Optimization of E-bike networks

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Abstract

Battery-assisted bicycles, or E-bikes, are part of a disruptive wave of transportation technology that uses electricity and rechargeable batteries to increase the velocity, the traveled distance and, as a consequence, the ridership. Biking and E-biking are globally recognized to have the potential to play an important role in the transition to a Net-Zero society. The widespread availability of E-bikes is significantly impacting several sectors of the tourist industry. Therefore, Touristic Administrations (TAs) now provide tourists with trail options and the corresponding charging infrastructure for E-bikers with different profiles. Our main objective is to provide TAs with a suitable decision-support tool that serves two purposes: 1) finding locations for charging stations by considering the difficulty and the cost of installing such stations in remote, often off-the-road locations; and 2) designing itineraries that are suitable for different categories of E-bikers. In the scientific literature, the first decision component has been mostly addressed in the context of electric cars, and it is not suitable for E-bikes. On the other hand, works on the second decision focused on muscular bikes, thus ignoring the first decision component. In this paper we aim at closing this gap. We formulate this problem as a mixed-integer linear program. We develop an efficient branch-and-cut algorithm and then provide a comprehensive computational experiment. We present a case study in the Asiago Sette Comuni Plateau in Italy, where the obtained charging stations and bike trails maximize a measure of attractiveness for three types of users

Keywords— E-bikes; Charging station location; Itinerary design; Touristic Districts

1 Introduction

The road-transportation sector represents one of the major contributing factors to the emission of Green-House Gases (GHG). The correlation between elevated GHG emission levels and climatic change has been scientifically proven. Several countries prioritized the electrification of the transportation system as a measure for the reduction of GHG emissions. This results in the rise of electric vehicles on the market, including short-range vehicles, such as scooters and bikes. In particular, the large diffusion of E-bikes is significantly impacting sectors of the tourist industry: activities that

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were previously exclusive to expert riders are now accessible to the general public, thus potentially diverting a considerable basin of tourists towards low-carbon activities.

One of the UN goals for sustainability within SDG 11 is to reach a sustainable transport and mobility system that is universal, efficient, and safe. Green biking and e-biking is globally recognized to be an important element to achieve this goal (see, for example Carra et al. 2023; Deenihan et al. 2013; Gazzola et al. 2018; Zhu 2022, and references therein). It has an impact on leisure activities, as well as in everyday commuting activities. The last years, especially in the post-pandemic scenario, have witnessed a significant increment in the market for muscular and E-bikes and a more widespread use of these transport means. City administrators are fostering this behavioural change by strengthening the biking network (construction of paths or reserved bike lanes), facilities (bike parking and charging stations), and services (E-scooter and E-bike sharing and rental). This trend is affecting the tourist industry too. For example, touristic regions that have been traditionally considered isolated and difficult to reach, can greatly take advantage from this increased interest in cycle tourism (Gazzola et al. 2018). Indeed, some of these regions offer bike park infrastructures to exploit the dense network of forest trails in order to attract riders of different types. Facilitating the adoption of bikes and E-bikes promotes touristic activities with lower environmental impact when compared with other activities requiring transportation modes such as cars and motorbikes (Carra et al. 2023). Moreover, the availability of a suitable trail network and biking infrastructure makes it possible to prioritize the access to environmentally fragile sites to users with lower environmental impact such as hikers and bikers. To this purpose, it worth noting that the rise of muscular and electric cycle tourism is a step towards the sustainable development of less accessible wide areas (Deenihan et al. 2013), thus contributing in mitigating the depopulation phenomenon in fragile areas.

Touristic Administrations (TAs), i.e., agencies tasked with managing the infrastructure and operations for tourists in a Touristic District (TD henceforth), that wish to develop a touristic offer for E-bikers must be able not only to propose a set of itineraries that can be attractive for several E-biker profiles, but also to plan a suitable charging infrastructure to cover the demand of energy for all riders. These two decision levels are strictly interrelated, as a poor choice on the charging infrastructure may considerably limit the quality and variety of itineraries offered to E-bikers.

Determining touristic itineraries according to user preferences is a considerably complex problem. Including decisions regarding the charging infrastructure makes the problem remarkably more complex. In fact, on the one hand TAs face budget limits. On the other hand, the cost of installing a charging station in a TD, especially in remote areas, can be significantly high. Indeed the power network is not available everywhere, and different supply systems may be needed. Moreover, depending on the itineraries, the location of chargers must be strategically defined, in order to maximize the coverage and the utility. In summary, the variety and extension of touristic bike trail areas gives TAs several degrees of freedom for the strategic (i.e., long-term) planning of the E-bike charging infrastructure, but budget constraints can hinder their effort to satisfy a wide range of trail users. Therefore it is of utmost importance that this be approached as an optimization problem.

The main focus of this paper is to provide TAs with a suitable decision-support tool that helps in making decisions about the design of a complete network of E-bike itineraries, together with a set of locations where charging stations should be installed. A given budget limit must be fulfilled. When designing the network of itineraries, the tool takes into consideration different categories of riders, and for each class it provides a set of dedicated itineraries maximizing a suitable user-specific measure of attractiveness.

The problem of determining touristic offers in terms of packages or itineraries according to the tourist's preferences is a complex problem and it has been addressed in the literature both by qualitative and quantitative methods (see the literature review in Zhu 2022, for example). A consistent stream of literature proposes the adoption of quantitative methods for the touristic route planning (see Giovannini et al. 2016, 2017; Malucelli et al. 2015; Piya et al. 2023; Ruiz-Meza and Montoya-Torres 2021, 2022; Zhao and Alfandari 2020; Zhu 2022, to mention a few). In particular, several contributions focused on the cycle tourist network design and routing (see, for example Černá et al. 2014; Malucelli et al. 2015; Zhu 2022). None of these works addressed the problem of determining the location of the charging stations.

Concerning the literature on the charging infrastructure design for Electric Vehicles (EVs), the literature on quantitative methods is rich, though the main focus is on E-cars (see Shen et al. (2019) for a survey). The context of charging infrastructure for E-bikes in a touristic environment is fairly different from the more general EV case: in the case of E-cars, the itinerary is defined according to a shortest-time logic, while for the cycle tourism the itineraries are expected to maximize the interest of the users. Moreover, roads form a dense and complex network, allowing E-cars flexible itineraries between two points. This characteristic, combined with the presence of a dense electric grid, allows the decision maker to choose the charger positions by adopting flow-capturing models. From a charging perspective, these approaches are inadequate for TDs, where the morphology of the territory and the distance from the electric grid force E-bikers to follow specific trails to reach possible charging points. Moreover, while E-cars have no alternatives other than recharging before running out of energy, E-riders can always use muscular power, or exploit downhills to reach a charger, in case of emergency. As a consequence, although the literature presents a wide variety of formulations and models, the location problem applied to TDs remains unexplored.

Summarizing, quantitative methods exist both in the literature addressing the touristic tour design, as well as the charging infrastructure design for EVs. However, the latter is not suitable to address the specificity of TDs considered in this work. Furthermore, we aim at simultaneously optimizing the design of touristic itineraries for E-bikers maximizing user-specific profiles, as well as determining optimal charger locations by accounting for a given budget constraint. To the best of our knowledge, this problem has not been addressed in the literature, and this is the main motivation of our work.

The contribution of this paper includes 1) the definition of a new mixed-integer linear optimization model (the E-Bike Charger Location Routing Problem, E-BCLRP) and 2) the application of the proposed model to a case study in the Asiago Sette Comuni Plateau, in Italy. The proposed model combines elements of two challenging problems from the literature, i.e., the Location Routing Problem and the Touristic Trip Design Problem. Because our optimization model contains a number of constraints that is exponential in the size of the problem, it would be prohibitive to solve real-world instances. We address this issue by developing a branch-and-cut algorithm that only introduces selected constraints while still solving the problem to optimality. The viability of the proposed method is demonstrated by a computational experiment on a real-world case study.

