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Robust Stackelberg Equilibrium Water Allocation Patterns in Shallow Groundwater Areas

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Key Points:

- An introduction of robust measure into multi-level programming balances solution robustness and model optimality
- There is a critical point that decision makers can achieve high objective values and strong robustness with low costs

Supporting Information:

Supporting Information may be found in the online version of this article.

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Abstract

It is challenging for decision-makers (DMs) to deal with uncertainties in multi-level agricultural water resource systems, where DMs independently make decisions but have different levels of power. In this paper, we model the multi-level agricultural water resources system under deep uncertainties as a Stackelberg game, use multi-level programming to solve equilibrium water allocation problems, and introduce robustness metrics into multi-level programming to balance solution feasibility and model optimality within uncertain environments. The approach is applied to a shallow groundwater area with three decision levels, pursuing, from the top level to the bottom one, high food production, fair water allocation, and increased economic benefit. The model generated a series of optimal equilibrium solutions with different robustness degrees. DMs can choose "rational" solutions according to their acceptable costs, oriented robustness degree, expected objective values, and advance risk assessment of uncertainties. Among these solutions, we capture a critical point with high objective values and strong robustness, where DMs can accomplish both objective optimality and solution robustness with a low cost. The proposed approach in this study provides a posterior decision support to consider solution robustness while designing policies in multi-level agricultural water resource systems under deep uncertainties.

1. Introduction

Many water resources management systems worldwide are characterized by complex governance schemes involving multiple institutional decision-makers (DMs) with different levels of power interacting nationally and internationally (Du et al., 2019; Lennox et al., 2011; Tantoh & Simatele, 2018). Typically, such systems are modeled using prescriptive water management models based on the optimization of control approaches and game theoretic descriptive models (Du et al., 2019; X. Li et al., 2023; D. Liu et al., 2020; Soliman, 2022). In agricultural water management systems, national administrative department, regional management department, and local farmers full participate in water resources distribution and use, have top-down goal expectations and water use restrictions, and interactively affect each DMs' goal achievement (F. Zhang et al., 2020; X. Zhang et al., 2022). This relation is conceptualized as a Stackelberg game, which captures participants' motivations, dynamic actions, and goal utilities in multi-level agricultural water resource management systems (Kicsiny et al., 2014; X. Zhang et al., 2022). A few researchers used the Stackelberg game theory to solve multi-level water allocation problems, however, still face a challenge to deal with inherent uncertainties.

The Stackelberg game is a dynamic game with full information (Anandalingam & Apprey, 1991). One DM acts as the leader and moves first, followed in sequence by the others—the "followers"—until a solution is reached that satisfies all DMs, called an equilibrium solution (S. Li et al., 2022; Paskseresht et al., 2020; Shih et al., 1996; Z. Tang, 2019). To solve this decentralized planning problem, one can use multi-level optimization and multi-level programming (MLP) techniques to transform the game process into mathematical algorithms, extending the Stackelberg game theory to widespread practical applications (Anandalingam & Apprey, 1991; Chen, He, et al., 2017; Chen, Lu, et al., 2017; Gupta & Gupta, 2022; Huang et al., 2022; Jia & Culver, 2006; Pramanik & Roy, 2007). In agricultural studies, the Stackelberg game has been already used mediumly. Kicsiny et al. (2014) modeled the relationship between the authority and producer sectors as a Stackelberg game and achieved water use rationalization in a drought emergency. Z. Xu et al. (2019) established the Stackelberg game-Nash-Cournot model between a local reservoir authority and sub-area managers and identified satisfactory water transaction patterns in agricultural water markets. Cabo et al. (2014) employed the Stackelberg game theory to describe the

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dynamic interaction between the water donor region and the water recipient region in interconnected river basins. They obtained the optimal water transfer amount and price in the equilibrium between water supply and demand.

The Stackelberg game theory and corresponding algorithms effectively depict characteristics and generate satisfactory solutions through computational programming. However, the gaming process may become complex when uncertainties affect systems, which is often the case. In a Stackelberg game with uncertainties, participants at different decision levels may hold varying risk attitudes and exhibit diverse behaviors when proposing schemes. The DMs may seek schemes with optimal performance within uncertain ranges or schemes with the minimum probability of undesirable outcomes. Traditionally, researchers maximize expected objective values (S. Guo et al., 2022; Y. Li et al., 2022; B. Liu et al., 2022; J. Xu et al., 2013; F. Zhang et al., 2020) or optimize objective values using a scenario-based approach, where uncertainty is well-characterized probabilistically or assigned likelihoods based on the expertise of DMs (A. Ciullo et al., 2023; Ning & You, 2019; L. Yu et al., 2020; F. Zhang et al., 2020). For example, F. Zhang et al. (2020) described random surface runoff as a P-III distribution, set three exceeding probabilities as low (75%), middle (50%), and high (25%) flow levels, and maximized expected objective values in multi-level water resource systems. The complexity of Stackelberg game theory increases when dealing with deep uncertainties whose distributions are uncertain. Deep uncertainties are usually expressed as interval values and fuzzy sets (Li et al., 2020; J. Tang et al., 2022; F. Zhang et al., 2020). Ma et al. (2020) described water availabilities as interval values under prespecified probability scenarios. They formulated satisfactory water allocation patterns in a bi-level decision-making structure of the agricultural water-energy nexus. L. Yu et al. (2020) considered crop-related parameters as interval values, regarded allowable maximum fertilizer and pesticide usage as fuzzy sets, and achieved optimal agricultural water resource management schemes under scenarios based on given credibility levels and the upper and lower bounds in agricultural water-energy-food nexus with three decision levels. Such approaches reach solutions easily, but require DMs to accurately propose decision preferences for risks prior to knowing consequences of solutions (Herman et al., 2015). Strict bounds are difficult to define for deep uncertainties, meaning solutions with narrow ranges for deep uncertainties are riskier, as they may ignore extreme and vulnerable points. Extensive studies have demonstrated a willingness to focus on solution deviations from projections, seek cost insurance that does not exceed expected damages, and search for a solution in favor of reducing the variance in potential outcomes, which is a robust solution (Ahmadvand & Sowlati, 2022; Perelman et al., 2002; Rouhani & Amin, 2022).

A solution insensitive to system errors or randomness is termed *robust* (Mulvey et al., 1995; Salazar et al., 2022; B. Xu et al., 2022). Robustness can be measured by multiple metrics (McPhail et al., 2018). Q. Sun et al. (2022) optimized the objective values according to the worst scenario and guaranteed the proposed solutions would remain feasible in all other scenarios. Mulvey et al. (1995) listed several quantification types of positive and negative variations relative to nominal values in measuring robustness. According to Ahmed and Sahinidis (1998), a decision is robust if the actual cost of realized scenarios remains "close" to the optimal expected cost for all possible scenarios. Some researchers quantify robustness via *regret* and *satisficing* measures (Giuliani & Castelletti, 2016; Herman et al., 2015): regret is defined as the difference between the solution's performance and the best solution; satisficing quantifies the difference between solution performance and a threshold of performance required. D. Yu et al. (2022) and X. Guo et al. (2022) adopted the difference between the actual objective values and the optimal objective value as an indicator of robustness to measure the DMs' regret degree. Instead of assigning preference weights beforehand, the robust framework provides a posteriori decision support. It involves more flexible evaluation forms to robustness, combines the expertise, knowledge, and preference of DMs in treating specific uncertainties, and considers the ability of DMs to withstand failure to meet a given performance threshold. Furthermore, robust solutions allow a tradeoff between maximizing model performance and giving certain variations space in an uncertain environment.

To our best knowledge, no studies have discussed robustness in the Stackelberg game or introduced robustness measures into the computational algorithm in multi-level water allocation problems. In this paper, we use a robust framework to characterize uncertainties in the context of multi-level agricultural water allocation. We introduce a robustness measure into the multi-level programming, letting DMs considering possible solution variations and costs while pursuing optimal objectives. We apply the approach in a shallow groundwater area, the Hetao Irrigation District (HID), China, with three contributions: (a) we examine the impacts of selecting robustness coefficients on the satisfactory degree of each objective; (b) we provide DMs a series of alternatives for choosing preferred robustness degrees; (c) we explore the utility of robustness measures in the Stackelberg game. We further discuss strategies for DMs to realize high objective values when facing uncertain water availabilities.

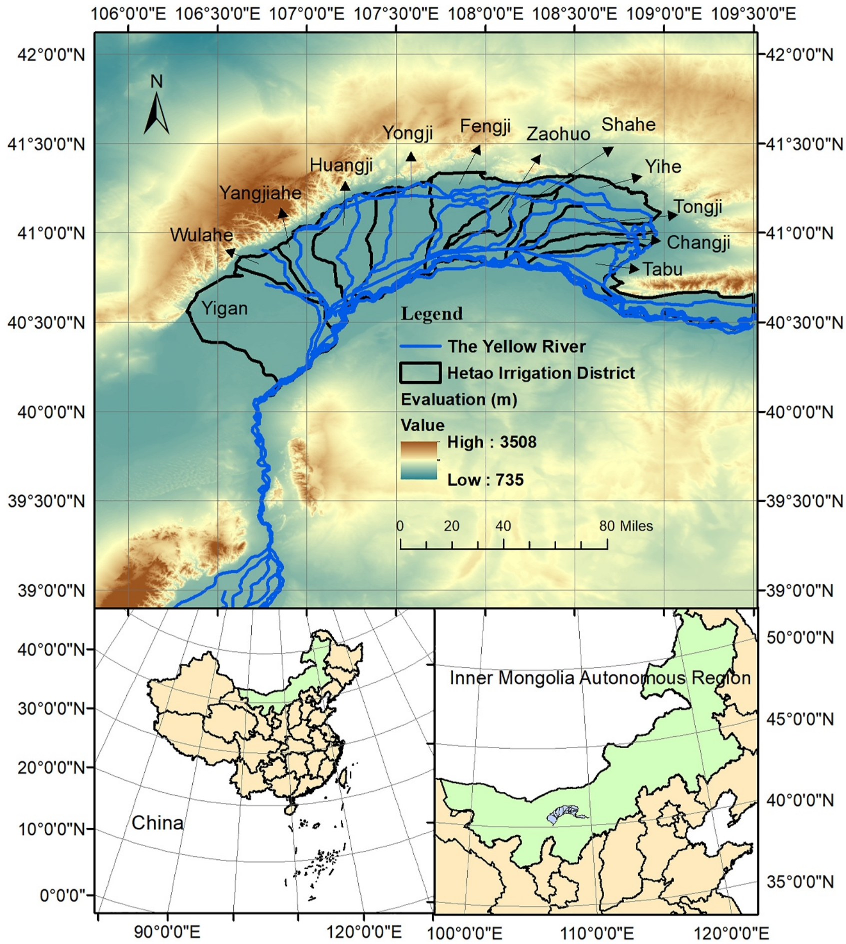

Figure 1. The Hetao Irrigation District.

