymbol{e}} &= -\frac{\omega_{J_2}}{2h^5}\left\{\left[1-\frac{5}{h^2}(\boldsymbol{k}\cdot\boldsymbol{h})^2\right]\mathbb{H} + 2(\boldsymbol{k}\cdot\boldsymbol{h})\mathbb{K}\right\}\cdot\boldsymbol{e} \\ &\quad - \sum_{j=1,2}\omega_j\left[\boldsymbol{w}_j\cdot(5\boldsymbol{e}\boldsymbol{h}-\boldsymbol{h}\boldsymbol{e})\cdot\mathbb{W}_j - 2\mathbb{H}\cdot\boldsymbol{e}\right] \end{aligned} \tag{15}$$

where

$$\omega_{J_2} = \frac{3nJ_2R_\oplus^2}{2a^2}, \quad \omega_j = \frac{3\mu_j}{4na_j^3(1-e_j^2)^{3/2}}, \tag{16}$$

$\mathbb{K}, \mathbb{H}, \mathbb{W}_j$ are corresponding cross-product dyadics of $\boldsymbol{k}, \boldsymbol{h}, \boldsymbol{w}_j$, defined as

$$\mathbb{A} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \tag{17}$$

and $n = \sqrt{\mu/a^3}$ the mean motion of the spacecraft.

The equations of motion show that $\dot{\boldsymbol{e}}$ vanishes for circular orbit. In this case, $\boldsymbol{h} = \boldsymbol{w}$, we have

$$\dot{\boldsymbol{w}} = -\omega_{J_2}(\boldsymbol{k}\cdot\boldsymbol{w})\mathbb{K}\cdot\boldsymbol{w} - \sum_{j=1,2}\omega_j(\boldsymbol{w}_j\cdot\boldsymbol{w})\mathbb{W}_j\cdot\boldsymbol{w} \tag{18}$$

The first term of the right-hand side drive the orbital plane of the spacecraft to regress around the Earth's north pole, while the latter terms drive the orbital plane to regress around the ecliptic north pole and the normal of the Moon's orbital plane, respectively.

The classical Laplace equilibrium is defined as the circular Laplace equilibria which is exactly the above case where $\boldsymbol{h}$ is in the plane spanned by $\boldsymbol{k}$ and $\boldsymbol{w}_s$, assuming the Moon's orbit lies in the ecliptic. The exact position can be described by the inclination of the Laplace equilibrium φ relative to the Earth equator. With the assumption above, the stationary condition becomes [8, 10, 9]

$$\tan 2\varphi = \frac{\sin 2\epsilon}{\cos 2\epsilon + (r_L/a)^5} \quad (19)$$

where r_L is the Laplace radius given by

$$r_L^5 = a^5 \frac{\omega_{J_2}}{\omega_m + \omega_s} \quad (20)$$

Eq. (19) has four solutions for φ in $[0, 2\pi)$. The one corresponding to the classical Laplace plane satisfy that $\varphi \to 0$ as $a \to 0$ and $\varphi \to \epsilon$ as $a \to \infty$. In this way, the classical Laplace plane coincides with the Earth equator at small altitude and with the ecliptic at high altitude, as shown in Fig. 1.

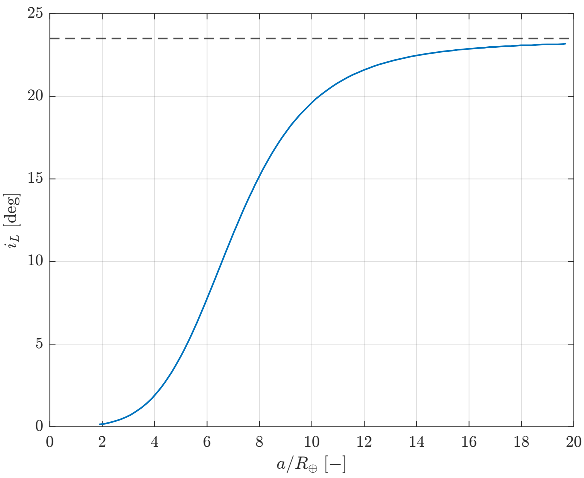

Fig. 1. Inclination of Laplace plane relative to the Earth equator.