The rest of the paper is organized as follows. We review the related literature in Section 2. Section 3 provides a detailed problem description (Section 3.1), the problem formulation (Sections 3.2 and 3.3), as well as the description of the solution algorithm (Sections 3.4). We present the case study in Section 4 and provide some concluding remarks in Section 5.

2 Literature

As previously mentioned, the problem tackled in this work has two main decision components: 1) to design a suitable charging infrastructure for the E-bikes, in particular, the location of the charging stations, and 2) to find a set of itineraries for E-bikers with different profiles and preferences.

The first decision component is often treated in the literature by demand coverage models. These models aim to plan the infrastructure so as to cover a discrete set of points representing the potential demand (see for example Dong et al. 2019; Lam et al. 2014; Zhu et al. 2016). The problem of locating E-bike chargers in a TD can be hardly reduced to the aforementioned EV case. Indeed, the demand is not fixed but it is moving in the network. Moreover, the fact that riders tend to select their trajectories according to their specific interests generates a variety of different possible itineraries.

As an alternative, the problem can be reduced to a capacitated facility location, where the facilities are the charging stations and the capacity is defined as the maximum number of vehicle that can be served. In this case the literature is quite wide (see for example Wu et al. 2006), though some adaptation to the specific EV case must be introduced. In our case, the limited flows and the flexibility in the use of E-bikes, makes the capacity issue negligible. For this reason we decided to omit this aspect in our models.

Other contributions relate the charging station location to the flows, rather than to fixed demand points, as for example Cavadas et al. (2015) based on the method proposed in Hodgson (1990). In the case for TDs, it is hard to capture the information about E-bike flows. Indeed the flows can be highly affected by the presence of charging stations, thus the itineraries should be defined in conjunction with the location itself.

The second decision component of the E-BCLRP, can be seen as a particular case of the Tourist Trip Design Problem, which is concerned with optimally selecting a certain number of touristic activities and their sequence for a given tourist profile. Typically, several constraints need to be fulfilled, such as budget, time and distance limits (Ruiz-Meza and Montoya-Torres 2022). A variety of quantitative approaches have been proposed for this problem. The interested reader is referred to Ruiz-Meza and Montoya-Torres (2021) and Ruiz-Meza and Montoya-Torres (2022), providing in-depth literature reviews on the topic.

In relation with the literature on EV routing, the problem of determining an itinerary between a given origin-destination pair minimizing the overall time, including the deviations and the charging time of an EV is known as the The Electric Vehicle Shortest Path Problem (EVSPP). The literature proposes many contributions (see for example Adler et al. 2016; Schiffer et al. 2018) where the key decision is where the EVs are routed and how much to charge at each charging station. These works achieve good results with both exact and heuristic approaches (see for example Froger et al. 2019). These algorithm are at the base of more complex routing problems involving EVs. However, seeking the shortest itinerary is not suitable for our problem, given the leisure setting. A tourist generally aims at maximizing the value of the visited places according to her interest, rather than minimizing the travel time. These types of problems have been studied for muscular bikes in Černá et al. (2014); Malucelli et al. (2015). While these works consider the connection between a single pair of origin-destination sites, in this paper we tackle the problem of defining a set of different itineraries between multiple origins and destinations.

When we simultaneously consider the location of charging stations and the itinerary definition, we fall into the class of the so called Location and Routing problems (Drexl and Schneider 2015). There are specialized contributions in the EV case: for example, Yang and Sun (2015) tackles a location-routing problem for EV battery swapping stations. Such a problem aims at combining the strategic covering problem (such as the location of charging stations in a network) with the routing design, finding a set of itineraries that minimize the total battery swapping station costs and the EVs traveling cost. However, the main focus is still on the travel cost, which is not an issue for our present work.

Our problem can be seen to be related to a network design problem, such as those presented in Giovannini et al. (2016, 2017) for muscular bikes, where the main issue is to select a set of connections to be reconditioned in order to open new itineraries meeting the interests of the users. In that case, though, given that the focus is on muscular bikes, the location of charging stations is not of concern.

In this paper we will build on the models developed in the above mentioned papers to include the E-bike charger location aspects in the definition of the itineraries.

3 The E-Bike Charger Location Routing Problem

This section formally describes the E-BCLRP, provides a mathematical formulation and gives details about the adopted solution method. In particular, Section 3.1 introduces some notation and provides a detailed description of the problem. Section 3.2 gives a MILP formulation for the particular case of E-BCLRP where each itinerary leg and attraction site can only be visited once. In Section 3.3 we show how this formulation can be adapted to handle the case of multiple visits. The solution method is presented in Section 3.4.

3.1 Problem description

The E-BCLRP considers the geographical area within the TD where biking trails are present and assumes that the TA has decision power over it. Relevant parameters of this region are assumed to be known, such as the location of the sites of attraction and the set of existing trail legs. Furthermore, we assume that the morphology of the region is completely known. This information is useful to quantify several other parameters such as the touristic attractiveness of a given trail leg, the corresponding energy consumption, the difficulty rate, the slope, etc. More formally, we assume that we are given a graph $G(N, A)$, where the node set $N = \{1, \ldots, n\}$ represents the sites of interest in the region, and the arc set $A = \{(i, j) \mid i, j \in N\}$ represents all the existing direct trail legs between any two sites $i, j \in N$.

We further assume that TAs identify several categories of e-bikers profiles. For example, we may consider categories such as sporty, gastronomic, classical, family, etc. Each category comes with a given set of features and preferences. Let U be the set of user types. For each arc $(i, j) \in A$ and each user type $u \in U$, we let e^u_{ij} and t^u_{ij} be the energy and the time consumption incurred by user type u when traversing arc (i, j), respectively. Furthermore, we define a^u_{ij} and d^u_i the attractiveness of traversing arc $(i, j) \in A$ and that of visiting node $i \in N$, respectively, for user type $u \in U$.

We consider the problem where TAs must design a set of itineraries that are optimized for each user type, and that ensures a suitable presence of charging infrastructure. In particular, we assume that TAs have identified a set of pairs $P = \{(o, d) \mid o, d \in N\}$ listing all the origin and destination pairs that should be linked by an itinerary. Moreover, we consider that any itinerary for user type $u \in U$ should not exceed a given maximum riding time T_u.

In order to account for e-bike battery levels, we assume a homogeneous e-bike type, with battery capacity E. E-bikes may charge their battery if a charging station is available at a given site $i \in N$.

Generally speaking, not all sites are suitable for charging, and we let v_i be a binary parameter with value 1 if it is possible to install a charging station in site $i \in N$, and 0 otherwise. We let c_i be the installation cost at site $i \in N$. We assume that users charge their bikes before departure, therefore bikes leave the origin fully charged.

The E-BCLRP aims to design one itinerary for each $(o, d) \in D$, and for each user type $u \in U$. Furthermore, the E-BCLRP must determine where to install charging stations among the potential sites with $v_i = 1$. The resulting itineraries must be energy feasible, i.e., the sequence of visited sites must include appropriately-located charging stations that make it feasible to reach the destination. The objective is to maximize the total attractiveness collected by the set of obtained itineraries, while minimizing the total cost of the installed charging stations.

3.2 Problem formulation: the single visit case

In this section we focus on the particular case of the E-BCLRP where, for each user type $u \in U$ and each origin-destination pair $p \in P$, each site $i \in N$ can be visited at most once, and each itinerary leg $(i, j) \in A$ can be traversed at most once. We will see in the next section how the proposed formulation can be easily adapted to the general case where more than one site visit and leg traversal are allowed.

