A SOFTWARE SUITE FOR CONJUNCTION ANALYSIS ASSESSMENT IN SPACE SURVEILLANCE AND TRACKING APPLICATIONS

Sergio Bonaccorsi(1), **Marco Felice Montaruli**(2), **Pierluigi Di Lizia**(3), **Moreno Peroni**(4), **Alessandro Panico**(4), **Marco Rigamonti**(4), **Francesco Del Prete**(4)

(1)*Politecnico di Milano, Via G. La Masa 34, 20156, Milan, Italy, sergio.bonaccorsi@polimi.it*
(2)*Politecnico di Milano, Via G. La Masa 34, 20156, Milan, Italy, marcofelice.montaruli@polimi.it*
(3)*Politecnico di Milano, Via G. La Masa 34, 20156, Milan, Italy, pierluigi.dilizia@polimi.it*
(4)*Via Pratica di Mare 45, 00071, Pomezia (RM), Italy*

ABSTRACT

The in-orbit overpopulation is currently fostering Space Surveillance and Tracking (SST) related applications, where identification and characterization of close approaches between pairs of space objects has become crucial. This paper presents a tool for conjunction analysis whose first module, which has proved to significantly speed up the process, includes a sequence of geometric and temporal filters to reduce the catalogue of possible colliding pairs. The remaining objects are propagated to calculate quantities such as Time to Closest Approach (TCA), Miss Distance (MD) and Probability of Collision (PoC) according to three different methods. For conjunctions with either a MD or a PoC that exceed alert levels, the algorithm allows an impulsive Collision Avoidance Maneuver (CAM) to be planned at specified maneuver epochs, given the MD and PoC thresholds to be met afterwards. The CAM is calculated through an analytical unperturbed approach that aims to satisfy the target PoC or MD. The results are then verified through a perturbed propagation, and MD and PoC after the maneuver are recalculated to check their compliance with the desired thresholds. Finally, the paper assesses the performance of ISOC in terms of synthetic and real data.

1 INTRODUCTION

In the last decades, in orbit population has become a major issue for space agencies and institutions worldwide. The two most populated regions are Low Earth Orbit (LEO) and Geostationary Orbit (GEO). Among orbiting objects, just a small fraction is represented by co-operative satellites and the main part is represented by space debris, which include inactive satellites, rocket bodies, and fragments of all sizes [1]. Space debris represent a threat to space activities. Indeed, their presence may jeopardize the operative mission of active satellites, given that the possible impact with a space debris ranges from cumulative erosion of satellite surface to the possible satellite destruction, with the generation of thousands of additional pieces of debris and inevitable environmental drawbacks and possible cascade effects. Therefore, different strategies have been implemented to guarantee safe operations, and an international commitment is currently taking place in the Space Surveillance and Tracking (SST) field. Europe deals with this topic through two programmes: the European Space Agency (ESA) Space Situational Awareness (SSA) programme [2] and the European Space Surveillance and Tracking (EUSST) framework [3]. The latter groups European national agencies and institutions and is in charge of carrying out the following services: conjunction analysis, fragmentation analysis

