



A comprehensive data-driven approach to estimate track longitudinal level from inertial measurements

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Abstract

Infrastructure managers rely on diagnostic trains that periodically measure track geometry and vehicle accelerations to ensure the safety of the railway network. Their runs are scheduled depending on the line priority, in order to safely monitor the evolution of track defects. However, sudden and unpredictable defect growth may happen and be missed between successive runs. Therefore, condition monitoring systems have been installed on in-service vehicles. In fact, these trains run every day along the same line, so they can provide additional information useful for maintenance practices. When trains run along conventional lines, their speed significantly changes depending on the line characteristics, and vehicle accelerations strongly depend on speed. Therefore, monitoring systems that rely on vehicle accelerations should carefully take this effect into account. In this paper, a methodology to estimate the track longitudinal level using bogie accelerations from an in-service vehicle is presented. The recorded accelerations were double-integrated to account for the speed variation, and a model-based strategy was adopted to reduce the filtering action of the primary suspension. Data were recorded during a two-year monitoring campaign along an Italian railway line. The methodology allowed for the estimation of the longitudinal level along specific track sections, considering statistical measures like the peak value. A maximum error of 1 mm was found between the estimated values and those measured by the diagnostic train (considering a defect with magnitude of 7.5 mm). Therefore, the results showed that it is possible to estimate the peak longitudinal level between the two rails using one single vertical accelerometer installed on the bogie of an in-service vehicle. The results of this research may be used to support the current maintenance strategy with daily estimations of track longitudinal level. It should be noted that specific attention was given only to this type of track geometry parameter, since it often drives maintenance operations. In the future, the possibility to extend the methodology to the estimation of different type of defects, like cross-level and twist, could be considered.

Keywords Rolling stock-based diagnostic system · Railway track monitoring · Railway infrastructure · Track condition · Condition-based maintenance · Predictive maintenance

1 Introduction

Maintenance operations are essential for ensuring the safety and reliability of railway lines and rolling stock. Generally, these maintenance activities are triggered based on the condition of the line. The actual condition can be obtained from the observation of diverse parameters that represent the current state of the line. These inspection activities are made, in general, using dedicated diagnostic vehicles or equipment, and are scheduled at regular intervals to manage and prevent

issues. However, the conventional approach of setting fixed intervals between observations may not ensure the optimal use of time and resources. By employing a more dynamic and data-driven strategy, maintenance efforts can be more effectively aligned with actual needs, thereby enhancing both efficiency and performance.

If the condition of rolling stock and infrastructure is well understood, maintenance operations can be tailored to the actual needs of these railway assets. This enables a transition from a scheduled maintenance strategy to a condition-based maintenance (CBM) approach. By adopting CBM, railway managers and operators can optimise resource utilisation. This approach not only enhances maintenance efficiency but also improves overall performance. Furthermore, with a thorough understanding of asset condition, predictive

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maintenance strategies can be implemented. These strategies allow for even greater optimisation of resource allocation, sustainability, and cost management. Ultimately, CBM and predictive maintenance together enable a more responsive and sustainable approach to maintenance management, ensuring that resources are allocated where they are needed most.

Focusing on railway infrastructure, tracks are typically inspected but not continually monitored. The primary purpose of inspections is to identify potential critical situations. Consequently, the intervals between consecutive inspections are designed to be sufficiently short to detect defects. However, these intervals are often too long to properly monitor the evolution of these defects over time. The lack of continual monitoring implies that, although inspections can identify issues, they may not provide sufficient data to observe the progression of defects and mitigate their deterioration over time. Track condition data are collected using diagnostic vehicles or track recording vehicles (TRV). These vehicles can record both acceleration data and track geometry parameters [1]. Transitioning from inspections to continual monitoring requires more frequent track surveying. However, increasing the frequency of TRV runs often faces feasibility challenges due to associated costs and availability constraints.

Equipping in-service trains with a reduced set of sensors to provide information about the state of the line in the intervals between TRV runs could facilitate the transition from inspections to continual monitoring [2].

Nowadays, modern railway vehicles are increasingly equipped with acceleration sensors specifically for the condition-based maintenance of the train itself [3]. These sensors enable the continual monitoring of critical components, allowing for the early detection of anomalies and the implementation of predictive maintenance strategies [4–6], thereby enhancing safety and operational efficiency. Additionally, the same sensors can also be used for the continual monitoring of the track.

Proposed applications include the monitoring of track irregularities with inertial sensors [7–13], detecting local track defects such as squats [14], and monitoring of insulated joints [15]. Sensors can be installed on various components of the vehicle, the specific configuration depending on the application and defect type. Specifically, research has shown that a reduced set of sensors mounted on the bogie, based on accelerometers or gyroscopes, allows to observe vertical and lateral irregularity [16–19]. A combination of both accelerometer and gyroscope has even been used to account for the filtering effect of the bogie wheelbase [20]. Other applications have shown the use of axle-box mounted accelerometers to monitor rail corrugation and roughness [21–23]. Finally, applications involving sensors mounted on the car body have also been proposed [24–27].

While accurate results have been achieved with current systems, they may not be practical for commercial applications, especially when considering a fleet of vehicles. In fact, for systems relying on signal histories collected by instrumented vehicles, the primary challenge lies in the substantial amount of data that must be managed, analysed, and aggregated to produce concise information about the condition of the track. This process can be resource-intensive and complex, making it less suitable for widespread commercial use. In this context, monitoring statistical measures offers a viable alternative. Statistical measures can simplify the data management process by focusing on key metrics that are easier to handle and interpret. For instance, current maintenance procedures often rely on peak and root-mean-square (RMS) values as parameters to determine the need for maintenance [28].

A more advanced approach would involve using statistical measures from in-service vehicles. For example, in [29], track degradation was monitored by assessing the standard deviation of track geometry measured by TRV. Previous studies from the authors [30–32] have demonstrated that statistical measures, derived from the standard deviation of acceleration measurements, can effectively monitor the condition of high-speed railway lines.

The use of these approaches for the continual monitoring of conventional lines poses additional challenges: monitoring methods must account for speed variation which can be significant due to the more frequent presence of station stops. Additionally, conventional trains often experience speed variability due to factors such as mixed traffic, varying track conditions, and operational restrictions in urban areas. In reference [33], the authors demonstrated that the standard deviation of the longitudinal level, obtained from vertical displacement of the bogie, is more effective than vertical acceleration, when significant speed variations are recorded along the line.

This paper presents the development of the methodology for estimating the track longitudinal level, in terms of peak value between the two rails in predefined spatial windows, with the aim of achieving its continual monitoring. First, the concept of the monitoring system is introduced. It is designed to operate with in-service trains in conjunction with the TRV. This integration facilitates continual monitoring of the longitudinal level. The system is based on the use of statistical measures that represent the state of the line in a given zone. Following this, the data processing methodology is described, consisting of three steps:

1. double integration of the measured accelerations to obtain vertical displacement;
2. compensation for the geometric filtering effect introduced by the bogie wheelbase;

- fusion of the data from the commercial vehicle with the crest factor of the longitudinal level measured by the TRV, to account for different types of defects, i.e. isolated and distributed defects.

Steps 1 and 2 are not intended as first estimation of the longitudinal level but represent signal processing aimed at minimising the influence of the vehicle speed and of the vehicle response to the different wavelengths.

The estimated peak values of longitudinal level are obtained through a multiple regression model that combines statistical measures to estimate the peak longitudinal level within a given section of the line.

The proposed solution is intended to complement TRV measurements, eventually leading to their rarefication. In this respect, particular emphasis is put on the capability to estimate with sufficient accuracy the peak value of the longitudinal level between the two rails, which drives current maintenance operations, and to trace its degradation over time.

2 Condition monitoring system

In this section, the system for monitoring the conditions of the track geometry is presented. Out of different parameters defined by the standards, the attention is here directed to monitoring the track longitudinal level, interpreted as the vertical deviation with respect to the reference running table [34].

The condition monitoring system shall be installed on an in-service vehicle to estimate the track longitudinal level by means of a predetermined regression model. Since commercial vehicles run with a much higher frequency on a given line with respect to a diagnostic train, the time elapsed between successive data acquisitions is greatly reduced, which allows the continual monitoring of the line conditions.

