Materials Research Proceedings 69 (2026) 874-879 https://doi.org/10.21741/9781644904251-154

Adaptive Augmentation of Incremental NDI Laws with Systematic $\mathcal{H}_\infty$ Synthesis

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Keywords: Flight Control System, Incremental Nonlinear Dynamic Inversion, Adaptive Control, Automatic Tuning

Abstract. Incremental Nonlinear Dynamic Inversion (INDI) is a promising control framework for systems characterized by significant nonlinearities and uncertainties, such as high-performance aircraft. However, the capabilities of INDI control are limited by its sensitivity to noise, delays, and modeling errors, which hinder its industrial adoption and certification. This paper presents an adaptive INDI extension with systematic synthesis capabilities, thereby enabling formal verification of performance and robustness, and paving the way toward certifiable INDI flight control systems.

Introduction

Given a control-affine nonlinear system $\dot{x} = f(x) + G(x)u$, the starting point of the INDI design is the linearization of the system about the previous time instant (x_0, u_0), obtaining

$$\dot{x} = \dot{x}_0 + F(x_0, u_0)\Delta x + G(x_0)\Delta u, \tag{1}$$

where $\Delta u = u - u_0$, $\Delta x = x - x_0$ are the input and state increments, respectively, $F(x_0, u_0) = \partial(f(x) + G(x)u)/\partial x$ evaluated at (x_0, u_0), $G(x_0)$ is the control effectiveness matrix and $\dot{x}_0 = f(x_0) + G(x_0)u_0$. The INDI control law is given by:

$$u = u_0 + \Delta u = u_0 + G^{-1}(x_0)(\nu - \dot{x}_0), \tag{2}$$

where ν is a virtual control input provided by an external controller [1] that achieves stable tracking of a reference signal x_{ref}.

When applied to the nonlinear plant, the control law approximately reduces the dynamics to a pure integrator, $\dot{x} = \nu$, which holds under the assumption of fast control update time and fast actuators, so that $\|F(x_0, u_0)\Delta x\| \ll \|G(x_0)\Delta u\|$. In the model-based INDI configuration, the On-Board Model (OBM)—a simplified dynamic model embedded in the flight control computer—is used to compute real-time estimates of the nominal state derivative, $\dot{x}_0$, and the control effectiveness matrix, $G(x_0)$. To enable this, both state and input feedback must be available from appropriate onboard sensors.

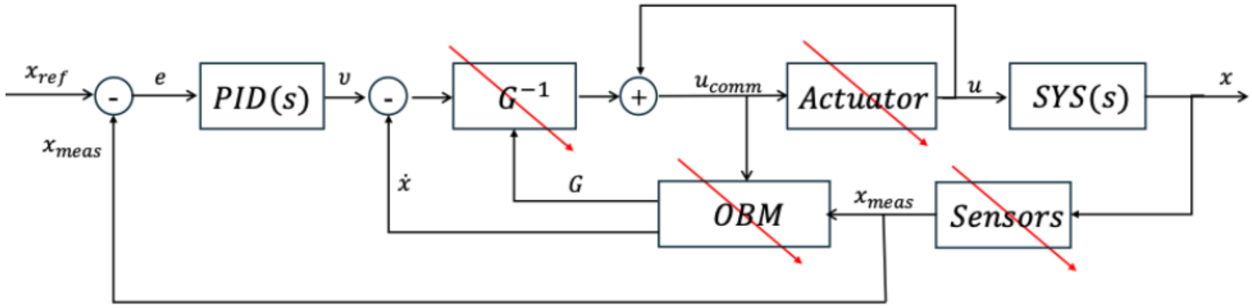

Fig. 1: Typical model-based INDI loop with uncertainty sources indicated with red arrows.