and re-entry prediction. These services exploit measurements obtained through ground-based sensors, which are optical telescopes (they provide highly accurate angular track), radars (in addition to angles, they provide either range or Doppler shift measurements or both) and lasers (they provide extremely precise range measurements) [4]. A key role is provided by the survey radars, which allow to characterize an unknown object orbit at the first detection [5].
Italy is involved in EUSST programme through Italian Space Agency (ASI), Astrophysics National Institute (INAF) and Italian Airforce (AM), and it is in charge of reentry and fragmentation services [6] [7] [8]. For this reason, efficient and reliable tools shall be designed to process observation data. Within this framework, the Flight Test Department of the Italian Air Force is responsible for the development of the system architecture that gather and process data and provide Space Situational Awareness to national military and civil users, as well as the EUSST consortium. Overall, the system has been designed with a web-based infrastructure in which space objects and relevant data populate a catalogue, whereas specific services and functions concur to create awareness about space events. The embedded software has been designed and implemented with an operational perspective and in partnership with national industry and academia. Specifically, the Collision Avoidance (CA) service is devoted to the conjunction analyses: it is in charge of assessing possible conjunctions among catalogued objects and, for those which overcome threshold quantities, a Conjunction Data Message (CDM) [9] is created, which groups the information of the satellites involved. Usually, the objects are distinguished in primary (the one which is manoeuvrable, for instance) and secondary (as space debris, for example) [10], and together they are often referred to as a pair. For them, the CDM reports the satellites state, both in terms of mean and covariance, the Time of Closest Approach (TCA), the related Miss Distance (MD), which is the distance at the closest approach, and the Probability of Collision (PoC).
This work describes the prototypal version of the conjunction analysis tool developed for the ISOC 3.0 Suite thanks to a collaboration involving the Italian Air Force, Leonardo Company and Politecnico di Milano. The software architecture has been designed to guarantee the highest performance in terms of computational times. From a general perspective, the tool involves three main modules, which ensure the accurate identification and characterization of possible conjunction events. First, a filtering sequence allows for the screening of all the pairs that do not meet certain criteria. Next, the remaining events are characterized through the calculation of relevant quantities such as the PoC, which is computed through the use of three different methods. Finally, based on some user requirements, the tool computes a Collision Avoidance Maneuver (CAM), which could be considered by the operator as a possible way to prevent the collision. Based on user's needs, it is possible to store the information calculated for the relevant events and provide some output files such as the CDM in the standard form as described by CCSDS (Consultative Committee for Space Data Systems). In addition to output file generation, it is also possible to take input files such as CDM, Two Line Elements (TLE), Orbit Ephemeris Message (OEM), and Orbit Parameter Message (OPM).
The paper is organized as follows. First, an overview of ISOC Suite is provided and the mathematical background of the different algorithms is discussed. Then, a numerical analysis is carried out to validate the tool. Finally, the algorithm performance is assessed through operational real case scenarios.

2 ITALIAN SST OPERATIONS CENTRE

ISOC was originally established in 2015 and operated by the military personnel of the Flight Test Department of the Italian Air Force. Currently, the operational activities are led by the the Space Situational Awareness Centre (C-SSA), whereas the Flight Test Department is responsible for Research and Development tasks.

The ISOC Suite is a complex system that was originally developed to support Space Surveillance and Tracking tasks, but it is currently evolving towards a broader awareness of the space scenario, to enhance the national security for both civil and military applications. ISOC is also included in the EUSST framework, supporting the service listed below:

- Re-entry (RE): prime responsible for the analysis of uncontrolled re-entry in low atmosphere for large and dangerous objects.
- Fragmentation (FG): prime responsible for the analysis of in-orbit fragmentation as consequence of satellite break-ups or collisions.
- Collision Avoidance (CA): in the past as cold redundant operational center for the analysis of the collision probability and geometry for conjunction events.

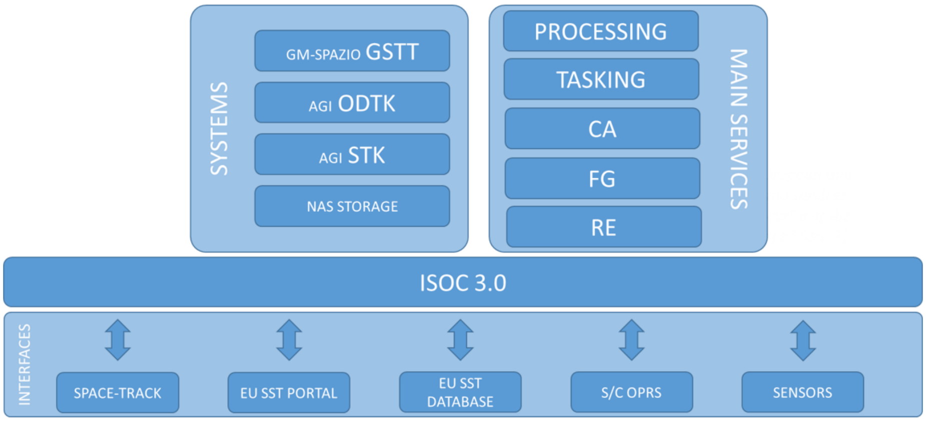


Figure 1: ISOC Architecture

ISOC Suite is used to support the above-mentioned services, whose high level architecture is represented in Figure 1. The main inputs of the suite are provided by national sensors, consortium observations, European observation catalogue along with available public sources. The inner part of the system is able to use also commercial on the shelf (COTS) and proprietary software. The system output are the services shown in the right part of Figure 1. A functional part of the entire system is the Collision Avoidance process, that could be assured by the suite described in this document.