In this context, it would be unfeasible to install a sophisticated measuring system because of the high cost of the equipment, its maintenance and the significant amount of data to be managed. Therefore, the designed measuring system to be installed on a commercial vehicle is based on a reduced setup of sensors. In the considered application, a vertical accelerometer (monoaxial MEMS accelerometer with measuring range $\pm 5g$, nominal sensitivity 540 mV/g, spectral noise $20 \mu g/\sqrt{\text{Hz}}$, sampled at 1000 Hz) is mounted on the bogie, in a position close to its geometrical centre. The sensor position follows the same setup installed on the Italian Frecciarossa 1000 high-speed train. Bogie mounted accelerometer appears preferable for the application, being subjected to lower acceleration levels than axlebox accelerometer, thus requiring smaller measuring range (better

signal-to-noise ratio), the installation and maintenance is simpler all resulting in increased reliability and availability.

In addition, on the specific vehicle, a global navigation satellite system (GNSS) is installed, and a dedicated geo-localisation algorithm runs on-board the train, with a map-matching procedure based on GNSS data and odometry [35].

To reduce the quantity of data transmitted, the use of statistical measures that represent the state of a given section of the line was already adopted and validated by the authors in [30–32] where a condition monitoring system for high-speed applications was presented. Specifically, given that maintenance operations are triggered when the magnitude of the defects exceeds predefined thresholds, the proposed monitoring system estimates the peak value of track longitudinal level adopting pre-built regression models. Different models were proposed in [30–32], relying on the standard deviation of bogie vertical acceleration as regressor. Both the acceleration standard deviation and the peak value of the longitudinal level were evaluated considering predefined sections of the line, each one 100 m long (computed by means of the purposely defined geo-localisation algorithm [35]).

Once the working principle of the condition monitoring system developed in [30–32] has been recalled, attention is devoted to the modifications required for its extension considering variable speed. The described condition monitoring system is visually presented in Fig. 1. Note that the scheme applies to both high-speed and conventional lines application, with constant or variable speed. Depending on the type of railway line, the pre-processing stage is modified, as later discussed.

Since the described monitoring system installed on a commercial vehicle is not directly measuring the track longitudinal level but the vertical acceleration of the bogie, it is necessary to determine the relation between the two quantities. To do so, a data pre-processing stage is established, that allows determining the statistical measures from the acceleration measurement. A detailed description of the pre-processing steps will be presented in Sect. 3. Then, these statistical measures can be related to the longitudinal level by means of new (purposely built) linear regression models.

A training phase is required to define the regression model, as highlighted in Fig. 1. To this aim, the data collected by the commercial train and the direct measurements of longitudinal level coming from the diagnostic train in the same period of time are used. Once the linear regression model has been defined, the operational phase follows, which allows estimating the longitudinal level based on the acceleration levels daily recorded by the in-service vehicle. Note that when new data from the diagnostic train are available, the regression model can be also updated.

Specifically, if the monitoring system is intended to be used along conventional lines, it should account for the fact that trains run at different speed depending on the line

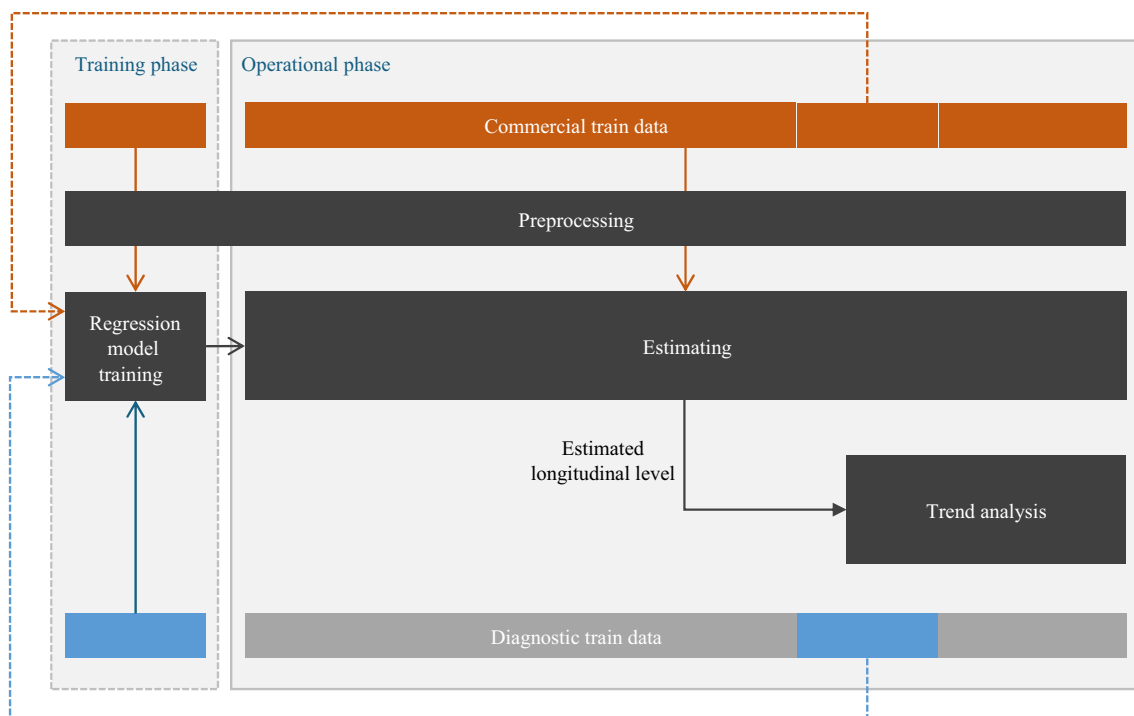


Fig. 1 Condition monitoring system

characteristics and they perform several steps to serve the railway stations. Thus, one of the major challenges for a condition monitoring system that relies on vehicle accelerations consists in properly accounting for the speed variation, as it is well known that acceleration levels have a huge dependency on the speed of the vehicle [36]. The second challenge regards a major drawback related to the adoption of a single-sensor setup, related to the geometrical filter introduced by the bogie geometry. To tackle both topics, acceleration signals are pre-processed before using them as input to the linear regression model. In the following section, the methodology adopted is explained in detail.

3 Methodology

In this section, the methodology developed to monitor the longitudinal level of the railway track from inertial measurements is presented. The main objective is the determination of the longitudinal level in D1 range that, according to the European standard [34], is defined as the vertical deviation of the running table of any rail including defects with wavelengths in the 3–25 m range. The attention is posed to this specific wavelength range as it is strictly related to running safety, and thus, they often cause track repair along railway lines.

The monitoring system under design must respond to two main requirements. At first, the system should be suitable

to be adopted along conventional lines; therefore, it needs to account for the speed variation along the railway line. Secondly, the system should be based on a reduced sensor setup, thus allowing an easy integration of the measuring system in the current commercial fleet.

To meet the requirements, a single sensor was selected and installed at the centre of the bogie to measure its vertical acceleration. For what concerns the effect of speed, it is well known that the vehicle response (and thus the acceleration level) is highly dependent on the vehicle speed. Therefore, to better understand the behaviour of the vehicle, a simplified multibody model was used to simulate the bogie vertical response. The adopted model is shown in Fig. 2a and consists of 6 degrees of freedom (DOFs), which are the bounce and pitch motions, respectively, of the car body, of the front and of the rear bogies. Primary and secondary suspensions were modelled as Maxwell elements to reproduce the rheological behaviour of the damper components. All model parameters were selected to reproduce the dynamic response of the vehicle to be instrumented.

The railway vehicle model was then used to calculate the frequency response functions (FRFs) relating the front bogie response to the track irregularity. To analyse the effect of the speed variation, the FRFs were calculated for different vehicle speeds, ranging from 100 to 300 km/h. The vertical response of the front bogie was computed in terms of both vertical acceleration and displacement (at the centre of the bogie, where the measurement system is installed).