The E-BCLRP is comprised of two main decision levels. The first is concerned with the choice of the sequence of sites and trail legs forming each itinerary. To this purpose, for each origin/destination pair $p = (o, d) \in P$ and each user type $u \in U$, we introduce the binary variable x_{ij}^{up} with value 1 if the itinerary from o to d associated with user type u traverses the trail leg $(i, j) \in A$, and 0 otherwise. Similarly, we let γ_i^{up} be a binary variable with value 1 if the itinerary from o to d associated with user type u passes by the site $i \in N$, and 0 otherwise. These two sets of variables are linked by the following constraints

$$\sum_{j|(i,j)\in A} x_{ij}^{up} = \gamma_i^{up} \qquad \forall\, p = (o, d) \in P,\ \forall\, i \in N \setminus \{o\}, u \in U, \tag{1}$$

$$\sum_{j|(j,d)\in A} x_{jd}^{up} = \gamma_d^{up} \qquad \forall\, p = (o, d) \in P,\ u \in U. \tag{2}$$

The next set of constraints conveys the idea that the x_{ij}^{up} variable expresses a flow from i to j, for each path $p \in P$ and for each user type u. Therefore, for every intermediate node i on the path from source to destination, the total flow entering i must be equal to the total exiting flow, while the flow balance, i.e., entering minus exiting, is negative (resp. positive) at the origin (resp. destination). These so-called flow-conservation constraints can be expressed as:

$$\sum_{j|(j,i)\in A} x_{ji}^{up} - \sum_{j|(i,j)\in A} x_{ij}^{up} = \delta_i^p \qquad \forall\, p = (o, d) \in P,\ \forall\, i \in N, \forall\, u \in U, \tag{3}$$

whose right-hand side is used to discern origin and destination from intermediate nodes: $\delta_i^p = -1$ if $i = o$, $\delta_i^p = 1$ if $i = d$, and 0 otherwise.

Any suitable optimization model must include all and only the solutions that are suitable for the problem; here, variables x must form paths from source to destination. This implies that solutions that do not amount to paths should be excluded. Flow conservation constraints do generate paths but do not exclude solutions where, for an origin/destination pair $p = (o, d)$ and user type u, the value of variables x_{ij}^{up} form a path from o to d plus one or more *subtours*, i.e., cycles in the graph that do not use any arc of the actual path. We must therefore add constraints that exclude subtours:

$$\gamma_v^{up} \leq \sum_{(i,j)\in A | i \notin N^p, j \in N^p} x_{ij}^{up} \qquad \forall\, p=(o,d) \in P, \forall\, v \notin N^p, \forall\, N^p \subset N, \forall\, u \in U, \tag{4}$$

where N^p denotes any subset of nodes containing both o and d. More specifically, given a subset $N_p \subset N$ and its complementary $N \setminus N_p$, such that N_p contains o and d, constraints (4) impose that the total flow from $N \setminus N_p$ to N_p must be at least 1, provided that a node $v \in N \setminus N_p$ is included in the itinerary (and consequently $\gamma_v^{up} = 1$). We note that these constraints are analogous to those introduced by Balas (1989) in the context of the Prize-collecting traveling salesman problem, but specialized for the case of searching for paths, as required for the design of itineraries.

To ensure that the riding time of each itinerary, defined as the total time spent riding and therefore excluding the possible recharge time, does not exceed the limit associated with a given user type, we also have:

$$\sum_{(i,j)\in A} t_{ij}^u x_{ij}^{up} \leq T^u \qquad \forall\, p \in P,\ \forall\, u \in U. \tag{5}$$

Note that, while these constraints may be easily modified to limit the total travel time, thus including charging times, this is not their intended purpose. In fact, charging time can be used by riders for other leisure activities, such as visiting a town or a mountain top.

The constraints presented so far ensure that variables x_{ij}^{up} and γ_v^{up} correctly describe itineraries connecting each origin to its destination. However, the E-BCLRP is also concerned with the choice of where to locate charging stations and the corresponding maximum E-bike battery level for each site in a given itinerary. For this, we introduce a set of binary variables y_i with value 1 if a charging station is installed in site $i \in N$. Furthermore, for each origin/destination $p \in P$, type of user $u \in U$ and site $i \in N$, we introduce a set of continuous variables $b_i^{up} \geq 0$ accounting for the upper bound on the energy stored by an E-bike upon arrival at node i, computed by assuming that the E-bike left the origin of the itinerary fully charged.

We ensure that the battery is fully charged at the beginning of the itinerary by imposing:

$$b_o^{up} = E \qquad \forall\, p=(o,d) \in P,\ \forall\, u \in U. \tag{6}$$

Moreover, given an origin-destination pair $p \in P$ and user type $u \in U$, the following set of constraints ensures the existence of a feasible charging policy for the obtained itinerary:

$$b_j^{up} \leq b_i^{up} - e_{ij}^u x_{ij}^{up} + E v_i y_i + (1 - x_{ij}^{up})E \qquad \forall\, p \in P,\ \forall\, (i,j) \in A, \forall\, u \in U, \tag{7}$$

$$b_j^{up} \leq E - e_{ij}^u x_{ij}^{up} \qquad \forall\, p \in P,\ \forall\, (i,j) \in A, \forall\, u \in U. \tag{8}$$

When $x_{ij}^{up} = 1$, constraints (7) ensure that that the battery levels of an E-bike at node j is bounded by the battery level at node i minus the battery consumption on arc (i,j) if no charging station is installed in i, and a trivial upper bound otherwise. When $x_{ij}^{up} = 0$, constraints (7) are not active. In conjunction with constraints (8), these constraints also ensure that the battery level is bounded by E when a charging station is visited.

The total cost for the installation of charging station should not exceed a given budget F. We enforce this by imposing the following constraints:

$$\sum_{i \in N} c_i y_i \leq F. \tag{9}$$

Finally, below is the objective function:

$$\max_{b\ y\ x\ \gamma} \quad \sum_{p\in P}\sum_{u\in U}\Big(\sum_{(i,j)\in A} a_{ij}^{u} x_{ij}^{up} + \sum_{i\in N} d_i^u \gamma_i^{up}\Big). \tag{10}$$

In the above expression, the first and the second term maximize the attractiveness collected by the set of visited itinerary legs and sites, respectively.

3.3 Extension to multiple visits

The model presented in the previous section designs itineraries between origin/destination pairs $p \in P$ in such a way that a given user type $u \in U$ can visit a site $i \in N$ at most once, and can traverse an itinerary leg $(i,j) \in A$ at most once. However, it is not uncommon for bikers to pass by the same site or itinerary leg more than once, especially if the site or the leg are particularly attractive, or because the geographical configuration requires it. In these cases it is reasonable to assume that the attractiveness for subsequent visits decreases with the number of visits (see, for example Giovannini et al. 2016).

The design of itineraries with multiple visits has been already addressed in the literature (Giovannini et al. 2016, 2017) for the case of muscular bikes. The proposed models typically assume a bound on the maximum number of visits and introduce a large amount of additional variables and constraints to track the number of visits to a given site or itinerary leg. The resulting problems are rather complex. Furthermore, these models are not able to account for the location of charging stations and the consequent battery charging levels.

This paper proposes an alternative modeling strategy, which mainly consists in suitably transforming the original graph $G(N, A)$ described in Section 3.2 into a new larger graph $\tilde{G}(\tilde{N}, \tilde{A})$. The advantage is that the relatively simple structure of the model in Section 3.2 is preserved. For simplicity, in the following we describe the proposed technique for the case of at most two visits at the same site or itinerary leg, which fits most practical cases. However, the procedure can be further generalized to the case of more than two visits, to the expense of dealing with larger graphs.

We first describe the graph transformation. The main idea is to have two copies of the original graph organized in two levels. Visiting a node or an arc in the lower level is interpreted as a first visit, while visiting a node or an arc in the upper level is interpreted as a second visit. Clearly, it may happen that an itinerary visits some nodes or arcs (in either direction) only once, and others twice. Therefore, we also need arcs connecting the two levels.