The orbit of an Earth satellite disturbed by the geopotential perturbation is usually described by referring to the Earth equator. However, the effect of the gravitational attraction of a third body is clearer and simpler when referred to its orbital plane,i.e., the lunar plane for the Moon and the ecliptic for the Sun. In the case of this paper, the analysis of the combined effects is simplified when referred to the Laplace plane, where both effects are comparable. We can achieve that by writing Eq. (15) into components in the frame referred to the Laplace plane, or writing the double averaged Hamiltonian Eq. (12) in terms of classical Keplerian elements relative to the Laplace plane.

The double averaged Hamiltonian can be written as

$$\overline{\overline{\mathcal{H}}} = \overline{\overline{\mathcal{H}}}(a, e, i, \Omega, \omega, \Omega_m(t), \omega_m(t)) \quad (21)$$

where the elements are all relative to the Laplace plane. To further simplify the model, one can average again the Hamiltonian over the right ascension of the ascending node (RAAN) of the spacecraft orbit as follows,

$$\overline{\overline{\overline{\mathcal{H}}}} = \int_0^{2\pi} \overline{\overline{\mathcal{H}}}\, \mathrm{d}\Omega = \overline{\overline{\overline{\mathcal{H}}}}(a, e, i, \omega, \omega_m(t)) \quad (22)$$

as $\Omega_m(t)$ is coupled with Ω. If we further drop the time dependent terms by assuming a fixed lunar perigee, we have $\overline{\overline{\overline{\mathcal{H}}}}(e, i, \omega)$ which is a single degree-of-freedom Hamiltonian, as the semimajor axis a is not affected by J_2 and lunisolar perturbations and the well-known Kozai parameter [13]

$$\Theta = (1 - e^2)\cos^2 i \quad (23)$$

remain constant since the z-component of the angular momentum is conserved. The triple averaged Hamiltonian and thus the disturbing potential can then be substituted into Lagrange planetary equations to get the triple averaged equations of motion.

The validity of the model can be verified by comparing with the double-averaged model as in Fig. 2 whose accuracy has already been proved by previous research [2, 3]. The figure shows that although there are some discrepancies, the triple averaged model follows well with the double averaged one. Fig. 3 shows the propagation using models relative to the equator [3]. By comparing the results in the two figures, it is evident that the accuracy of triple averaged model is largely improved by changing the reference plane from equator to the Laplace plane.

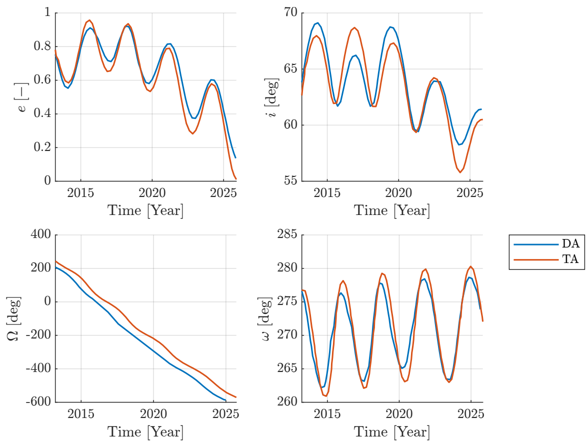

Fig. 2. Evolution of orbital elements relative to the Laplace plane.

3. Disposal manoeuvre design technique

The objective of our problem is to give an impulsive manoeuvre to the spacecraft at some point so that the orbit of the spacecraft evolves towards an Earth re-entry under the effects of natural perturbation.

The manoeuvre is modeled as

$$\boldsymbol{\Delta v} = \begin{bmatrix} \Delta v_T \\ \Delta v_N \\ \Delta v_H \end{bmatrix} = \Delta v \begin{bmatrix} \cos\alpha\cos\beta \\ \sin\alpha\cos\beta \\ \sin\beta \end{bmatrix}, \quad (24)$$

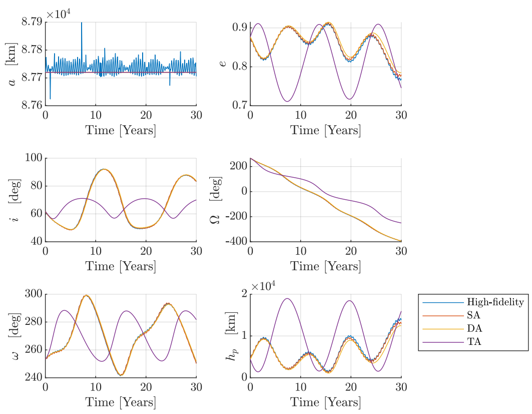

Fig. 3. Evolution of orbital elements relative to the Equator [3].