Finally, this work offers a posterior decision support that measures robustness and costs of changing alternatives in multi-level agricultural water resources management systems.

2. Study Area

The HID, located in the western region of China's Inner Mongolia Autonomous Region (Figure 1), is a semi-arid region with a temperate continental monsoon climate (W. Sun, 2014), with average precipitation of less than 250 mm per year and average evaporation of 2,011–2,300 mm/year (Xue et al., 2018). To ensure water demand in this region, since 2000, the government has diverted an average of 2.324–4.036 billion m^3 of water annually from the Yellow River (between April and September) to satisfy local drinking, industrial, and agricultural water demand. The HID is an important region for grain harvesting in China, with over 80% of the district's agricultural area devoted to growing wheat, maize, and sunflower. The local canal system and land use distributions, as well as soil characteristics and administrative division, separate the HID into 12 water supply response units (WSRU), namely (from west to east): Yigan, Wulahe, Yangjiahe, Huangji, Yongji, Fengji, Zaohuo, Shahe, Yihe, Tongji, Changji, and Tabu (Luo et al., 2021). In HID, agricultural water accounts for 80%–90% of the diverted water volume, but the exact amount is highly uncertain, as it is influenced by hydroclimatic conditions and national policies. In the last 40 years, the irrigation has caused groundwater depth to increase to 1.5–3 m. Though variable in quantity, groundwater is the second main water source for growing crops. The frequent salt-water exchange between soil and groundwater improves the salt content in shallow soil layers and increases the risk of soil salinization and crop yield reduction (X. Zhang et al., 2021). To mitigate this risk, a comprehensive water use strategy is needed to balance the regulation of soil salt, efficient and environmentally conscious water use, and goal achievement at each decision level (X. Zhang et al., 2022).

In HID, the regional water governance is organized in three tiers of responsibility: national government managers, local government managers, and local farmers (X. Zhang et al., 2022).The *first-level* DMs, national government managers, allocate water resources among multiple users along the Yellow River. They supervise the impact of groundwater depth decline on the environment and lead the water diversion project to maintain food security and the well-being of local residents. They have the most decision-making power in determining water diversion and groundwater amount for local use. Their goal is to maximize total food production in this area. The *second-level* DMs, local government managers, are responsible for allocating water to each WSRU after receiving the total allowable water diversion and groundwater amount from the first-level DMs. The local government managers strive to ensure a fair distribution of water resources. The *third-level* DMs, local farmers, address local water use problems and independently determine the amount of water diversion and groundwater irrigation each crop receives in each growth period. Their goal is to maximize the economic benefits of the planted crops.

3. Methodology

3.1. Optimization Model

As mentioned above, the HID follows a water resources system with three decision-making levels. The *first-level* DMs pursue maximizing total food production, which can be expressed mathematically as the following objective function:

$$p_1^* = \arg\max_{p_1 \in Z_1} f_1(p_1) = \arg\max_{p_1 \in Z_1} \left[\sum_{i=1}^{12} \sum_{j=1}^{2} (A_{ij} \cdot Y_{aij}) \right] \tag{1}$$

where p_1 is the total water diversion and total groundwater allocated to HID, the decision variable of the *first-leve*l DMs; Z_1 is the solution set of p_1; the subset p_1^* of Z_1 is denoted that maximizes the objective of the *first-level* DMs, $f_1(p_1)$; $i \in [1, 12]$ identifies a given water supply response unit (WSRU; namely: Yigan, Wulahe, Yangjiahe, Huangji, Yongji, Fengji, Zaohuo, Shahe, Yihe, Tongji, Changji, and Tabu); and, $j = 1$ and $j = 2$ represent, respectively, wheat and maize, the two food crops produced in the region; A_{ij} is the planting area of crop j in WSRU i; Y_{aij} is the actual yield of crop j in WSRU i.

In the *second-level*, we measure water allocation fairness via the Gini coefficient (Gini, 1913), which ranges between 0 and 1: the smaller the magnitude of the coefficient, the higher the allocation fairness. The goal of second-level DMs, therefore, is to minimize the Gini coefficient:

$$p_2^* = \arg\min_{p_2 \in Z_2} f_2(p_2) = \arg\min_{p_2 \in Z_2} \left\{ \sum_{k=1}^{12} \left(\frac{1}{12} \cdot R_{WQk} \right) + 2 \sum_{k=1}^{12} \left[\frac{1}{12} \cdot (1 - AR_{WQk}) \right] - 1 \right\} \tag{2}$$

where p_2 is the water diversion and groundwater allocated to each WSRU, the decision variables of the *second-leve*l DMs; Z_2 is the solution set of p_2; the subset p_2^* of Z_2 is denoted that minimizes the objective of the *second-level* DMs, $f_2(p_2)$; R_{WQ} is the ratio of water allocation (the sum of water diversion and groundwater allocated in one WSRU) to the water demand, reflecting the water distribution ratio in each WSRU; to calculate the Gini coefficient in HID, the R_{WQi} is ordered from smallest to largest, labeled by k, becoming R_{WQk}; AR_{WQk} is the cumulative value of R_{WQk}.

In the *third-level*, DMs' goal is to maximize the economic benefits of the planted crops. Their objective function can be expressed mathematically as the following:

$$p_3^* = \arg\max_{p_3 \in Z_3} f_3(p_3) = \arg\max_{p_3 \in Z_3} \left[\sum_{i=1}^{12} \sum_{j=1}^{3} (pr_{\text{crop}j} \cdot A_{ij} \cdot Y_{aij}) - \text{CW} - \text{CEC} \right] \tag{3}$$

where p_3 is the water diversion and groundwater allocated to each crop in each crop growth period, the decision variables of the *third-leve*l DMs; Z_3 is the solution set of p_3; the subset p_3^* of Z_3 is denoted that maximizes the objective of the *third-level* DMs, $f_3(p_3)$; $pr_{\text{crop}j}$ is the price of crop j in yuan/kg; CW is the cost of using water

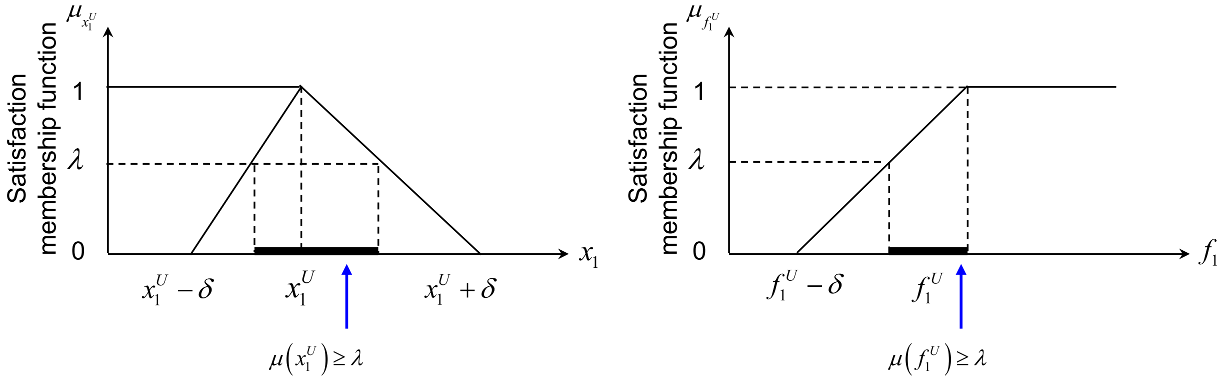

Figure 2. Examples of satisfaction membership functions. Here, we assume x represents the decision variable and f represents the objective. The left illustration is the triangle satisfaction membership of a decision variable of an upper-level decision maker (x_1^U). The right illustration is a trapezoid satisfaction membership of an upper-level DM's goal value (f_1^U). In both illustrations, δ is the leeway; $\mu(x_1^U)$ is the satisfaction degree (SD) of x_1^U; and λ is the SD.

diversion; and CEC is the cost of using groundwater, including energy fees for extracting and purifying high-salt-content groundwater. A more detailed description of each term is given in Supporting Information S1.

This study uses the Jensen crop production function, mapping evapotranspiration into crop yield (X. Zhang et al., 2022). The physical-based crop water consumption model follows the law of mass conservation, taking as input the movement of water or salt between precipitation, water diversion, crop, soil, groundwater, and drainage. This study restricts the soil water to be between wilting point and saturated soil water content, the total water use to be less than the available water supply, and the food production to be more than the necessary amount to ensure the food security. A detailed description of above physical-based crop water consumption and constraints is given in Supporting Information S1. In constraints, the available water diversion and groundwater resource are both uncertainties expressed as fuzzy numbers.