The combined effects of phase lag due to sensor dynamics, delays, actuator dynamics, and model estimation errors (Fig. 1) result in a deviation from ideal integrator behavior. The influence of these non-idealities on the INDI loop dynamics can be explicitly characterized by introducing perturbations in both the state function f and the control effectiveness G as follows:

$$\dot{x} = f(x) + \Delta f(x) + \big(G(x) + \Delta G(x)\big)u. \tag{3}$$

In [2], perturbations are matched in the actuator channel and compensated via an $\mathcal{L}_1$ adaptive controller in augmentation on the inner loop. This approach, however, results in a time-consuming tuning procedure, as the number of the adapted parameters equals the number of control channels. To overcome this issue, a novel adaptive architecture which augments the virtual control signal is presented, where an automatic tuning based on performance and robustness considerations is performed.

Adaptive Augmentation

The perturbation term of Eq. 3 is lumped into a single virtual disturbance $\sigma = \Delta f(x) + \Delta G(x)G^{-1}(x)\big(v - f(x)\big)$. Therefore, the INDI dynamics can be rewritten as $\dot{x} = v + \sigma$, i.e., a pure integrator with an input disturbance, recasting the whole problem into a classical linear disturbance rejection framework.

We solve the control problem via an adaptive augmentation approach by implementing an Extended State Observer (ESO) [3] that estimates the virtual disturbance and provides a control contribution v_{ad} equal and opposite to its estimate $\hat{\sigma}$ (Fig. 2).

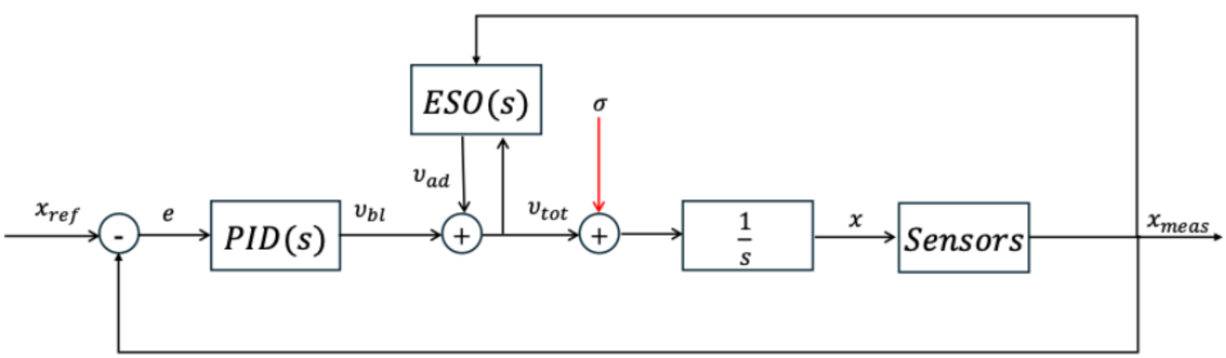

Fig. 2: Adaptive augmentation control architecture.

The considered ESO is described by

$$\dot{\hat{x}}(t) = \nu(t - \Delta T) + \hat{\sigma}(t) + L_1\big(x_{meas}(t) - \hat{x}(t)\big), \tag{4a}$$

$$\dot{\hat{\sigma}}(t) = Proj\{L_2\big(x_{meas}(t) - \hat{x}(t)\big)\}, \tag{4b}$$

$$\nu_{ad}(t) = -\hat{\sigma}(t), \tag{4c}$$

where x_{meas} is the state value measured by sensors, while L_1 and L_2 are positive definite ESO tunable gains. ΔT denotes a time shift applied to the predictor input. The inclusion of this term aims at compensating for the time misalignment between the predictor's state estimate $\hat{x}$ and the measured state x_{meas} introduced by sensor dynamics and delays. Numerical results show that this modification improves stability margins. Finally, a projection operator $Proj(\cdot)$ is applied to the virtual disturbance estimate to prevent drifting. One can prove that, with a proper tuning of the ESO gains, $\hat{\sigma}(t) \approx \sigma(t)$ and thus pure integral behavior of the INDI loop is approximately recovered.

Structured $\mathcal{H}_\infty$ Synthesis

The resulting proposed architecture (Fig. 2) is suitable for structured $\mathcal{H}_\infty$ synthesis [4] due to its simple structure and linearity in the parameters of the controller. Structured $\mathcal{H}_\infty$ solves a non-convex optimization problem to minimize the $\mathcal{H}_\infty$ norm of selected closed-loop transfer functions while respecting constraints on the controller's structure. These transfer functions represent key performance metrics such as tracking error, disturbance amplification, and control effort. Since the designer focuses on particular frequency ranges, it is common practice to weight each of these functions in the frequency domain. The designer can then tune the control system by adjusting these weighting functions to shape desired performance.