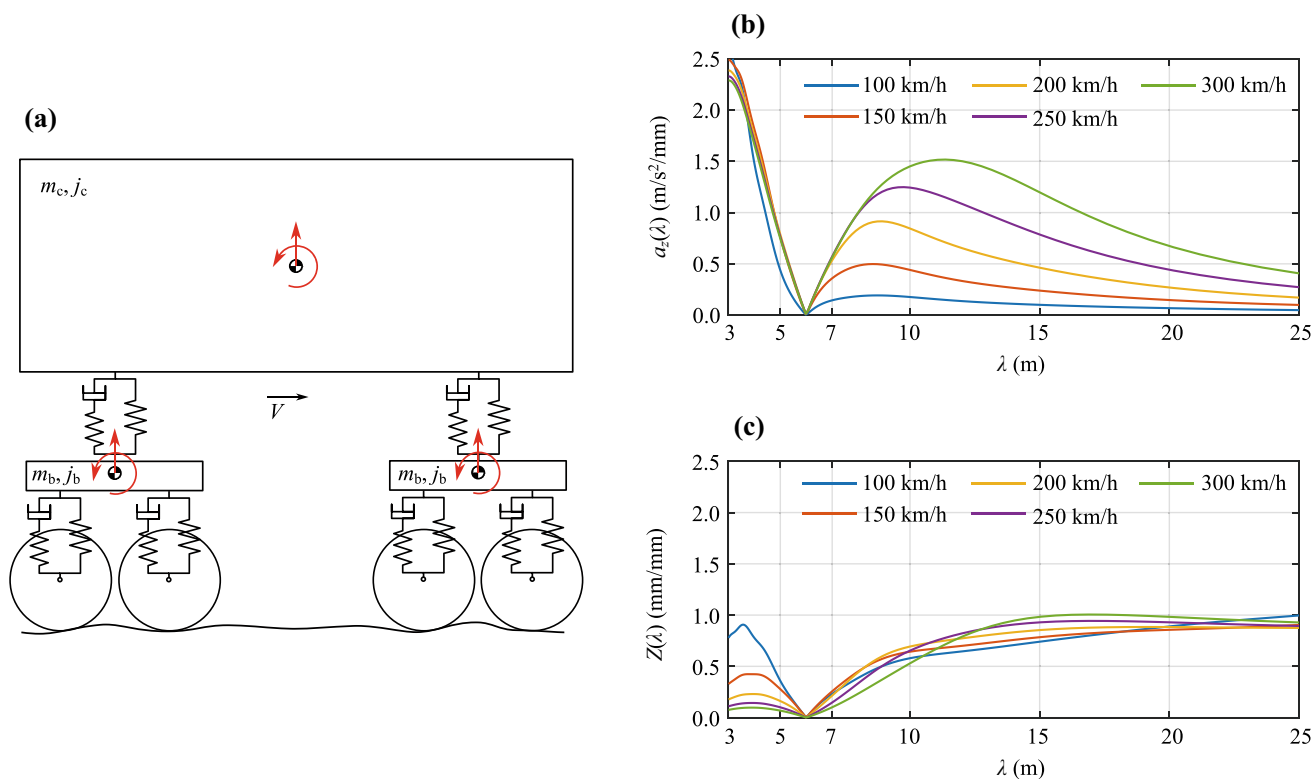


Fig. 2 Frequency responses of the front bogie centre to track longitudinal level as a function of the wavelength λ of track irregularity: **a** vehicle model travelling at constant speed V , composed by two bogies (with mass m_b and moment of inertia J_b) and the carbody (with mass m_c and moment of inertia J_c); **b** FRF of the vertical acceleration a_z ; **c** FRF of the vertical displacement Z

Figure 2b and c, respectively, shows the FRFs in terms of bogie vertical acceleration and displacement, for the different vehicle speeds. The FRFs are presented as a function of the wavelength of the track irregularity consistent with the D1 range.

The FRFs shown in Fig. 2b and c highlight how the dynamic response of the bogie (both in terms of vertical acceleration and displacement) depends on the travelling speed of the vehicle. In addition, it is possible to observe the geometrical filter introduced by the bogie wheelbase (3 m in the considered case), which makes the response of the system go to zero for irregularities with a wavelength twice the wheelbase of the bogie. This result suggests that it will not be possible to observe defects with wavelengths close to 6 m. In the following Sect. 3.2, the strategy adopted to cope with this vehicle characteristic will be presented. However, it can be here anticipated that the railway line under analysis does not show significant contributions of such wavelength to the track longitudinal level.

In the following, an in-depth analysis of the above results will be provided, focusing on the definition of the pre-processing steps required to design the condition monitoring system for conventional lines.

3.1 Data pre-processing—step 1: compensation of the speed dependency by double integration of the acceleration signals

The comparison of the FRFs shown in Fig. 2b and c allows demonstrating that bogie accelerations are affected by vehicle speed more than bogie displacement, as we already analysed also in [33]. This is particularly evident for wavelengths longer than two times the bogie wheelbase (6 m in this case).

This result suggests that bogie vertical displacements can be suitable regressors for estimating statistical measures of the track longitudinal level in those applications where the train speed may significantly vary along the railway line, and also from run to run. With reference to the measurement system installed on the vehicle, it is reminded that a bogie vertical acceleration was considered. Therefore, the first step of the data processing consists in the double integration of the measured acceleration signal.

To reduce the computational effort, the sensor node processes the data in the frequency domain. For the considered 100 m section of the line, at first, the fast Fourier transform (FFT) of the vertical acceleration is computed. Then, the double integration of the signal is performed in frequency

domain. The signal is then filtered into the D1 range computing the standard deviation of vertical displacement s_{VD} from Eq. (1),

$$s_{VD} = \sqrt{\sum_k \frac{1}{2} |Z_k|^2}, \quad (1)$$

where Z_k is the k -th frequency component of the vertical displacement considering the single-side spectrum, and k boundaries depend on the vehicle speed to achieve D1 range filtering.

This procedure is iterated for each 100 m section of the railway line, these sections being defined according to the geo-localisation algorithm running in real-time within the sensor node. This way, a set of statistical measures are collected and can be adopted to estimate the track longitudinal level.

At last, the linear regression model to estimate the peak longitudinal level between the two rails ($L_{\max,VD}$) considering s_{VD} as regressor is realised, as reported in Eq. (2):

$$L_{\max,VD} = \alpha_1 + \alpha_2 \cdot s_{VD}, \quad (2)$$

where α_1 and α_2 are the coefficients of the linear regression model.

To test the methodology, experimental data recorded by a TRV along an Italian railway line were adopted. A monitoring period of 24 months has been considered, with the train running along an Italian railway line 180 km long. In the considered period, the TRV completed 25 runs recording both bogie accelerations and track geometry. The data processing steps previously described have been followed, dividing the railway line in sections of 100 m and computing s_{VD} . For every section, also the peak longitudinal level (L_{\max}) can be computed, considering the longitudinal level directly measured by the TRV. The two information are then gathered in a dataset (L_{\max}, s_{VD}) collected at the same kilometric position. Finally, Eq. (2) was used to fit the data with a linear regression model. Totally, 17 runs out of 25 were considered to train the linear regression model of Eq. (2). This way, different conditions for both rails and wheel profiles are included (i.e. new and worn), providing statistical relevance to the regression model.

To verify the model capability to represent the phenomenon under study, reference is made to the coefficient of determination R^2 . A value of $R^2 = 0.7$ is reached, which demonstrates a rather high degree of correlation between the two variables, confirming the possibility to adopt s_{VD} to estimate L_{\max} . In addition, the value assumed by R^2 turns out to be comparable with the one achieved in [30] for the estimation of L_{\max} along high-speed lines. In that case, we achieved $R^2 = 0.72$ using the standard deviation of bogie vertical acceleration as a regressor since the analysis was carried out at constant vehicle speed.