3.2. Stackelberg Game

The water allocation system in HID can be modeled as a Stackelberg game, a dynamic game with full information. Upper-level DMs impose restrictions on the decision-making of lower-level DMs—the actions of whom, in turn, could affect the goals of upper-level DMs. This study assumes that each DM is rational people who optimize their goals and are not affected by external factors. The decision-making process and structure has the following features (Shih et al., 1996):

1. Each DM's demand can be expressed by its objective function, constraints, and feasible decision space.
2. The DMs in each level independently make decisions and optimize their own goal.
3. The decisions are made sequentially, from the top to the bottom DM.
4. All DMs partake in an interactive decision process. Upper-level DMs are the first to set their decision goals or decision variables. They then ask the lower-level DMs for their optimal decision goal or decision variables that can be optimized independently. Once the lower-level DMs submit their decision goals and variables, the upper-level DMs can further modify their goals and decision variables. This process is repeated until all DMs are satisfied with a solution, called the *equilibrium solution.*

The decision-making process described above can be modeled mathematically using the multi-level programming method (MLP) (Shih et al., 1996). In the MLP, upper-level DMs allow some leeway in their proposed goals and decision variables, providing a larger decision space for the lower-level DMs to realize their goals. However, allowing some leeway doesn't violate the non-cooperated nature of multi-level programming, which is the difference between cooperative multi-objective optimization (X. Zhang et al., 2022). The detailed description and mathematical functions for this decision process are given in Text S3 in Supporting Information S1. In this study, the leeway is expressed by fuzzy satisfaction membership functions (Figure 2), where the satisfaction degree (SD) (λ) is a measure of the distance between actual and ideal values ($\lambda \in [0, 1]$, $\lambda = 1$ being the ideal value). We express satisfaction memberships of three-level objectives as trapezoids. For decision variables, we consider the satisfaction membership of total groundwater use—one of the decision variables of first-level DMs—as a triangle. We do not set strict preferences for total water diversion, the water allocation in any WSRU, or the

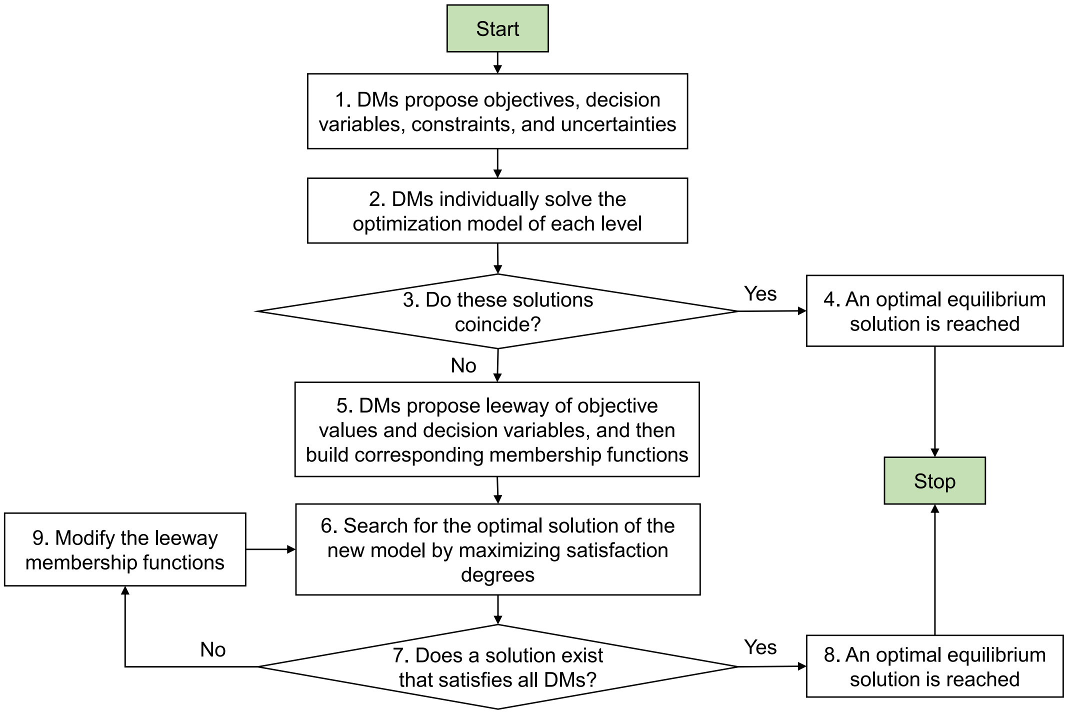

Figure 3. The decision-making process of a Stackelberg game with uncertainties.

irrigation for any crops, whose satisfaction degrees are all set to 1. Since the DMs reject unsatisfactory solutions during the interaction process, all the solutions within the given leeway are satisfactory to DMs (Shih et al., 1996; X. Zhang et al., 2022).

In a Stackelberg game with uncertainties (Figure 3), DMs must consider solution feasibility and costs of infeasible solutions. DMs at different levels may be more or less optimistic or conservative regarding their risk evaluations to uncertainties. Under uncertainties, it becomes more complex for DMs to establish an appropriate leeway, and considerable time may be spent searching for the most "reasonable" values of uncertain parameters that satisfy each DM. Uncertainties in a Stackelberg game significantly increase the time it takes to achieve the equilibrium solution.

3.3. Uncertainties Measurement

In this study, we consider two uncertainties of water diversion and groundwater availability. However, the collected data cannot form a distribution curve. We express these uncertainties as fuzzy numbers (show in Table S2 in Supporting Information S1).

A fuzzy number is a mathematical concept that extends the notion of a traditional number to handle situations with uncertainty or ambiguity (B. Liu, 2000; Soncini-Sessa et al., 2007). A fuzzy number refers not to a single value but to a set of possible values. The membership function represents the degree to which a value belongs to this set, and it assumes values ranging from 0 (completely not a member) to 1 (completely a member). According to the theory of Zadeh (1978) and Jiménez et al. (2007), if $\tilde{A}$ is a fuzzy number, its membership function $\mu_{\tilde{A}}: \Omega \rightarrow [0,1]$ is expressed as in Equation 4. The *α-cut* value of a fuzzy number $\tilde{A}$ is defined as $\widetilde{A_\alpha} = \{x \in \Omega | \mu_{\tilde{A}}(x) \geq \alpha\}$. Since the fuzzy degree of $\tilde{A}$, $\mu_{\tilde{A}}$, is upper semi-continuous, *α-cut* values are closed and bounded intervals. It can be expressed by $\widetilde{A_\alpha} = [f_A^{-1}(\alpha), g_A^{-1}(\alpha)]$. If $f_A(x)$ and $g_A(x)$ are both linear functions, $\tilde{A}$ is a trapezoidal fuzzy number, $\tilde{A} = (A_1, A_2, A_3, A_4)$. If $A_2 = A_3$, $\tilde{A}$ is a *triangular* fuzzy number (Figure 4).

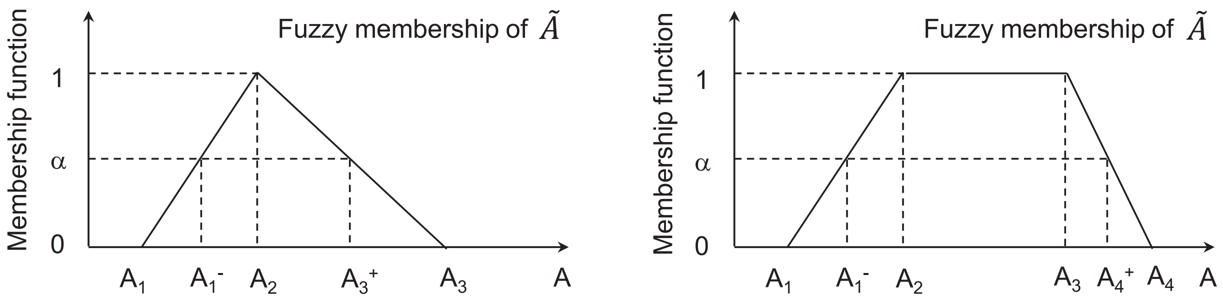

Figure 4. Examples of trapezoidal and triangular fuzzy numbers.

$$\alpha = \mu_{\tilde{A}}(x) = \begin{cases} 0 & \forall x \in (-\infty, A_1], \\ f_A(x) & \text{increasing on } [A_1, A_2] \\ 1 & \forall x \in [A_2, A_3] \\ g_A(x) & \text{decreasing on } [A_3, A_4] \\ 0 & \forall x \in [A_4, +\infty), \end{cases} \tag{4}$$

The possibility and necessity measures can be used to evaluate constraints with fuzzy numbers (Inuiguchi & Ramik, 2000; B. Liu & Liu, 2002). The possibility measure evaluates to what extent it is possible that a non-deterministic variable is restricted by the fuzzy number $\tilde{A}$. The necessity measure evaluates how certain it is that the non-deterministic variable is restricted by the fuzzy number $\tilde{A}$. Let g be a crisp (non-fuzzy) number that is not greater than the fuzzy number $\tilde{A}$. The possibility and necessity degrees of g not being greater than $\tilde{A}$ can be expressed via Equations 5 and 6 (and as shown in Figure 5). If the possibility or necessity levels are given in advance, constraints with fuzzy numbers become constraints with deterministic parameters.

$$\mathrm{Pos}\left(g \le \tilde{A}\right) = \begin{cases} 1, & g \le A_2 \\ \dfrac{A_3 - g}{A_3 - A_2}, & A_3 \le g \le A_2 \\ 0, & g \ge A_3 \end{cases} \tag{5}$$

$$\mathrm{Nes}\left(g \le \tilde{A}\right) = \begin{cases} 1, & g \le A_1 \\ \dfrac{A_2 - g}{A_2 - A_1}, & A_1 \le g \le A_2 \\ 0, & g \ge A_2 \end{cases} \tag{6}$$

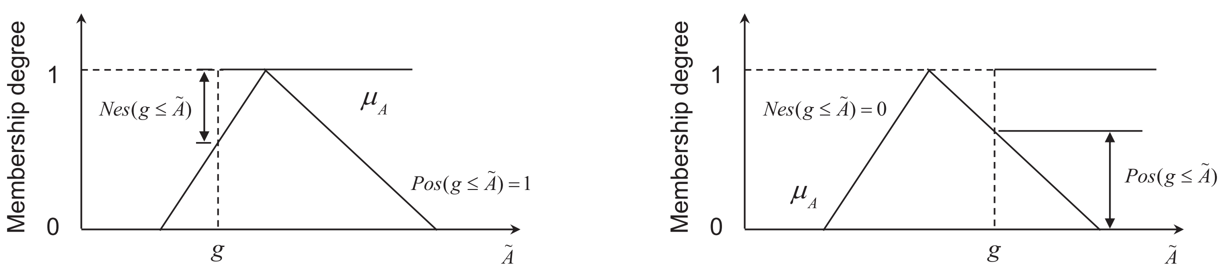

Figure 5. Possibility and necessity degrees of $g \le \tilde{A}$.