More formally, we let $\tilde{N} = N \cup N'$, where $N = \{1, \ldots, n\}$ represents the lower level and is defined as in Section 3.2, while $N' = \{n+1, \ldots, 2n\}$ is the upper level. For all $i \in N$, the node $i+n \in N'$ is a copy of i in the second level, and it represents the second visit to the same site i. Similarly, consider the set of arcs of the first level $A \subset N \times N$, as defined in Section 3.2. For each arc $(i,j) \in A$ we introduce a copy in the second level and another two arcs connecting the two levels. Specifically, we let the set of arcs between nodes of the second level be $A' = \{(i+n, j+n) \mid (i,j) \in A\}$. The third group of arcs A'' links the two levels and we have $A'' = \{(i, j+n) \mid (i,j) \in A\} \cup \{(i+n, j) \mid (i,j) \in A\}$. We then let $\tilde{A} = A \cup A' \cup A''$. Figure 1 shows the two-layer configuration for two nodes i and j connected by an arc $(i,j) \in A$. Note that if arc (j,i) also exists, the two-layered graph will have eight arcs between i, j, and their upper-level counterparts.

The energy and time consumption of an arc $(i,j) \in A$ is the same for all its copies, hence for each $(i,j) \in A$ we have $e_{ij}^u = e_{i+n,j+n}^u = e_{i,j+n}^u = e_{i+n,j}^u$. Similarly, for each $(i,j) \in A$, we have $t_{ij}^u = t_{i+n,j+n}^u = t_{i,j+n}^u = t_{i+n,j}^u$. As previously mentioned, we assume that the second visit is less

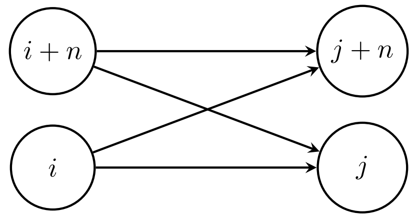

Figure 1: Two-layered structure for arc (i, j).

attractive than the first visit. Therefore, for each node $i \in N$, we have that $d_i^u \geq d_{i+n}^u$. A similar concepts holds for the attractiveness of the arcs. Arcs in A and in A'' represent first visits, thus for each $(i, j) \in A$, we have $a_{ij}^u = a_{i,j+n}^u = a_{i+n,j}^u$. Arcs in A' represent second visits, therefore we have $a_{ij}^u \geq a_{i+n,j+n}^u$. This concludes the description of the graph $\tilde{G}(\tilde{N}, \tilde{A})$ and its attributes.

We now specify the few modifications needed to adapt the model in Section 3.2 to the new graph $\tilde{G}(\tilde{N}, \tilde{A})$. First, with the exception of constraints (9), which can be kept as they are, any occurrence of N and A should be substituted by $\tilde{N}$ and $\tilde{A}$. Then, to correctly account for the fact that installing a charging station in node $i \in N$ is equivalent to also installing it in its copy, we impose:

$$y_i = y_{i+n} \qquad \forall i \in A. \tag{11}$$

The following constraints ensure that the second traversal between i and j must use arcs of the second level:

$$x_{ij} + x_{i,j+n} + x_{i+n,j} \leq 1 \qquad \forall (i, j) \in A. \tag{12}$$

Finally, it can be shown that, due to recharging restrictions, it could be optimal in some particular cases to traverse an arc belonging to the second level of the graph without passing for the corresponding arc of the first level. If this happens, we need to be sure that the attractiveness attributed to the second level arc corresponds to the correct first-visit value. To this purpose, for each origin/destination pair $p \in P$, each user type $u \in U$ and itinerary leg $(i, j) \in \tilde{A}$, we introduce a new set of non-negative continuous variables q_{ij}^{up} representing the value of the true attractiveness. For all arcs representing first visits, i.e., in A or in A'', we simply impose q_{ij}^{up} to be equal to the original attractiveness value:

$$q_{ij}^{up} = a_{ij}^u x_{ij}^{up} \qquad \forall (i, j) \in A \cup A''. \tag{13}$$

For the arcs in A' we impose:

$$q_{i+n,j+n}^{up} \leq a_{i+n,j+n}^u x_{i+n,j+n}^{up} + (a_{ij}^u - a_{i+n,j+n}^u)(1 - x_{ij}^{up}) \qquad \forall (i, j) \in A, \tag{14}$$

$$q_{i+n,j+n}^{up} \leq a_{ij}^u x_{i+n,j+n}^{up} \qquad \forall (i, j) \in A. \tag{15}$$

The above constraints ensure that if the second-level arc $(i+n, j+n)$ is visited, but the corresponding first level arc (i, j) is not, we correctly associate the first-visit attractiveness to it, i.e., a_{ij}^u. In all other cases, the attractiveness associated to arc $(i + n, j + n)$ is $a_{i+n,j+n}^u$. It is worth noting that constraints (14) can be strengthened as follows:

$$q_{i+n,j+n}^{up} \leq a_{i+n,j+n}^u x_{i+n,j+n}^{up} + (a_{ij}^u - a_{i+n,j+n}^u)(1 - x_{ij}^{up} - x_{i,j+n}^{up} - x_{i+n,j}^{up}) \qquad \forall (i, j) \in A. \tag{16}$$

The above expression is valid because, as stated in constraints (12), $x^{up}_{ij} + x^{up}_{i,j+n} + x^{up}_{i+n,j} \leq 1$. Examples of optimal paths that traverse one arc more than once are given in the next section. Finally, the objective function needs to be modified as follows:

$$\max_{b\ y\ x\ \gamma} \quad \sum_{p\in P}\sum_{u\in U} \Big(\sum_{(i,j)\in\tilde{A}} q^{up}_{ij} + \sum_{i\in\tilde{N}} d^u_i \gamma^{up}_i \Big). \tag{17}$$

3.4 The branch-and-cut algorithm

The models presented in Sections 3.2 and 3.3 are both Mixed Integer Linear Programs (MILPs). This type of mathematical programs is well studied, and commercial solvers can tackle MILPs of considerably large size. When addressing our models, however, constraints (4) represent a major challenge: there is one such constraint for each subset of N (and for each user type), hence including them all would yield a model with a number of constraints that grows exponentially with the number n of sites. As a consequence, merely entering these models in a commercial solver is not practical, even for small problem instances. We thus resorted to a Branch-and-Cut Algorithm (BCA) (Padberg and Rinaldi 1991). We now illustrate the BCA applied to the model in Section 3.2. The technique trivially applies also to the model in Section 3.3.

BCAs proceed as follows. First, integer restrictions are relaxed to obtain the linear relaxation of the original problem. In our case, integer restrictions are substituted by the following inequalities:

$$0 \leq x^{up}_{ij} \leq 1 \qquad \forall\ p \in P,\ u \in U,\ (i,j) \in A, \tag{18}$$

$$0 \leq \gamma^{up}_{i} \leq 1 \qquad \forall\ p \in P,\ u \in U,\ i \in N, \tag{19}$$

$$0 \leq y_i \leq 1 \qquad \forall\ i \in N. \tag{20}$$

The resulting linear program is then solved by the Cutting planes algorithm (Gilmore and Gomory 1961). This technique is an iterative procedure that generates constraints (4) dynamically. Each iteration of the Cutting plane technique solves a so-called Master Problem (MP), which is a continuous relaxation of the original problem that only contains a relatively small number of constraints (4). The solution obtained from the MP is then checked for feasibility with respect to constraints (4) via a so-called Separation Problem (SP). If the solution to the SP indicates that the current solution violates some of the constraints in (4), they are added to the MP, and the process is iterated. Otherwise, the current MP is optimal. Integrality constraints are then progressively recovered by implicit enumeration.