Once the regression model of Eq. (2) has been realised, the attention can be posed to its practical application. In the following, a specific 100 m section of the railway line is considered, and the peak value of the longitudinal level is represented as a function of time. In Fig. 3a, three markers are used to represent the data: L_{\max} directly measured by the TRV is presented as blue dots; $L_{\max,VD}$ estimated by the regression model of Eq. (2) is reported with yellow squared markers; in addition, $L_{\max,VA}$ is shown with red triangles. The latter is computed considering the standard deviation of the bogie vertical acceleration s_{VA} , making use of a regression model similar to the one of Eq. (2). $L_{\max,VA}$ has been reported to exemplify the improvement in estimation accuracy that is reached when the standard deviation of the vertical displacement is considered instead. For completeness, Fig. 3b shows the train speed for every run.

It is here reminded that 25 train runs were recorded in the overall monitoring period. However, considering the data reported in Fig. 3a, only 18 data points are available for the considered track section. In fact, the acquisition system was not always active for the entire train travel. If reference is made to the direct measurements taken by the TRV (blue dots), it turns out that the defect magnitude shows no significant evolution over time in the considered section of the line. Therefore, no maintenance operations were performed during the entire period.

We focus now the attention onto the estimations made using the linear regression models. The results presented in Fig. 3a show that the estimations made using the bogie vertical acceleration ($L_{\max,VA}$, reported as red triangles) strongly

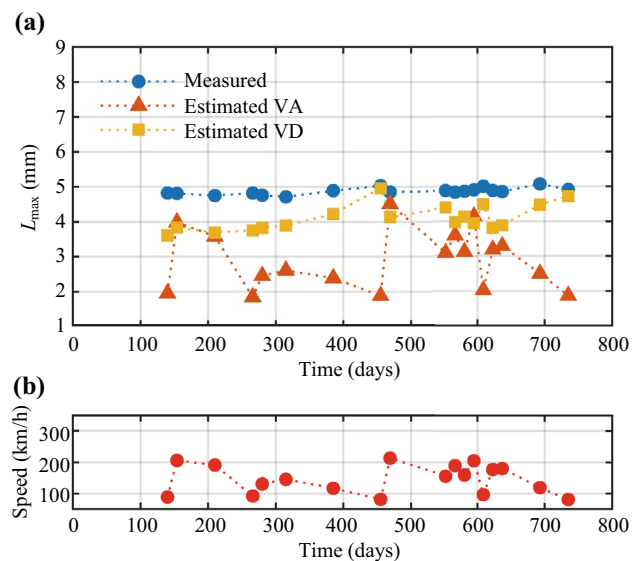


Fig. 3 Comparison of the peak longitudinal level measured by the diagnostic train (L_{\max}) and estimated by different regression models: **a** L_{\max} , $L_{\max,VA}$, and $L_{\max,VD}$; **b** train speed

depend on the vehicle speed. In fact, the trend of the red markers resembles the one of the vehicle speeds shown in Fig. 3b. On the other hand, the estimations achieved considering the bogie displacement ($L_{\max,VD}$, highlighted as yellow squared markers) better follow the direct measurements (blue dots). This result proves that the adoption of the standard deviation of the bogie displacement as regressor mitigates the effect of vehicle speed on the estimation of the peak longitudinal level of track irregularity. However, it is possible to observe that there is still a general underestimation of the defect magnitude that can be associated to the filtering action introduced by the bogie wheelbase. Thus, a second data processing step is introduced, as discussed in the following section.

3.2 Data pre-processing—step 2: compensation of bogie filtering

As can be observed in Fig. 2, the response of the bogie goes to zero for the 6 m wavelength, as a consequence of the filter introduced by the bogie wheelbase. In fact, the bounce motion is not excited when the bogie encounters a vertical track irregularity with a wavelength $\lambda = 2l/n$, where l is the bogie wheelbase and n is a positive odd integer. This behaviour is schematically represented in Fig. 4a, where the displacement of the bogie centre for different track

irregularities is shown. It is worth noting that the bogie filter not only affects the 6 m wavelength (which is completely filtered out), but also the ones immediately before and after. In fact, the amplitude of these contributions is significantly reduced, as visible in Fig. 4b that shows the same FRFs of Fig. 2c and also includes an additional curve, obtained averaging the FRFs at different speed. As a result of this filtering effect, an accelerometer mounted in correspondence of the bogie centre is not able to record specific wavelengths. Therefore, the capability of the system to monitor the vertical track irregularity would be reduced if defects with wavelengths shorter than 7 m are present.

To reduce the effect of the bogie filter, a model-based approach is adopted. The proposed method relies on a weighting function, which is applied to correct the magnitude of the frequency components of vertical displacement (obtained from the integration of the measured vertical acceleration). The weighting function is derived from the reciprocal of the FRF relating the bogie displacement to the track irregularity. The designed weighting function is shown in Fig. 4c with a red solid line, and it is represented for wavelength longer than 7 m. This way, it is possible to compensate for the bogie filtering, avoiding the asymptotic behaviour that characterises the 6 m wavelength. For completeness, a dashed line is used to show the weighting function in the 3–6 m range. It is worth mentioning that a unitary

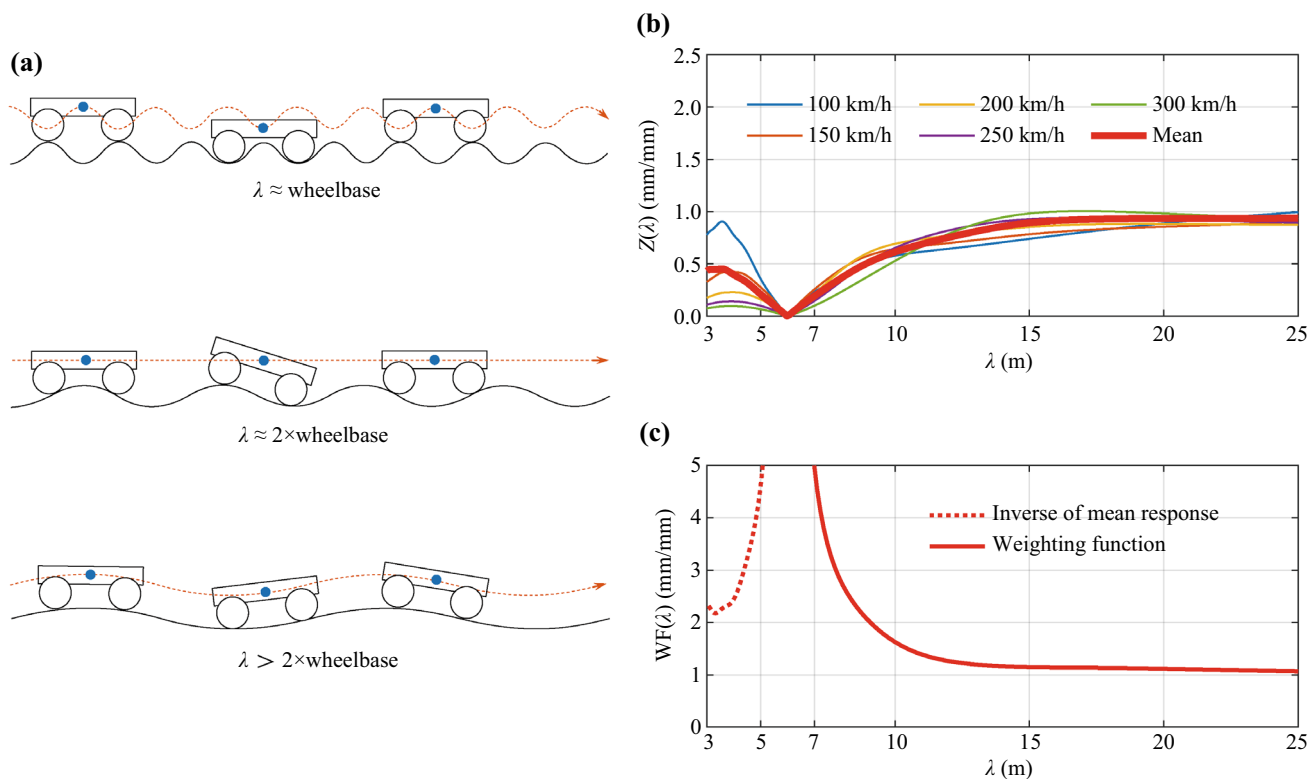


Fig. 4 Bogie filtering effect: a schematic representation; b FRF of vertical displacement for different speeds; c weighting function

weight is considered in this range. This choice, although arbitrary, is based on the experimental evidence that very few defects having such wavelengths are found in the considered railway line, as visible in Fig. 5. In fact, the spectrogram of the longitudinal level recorded by the TRV along the entire railway line exhibits minimal contributions for wavelengths shorter than 6 m. This observation implies that irregularities of such small wavelengths do not significantly contribute to the overall track longitudinal level quality.