3.4. Robust Possibilistic Optimization

A solution to an optimization problem is said to be robust if it achieves both *feasibility robustness* and *optimality robustness*. As defined by Mulvey et al. (1995), "*Feasibility robustness requires that the solution remains "almost" feasible for all possible uncertain values. The optimality robustness means that the objective value remains close to the optimal value for any realization of uncertain parameters*". In this study, the uncertain water diversion and groundwater resources are resource-based uncertainties. The optimization model with fuzzy numbers on the right-hand side of constraints is expressed as Equation 7.

$$\begin{cases} \underset{x \epsilon X}{\text{maximize}} & f(x) \\ \text{subject to} & Ax \leq \tilde{b} \\ & x \geq 0 \end{cases} \tag{7}$$

where x is the decision variable; X is the solution set of x; $f(x)$ is the objective function; A is the coefficient in the constraint; $\tilde{b}$ is the fuzzy number in the right-hand side of the constraint.

$$\begin{cases} \underset{x \epsilon X}{\text{maximize}} & f(x) - \beta[(1-\alpha)\, b_2 + \alpha b_1 - b_1] \\ \text{subject to} & Ax \leq (1-\alpha)\, b_2 + \alpha b_1 \\ & x \geq 0 \\ & 0.5 \leq \alpha \leq 1 \end{cases} \tag{8}$$

In this study, we use the necessity measure (Equation 6) to transform uncertain constraints into deterministic ones. We assume that DMs at all three levels reach an agreement on the attitude to uncertainties at the end of the interaction process. Thus, the term $Ax \leq \tilde{b}$ in Equation 7 becomes $Ax \leq (1-\alpha)b_2 + \alpha b_1$ in Equation 8. The α (>0.5 in this study) is the confidence level of the fuzzy parameter. We measure the robustness by the difference between the value of b used in this study, $[(1-\alpha)b_2 + \alpha b_1]$, and the worst-case value, b_1, which is expressed as $[(1-\alpha) b_2 + \alpha b_1 - b_1]$. The β is the weight (importance) of the robustness term against the objective value and is the robustness coefficient (RC) in this study, representing the DMs' requirement for solution robustness. DMs can select the β according to their experience, financial situation, and risk-bearing capacity. DMs can see tradeoffs between robustness and costs by exploring the sensitivities of the model to the β. The term $\beta \cdot [(1-\alpha) b_2 + \alpha b_1 - b_1]$ controls the feasibility robustness. This approach considers α as an independent variable, which is optimized along with other decision variables. Optimizing α avoids the subjective judgment of DMs on the best confidence level of constraints with non-deterministic parameters. Using the necessity measure to measure robustness could provide reliable values in robust optimization (Fu & Kapelan, 2011; Pishvaee et al., 2012).

Referring to the mathematical expression of the Stackelberg game (Text S3 in Supporting Information S1) and the robust theory above, in this study, the objective function considering robustness can be expressed as Equation 9:

$$\max = 100\,\lambda - \beta \cdot \sum_{t=1}^{T} [(1-\alpha)\, W_{ss\alpha t2} + \alpha W_{ss\alpha t1} - W_{ss\alpha t1}] - \beta \cdot \sum_{i=1}^{I} [(1-\alpha)\, Q_{ss\alpha i2} + \alpha Q_{ss\alpha i1} - Q_{ss\alpha t1}] \tag{9}$$

where λ is scaled by a factor of 100 to keep its dimensions consistent with the robustness measurements of fuzzy water availabilities. The second and third terms of Equation 9 evaluate the difference between the water diversion/groundwater values used in this study and the worst-case water diversion/groundwater values. In this study, the trapezoid fuzzy available water diversion at growth period t is expressed as ($W_{ss\alpha t1}$, $W_{ss\alpha t2}$, $W_{ss\alpha t3}$, $W_{ss\alpha t4}$), in m^3, while the trapezoid fuzzy available groundwater at the growth period t in WSRU i is expressed as ($Q_{ss\alpha t1}$, $Q_{ss\alpha t2}$, $Q_{ss\alpha t3}$, $Q_{ss\alpha t4}$), in m^3. The RC, β, is given by DMs in advance. In this study, we will input $\beta = 0$, 0.01, 0.05, 0.1, 0.2, 0.5, 1, 2, and 3 into the model to calculate the model sensitivities to RCs.

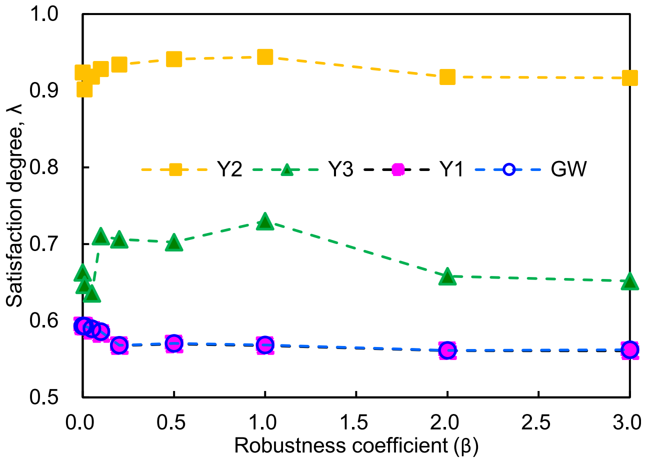

Figure 6. Satisfaction degree (λ) changes with increasing robustness coefficient (β). Y1 is the λ of total food production (objective of first-level decision-makers (DMs)); Y2 is the λ of Gini coefficient (objective of second-level DMs); Y3 is the λ of economic benefit (objective of third-level DMs); GW is the λ of groundwater usage (one decision variable of first-level DMs).

3.5. Solving Steps

Based on the above, we solve the robust optimization of the Stackelberg game through the following steps:

1. Build the objective function and constraints of each decision level in the agricultural water management systems. Establish the physical-based model.
2. Build satisfaction membership functions of objectives and decision variables in each decision level. Set the objective function of maximizing SD in the Stackelberg game.
3. Analyze the data size required in the physical-based model and optimization problem. Express the uncertain data as fuzzy numbers.
4. Use the necessity measure to transform fuzzy parameters in constraints to deterministic parameters associated with confidence level, α.
5. Modify the objective function of maximizing SD as the one considering robustness.
6. Input the preferred the RC, β, which could be 0, 0.01, 0.05, 0.1, 0.2, 0.5, 1, 2, and 3 in this study, to the objective function of the step 5. Optimize the optimization model using a genetic algorithm.
7. If input only one preferred β, the optimal equilibrium water allocation solution and corresponding parameters will be obtained.
8. If input many values of the β, the sensitivity of optimal equilibrium solutions and corresponding parameters to the β will be obtained. These results are the reference for DMs assessing effectiveness and costs when considering robustness.

4. Results

4.1. Optimal Equilibrium Solutions

The model generates equilibrium solutions and displays the relationship between SD and RC (β) after interactions. This relationship shows how the equilibrium SDs change with the β (Figure 6) and is valuable for decision-makers (DM) to observe fluctuations in objective values and help to choose the β. As shown in Figure 6, the three-level objectives show varying difficulties in achieving high satisfaction degrees. The SD (λ) of water allocation fairness (yellow line) is the one that can easily reach high values. The λ of total economic benefits (green line) is more sensitive to the β and fluctuates when the RC increases. The λ of total food production and groundwater usage (blue and purple lines, overlapping) are at the bottom of the figure. This may be either because the first-level DMs proposed a narrow leeway, or because achieving high food production has a lower marginal benefit in this model. Since the given leeway of total food production is within the large range, we believe that during the gaming process, the high food production has a lower marginal benefit than the other two objectives through using the MLP. The first-level DMs can raise the minimum accepted food production to improve the total food production. In Figure 6, the λ of total food production and groundwater use almost coincide, appearing as one line. It indicates a positive impact of groundwater on crop yield and more groundwater is needed to improve food production particularly under limited water availability (high robustness requirement).

4.2. Robustness

In Figure 7, the solution robustness (blue line) improves first rapidly and then slowly, with a critical point at RC = 0.2. At this point, the solution is 62.63% more robust than that at RC = 0, at a low cost (0.60) of solution variation (or infeasibility) (Figure 7). The total usage of water diversion decreases by 13.96%, and the groundwater usage declines by 6.23% (Figure 9). The point RC = 0.2 is also a critical value of changing the trend of the confidence level (α) of fuzzy parameters and maximum satisfaction degrees (λ) (Figure 8). These results offer DMs alternatives with solution robustness, possible costs, confidence levels of fuzzy parameters, and objective values. DMs know how these aspects change and then can choose the preferred solution after a self-evaluation. This decision process is the difference between robust optimization and other optimization methods, where DMs should first choose a confidence level of uncertain parameters, then get reactive alternatives under different

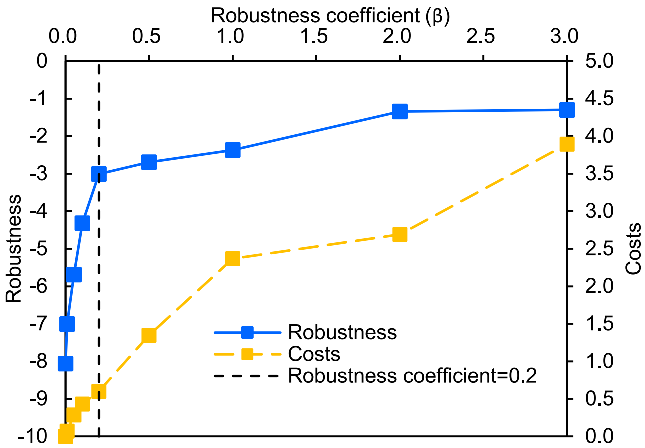

Figure 7. The robustness and costs changes with the robustness coefficient (RC) (β). The RC β can be regarded as the cost per unit of variation. This study takes the product of RC β and variation as the total costs decision-makers pay for the solution robustness.

confidence levels, but don't know costs induced by changing infeasible schemes. In this study, the results of sensitivity analysis provide a decision base for making "rational" decisions.