For the case of the E-BCLRP, the SP is similar to what is presented in Balas (1989), but it requires to be adapted to the case of searching for paths, instead of cycles. In particular, let $\bar{x}^{up}_{ij}$ be the current MP solution, and for each $u \in U$ and $p \in P$ let us consider the capacitated graph $\tilde{G}^{up}(N, A)$, where the capacity of each arc $(i, j) \in A$ is given by the value $\bar{x}^{up}_{ij}$. For each $v \in N$ different from o and d, the SP searches for the maximum flow between v and d. This is equivalent to identifying two partitions of N, namely N^p, which contains o and d, and its complement $N \setminus N^p$, containing v, with a minimum cut, i.e., with the minimum value of

$$\sum_{(i,j)\in A: i\notin N_p, j\in N_p} \bar{x}^{up}_{ij}. \tag{21}$$

Then, for each v and the above choice of N_p, the SP checks if constraints (4) are violated, and if violated, the corresponding cut is added to the MP. The process is iterated for each MP solution, for each $u \in U$ and $p \in P$, until no violated inequality is found. The solution method then normally proceeds to the branching phase of the implicit enumeration procedure.

4 An application to the Asiago Plateau resort

We have first applied the algorithm described in the previous section to a set of randomly generated instances with a number of nodes ranging from 15 to 50 to validate our approach. In order to simulate realistic conditions of a touristic area, we randomly generated mountain profiles, locations at various altitudes and bike trails between them, all with varying attractiveness, then ran our model on each. This validation step was necessary to ensure that the optimization algorithm only finds viable solutions, that the attractiveness is correctly computed for all optimal solutions found, and that the algorithm is suitable for instances of large-scale instances. Interested readers are referred to Massetti (2022) for details.

In this section, however, we report on applying our algorithm to a network of bicycle paths and roads in the Asiago Sette Comuni Plateau, located in the province of Vicenza, Italy. Stretching between Bassano del Grappa, a small city at the south-east, and the border with the Trentino-Alto Adige region, the network spans an area of about 50km by 40km. This territory is a true paradise for cycling lovers and mountain-bike excursionists as it offers a wealth of bike routes, with various degrees of difficulty. Until a few years ago, discovering the Plateau by mountain bike was an experience that only trained cyclists could enjoy, because of the long and difficult routes and the hard climbs. Lately, however, the area became accessible to less proficient cyclists thanks to the advent of E-bikes. The area boasts several bike rental facilities that provide E-bikes at the trailheads, thus spurring the growth of bicycle tourism in this region.

4.1 Construction of the trail network

We identified 28 nodes that are connected to one another through bicycle trails or normal roads and constructed a network that is depicted, with an underlying map, in Figure 2. All nodes are either points of interest for touristic, gastronomic, or sport-related purposes (e.g. mountain tops, town centers, viewpoints, etc.). They are numbered from 0 to 27 and listed in Table 1. We have made the assumption that there are three types of locations in this trail network: (a) locations, which we define of type T1, where a CS can be connected to the power grid; (b) locations that are far removed from the power grid, where a more sophisticated CS must be installed as it requires to generate its own power through, for instance, photovoltaic panels or a wind turbine; we call this type T2; and finally (c) locations where a CS cannot be installed.

CSs of types T1 and T2 are available on the market with different prices. Instead of using an absolute value for the CS cost c_i at each location and for the budget F, we have assigned a cost of c for CSs of type T1 and a cost of $2c$ for those of type T2, and assign the budget F a multiple of c. Values of F that are not multiples of c are redundant as they can be strengthened to $c\left\lfloor\frac{F}{c}\right\rfloor$ in constraint (9) due to integrality constraints on variables y_i. Column "Type" in the table contains the CS type as described above. Locations where no charger can be installed are marked by "–". For these locations it is sufficient to set $y_i = 0$.

Attractiveness of nodes and edges was set by taking into account the type of locations and connection: for instance, an edge corresponding to a steep climb has a lower attractiveness for tourists and gastronomical users, but higher for sporty users, while village centers with renowned restaurants have a higher attractiveness for the gastronomical users than the remaining two. Some nodes and edges have the same attractiveness irrespective of the user, for instance mountaintop or

Node	Location	Type	Node	Location	Type
0	Monte Cengio	–	14	Passo Vezzena	T1
1	Sp 349	T1	15	Gallio	T1
2	Cima del Porco	–	16	Monte Fior	–
3	Bassano del Grappa	T1	17	Stoner	T1
4	Malga Col dei Remi	T2	18	Enego	T1
5	Tresché Conca	T1	19	Monte Lisser	–
6	Roncalto – Mela	T1	20	Passo della Forcellona	–
7	Cima Echar	–	21	Malghe Mandrielle e Buson	T2
8	Sasso	T1	22	Monumenti	T2
9	Monte Valbella	–	23	Monte Ortigara	–
10	Asiago	T1	24	Cima della Caldera	–
11	Roana	T1	25	Anepoz	–
12	Malga Erio	T2	26	Primolano	T1
13	Monte Verena	–	27	Rifugio Campo Muletto	T2

Table 1: Locations of interest in the Asiago Sette Comuni Plateau.

scenic routes.

The chosen origin-destination pairs were formed from prior knowledge about known trails spanning multiple edges of our network and providing a good balance of routes for any type of users. Their collective attractions make a few spots viable to be origin or destination of a given route. Moreover, we considered round trip tours starting from the most popular locations. These pairs are as follows:

1. Asiago (10) to Bassano del Grappa (3);
2. Bassano del Grappa (3) to Asiago (10);
3. Primolano (26) to Asiago (10);
4. Asiago (10) to Roana (11);
5. Roana (11) to Gallio (15);
6. Passo Vezzena (14) to Asiago (10);
7. Round trip from/to Asiago (10);
8. Round trip from/to Roana (11);
9. Round trip from/to Enego (18);
10. Round trip from/to Bassano del Grappa (3);
11. Round trip from/to Passo Vezzena (14);

4.2 Battery consumption and power requirements

A model for the E-bike battery consumption is provided below. We consider several factors to determine how much power is consumed in a given trail leg: the e-bicycle dynamics, the terrain type, the weight and physical preparation of the cyclist under analysis. For the sake of simplicity, we have condensed this analysis by taking into account the main factors of power consumption in

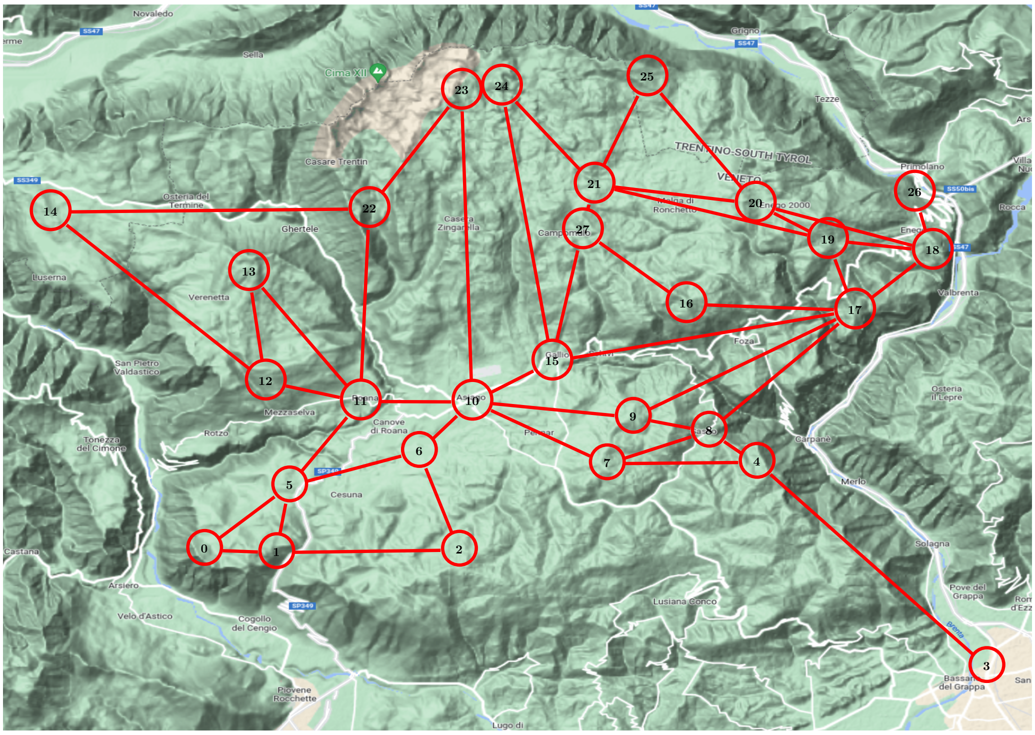

Figure 2: Map of the Asiago Plateau with graph of interest points in overlay. All edges in the graph represent bicycle trails, off-road trails, or normal road, in all cases bidirectional.