This tuning framework is more transparent and systematic for the designer than the standard manual tuning typically employed in the context of $\mathcal{L}_1$ adaptive control [2].

The presented approach is tested on a longitudinal autopilot of a nonlinear aircraft model in MATLAB/Simulink [5]. The autopilot consists of a cascade of three loops: the outer controls the climb angle γ, the middle the vertical acceleration a_z and the inner the pitch rate q (Fig. 3).

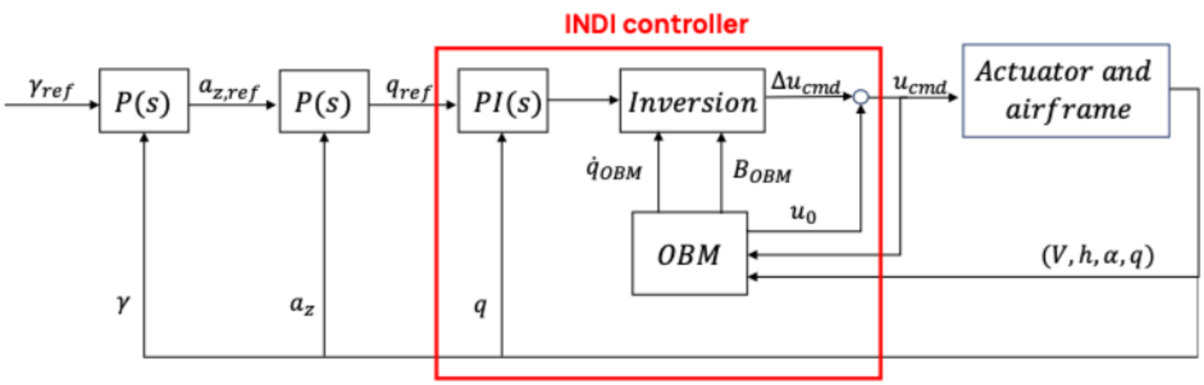

Fig. 3: Longitudinal autopilot baseline control system.

The baseline INDI controller adopted for the inner loop is augmented with the ESO architecture. Structured $\mathcal{H}_\infty$ synthesis is performed using three frequency-domain weighting functions, focusing on tracking performance, disturbance rejection capabilities, and control penalty:

$$W_{track}(s) = 0.1(s + 150\,)/(s + 1.5), \tag{5a}$$

$$W_{dist}(s) = 0.067(s + 15)/(s + 0.013), \tag{5b}$$

$$W_{act}(s) = 10(s + 0.15)/(s + 1500). \tag{5c}$$

The presented solution is then compared to the baseline INDI controller of Fig. 3 and to the $\mathcal{L}_1$ adaptive plant augmentation presented in [6].

The sensitivity functions in Fig. 4 show that the ESO augmentation retains most of the low-frequency disturbance rejection capability of the $\mathcal{L}_1$ augmentation presented in [6] while smoothing out the high-frequency peak.