In this study, DMs have to face risks caused by uncertain available water. This study uses the confidence level α to represent DMs' risk tolerance (Equations 6, 8, and 9). The α measures how certain it is that the fuzzy available water restricts water use constraints. The higher the α, the more conservative the attitude that DMs hold. If DMs select a low value of α, DMs have a higher risk tolerance and face a wide range of solution fluctuation, which means the solution has low robustness. The confidence level (green line in Figure 8) and the robustness (blue line in Figure 7) change similarly, increasing rapidly and then steadily. These two values change consistently and have similar meanings when used.

However, we consider the confidence level α (risk tolerance) and RC β (robustness) to have some differences. We distinguish these two parameters as follows:

In most research, DMs determine the α, DMs' attitudes toward risk, beforehand to transform uncertain parameters to deterministic parameters when solving optimization problems with uncertainties. In this study, we introduce the robustness measure into the model and regard α as a decision variable optimized in line with other variables. The β is given in advance and balances the objective optimality and model robustness (Equations 8 and 9). In this way, the model could achieve a compromised but satisfactory solution under the given decision space. The α here plays a functional role that determines the water amount and assists the model in finding the equilibrium solution.

The RC, β, has a broader meaning than α. The large β means lower tolerance to solution infeasibility, preferring conservative solutions and uncertain parameters approaching the worst-case value. The β represents DMs' attitudes toward uncertainties, risks, and constraint infeasibilities. The β already contains the original meaning of α. Many researchers consider the β as the cost of robustness and obtain a satisfactory solution by minimizing the total cost. Thus, DMs can use the β to measure the effects of uncertainties on objective values.

4.3. Water Allocation Fairness

As shown in Figure 10, in the Stackelberg game, the equilibrium solutions vary nonlinearly with the RC β. The SD of the Gini coefficient climbs first, declines slightly, and finally plateaus, indicating the Gini coefficient reaches its best value in the middle-value range of the β, and poor water allocation fairness happens in both the small-value and large-value ranges of the RC. In the small-value and large-value ranges of the β, the model improves the total food production or economic benefits because of their higher marginal effectiveness at the expense of water allocation equity. For example, in the large-value range of the β, the total water availability is close to the worst-case scenario. The irrigation water is allocated more to WSRUs with high water use efficiencies. In this case, this water allocation pattern improves the SD of food production but decreases the SD of the water allocation fairness.

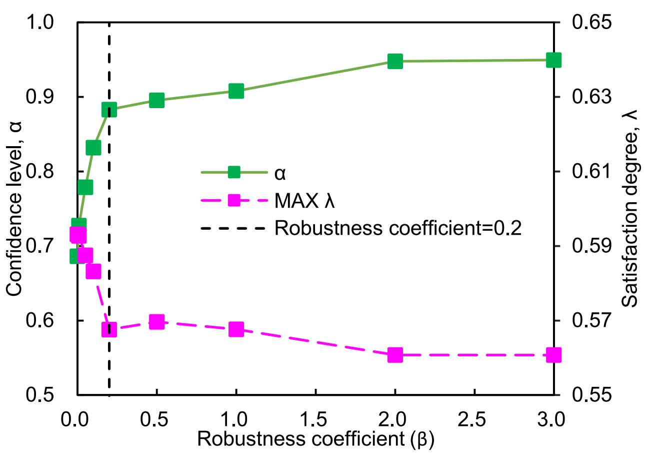

Figure 8. The maximum satisfaction degree (λ) and confidence level (α) of constraints change with the robustness coefficient (β). The maximum λ here refers to the objective functions of the Stackelberg game (the first term of Equation 9).

In the range $\beta \in [0, 1.0]$, the water allocation proportion is sensitive to the variation of β. The allocation proportion of the total water availability, water diversion, and groundwater varies greatly as the β improves. The groundwater allocation fluctuates most drastically because the groundwater resource amount is smaller than the water diversion, and people can use it more flexibly. In the range $\beta \in [1.0, 2.0]$, the water allocation proportions show progressively less marked differences among the 12 WSRUs. In the range $\beta \in [2.0, 3.0]$, the water allocation proportion in each WSRU remains stable.

The water allocation proportions of total water availability all reach over 60%. The water diversion from the Yellow River satisfies 40%–65% of the total

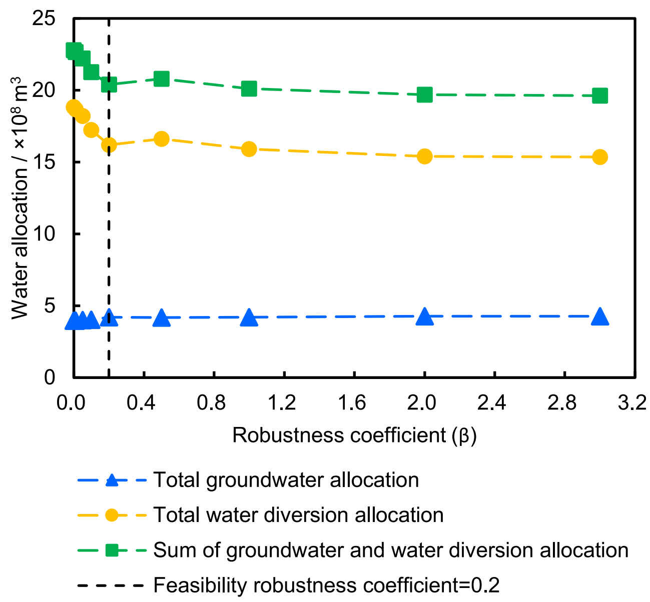

Figure 9. The changing trend of water allocation with robustness coefficient (β).

crop water demand, and the groundwater resource supplies 15%–30% of water for growing crops.

4.4. Farmers' Strategies

As results shown in Figure 11, local farmers should meet crop water demand in the order of wheat > maize > sunflower. The water allocation proportion increases with increasing β in the wheat field, decreases by nearly 50% in the maize field from $\beta = 0$ to $\beta = 3$, and remains within [40%, 60%] in the sunflower field. In Figure 12, the wheat yield in the 12 WSRUs is all close to the maximum value, while the maize yield in some WSRUs is slightly lower than 100%. As an economic crop, the maize yield in the eastern WSRUs (over 80% of the maximum) is higher than in the western WSRUs (50%–80% of the maximum).

We now take the Yangjiahe WSRU as an example to discuss the local farmers' strategies for efficient water use (Figure 13). In April and May, farmers can allocate almost all the water diversion to wheat and maize fields (Figure 13, panels *a* and *b*). A large amount of irrigation considerably improves the groundwater depth and soil water content in the fields. In the wheat field, the groundwater depth rises to 1.73 m ($\beta = 0$), 2.05 m ($\beta = 0.2$), and 2.13 m ($\beta = 3$) (Figure 13, panel *d*), and the relative soil water content reaches 97.76% ($\beta = 0$), 95.03% ($\beta = 0.2$), and 100% ($\beta = 3$) in May (Figure 13, panel *g*). In the maize field, the groundwater depth rises to 1.86, 1.98, and 2.12 m, respectively (Figure 13, panel *e*), and the relative soil water content reaches 100% for all values of β (Figure 13, panel *h*). The expanded groundwater resource can replenish the soil water lost through groundwater evaporation in the next growth stages. In the wheat and maize fields, crop water consumption and evapotranspiration remain high in June and July, when is a critical water demand period for sunflowers, and more water is allocated to the sunflower field (Figure 13, panels *j* and *k*). The sunflower fields have a varied irrigation pattern, with lower robustness than the other two food crops as the β changes. In the Yangjiahe WSRU, groundwater allocation follows the order maize > wheat > sunflower (Figure 13, panels *a*–*c*).

As time passes, the salt accumulates in the shallow soil layers, and its electric conductivity increases, making it necessary to eliminate the salt from the shallow root zone at the end of the growth period.

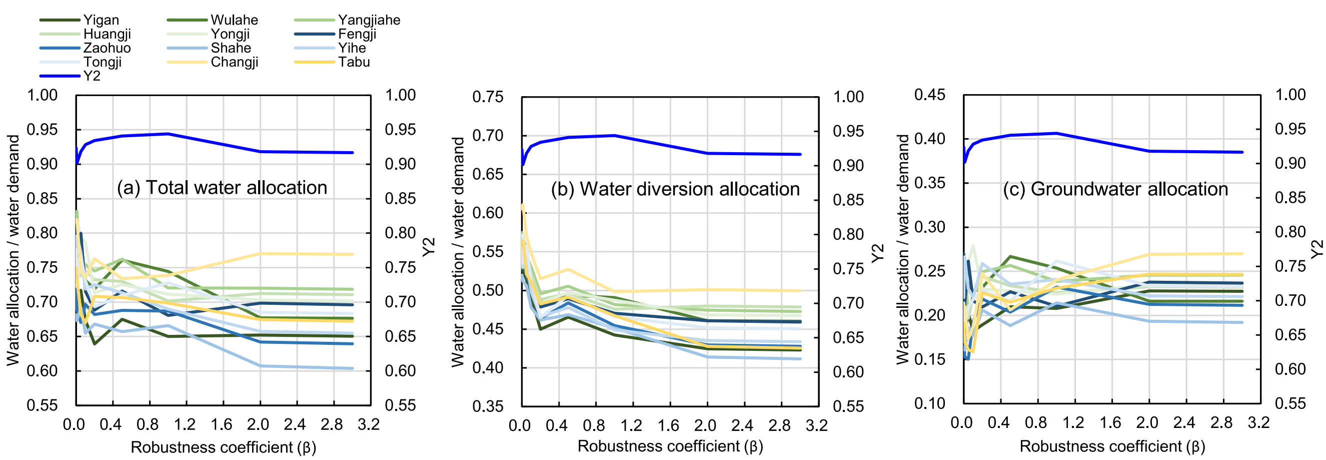

Figure 10. The water allocation proportion (water allocation/water demand) and the satisfaction degree (SD) of the Gini coefficient in the 12 WSRUs. Y2 is the SD of the Gini coefficient. (a) The proportion of total water allocation to water demand; (b) the proportion of water diversion allocation to water demand; (c) the proportion of groundwater allocation to water demand. In this study, the small-value, middle-value, and large-value ranges are roughly [0, 0.4], [0.4, 1.2], and [1.2, 3.0].