Once the principle behind the compensation of bogie filtering has been presented, the detailed implementation is addressed. Even though the bogie vertical displacement is considered, it is possible to observe a dependency of the bogie response with vehicle speed, especially for wavelength shorter than 7 m. In fact, several FRFs are shown in Fig. 4b which are associated to vehicle speed ranging from 100 to 300 km/h. To account for this dependency, the average response function $\bar{Z}(\lambda)$ has been computed (reported as a solid red line) and adopted in the following data processing steps. $\bar{Z}(\lambda)$ is calculated according to Eq. (3):

$$\bar{Z}(\lambda) = \sum_{V_{\text{train}}} Z_V(\lambda), \tag{3}$$

where Z_V is the response of the vehicle at speed $V \in V_{\text{train}} = \{100, 150, 200, 250, 300\}$ km/h.

The weighting function (WF), denoted by $W(\lambda)$ here, corresponds then to the reciprocal of the mean response function $\bar{Z}(\lambda)$, according to Eq. (4):

$$W(\lambda) = \frac{1}{\bar{Z}(\lambda)}. \tag{4}$$

By evaluating the weighting function $W(\lambda)$ at the wavelengths λ_k of interest, the weights w_k corresponding to each frequency component of the vertical displacement $Z_{k,D1}$ are obtained, as detailed by Eq. (5). The conversion between frequency and wavelength is made considering the average speed \bar{v} of the vehicle computed over the 100 m window used to analyse the signals. This is based on the reasonable

assumption that the speed variation within each 100 m section is negligible.

$$w_k = W\left(\lambda_k = \frac{\bar{v}}{f_k}\right). \tag{5}$$

The magnitude of each frequency component of the vertical displacement $Z_{k,D1}$ is thus corrected using the corresponding weight w_k , as per Eq. (6).

$$|Z_{k,\text{corr}}| = w_k |Z_{k,D1}|. \tag{6}$$

Once the weighed displacement is computed, the statistical measures can be calculated. Specifically, the standard deviation of the compensated vertical displacement $s_{\text{VD+WF}}$ is calculated according to Eq. (7):

$$s_{\text{VD+WF}} = \sqrt{\sum_k \frac{1}{2} |Z_{k,\text{corr}}|^2}. \tag{7}$$

Finally, the estimated peak longitudinal level between the two rails $L_{\text{max,VD+WF}}$ is obtained, adopting $s_{\text{VD+WF}}$ as the regressor and using the linear regression model of Eq. (8):

$$L_{\text{max,VD+WF}} = \beta_1 + \beta_2 \cdot s_{\text{VD+WF}}, \tag{8}$$

where β_1 and β_2 are the coefficients of the linear regression model fitted using the peak value of the longitudinal level (measured from the diagnostic train), and the standard deviation of vertical displacement $s_{\text{VD+WF}}$ (computed adopting the compensation method). The regression model is characterised by a coefficient of determination $R^2 = 0.73$. Therefore,

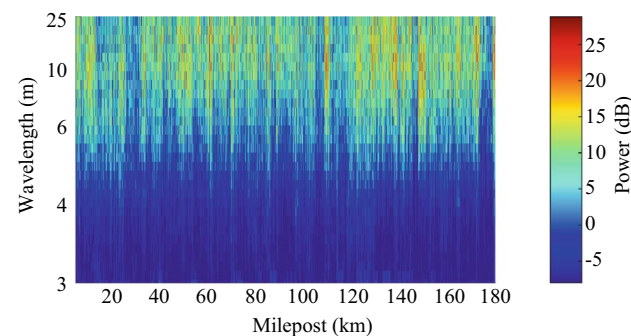


Fig. 5 Spectrogram of track longitudinal level

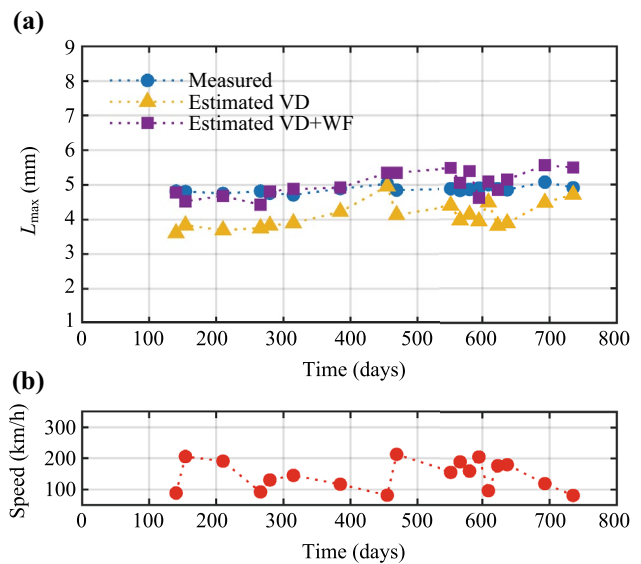


Fig. 6 Comparison of the peak longitudinal level measured by the diagnostic train (L_{max}) and estimated by the regression models: **a** L_{max} , $L_{\text{max,VD}}$, and $L_{\text{max,VD+WF}}$; **b** train speed

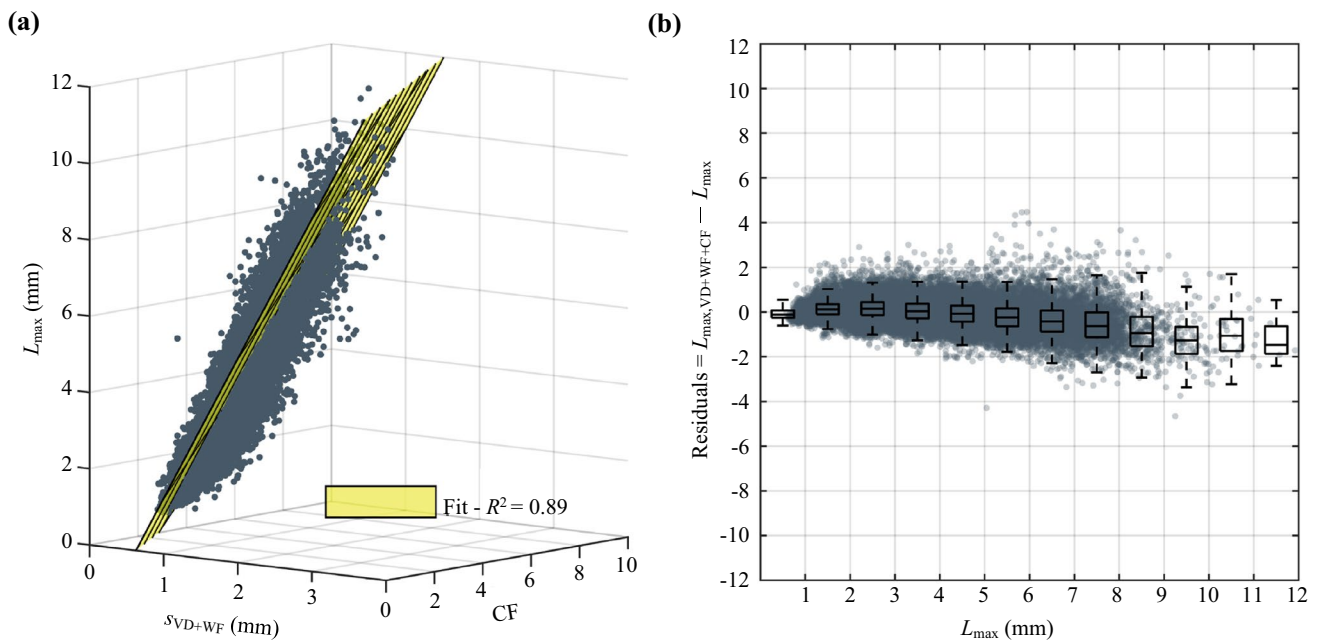


Fig. 7 Regression model $L_{\max,VD+WF+CF}$: **a** fitting plane representing the regression model; **b** residuals of the fitted model

an improvement has been reached if compared to $R^2 = 0.7$ that characterises the regression model of Sect. 3.1 (based on just double integration).