an E-bike. The literature provides more detailed estimates on energy consumption in bicycle riding, both battery-assisted and not, see e.g. Giordano et al. (2022).

Because the European classification standards (European Union 2013) dictate that an E-bike can provide a maximum rated power of 250 W, and given that this decreases until the maximum speed of $v = 25$km/h is reached, we consider v to be the maximum cruise speed for all types of user. We also assume that the E-bike is of the Pedelec Drive type, i.e., the motor is activated by the cyclist's pedalling effort and it is switched off as the cyclist stops pedalling. Finally, assuming that the maximum velocity on the cyclist network is 25km/h uphill and 35km/h downhill, we can model the E-bike energy consumption as follows.

When riding uphill, the power required by the electric motor, which we denote as P_{ebike}, is equal to the total power P_{Tot} necessary to keep a constant speed of 25km/h minus the power generated by the cyclist, P_{user}. Specifically, the total power P_{Tot} has three components: the power to overcome drag P_{drag}, friction P_{friction}, and gravity P_{hill}, see e.g. Lim (2004):

$$P_{\text{Tot}} = P_{\text{drag}} + P_{\text{friction}} + P_{\text{hill}}, \tag{22}$$

where: $P_{\text{drag}} = \frac{1}{2} C_d A \rho v^3$ with C_d being the aerodynamic drag coefficient, A the frontal area exposed to drag (estimated in 0.6m^2), $\rho = 0.4\text{kg/m}^3$ the air density, and v the velocity; $P_{\text{friction}} = gMR_c v$ where $g = 9.81\text{m/s}^2$, M the rider+bike mass, and R_c the rolling friction coefficient, estimated in 0.0125 for mountain bikes; $P_{\text{hill}} = gMv\sigma$ where σ is the slope multiplier that makes σv the actual gain in altitude per unit of time.

The parameter P_{user} is a function of the user type, essentially depending on its physical preparation: while professional cyclists can generate 200W to 300W consistently for a few hours, amateur athletes can reach 120W. Based on this value, we also assume that the tourist cyclist can produce 80W, the gastronomic one 60W, and the sporty user 100W.

Battery consumption is therefore determined by the power extracted from the E-bike, $P_{\text{ebike}} = P_{\text{Tot}} - P_{\text{user}}$, but we use a higher value $P_{\text{real}} = \frac{1}{\eta} P_{\text{ebike}}$ with $\eta = 0.8$ to account for efficiency losses that have been overlooked in our condensed analysis before. This yields an estimation of the total energy consumption e_{ij} over a given leg (i, j) depending on the riding time t_{ij}: $e_{ij} = P_{\text{real}} t_{ij}$.

The above considerations do not apply to downhill riding, where the required power to maintain cruise speed is null; note also that we do not consider energy recuperation by downhill riding and braking that occurs in some E-bike models. More details about the experimental setup and results can be found in the fourth author's Master's thesis (Massetti 2022).

4.3 Computational setup

We have used a computer equipped with an Intel i7-10750H processor with 12 cores running at clock frequency 2.6GHz, 32GB of RAM memory, and Linux operating system with kernel 5.15 for all our experiments. The computer has Python 3.9.12 installed. The model was created using the Python-MIP module for Python (Santos and Toffolo 2022) and solved using the Branch-and-Cut algorithm of Gurobi (Gurobi Optimization, LLC 2022).

We wrote a Python script to create the model described in the previous section. The initial model contains variables, objective function, and constraints as described in Sections 3.2 and 3.3, with the exclusion of constraint class (4), which contains a number of constraints that is exponential in the size of the trail graph G. As reported in Section 3.4, rather than generating all such subtour elimination constraints at the beginning, we employ a common feature in BCA implementations such as Gurobi that is callbacks: the solver allows for entry points where the user adds one or more violated inequalities, both at the initial phase before the BCA and during the BCA itself. All other parameters of both Python-MIP and Gurobi were left at default value. In all the experiments, we set a time limit of 8 hours to the computation time. After the expiration of the time limit the models returns the best solution found and an upper bound estimate of the optimal solution.

4.4 Experimental results

We solved the problem with different budget limits F to understand how the total attractiveness of a set of trails depends on the budget. Per our assumption in Section 4.1, the value of F is a multiple of the cost c that we have assumed to hold for the CS of the simpler type. Our chosen values of F range in the set $\{c, 2c, 4c, 8c, 16c\}$; for instance, $16c$ corresponds to a combination ranging between 16 CSs of type T1 and 8 of type T2. For all of our experiments, regardless of the budget limit the computation always stopped after a time limit of two hours imposed on the solver, with a gap around 5% between the best found solution and the upper bound estimate of the optimal solution.

Figure 3 shows the number and the placement of the charging stations with the different budget settings. In the five considered instances, we always exploit all the budget, with the exception of the $16c$ case, where only $10c$ out of the total budget is used for installing four cheap chargers and three expensive ones. This is due to the fact that the model generates only one itinerary per user type. Thus, once the best options per user are covered by the chargers, apparently there is no advantage in having extra chargers, even if the budget would allow for it. Note that the solutions are almost incremental. Indeed, almost all chargers activated for a given budget limit are also present when the budget is doubled. The only exceptions are the charger in node 8 that moves to node 4 when the budget increases above $2c$, and the charger in node 1, which is not activated for budget equalling

8c or 16c.

In Figure 4 we reported the value in terms of attractiveness of the best solution found with the different budget limits, including budget 0€, which is the solution obtained when no charging station is allowed. When the budget is 0€, it is not possible to find an itinerary for some types of users, in particular the gastronomic user in the origin-destination pair involving a large total elevation, as for example the pair 3-10. The plot shows how the value of the solution rapidly increases with the addition of few chargers, while it stabilizes above the 4c threshold. This result points out how the decision on where to install the chargers is crucial, even for a small investment.

The total attractiveness as a function of budget depicted here clearly depends on the cost ratio of CSs of type T2 and T1, and it is possible that different optimal solutions and CS locations would arise from a 3:1 or k:1 cost ratio. We do not provide results for cost ratios other than 2:1, but the fact that with $F = 16c$ one obtains an optimal solution that only costs 10c (see above) seems to indicate that different cost ratios would not radically change the optimal solutions or the attractiveness/budget profile.

Figures 5 and 6 compare the best found itineraries for each type of user obtained without chargers and with the seven chargers resulting from the solution of the 16c case. We considered the case of origin-destination pair 26-10 and the round trip starting from 10. It is remarkable that the presence of the chargers allows the users to reach sites that would be otherwise unreachable, such as scenic spot 20 (Passo della Forcellona). From the quantitative point of view, the increment in the attractiveness when increasing the budget from 0 to 16c is very significant, and even more impressive is the increment in the total elevation that is possible by exploiting the chargers.