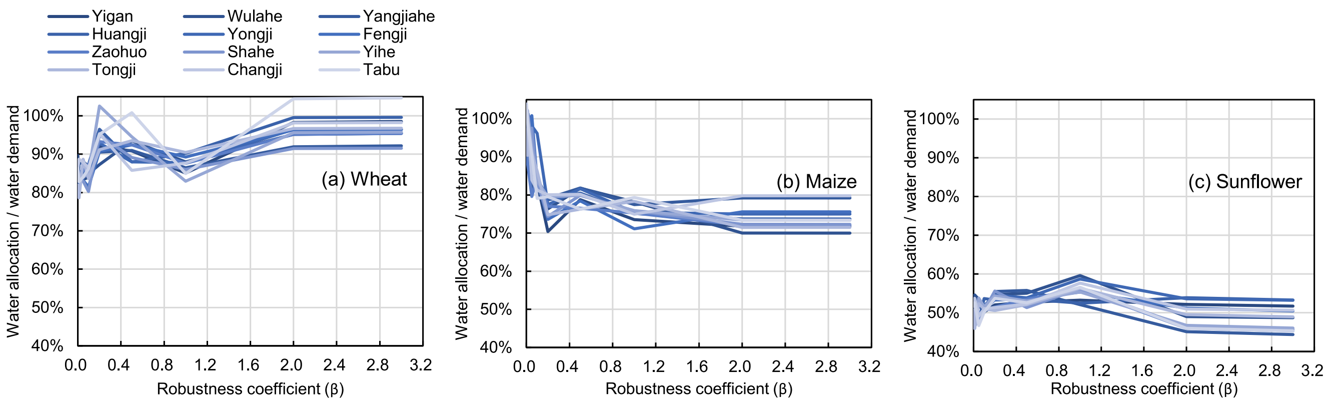

Figure 11. The water allocation proportion of each crop for (a) wheat; (b) maize; (c) sunflower.

5. Discussion

5.1. The Location of Uncertain Parameters

We further illustrate the terms "*feasibility robustness*" and "*optimality robustness*" mentioned in the article by Mulvey et al. (1995) as the terms relate to the location of uncertain parameters in the model: *"The optimality robustness means that the objective value remains close to the optimal value for any realization of uncertain parameters. […] The feasibility robustness requires that the solution remains "almost" feasible for all possible uncertain values"*.

If uncertain parameters are in objective functions, uncertainties cannot influence the decision space of independent variables (Ahmadvand & Sowlati, 2022); in other words, the solutions remain feasible under uncertain environments, and DMs do not need to measure the infeasibilities of solutions caused by uncertain data. However, uncertain parameters influence objective values. The comprised value of uncertain parameters may be not the optimal value in the real world where uncertain parameters are known. DMs should measure the *optimality robustness*, which is a sensitive evaluation of desired goals under uncertain conditions.

If uncertain parameters are on the left- or right-hand side of constraints, uncertainties can change the decision space of independent variables and further influence objective values. The change of the decision space leads to a problem of whether the proposed solutions remain feasible once uncertain parameters are known. In this situation, DMs can measure the impact of the RC on solution variability and choose acceptable/preferred levels of solution infeasibility to avoid unbearable costs (Figures 7 and 8). This is the so-called "feasibility robustness." Uncertain parameters in constraints can change objective values. DMs can measure the variability of objective values with uncertainties, so-called "optimality robustness." In real applications, whether to use feasibility robustness or optimality robustness depends on the given situation and DMs' attitudes. DMs might even select only one constraint with the most concern as the robustness measurement.

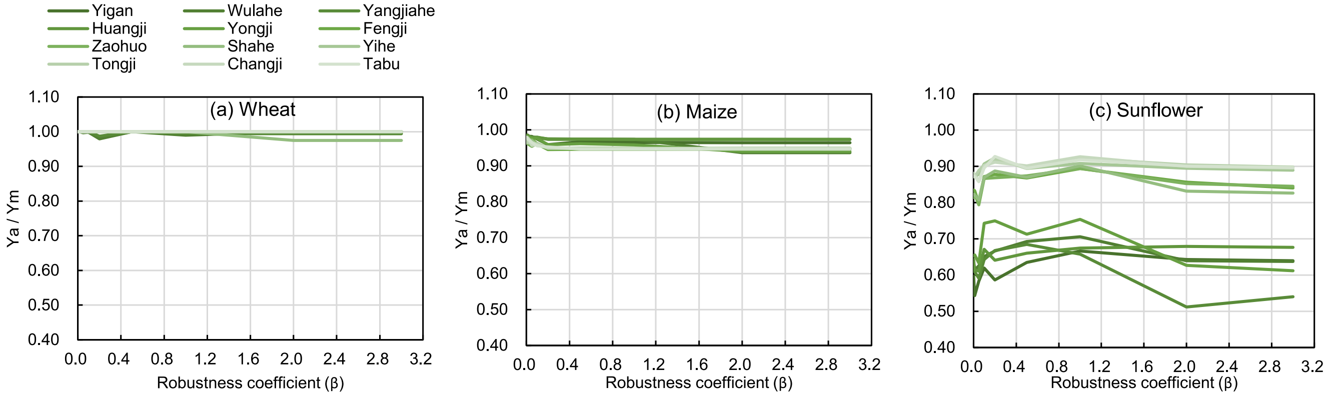

Figure 12. The ratio of actual crop yield (Y_a) to the maximum crop yield (Y_m) for (a) wheat; (b) maize; (c) sunflower.

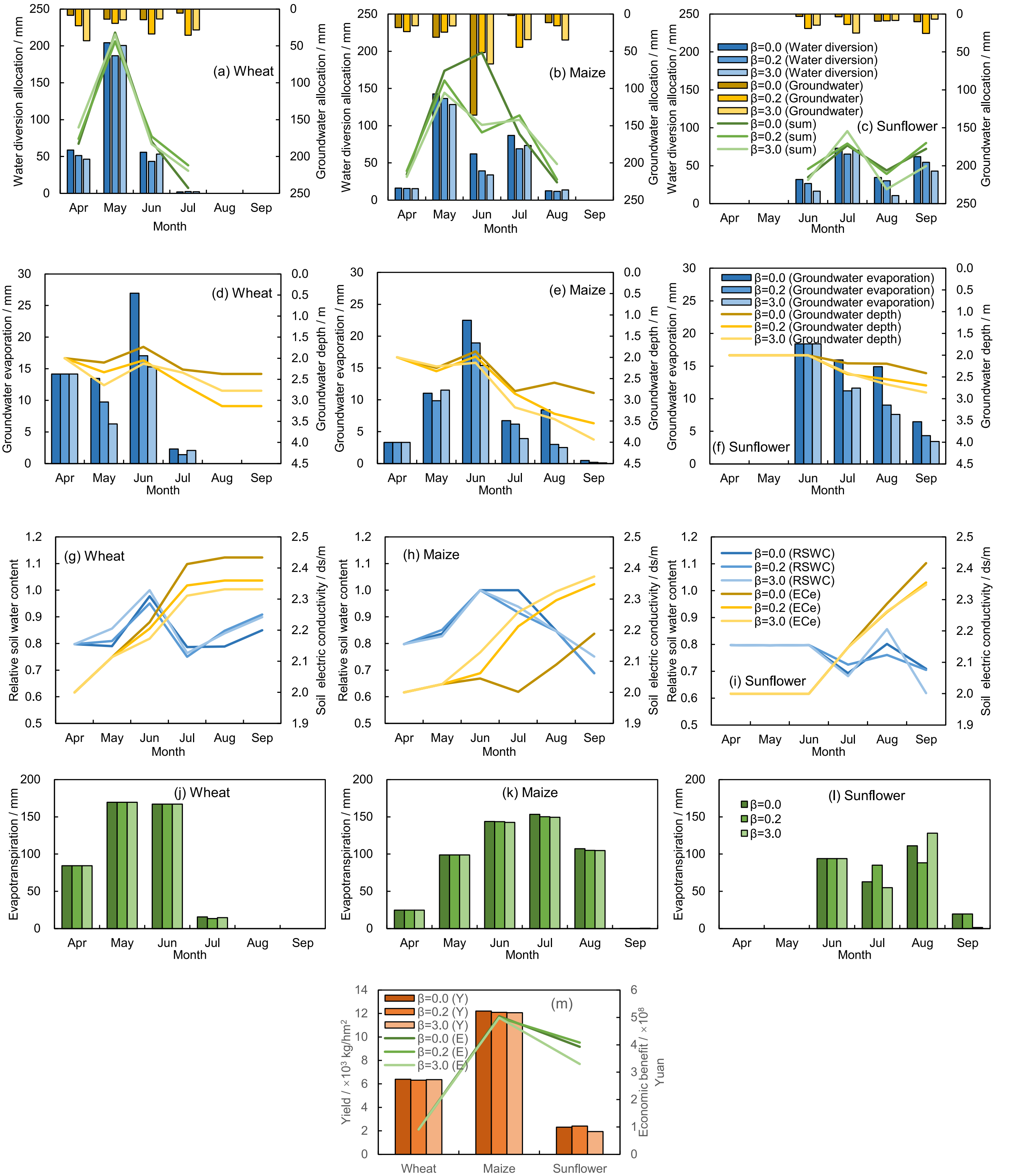

Figure 13. The crop growth process under three scenarios (robustness coefficients $\beta = 1.0$, $\beta = 0.2$, and $\beta = 3.0$). Water allocation in (a) wheat field; (b) maize field; (c) sunflower field. Groundwater depth and groundwater evaporation in (d) wheat field; (e) maize field; (f) sunflower field. Relative soil water content and electric conductivity in (g) wheat field; (h) maize field; (i) sunflower field. Evapotranspiration of (j) wheat; (k) maize; (l) sunflower. Yield and economic benefits of each crop (m).

5.2. Uncertainties in a Stackelberg Game

In a Stackelberg game, uncertain data may exist at each decision level. Each DM has to face the uncertain data in their sub-model when optimizing their objective functions. During the interactive process, DMs potentially pass the uncertainties to other decision levels through multiple feedbacks of "pass" or "reject." Finally, every uncertainty in the model is sustained by all DMs. However, it is the top-level DM, the first one proposing their goals and leeway, who must consider all possible uncertainties in this full-information game. Otherwise, it becomes difficult for lower-level DMs to realize the goal proposed by upper-level DMs within a narrow decision space. Conversely, lower-level DMs receive the expected goals, decision space, and leeway from the upper-level DMs, which contain implicit uncertainties from the upper-level sub-model. Therefore, the lower-level DMs make decisions based on the uncertainties given by the upper-level DMs. Even if the upper-level DMs may miss some uncertainties from the lower level, the upper-level DMs can infer this information from the lower-level DMs' solutions and make modifications accordingly.