To better highlight the benefits resulting from the compensation of the bogie filtering, the same defect of Fig. 3 is reanalysed. In Fig. 6a, the previous estimation $L_{\max,VD}$ (obtained from the s_{VD}) is reported with yellow squared markers, and the updated estimation $L_{\max,VD+WF}$ is shown using purple diamond markers. As a reference, also the measured L_{\max} is reported using blue dots. It is clear that the estimation error is significantly reduced, being the estimations reported as purple markers well superimposed to the measured data (blue dots).

The results of Fig. 6 show a significant improvement of the condition monitoring system, with the estimation error assuming values generally smaller than 1 mm. To complete the analysis, the last step of data processing is addressed in the following section, devoted to the distinction of the typology of the defect under analysis.

3.3 Data pre-processing—step 3: accounting for the isolated or cyclic nature of the defects

As already shown in a previous work [32], the nature of the defect plays a role in the estimation of the L_{\max} . In fact, both isolated and distributed defects can be found along the railway line. A distributed defect occurs as a series of successive defects inside the considered 100 m window. It is clear that significantly different acceleration levels would be measured by the instrumented vehicle depending on the

type of defect, even assuming it to show the same magnitude (given the dynamic excitation of the system).

To distinguish the defect typology, in [32] the authors proposed to adopt the crest factor (CF), the ratio between peak and RMS value, which defines how severe peaks are in the considered signal. C_L is introduced as an additional regressor, obtaining it directly from the longitudinal level measured by the TRV, which allows to define a multiple linear regression model, according to Eq. (9).

$$L_{\max,VD+WF+CF} = \gamma_1 + \gamma_2 \cdot s_{VD+WF} + \gamma_3 \cdot C_L \tag{9}$$

Compared to Eq. (8), three coefficients γ_1, γ_2 and γ_3 are now adopted, which define the plane that best fit the available data. Moreover, Eq. (9) in fact introduces a data fusion between processed dynamic measurements on the commercial train and available geometry data from TRV measurements.¹

With reference to the model in Eq. (9), Fig. 7a presents the observed L_{\max} values plotted against the regressor s_{VD+WF} and the regressor C_L . The fitted model is also displayed as a yellow surface. Figure 7b shows the residuals, depicted as

¹ In the current methodology exploitation, vehicle dynamics and track geometry data are collected during the same run. In view of the actual implemation of the methodology, acceleration data is available more frequently, e.g. on a daily basis, while geometry data is at disposal only after TRV runs (on fortnightly or monthly basis, depending on the line class). As a consequence, the C_L would be kept constant between successive TRV runs and updated as soon as new data are available.

blue dots, alongside a boxplot that summarises the dispersion of the data within 1 mm bins. The regression model is characterised by a coefficient of determination $R^2 = 0.89$, that is significantly higher than the 0.73 value obtained in Sect. 3.2. In addition is also aligned to the value reached along high-speed lines, where $R^2 = 0.89$ was recorded [32], confirming the strategy to be promising.

The spread of the data around the model is limited, as further demonstrated by the residuals shown in Fig. 7b. The boxplots indicate that data dispersion remains contained; however, a slight bias emerges, increasing with the amplitude of the defect. This can be attributed to the reduced number of samples available for the higher defect amplitudes.

The proposed solution will thus rely on data, respectively, collected:

- on a daily basis, processing the acceleration data from the instrumented commercial train, to compute s_{VD+WF} according to the data processing steps described in Sects. 3.1 and 3.2;
- once the TRV performs the track inspection (once every 2 to 4 weeks depending on the priority level of the considered railway line), to compute the crest factor C_L . This parameter was proved to slowly evolve in [32], so that the reasonable assumption to keep it constant between successive TRV runs is made.

The described strategy relies in a data fusion technique between TRV and commercial train. On the one hand, this can be regarded as a compromise, given that the condition monitoring system is no more completely based on data from a commercial vehicle. On the other hand, it is worth reminding that the designed solution aims at supporting the current maintenance strategy, providing accurate estimations of the longitudinal level. Therefore, priority is given to maximising the accuracy of the estimation of track defect severity, rather than to the definition of a fully autonomous system.

3.4 Analysis of the different steps' contribution

Once all the steps of data processing have been described, attention is paid to the performance of the condition monitoring system. To this aim, a comparison among the different steps is proposed, considering the percentage error that characterise the estimations of L_{max} . In Fig. 8, the methodology designed for high-speed applications (based only on bogie vertical acceleration [30]) is considered as reference. Then, the updated methodologies relying, respectively, on vertical displacement (Sect. 3.1), weighting function (Sect. 3.2) and crest factor (Sect. 3.3) are also reported in Fig. 8. Along the x-axis, the peak value of longitudinal level has been aggregated in bins of 1 mm, and all data recorded along the entire railway line (180 km long) during the whole monitoring

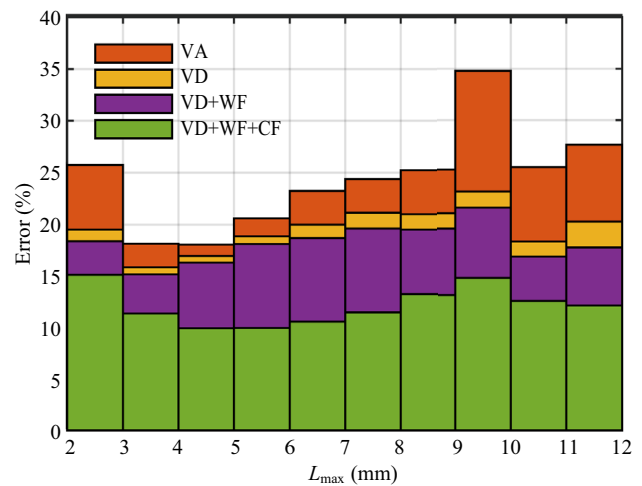


Fig. 8 Comparison of the percentage error calculated for each step of the methodology. Data collected along the entire railway line (180 km long) during the whole monitoring period (25 runs)

period (25 runs) have been considered to gain statistical relevance.

The results presented in Fig. 8 show the performance improvement in the whole range of longitudinal level, at each step of the methodology. To exemplify, let us consider the 9–10 mm range, 10 mm being a typical threshold adopted to define alert for possible maintenance needs, where the highest initial error was registered, with values up to 34.8% if the bogie vertical acceleration is adopted as regressor (VA, red histogram in Fig. 8). The adoption of vertical displacement allows for a significant reduction of the error that drops to 23.1% (VD, yellow histogram). This result clearly demonstrates that double integration is effectively managing the effect of vehicle speed (Fig. 3). It is worth mentioning that the data considered in this work are actually recorded along an Italian high-speed line. As such, the speed variation is not as relevant as it could be along conventional lines, where many accelerations and braking phases are generally occurring to allow serving the railway stations. In the end, an even higher beneficial effect could be expected when the methodology will be applied along conventional lines. It is worth mentioning a minimum running speed should be set for the estimation procedure to be effective, given that it relies on accelerometer measurements that may suffer from reduced irregularity excitation and poor signal to noise ratio. In the current evaluation, the lower speed limit is set to 60 km/h.

The second step of the methodology (VD + WF, purple histogram) slightly reduces the estimation error that passes from 23.1% to 21.6% in the 9–10 mm range. Although an improvement can be still observed, it turns out to be less relevant than the previous one. This result could be related to the wavelengths of the defect that typically populate the

considered railway line, that are in the order of 9–12 m (as visible in the spectrogram reported in Fig. 5). In this range, the WF presented in Fig. 4 shows indeed a gain which is almost constant and equal to the unitary value. For future applications, it could be interesting to test the effect of the WF in those lines where many defects of shorter wavelength (and thus filtered by the bogie dynamics) are present, to verify if and to what extent this step of the methodology would be beneficial.