where $\Delta v, \alpha, \beta$ are the magnitude, in-plane, and out-of-plane angle of the maneuver, respectively. The variations of the Keplerian elements due to the manoeuvre are computed with Gauss' planetary equations as follows,

$$\begin{aligned}
\Delta a &= \frac{2}{n\sqrt{1-e^2}}\sqrt{1+2e\cos f_m + e^2}\Delta v_T \\
\Delta e &= \frac{\sqrt{1-e^2}}{na\sqrt{1+2e\cos f_m + e^2}}[2(\cos f_m + e)\Delta v_T \\
&\quad -\sqrt{1-e^2}\sin E_m \Delta v_N] \\
\Delta i &= \frac{r\cos u_m}{na^2\sqrt{1-e^2}}\Delta v_H \\
\Delta\Omega &= \frac{r\sin u_m}{na^2\sqrt{1-e^2}\sin i}\Delta v_H \\
\Delta\omega &= \frac{\sqrt{1-e^2}}{nae\sqrt{1+2e\cos f_m + e^2}}[2\sin f_m \Delta v_T \\
&\quad +(\cos E_m + e)\Delta v_N] - \cos i \Delta\Omega \\
\Delta M &= -\frac{1-e^2}{nae\sqrt{1+2e\cos f_m + e^2}}[(2\sin f_m \\
&\quad + \frac{2e^2}{\sqrt{1-e^2}}\sin E_m\right)\Delta v_T + (\cos E_m - e)\Delta v_N]
\end{aligned} \tag{25}$$

in which f_m is the true anomaly where the maneuver is applied, E_m is the corresponding eccentric anomaly given by

$$\tan\frac{E_m}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{f_m}{2}, \tag{26}$$

and $u_m = \omega + f_m$.

The Keplerian elements right after the maneuver are given by

$$kep_{post} = kep_{pre} + \Delta kep, \tag{27}$$

The cost function of optimisation is defined by a weighted sum of the terminal error and magnitude of the maneuver,

$$J = \max\left(\frac{h_{p,min} - h_{p,target}}{h_{p,target}}, 0\right) + w\Delta v \tag{28}$$

where w is weight based on mission scenarios.

A test case of a HEO mission is given in Fig. 4, where the disposal manoeuvre window is divided into 30 points which are then used as initial conditions of the optimisation process. The figure shows that the effective manoeuvres lie in the tangential direction of the orbit, and most of the manoeuvres are at the orbit perigee.

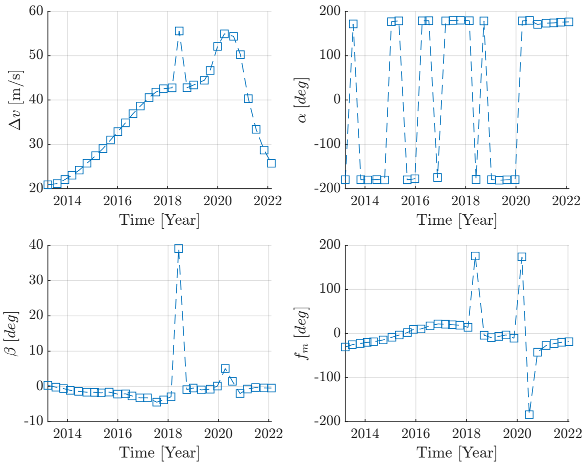

Fig. 4. Magnitudes, in-plane and out-of-plane angles, and corresponding true anomalies of the manoeuvres.

4. Conclusion

This research focuses on the PMD design of a spacecraft targeting an Earth re-entry. The natural orbital perturbation is exploited to save the propellant needed for the PMD process and is enhanced with impulsive manoeuvres. The dynamics of a spacecraft orbit considering the J_2 perturbation and lunisolar attractions is formulated by a semi-analytical model, and is averaged over mean anomaly of the spacecraft, over mean anomaly of a perturbing body, and over the RAAN of a spacecraft. The Laplace plane, a balanced position of orbital plane precession induced by J_2 and third-body perturbation, is proposed as the reference plane of the dynamics, which improved largely the accuracy of the triple averaged model.

The averaged orbital elements of a spacecraft can be computed from the Hamiltonian and the Kozai parameter for given initial conditions and the maximum eccentricity related to the re-entry condition can be computed by only analysing the Hamiltonian after the manoeuvre, without numerical orbit propagation. In this fashion, the disposal manoeuvre, minimising terminal errors and propellant consumption, could be optimised with much less computational effort. For future work, The preliminary results obtained with the averaged model can then be used as a first guess and then be refined with high-fidelity models.

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