6. Conclusions

This paper contributes a robust framework in the multi-level agricultural water management system. We modeled the multi-level agricultural water management system in a shallow groundwater area as a Stackelberg game and used the multi-level programming to solve the multi-level water allocation problem. We expressed the uncertain data in the system as fuzzy numbers and convert fuzzy numbers to deterministic values by the necessity measure. The introduction of the robust framework into the multi-level programming generated a series of alternatives with multiple robustness coefficients. These solutions formulate a decision-making base containing the robustness of equilibrium solution, the optimal confidence level of fuzzy numbers, the maximum SD, and the cost of maintaining certain robustness level. As the results reported, the objectives in different levels show different difficulties in achieving high satisfaction degrees, with the first-level goal of food security being the most difficult one. We suggest the first-level DMs better ensure the food security through an appropriate increase in the minimum food production restrict placed on the lower DMs. The bottom DMs (local farmers) can prioritize ensuring water demand in critical crop growth periods to realize efficient water use and high crop production. In the optimal equilibrium solutions, the food crops (wheat and maize) have more robust water allocation schemes than the economic crop (sunflowers) as uncertain data changes. From the results, the robustness measure performs well in reducing variations of equilibrium solutions in the multi-level system. DMs can achieve a balance of the solution feasibility and objective optimality under the given RC.

Data Availability Statement

The data on the available water resources and land utilization in the HID were collected from the yearbooks of Bayannur (Bayannur Bureau of Statistics, 2012). The parameters for the crop growth model are available in the research papers on field experiments conducted in the Hetao Irrigation District (Hou et al., 2016; Jiang et al., 2019; Xue et al., 2017; Yue, 2004; X. Zhang et al., 2021). Meteorological data were collected from local meteorological stations and the website of the China Meteorological Data Service Center (http://data.cma.cn/). The related detailed methods and calculation can be found in Supporting Information S1. Other data can be found in Data Set S1.

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References

Ahmadvand, S., & Sowlati, T. (2022). A robust optimization model for tactical planning of the forest-based biomass supply chain for syngas production. *Computers & Chemical Engineering*, *159*, 107693. https://doi.org/10.1016/j.compchemeng.2022.107693

Ahmed, S., & Sahinidis, N. V. (1998). Robust process planning under uncertainty. *Industrial & Engineering Chemistry Research*, *37*(5), 1883–1892. https://doi.org/10.1021/ie970694t

Anandalingam, G., & Apprey, V. (1991). Multi-level programming and conflict resolution. *European Journal of Operational Research*, *51*(2), 233–247. https://doi.org/10.1016/0377-2217(91)90253-R

Bayannur Bureau of Statistics. (2012). Bayannur statistical yearbook [Dataset]. *China Meteorological Data Sharing Service System*. https://hess.copernicus.org

Cabo, F., Erdlenbruch, K., & Tidball, M. (2014). Dynamic management of water transfer between two interconnected river basins. *Resource and Energy Economy*, *37*, 17–38. https://doi.org/10.1016/j.reseneeco.2014.03.002

Chen, Y., He, L., Guan, Y., Lu, H., & Li, J. (2017). Life cycle assessment of greenhouse gas emissions and water-energy optimization for shale gas supply chain planning based on multi-level approach: Case study in Barnett, Marcellus, Fayetteville, and Haynesville shales. *Energy Conversion and Management*, *134*, 382–398. https://doi.org/10.1016/j.enconman.2016.12.019

Chen, Y., Lu, H., Li, J., Ren, L., & He, L. (2017). A leader-follower-interactive method for regional water resources management with considering multiple water demands and eco-environmental constraints. *Journal of Hydrology*, *548*, 121–134. https://doi.org/10.1016/j.jhydrol.2017.02.015

Ciullo, A., Domeneghetti, A., Kwakkel, J., De Bruijin, K. M., Klijin, F., & Castellarin, A. (2023). Belief-informed robust decision making (BIRDM): Assessing changes in decision robustness due to changing assumptions in the distribution of deeply uncertain variables. *Environmental Modelling & Software*, *159*, 105560. https://doi.org/10.1016/j.envsoft.2022.105560

Du, W., Fan, Y., Liu, X., Park, S. C., & Tang, X. (2019). A game-based production operation model for water resource management: An analysis of the South-to-North Water Transfer Project in China. *Journal of Cleaner Production*, *228*, 1482–1493. https://doi.org/10.1016/j.jclepro.2019.04.351

Fu, G., & Kapelan, Z. (2011). Fuzzy probabilistic design of water distribution networks. *Water Resources Research*, *47*(5), W05538. https://doi.org/10.1029/2010WR009739

Gini, C. (1913). Variability and mutability. In *Studi economic-giuridici fac*. Giurisprudenza University of Cagliari, A. III, Part II. https://doi.org/10.1111/j.2397-2335.1913.tb03065.x

Giuliani, M., & Castelletti, A. (2016). Is robustness really robust? How different definitions of robustness impact decision-making under climate change. *Climatic Change*, *135*(3–4), 409–424. https://doi.org/10.1007/s10584-015-1586-9

Guo, S., Zhang, F., Engel, B. A., Wang, Y., Guo, P., & Li, Y. (2022). A distributed robust optimization model based on water-food-energy nexus for irrigated agricultural sustainable development. *Journal of Hydrology*, *606*, 127394. https://doi.org/10.1016/j.jhydrol.2021.127394

Guo, X., Guo, Q., Chen, Y., Valipour, E., & Nojavan, S. (2022). Regret-based management of wind-solar-thermal generation company under uncertainties: A novel stochastic p-robust optimization approach. *Sustainable Energy Technologies and Assessments*, *52*, 102313. https://doi.org/10.1016/j.seta.2022.102313

Gupta, R., & Gupta, J. (2022). Future generation communications with game strategies: A comprehensive survey. *Computer Communications*, *192*, 1–32. https://doi.org/10.1016/j.comcom.2022.05.024

Herman, J. D., Reed, P. M., Zeff, H. B., & Characklis, G. W. (2015). How should robustness be defined for water systems planning under change? *Journal of Water Resources Planning and Management*, *141*(10), 04015012. https://doi.org/10.1061/(ASCE)WR.1943-5452.0000509

Hou, Q., Wang, H., & Yun, W. (2016). Research on water production function of spring in Hetao Irrigation District (ID) of Inner Mongolia based on Jensen model [Dataset]. *Agricultural Research in the Arid Areas*, *34*(3), 84–89. (in Chinese). https://doi.org/10.7606/j.issn.1000-7601.2016.03.13

Huang, Y., Wang, Y., & Liu, N. (2022). A two-stage energy management for heat-electricity integrated energy system considering dynamic pricing of Stackelberg game and operation strategy optimization. *Energy*, *244*(A), 122576. https://doi.org/10.1016/j.energy.2021.122576

Inuiguchi, M., & Ramik, J. (2000). Possibilistic linear programming: A brief review of fuzzy mathematical programming and a comparison with stochastic programming in portfolio selection problem. *Fuzzy Sets and Systems*, *111*(1), 3–28. https://doi.org/10.1016/S0165-0114(98)00449-7

Jia, Y., & Culver, T. B. (2006). Robust optimization for total maximum daily load allocations. *Water Resources Research*, *42*(2), W02412. https://doi.org/10.1029/2005WR004079

Jiang, Y., Xiong, L., Yao, F., & Xu, Z. (2019). Optimizing regional irrigation water allocation for multi-stage pumping-water irrigation system based on multi-level optimization-coordination model [Dataset]. *Journal of Hydrology X*, *4*, 100038. https://doi.org/10.1016/j.hydroa.2019.100038

Jiménez, M., Arenas, M., Bilbao, A., & Rodríguez, M. V. (2007). Linear programming with fuzzy parameters: An interactive method resolution. *European Journal of Operational Research*, *177*(3), 1599–1609. https://doi.org/10.1016/j.ejor.2005.10.002

Kicsiny, R., Piscopo, V., Scarelli, A., & Varga, Z. (2014). Dynamic Stackelberg game model for water rationalization in drought emergency. *Journal of Hydrology*, *517*, 557–565. https://doi.org/10.1016/j.jhydrol.2014.05.061

Lennox, J., Proctor, W., & Russell, S. (2011). Structuring stakeholder participation in New Zealand's water resource governance. *Ecological Economics*, *70*(7), 1381–1394. https://doi.org/10.1016/j.ecolecon.2011.02.015

Li, M., Fu, Q., Singh, V. P., Liu, D., & Li, J. (2020). Optimization of sustainable bioenergy production considering energy-food-water-land nexus and livestock manure under uncertainty. *Agricultural Systems*, *184*, 102900. https://doi.org/10.1016/j.agsy.2020.102900

Li, S., Zhang, L., Nie, L., & Wang, J. (2022). Trading strategy and benefit optimization of load aggregators in integrated energy systems considering integrated demand response: A hierarchical Stackelberg game. *Energy*, *249*, 123678. https://doi.org/10.1016/j.energy.2022.123678

Li, X., Ma, W., Shi, H., Cheng, Z., & Lei, W. (2023). A game theoretic approach for evaluating high-water consumption manufacturing strategies under endogenous government financial subsidies. *Computers & Industrial Engineering*, *176*, 108907. https://doi.org/10.1016/j.cie.2022.108907

Li, Y., Wang, B., Yang, Z., Li, J., & Chen, C. (2022). Hierarchical stochastic scheduling of multi-community integrated energy systems in uncertain environments via Stackelberg game. *Applied Energy*, *308*, 118392. https://doi.org/10.1016/j.apenergy.2021.118392

Liu, B. (2000). Dependent-chance programming in fuzzy environments. *Fuzzy Sets and Systems*, *109*(1), 97–106. https://doi.org/10.1016/S0165-0114(97)00384-9