5 Conclusions

The deployment of charging stations is a milestone enabling the implementation of touristic E-bike networks. The diffusion of E-bike networks contributes channeling an important sector of the touristic industry towards net-zero objectives. Moreover, it contributes to the long-term and wider effect of activating a positive synergy towards functional cycling, i.e., cycling as a mean of transport, even in those countries without a well-established cycling culture. Since regular cyclists are more willing to commute by bike than others, promoting cycle tourism and enlarging the cyclist community indirectly through e-bikes fosters the use of bike as a means of transport, as the case of the Great Western Greenway in Ireland, described in Deenihan et al. (2013), suggests.

The decision on where to place the E-bike chargers becomes thus a crucial problem to maximize the impact on potential users complying with budget restrictions. In this paper we proposed an optimization approach to this decision support problem. The problem is not only about deciding where to place the chargers, complying with budget constraints, but also about the definition of potential itineraries for different classes of users. This second component is intended to capture the users' needs so as to provide a solution that maximizes the utility for users. The resulting optimization problem is new in the optimization literature and it is computationally challenging. The proposed model applied to the study case of a touristic district provides interesting insights, and demonstrate the practical applicability of the approach to real-life contexts.

6 Acknowledgements

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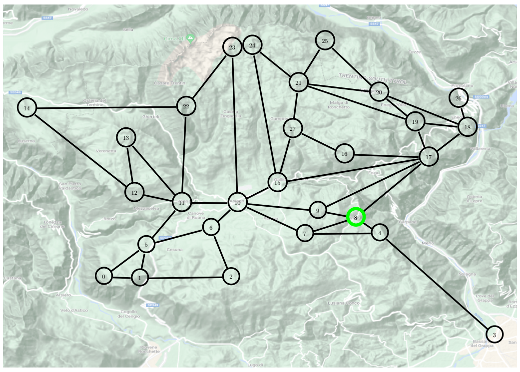
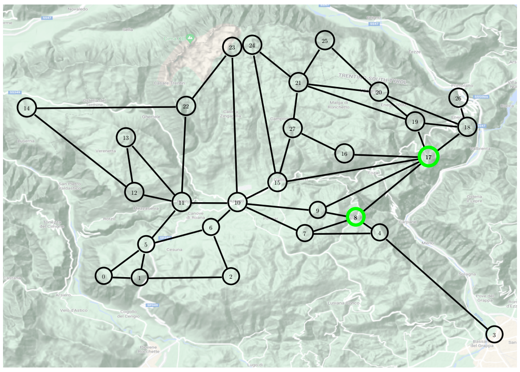
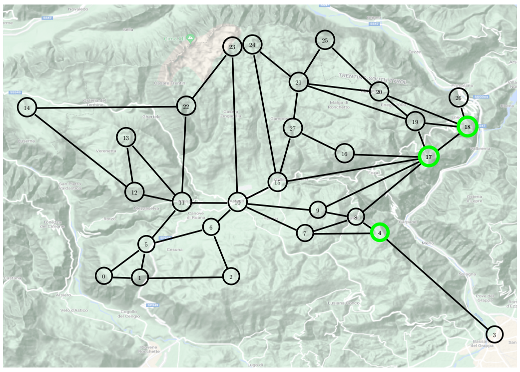


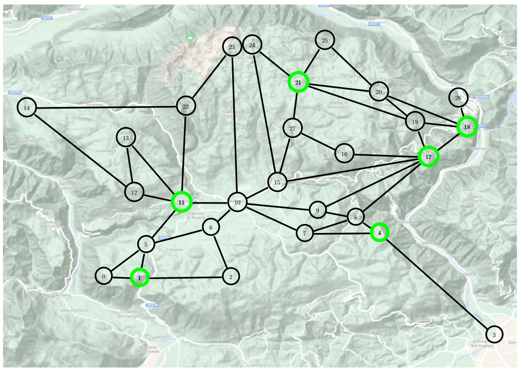
(a) $F = c$



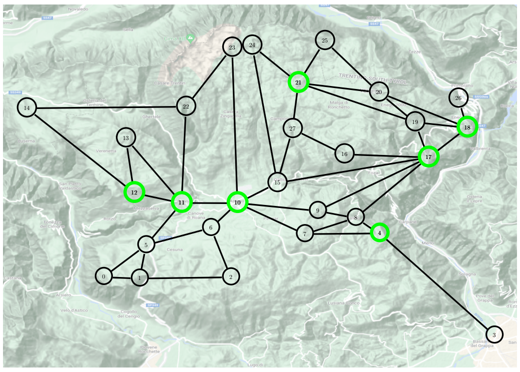
(b) $F = 2c$



(c) $F = 4c$



(d) $F = 8c$



(e) $F = 16c$

Figure 3: Charger location configurations for different values of the budget F

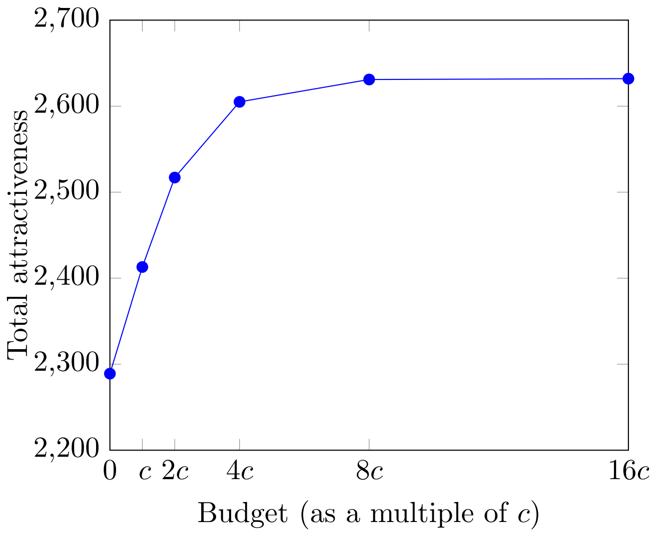


Figure 4: Total attractiveness plotted for several values of the budget F.

Next-Generation EU (Piano Nazionale Di Ripresa E Resilienza (PNRR)—Missione 4 Componente 2, Investimento 1.4—D.D. 1033 17/06/2022, CN00000023). This manuscript reflects only the author's views and opinions; neither the European Union nor the European Commission can be considered responsible for them.

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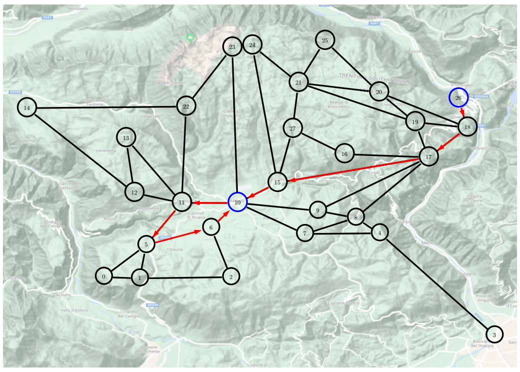
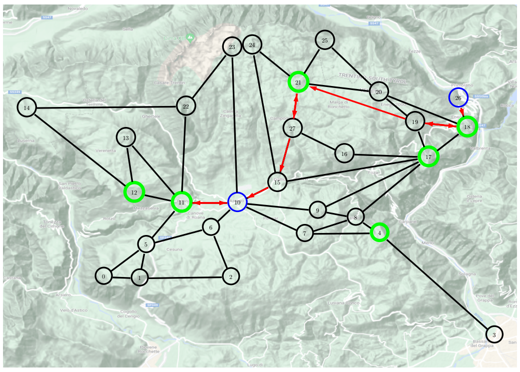
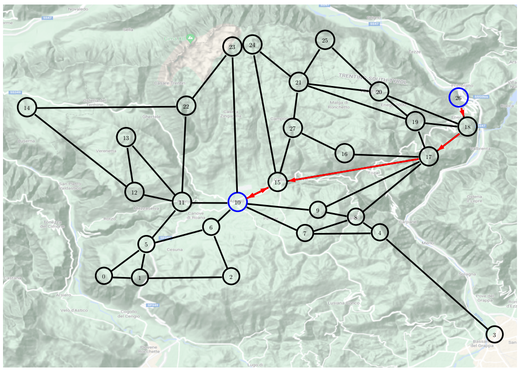


