Community detection for undergraduate mathematical views

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Beliefs are propositions about a certain topic that are regarded as true (Philipp, 2007), they are individually held (Erens & Eichler, 2019) and tend to form clusters as they "always come in sets or groups, never in complete independence of one another" (Green, 1971, p. 41). A belief is seldom isolated, but it is connected to other beliefs, which are part of the same cluster of beliefs. According to Green (1971), belief clusters are coherent families of beliefs across multiple contexts. For this reason, beliefs can be understood in terms of views of mathematics (Grigutsch, Raatz & Törner, 1998), namely as epistemological beliefs about mathematics, its teaching and learning. According to Grigutsch et al. (1998), it is possible to outline four different views: a process-oriented (P) view that represents mathematics as a creative activity consisting of solving problems using different and individual ways; an application-oriented (A) view that represents the utility of mathematics for real world problems as the main aspect of the nature of mathematics; a formalist view (F) that represents mathematics as characterised by a strongly logical and formal structure; a schema-oriented (S) view that represents mathematics as a set of calculation rules and procedures to apply for routine tasks. Erens and Eichler (2019) operationalised these four views into a Likert-scale questionnaire made of 24 statements, each of which is assigned to a specific view. Examples of statements are: "mathematics helps to solve tasks and problems that originate from daily life" (A), "logical strictness and precision are very essential aspects in mathematics" (F), "in order to comprehend and understand mathematics, one needs to create or (re-)discover new ideas" (P), "doing mathematics demands a lot of practice in adherence and applying to calculation rules and routines" (S).

In this poster, we aim at showing a methodology for clustering these statements and checking whether the four categories defined for the aforementioned four views are a posteriori confirmed by a survey administered to 93 students enrolled in the third year of undergraduate studies in mathematics at the University of Torino (more details can be found in Andrà, Magnano, Brunetto & Tassone, submitted). On one hand, we claim that the need for an a posteriori analysis resides in the importance to verify the reliability of the measurement tool. On the other hand, the methodology shown here represents a novelty, as it is based on network analysis.

A network is a set of nodes connected to each other through an edge. Indeed, a network N is a pair (V, L) where V is the set of nodes and L is a subset of the Cartesian product $V \times V$. There are two mathematical tools that allow one to analyze a network: (i) the graphical representation of the network, and (i) the adjacency matrix, which describes the connections between nodes as its component $a_{ij}>0$ if the nodes i and j are connected by a link, otherwise is it null.

One of the potentialities of network analysis is the possibility to use clustering tools that do not require particular metric definitions. These techniques are called community detection as the goal is to identify subnets of nodes characterized by relatively large internal connectivity, namely nodes that tend to connect much more with other nodes in the group than the rest of the network. To this end, we used the so-called "Louvain method" which is based on the optimization of a quantity called modularity (Q). Given a partition into k sub-graphs $\{C_1, C_2, . . . , C_P\}$ of the network, the modularity (Q) is the normalized difference between the total weight of the links inside the sub-graphs C_k and the expected value of total weight in the randomized "null network model" (Newman, 2010).

It is also important to recall that a network can be built as a projection of a bipartite network, namely one made up of two distinct classes of nodes V_n and V_m, and links can only connect nodes of different classes. This is the case of the data analysed in our study, since the network has been built with nodes identified by both the 93 students and the 24 items. A student-node is connected to an item-node with an edge weighted 1, 2, 3, 4, or 5 depending on the rank assigned to it on a 5-point Likert scale. Student-nodes are not directly connected to each other, as well as item-nodes. Another typical example of a network of this sort is the Amazon network, where consumers are linked to each product if they have purchased it at least once. However, it is possible to define a link between consumers if they have purchased the same product, or a link between products if they were purchased from at least one same consumer. This method allows one to create a similarity metric between participants or between questions in the context of questionnaires (Brunetto, 2017).

In our poster, we confirm an almost perfect correspondence between a priori classification of Erens and Eichler's (2019) work and our investigation on a sample of undergraduate mathematics students a posteriori, and we extend clustering to other aspects of mathematics teaching and learning.

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