Liu, B., Ge, X., Liu, B., & Yuan, X. (2022). Distributionally robust optimization of non-fossil fuels processing network under uncertainty. *Chemical Engineering Journal*, *445*, 136756. https://doi.org/10.1016/j.cej.2022.136756

Liu, B., & Liu, Y. K. (2002). Expected value of fuzzy variable and fuzzy expected value models. *IEEE Transactions on Fuzzy Systems*, *10*(4), 445–450. https://doi.org/10.1109/TFUZZ.2002.800692

Liu, D., Ji, X., Tang, J., & Li, H. (2020). A fuzzy cooperative game theoretic approach for multinational water resource spatiotemporal allocation. *European Journal of Operational Research*, *282*(3), 1025–1037. https://doi.org/10.1016/j.ejor.2019.10.018

Luo, B., Zhang, F., Liu, X., Pan, Q., & Guo, P. (2021). Managing agricultural water considering water allocation priority based on remote sensing data. *Remote Sensing*, *2021*(13), 1536. https://doi.org/10.3390/rs13081536

Ma, Y., Li, Y. P., Huang, G. H., & Liu, Y. R. (2020). Water-energy nexus under uncertainty: Development of a hierarchical decision-making model. *Journal of Hydrology*, *591*, 125297. https://doi.org/10.1016/j.jhydrol.2020.125297

McPhail, C., Maier, H. R., Kwakkel, J. H., Giuliani, M., Castelletti, A., & Westra, S. (2018). Robustness metrics: How are they calculated, when should they be used and why do they give different results? *Earth's Future*, *6*(2), 169–191. https://doi.org/10.1002/2017EF000649

Mulvey, J. M., Vanderbei, R. J., & Zenios, S. A. (1995). Robust optimization of large-scale systems. *Operations Research*, *43*(2), 264–281. https://doi.org/10.1287/opre.43.2.264

Ning, C., & You, F. (2019). Optimization under uncertainty in the era of big data and deep learning: When machine learning meets mathematical programming. *Computers & Chemical Engineering*, *125*, 434–448. https://doi.org/10.1016/j.compchemeng.2019.03.034

Paksseresht, M., Mahdavi, I., Shirazi, B., & Mahdavi-Amiri, N. (2020). Co-Reconfiguration of product family and supply chain using leader–follower Stackelberg game theory: Bi-Level multi-objective optimization. *Applied Soft Computing Journal*, *91*, 106203. https://doi.org/10.1016/j.asoc.2020.106203

Perelman, L., Housh, M., & Ostfeld, A. (2002). Robust optimization for water distribution systems least cost design. *Water Resources Research*, *49*(10), 6795–6809. https://doi.org/10.1002/wrcr.20539

Pishvaee, M. S., Razmi, J., & Torabi, S. A. (2012). Robust possibilistic programming for socially responsible supply chain network design: A new approach. *Fuzzy Sets and Systems*, *206*, 1–20. https://doi.org/10.1016/j.fss.2012.04.010

Pramanik, S., & Roy, T. K. (2007). Fuzzy goal programming approach to multi-level programming problems. *European Journal of Operational Research*, *176*(2), 1151–1166. https://doi.org/10.1016/j.ejor.2005.08.024

Rouhani, S., & Amin, S. H. (2022). A robust convex optimization approach to design a hierarchical organ transplant network: A case study. *Expert Systems with Applications*, *197*, 116716. https://doi.org/10.1016/j.eswa.2022.116716

Salazar, J., Castelletti, A., & Giuliani, M. (2022). Multi-objective robust planning tools. In *Oxford research encyclopedias, environmental science*.

Shih, H. S., Lai, Y. J., & Lee, E. S. (1996). Fuzzy approach for multi-level programming problems. *Computers & Operations Research*, *23*(1), 73–91. https://doi.org/10.1016/0305-0548(95)00007-9

Soliman, A. (2022). Prescriptive drought policy and water supplier compliance. *Ecological Economics*, *197*, 107429. https://doi.org/10.1016/j.ecolecon.2022.107429

Soncini-Sessa, R., Castelletti, A., & Weber, E. (2007). *Integrated and participatory water resources management: Theory*. Elsevier.

Sun, Q., Wang, R., Chen, S., Wu, C., Wei, Z., & Zang, H. (2022). Nash–cournot power market model with a high penetration of prosumers: A distributionally robust optimization approach. *Journal of Cleaner Production*, *353*, 131565. https://doi.org/10.1016/j.jclepro.2022.131565

Sun, W. (2014). *The analysis of difference scales diversity law of irrigation water efficiency and water saving potential in Hetao Irrigation area of Inner Mongolia*. Inner Mongolia Agricultural University. (in Chinese).

Tang, J., Fu, C., Mi, C., & Liu, H. (2022). An interval sequential linear programming for nonlinear robust optimization problems. *Applied Mathematical Modelling*, *107*, 256–274. https://doi.org/10.1016/j.apm.2022.02.037

Tang, Z. (2019). Solving Stackelberg equilibrium for multi objective aerodynamic shape optimization. *Applied Mathematical Modelling*, *72*, 588–600. https://doi.org/10.1016/j.apm.2019.03.036

Tantoh, H. B., & Simatele, D. (2018). Complexity and uncertainty in water resource governance in Northwest Cameroon: Reconnoitering the challenges and potential of community-based water resource management. *Land Use Policy*, *75*, 237–251. https://doi.org/10.1016/j.landusepol.2018.03.044

Xu, B., Sun, Y., Huang, X., Zhong, P., Zhu, F., Zhang, J., et al. (2022). Scenario-based multi objective robust optimization and decision-making framework for optimal operation of a cascade hydropower system under multiple uncertainties. *Water Resources Research*, *58*(4), e2021WR030965. https://doi.org/10.1029/2021WR030965

Xu, J., Tu, Y., & Zeng, Z. (2013). Bilevel optimization of regional water resources allocation problem under fuzzy random environment. *Journal of Water Resources Planning and Management*, *139*(3), 246–264. https://doi.org/10.1061/(ASCE)WR.1943-5452.0000248

Xu, Z., Yao, L., Zhou, X., Moudi, M., & Zhang, L. (2019). Optimal irrigation for sustainable development considering water rights transaction: A Stackelberg-Nash-Cournot equilibrium model. *Journal of Hydrology*, *575*, 628–637. https://doi.org/10.1016/j.jhydrol.2019.05.063

Xue, J., Guan, H., Huo, Z., Wang, F., Huang, G., & Boll, J. (2017). Water saving practices enhance regional efficiency of water consumption and water productivity in an arid agricultural area with shallow groundwater [Dataset]. *Agricultural Water Management*, *194*, 78–89. https://doi.org/10.1016/j.agwat.2017.09.003

Xue, J., Huo, Z., Wang, F., Kang, S., & Huang, G. (2018). Untangling the effects of shallow groundwater and deficit irrigation on irrigation water productivity in arid region: New conceptual model. *Science of the Total Environment*, *619–620*, 1170–1182. https://doi.org/10.1016/j.scitotenv.2017.11.145

Yu, D., Wu, J., Wang, W., & Gu, B. (2022). Optimal performance of hybrid energy system in the presence of electrical and heat storage systems under uncertainties using stochastic p-robust optimization technique. *Sustainable Cities and Society*, *83*, 103935. https://doi.org/10.1016/j.scs.2022.103935

Yu, L., Xiao, Y., Zeng, X. T., Li, Y. P., & Fan, Y. R. (2020). Planning water-energy-food nexus system management under multi-level and uncertainty. *Journal of Cleaner Production*, *251*, 119658. https://doi.org/10.1016/j.jclepro.2019.119658

Yue, W. (2004). *Study on the mechanism of consumption in Yichang irrigation sub-district of irrigation district of Inner Mongolia along the Yellow River (Dataset)*. Wuhan University (in Chinese).

Zadeh, L. A. (1978). Fuzzy sets as a basis for a theory of possibility. *Fuzzy Sets and Systems*, *1*, 3–28. https://doi.org/10.1016/S0165-0114(99)80004-9

Zhang, F., Guo, S., Liu, X., Wang, Y., Engel, B. A., & Guo, P. (2020). Towards sustainable water management in an arid agricultural region: A multi-level multi-objective stochastic approach. *Agricultural Systems*, *182*, 102848. https://doi.org/10.1016/j.agsy.2020.102848

Zhang, X., Guo, P., Zhang, F., Liu, X., Yue, Q., & Wang, Y. (2021). Optimal irrigation water allocation in Hetao Irrigation District considering decision makers' preference under uncertainties [Dataset]. *Agricultural Water Management*, *246*, 106670. https://doi.org/10.1016/j.agwat.2020.106670

Zhang, X., Li, C., & Guo, P. (2022). Stackelberg equilibrium water allocation patterns in shallow groundwater area under uncertainties. *Journal of Hydrology*, *612*, 128155. https://doi.org/10.1016/j.jhydrol.2022.128155

References From the Supporting Information

Allen, R. G., Pereira, L. S., Raes, D., & Smith, M. (1998). Crop evapotranspiration. Guidelines for computing crop water requirements. In *Food and agriculture organization of the united Nations, Rome, Italy (FAO irrigation and drainage paper no. 56)*.

Gao, X., Huo, Z., Xu, X., Qu, Z., Huang, G., Tang, P., & Bai, Y. (2018). Shallow groundwater plays an important role in enhancing irrigation water productivity in an arid area: The perspective from a regional agricultural hydrology simulation. *Agricultural Water Management*, *208*, 43–58. https://doi.org/10.1016/j.agwat.2018.06.009

Goerigk, M., Lendl, S., & Wulf, L. (2022). Two-Stage robust optimization problems with two-stage uncertainty. *European Journal of Operational Research*, *302*(1), 62–78. https://doi.org/10.1016/j.ejor.2021.12.046

Li, X., Zhang, C., & Huo, Z. (2020). Optimizing irrigation and drainage by considering agricultural hydrological process in arid farmland with shallow groundwater. *Journal of Hydrology*, *585*(2), 124785. https://doi.org/10.1016/j.jhydrol.2020.124785

Pen, S., Zhang, Y., & Shen, J. (1992). An optimal irrigation model for conjunctive operation of canals and wells in Hetao Irrigation District. *Advances in Water Science*, *3*, 199–206. (in Chinese).