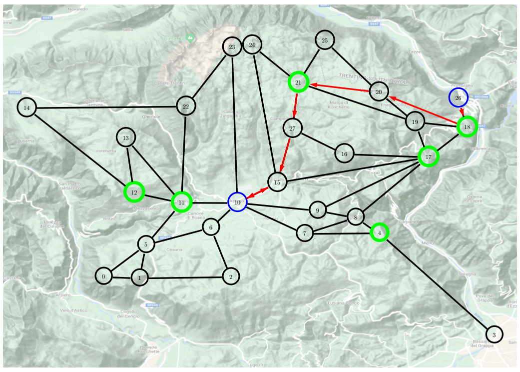
(a) Tourist user. Attractiveness: 40. Total elevation: 920m



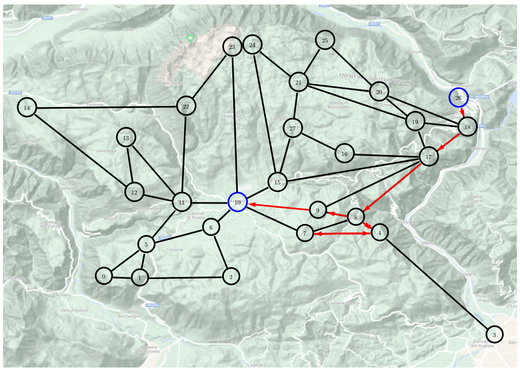
(b) Tourist user. Attractiveness: 99. Total elevation: 2800m



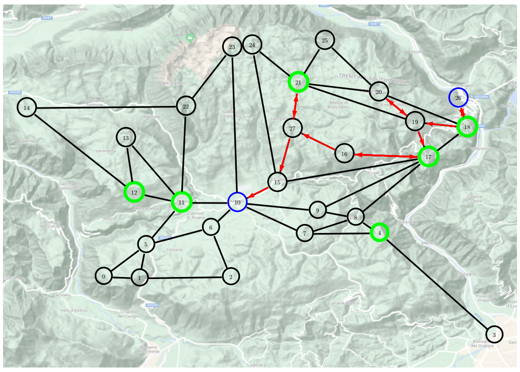
(c) Gastronomic user. Attractiveness: 59. Total elevation: 840m



(d) Gastronomic user. Attractiveness: 71. Total elevation: 1580m

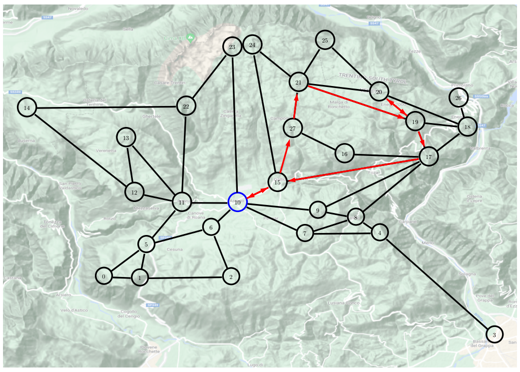


(e) Sporty user. Attractiveness: 81. Total elevation: 1340m

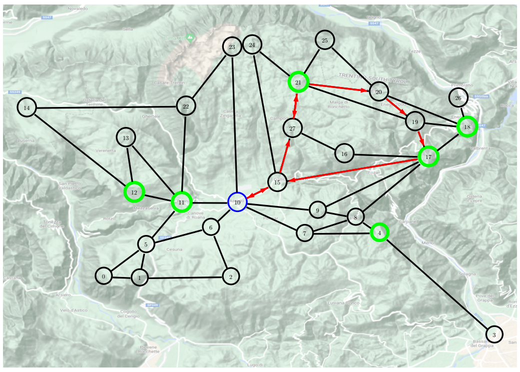


(f) Sporty user. Attractiveness: 118. Total elevation: 3750m

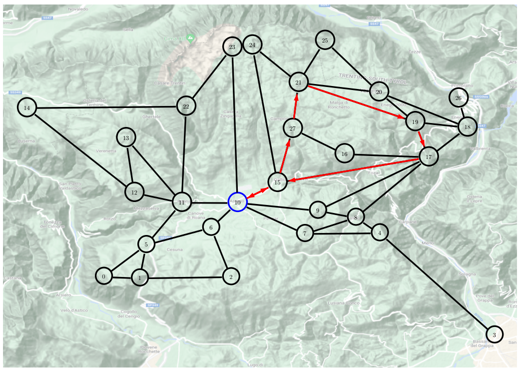
Figure 5: Optimal trails from 26 (Primolano) to 10 (Asiago) for two values of the budget: 0 (left) and 16c (right).



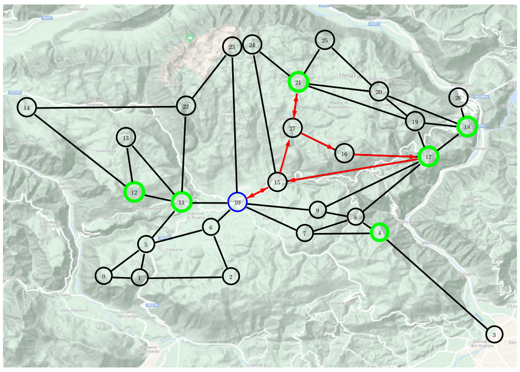
(a) Tourist user. Attractiveness: 82. Total elevation: 1090m



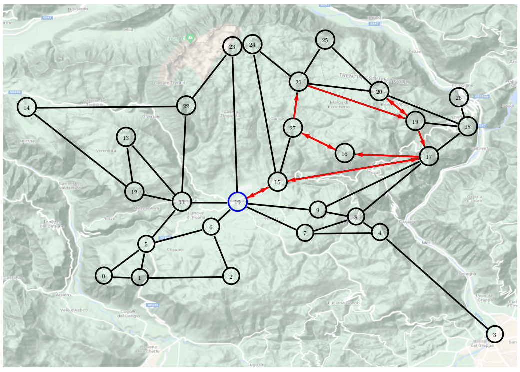
(b) Tourist user. Attractiveness: 89. Total elevation: 1270m



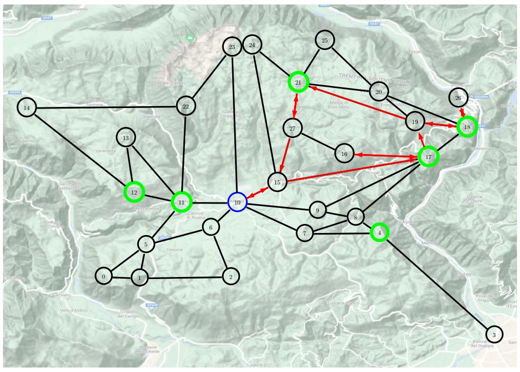
(c) Gastronomic user. Attractiveness: 49. Total elevation: 910m



(d) Gastronomic user. Attractiveness: 53. Total elevation: 1000m



(e) Sporty user. Attractiveness: 99. Total elevation: 1090m



(f) Sporty user. Attractiveness: 109. Total elevation: 3260m

Figure 6: Optimal trails for the round trip 10-10 (Asiago) for two values of the budget: 0 (left) and 16c (right).

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