Finally, the adoption of multiple regression including the crest factor allows reducing the estimation error down to 14.7% in the considered range (VD + WF + CF, green histogram of Fig. 8). This result further confirms the outcomes presented in [32], showing the benefits arising from data fusion techniques with direct longitudinal level measurements from the TRV.

Similar comments could be made also for the other L_{\max} ranges shown in Fig. 8, confirming the capability of each step of the methodology to lead to an accurate condition monitoring system for railway track longitudinal level. In the next section, the designed methodology will be applied to other 100 m sections of the line, to estimate L_{\max} .

4 Results

In this section, the results of applying the methodology for estimating the longitudinal level are presented. Specific sections of the railway line were selected based on the observation of defects that evolved over the period covered by the experimental campaign. At this stage of the research, the condition monitoring system is not installed on a commercial train yet. Therefore, in order to test the methodology proposed in this paper, we adopted acceleration measurements collected from the same diagnostic train that inspects the examined railway line. Given that the considered TRV is an out of service high-speed vehicle equipped with all the necessary measurement system, no significant differences (in terms of vehicle accelerations) are expected as a result of this choice. In addition, it is worth noting that the railway line under analysis is an Italian high-speed line. Therefore, vehicle speed is expected to often reach the maximum allowed one. Despite this aspect, as it will be shown in the following, significant speed variability has been observed during the entire monitoring period. This allows verifying the benefits of each step of the methodology, which is designed for conventional lines where significant speed changes are observed. In fact, to evaluate the effectiveness of the proposed methodology, the estimated longitudinal level is compared to the longitudinal level measured by the TRV.

Figure 9 illustrates the estimated peak longitudinal levels obtained for two sections of the railway line, employing the three stages of the methodology presented in Sect. 3. In

Fig. 9a and b, the results corresponding to a defect are shown (named as defect I) while Fig. 9c and d displays the results corresponding to a second defect (defect II). Four series of data are presented in Fig. 9a and c: measured L_{\max} (blue markers), $L_{\max,VD}$ (yellow markers) estimated from the s_{VD} , $L_{\max,VD+WF}$ (purple markers) estimated from the s_{VD+WF} , and $L_{\max,VD+WF+CF}$ (green markers) estimated from the multiple regression model using both s_{VD+WF} and the C_L as regressors. Together with longitudinal level results, mean vehicle speeds for each the test runs are shown in Fig. 9b and d, respectively, for defects I and II.

Focusing first on the series of measured data, the evolution trends observed for defects I and II are characterised by an initial phase of sustained increase followed by a maintenance intervention. The maintenance intervention is highlighted by the abrupt reduction in the peak values, occurring around day 275 for the defect I and around day 500 for the defect II. After the maintenance, the amplitude of the defect remains relatively stable for approximately 200 days. Subsequently, in the case of defect I, for which more data are available after the maintenance intervention, a slowly increasing trend is observed.

Regarding the speeds recorded for each trial, it can be observed that the diagnostic train travelled at significantly different velocities in different runs. A notable variability is observed in Fig. 9 for defect II. As shown in Fig. 9d, the diagnostic train travelled at speeds varying in a broad range from 200 to 300 km/h.

The significant speed variability observed during the series of acquisitions does not have a considerable impact in the results, demonstrating the benefit of using vertical displacement instead of vertical acceleration as regressor. This leads to an enhanced solidity of the method.

As can be observed from the results concerning the series of estimated peak values in Fig. 9, the methodology developed in this work allows to successfully estimate the peak value of the longitudinal level in a 100 m section. The results show that then trend is well captured by all three estimation methods (VD, VD + WF, VD + WF + CF), while the estimation of magnitude improves with the addition of each step to the methodology.

In this regard, even if the trend is already captured well when using just the vertical displacement with filter compensation, an error is still observed between the measured and estimated value for both the defects considered. In both cases, this difference is observed in the latter part of the time series, after the maintenance action. In the case of defect I, the actual longitudinal level after maintenance is indeed underestimated by a quantity close to 1 mm. On the other hand, defect II is overestimated by a similar amount. The final step, incorporating the crest factor of geometry as a second regressor to account for the nature of the defect, significantly improves estimation accuracy in terms of magnitude

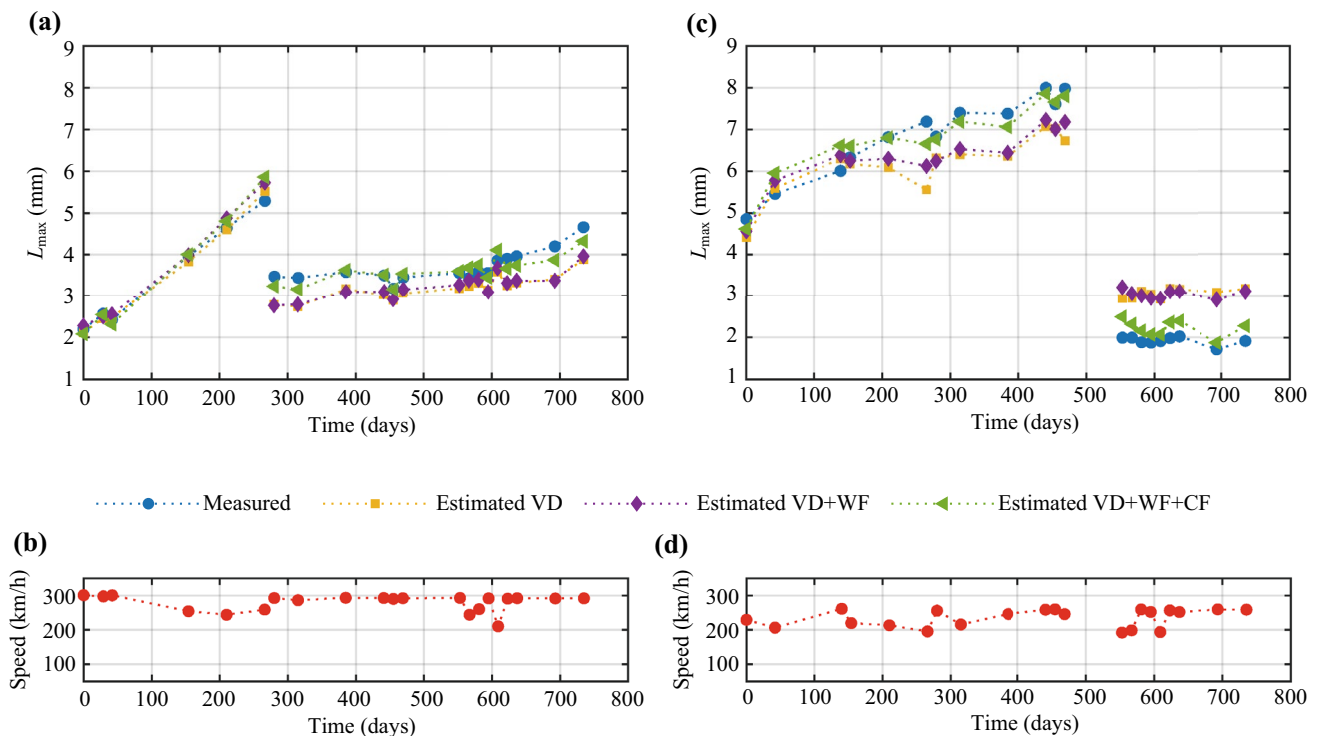


Fig. 9 Measured vs. model-predicted peak longitudinal level (L_{\max}) for defects I and II: **a** L_{\max} and **b** train speed at defect I; **c** L_{\max} and **d** train speed at defect II. Measurements were obtained from the TRV; predictions are from the regression models

of the defect. In fact, the green series is much better superimposed with the measured values, and the deviation from the level measured by the TRV is reduced to less than 0.4 mm.