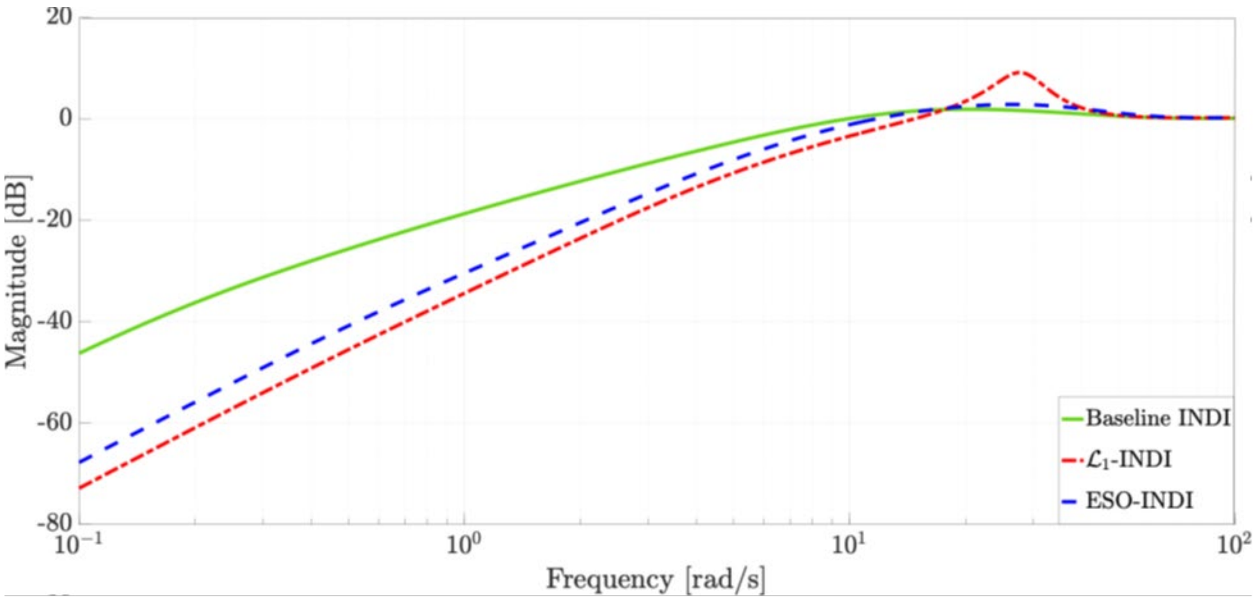

Fig. 4: Sensitivity functions of the INDI architectures.

Moreover, the Bode diagrams of the open-loop transfer function in Fig. 5 show that the ESO solution can recover and expand the stability margins of the baseline INDI controller of Fig. 3. Refer to Fig. 5: the light blue controller prioritizes an ample phase margin, while the dark blue controller prioritizes closed loop bandwidth. The designer can easily regulate the trade-off between the two by modifying the weighting functions of the $\mathcal{H}_\infty$ synthesis.

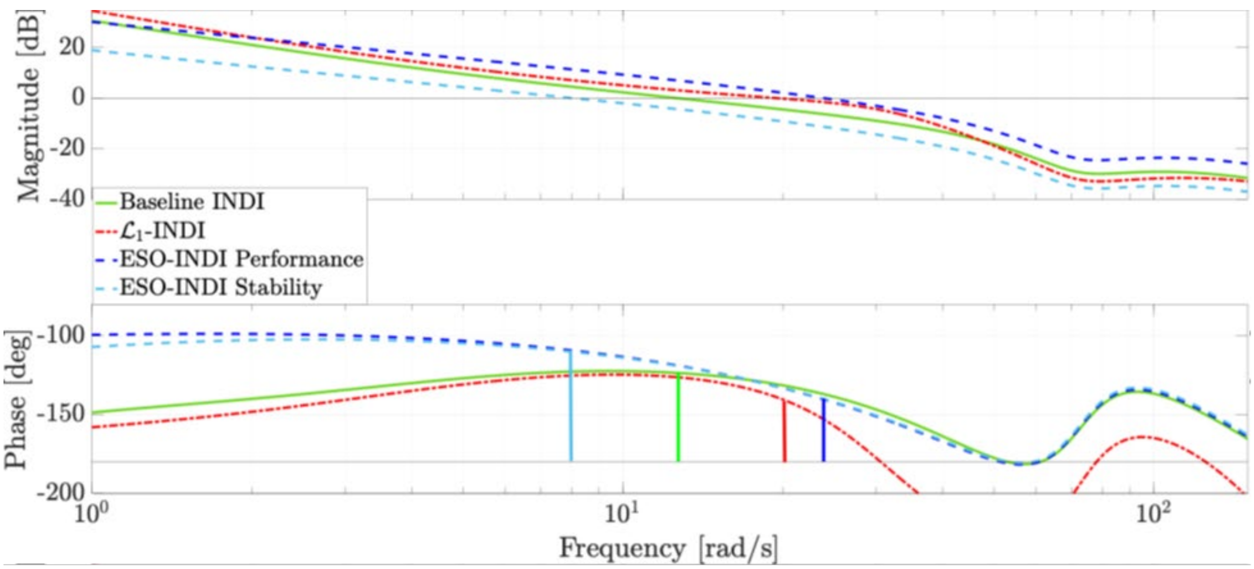

Fig. 5: Open-loop Bode diagrams.