In Fig. 10, the estimated peak values $L_{\max,VD+WF+CF}$, obtained with the complete methodology, are presented for other three different defects (named III, IV and V). These defects were already analysed in [32] with a different methodology. Defects III and IV, shown in Fig. 10a and c, respectively, are characterised by their differing natures: Defect III corresponds to a distributed defect, whereas defect IV is an isolated one. Defect V, shown in Fig. 10e, corresponds to an isolated defect with a particular trend characterised by a piecewise evolution.

As in the defects depicted in Fig. 10, the rate of evolution in these three cases is also well estimated. The analysis shows consistent accuracy across different scenarios, reflecting the solidity of the methodology. In Fig. 10a, three phases can be identified for the evolution of defect III, delimited by two maintenance interventions: the first at around day 350 and the second at around day 500. As can be observed, the peak values estimated using the methodology proposed in this work correspond closely with the results for each period. The rate of evolution is accurately represented, and the reduction in peak values following both maintenance interventions is effectively captured.

Similar considerations can be made for defect IV in Fig. 10c. However, in this case, only two distinct periods are observed, with one single maintenance intervention occurring around day 350. This intervention is also reflected in the estimated values, showing a reduction in the peak value of longitudinal level from approximately 7.5 to 3 mm. The estimated values align closely with the actual results for each period.

The case of defect V shows instead some discrepancies. The results corresponding to the initial part before the change in the evolution rate, as well as the period after the maintenance intervention, are aligned with the measured values. However, a shift of approximately 1 mm is observed in the second part of the time series, from day 150 to 350, prior to the maintenance intervention. During this period, the estimated values are consistently 1 mm lower than the measured values. Despite this discrepancy, the overall trend, specifically the rate at which the defect evolves, is accurately captured by the methodology.

In summary, the methodology accurately captures the rate of evolution, correctly reflects the reduction in peak values following maintenance interventions and successfully estimates the magnitude of the defect in most conditions. Even when the magnitude is not perfectly reproduced and a minor deviation in the values are observed, the general

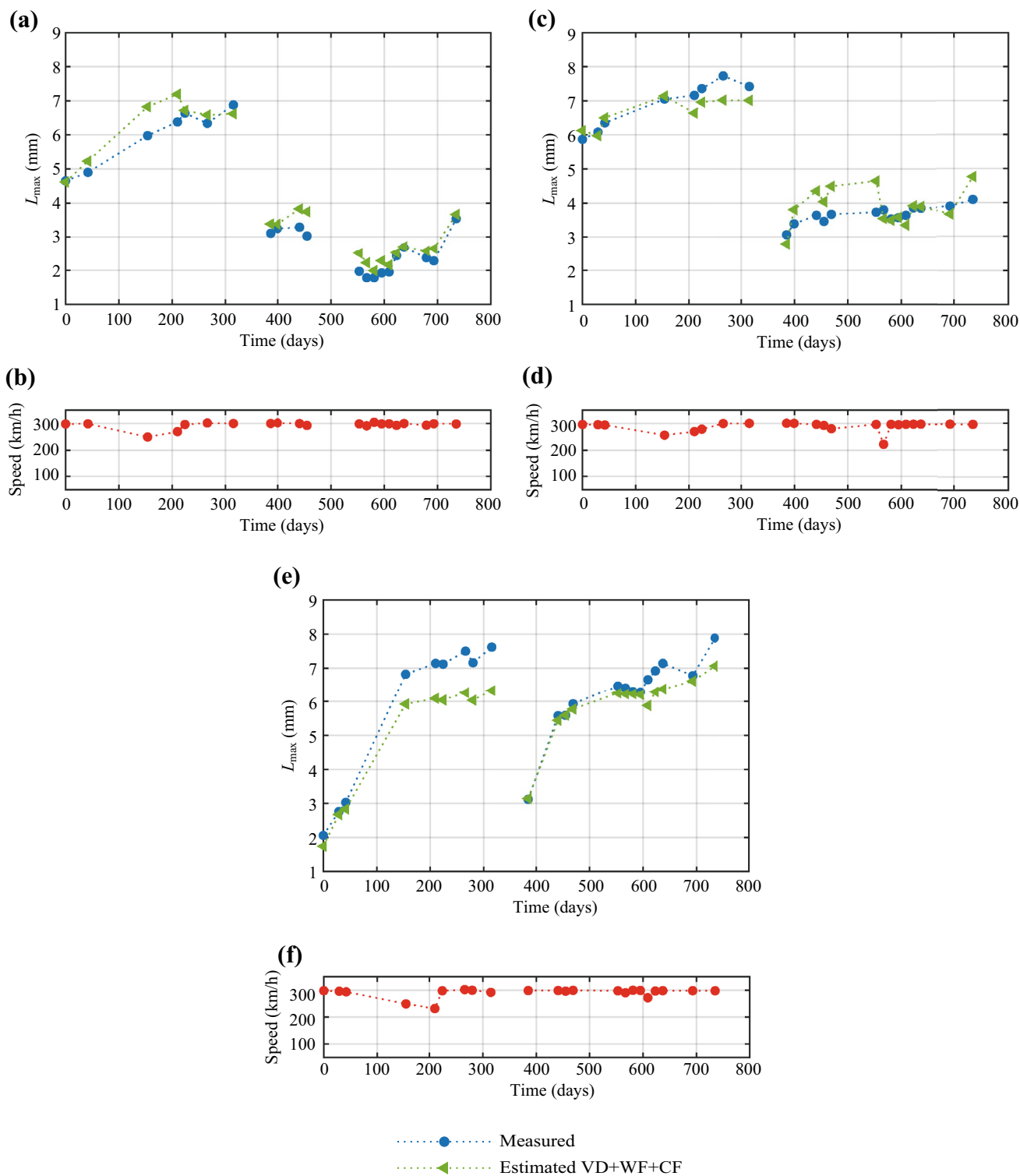


Fig. 10 Measured vs. model-predicted peak longitudinal level (L_{max}) for defects III, IV and V: **a** L_{max} and **b** train speed at defect III; **c** L_{max} and **d** train speed at defect IV; **e** L_{max} and **f** train speed at defect V. Measurements were obtained from the TRV; predictions are from the regression models

behaviour and progression of the defect are still effectively represented. Finally, even if defects show different natures, the methodology effectively handles these differences. This

adaptability demonstrates the flexibility and solidity of the approach, ensuring reliable performance across diverse scenarios.

5 Conclusions

In this work, a methodology was developed for estimating the longitudinal level of a railway track using inertial measurements from a single-sensor setup. The monitoring system was designed to estimate the peak longitudinal level for track segments having 100 m length. The procedure uses the standard deviation of bogie vertical displacement as a regressor of the peak longitudinal level between the two rails of the track. Additionally, the system incorporates a strategy to compensate for the filtering effect introduced by the bogie. Data fusion with less frequent information from TRV runs is applied to account for the co-existence of distributed and isolated track defects.

Vertical displacement was shown to be less dependent on vehicle speed than vertical acceleration, making the former quantity particularly useful for monitoring conventional railway lines where vehicle speed is not constant. Vertical displacement was obtained by double integrating vertical acceleration. Integration was performed in the frequency domain.

A method was successfully introduced to mitigate the filtering effect caused by the bogie wheelbase. This method involves adjusting the amplitude of each frequency component of the vertical displacement signal. The compensation is made using a weighting function derived from the reciprocal of the frequency response function of the vehicle.

The monitoring system was developed to estimate the peak longitudinal level for track segments of 100 m. To this end, a linear regression model was successfully trained to estimate the peak values of longitudinal level. The results show that the model is able to capture not only the evolution trend of the defects but also the magnitude, especially when the regression model adopts the crest factor of the measured longitudinal level (from TRV) as a secondary regressor. A coefficient of determination $R^2 = 0.89$ was found for this case, showing a quality of the estimate similar to the one found for high-speed trains running at constant speed, as presented in [32]. This achievement is particularly significant considering the wide range of speeds considered (from 60 to 300 km/h). Additionally, the percentage error was reduced to less than 15% in the whole range of measurement.

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