Simulation Results

Performance of the adopted solution are evaluated under model uncertainties (inertia and aerodynamics), actuator degradation (−20% of effectiveness), sensor dynamics (with a bandwidth of 34 rad/s) and a sinusoidal external disturbance. As can be seen in Fig. 6, the ESO architecture shows sensibly improved performance compared to the baseline INDI controller of Fig. 3.

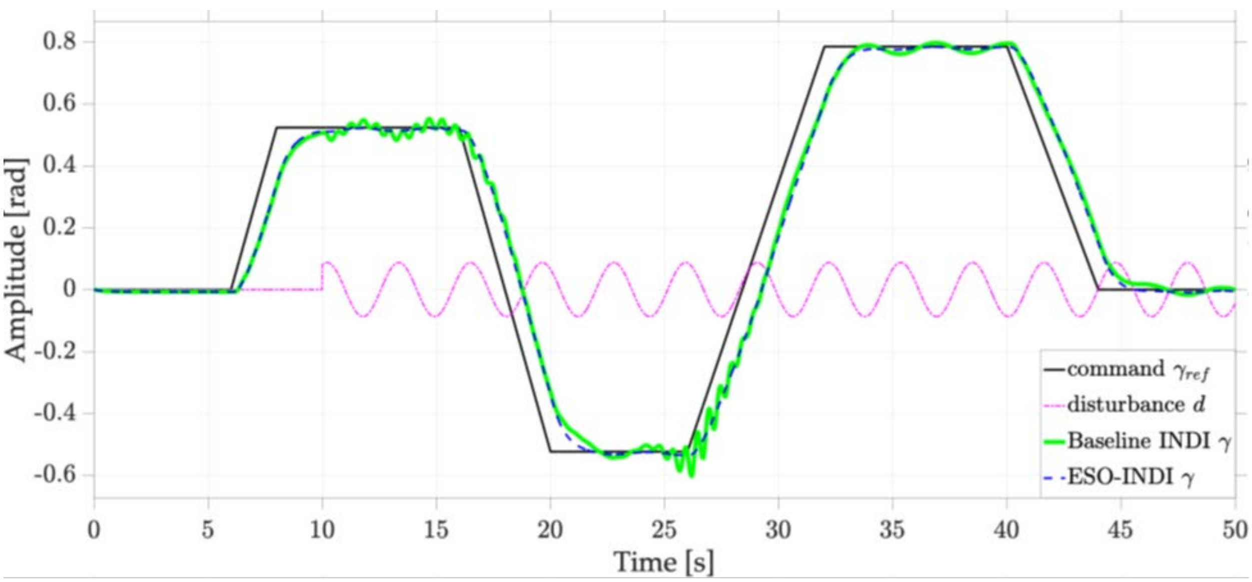

Fig. 6: Tracking of a reference climb angle.

Conclusions and Future Developments

Validation on a longitudinal autopilot case study shows that the $\mathcal{H}_\infty$-tuned INDI–ESO controller surpasses the performance and stability margins of a classical INDI or $\mathcal{L}_1$-adaptive scheme when sensor phase lag is present. The architecture also reduces manual tuning effort.

Several promising extensions can build on this foundation. First, disturbance estimation could be improved by incorporating Gaussian radial basis functions or neural network hybrids within the ESO, providing richer function approximation capabilities. Second, online delay identification algorithms could be embedded in the control loop to adaptively refine the ESO's delay estimate under variable computational or transmission latencies. Finally, a natural progression is to extend

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the ESO–INDI framework to a multivariable flight control framework, developing vectorial disturbance observers and structured $\mathcal{H}_\infty$ controllers that explicitly handle pitch–roll–yaw coupling, enabling a unified autopilot for both longitudinal and lateral–directional dynamics.

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