3 ALGORITHMS

3.1 Catalogue Screening

This module is essential to detect approaches between one or more primaries and a large number of secondaries, typically the entire space catalogue. Performing this analysis without a filtering sequence is extremely expensive in terms of computational resources. After defining a time interval for the analysis, and given the orbital and physical information of the primaries and secondaries of interest, a preliminary step consists in a pre-process of the input data by performing orbital propagation of the objects states at the initial time defined by the user. Then, the flow is divided into two stages: the use of a filtering sequence and the calculation of typical conjunction quantities for each remaining pair.

3.1.1 Filtering Sequence

The filtering sequence is based on the work of Hoots et al. [11], and it relies on the idea that a primary and a secondary may be far from being involved in a collision event because of geometrical considerations and absence of overlaps between relevant time intervals.
Two objects could be on orbits that do not allow them to be closer than a certain distance. In this case the Apogee-Perigee filter (*AP filter*) and the Orbit Path filter (*OP filter*) are responsible for the elimination of the discarded pairs. The Time filter (*T filter*) adds the information of the actual position of the space objects on their orbits, eliminating the pair if they do not pass in a region of relative closeness at the same time. The order of the filters within the sequence is uniquely driven by the computational cost of the related operations.
After this sequence, a filtered version of the original catalogue of secondaries is obtained for each primary, and passed to the one-to-one analysis, which calculates the TCA and the corresponding peculiar quantities of a conjunction event.

Apogee-Perigee Filter

The *AP filter* is the first of the series and has a very low computational cost. For each object it computes apogee and perigee and then, for each pair, it simply verifies the following geometrical criterion:

$$q - Q > D \tag{1}$$

Where q is the larger of the two perigees, Q the smaller of the two apogees and D is a threshold distance selected by the user. Based on the work of Alfano and Finkleman [12], a suggested value for this threshold is 10 km. Generally, the more accurate the orbital data, the smaller the threshold can be set.

Time Filter

In the filtering process, it is fundamental to consider the actual position of the two space objects on their orbits, going beyond simple geometric considerations. This means that primary and secondary must cross a region of relative closeness at the same time to be eligible for a possible collision.
The main idea is therefore to verify the overlap of time intervals defined according to the passage of the primary and the secondary in these regions. In general, a satellite will be in a region of vulnerability for a short period of time before and after it flies through the line of intersection of the two orbital planes. Time windows are therefore defined by selecting two orbital positions around the line of nodes and, through the Kepler equation, they are converted to a time interval. By adding the orbital period, a series of time windows for both the primary and the secondary is obtained. The next step is to verify the presence of overlap between them to state if the pair will pass the filter or not.
For the computation of the time windows, the algorithm presented by Hoots et al. [11] is implemented. This method fails in the case of co-planar and near co-planar pairs. Therefore, for those specific cases, an alternative method is used, which exploits the time rate of change of the relative position between the two objects to find the region of closeness, by identifying the change of slope corresponding to a local minimum. In particular, the switch from negative to positive values of this quantity is found.
The line of intersection between the two orbital planes moves with time, and so the tool calculates its first position with the input data, and then uses an accurate propagation method to update the knowledge of the state of the objects to finally calculate a more accurate version of the subsequent positions of the line of intersection.

Orbit Path Filter

The *OP filter* is based on the computation of the Minimum Orbital Intersection Distance (MOID). This quantity is calculated for each pair, and then it is compared to a threshold according to the following criterion:

$$MOID > D \tag{2}$$

Where D is a threshold distance selected by the user. Based on the work of Alfano and Finkleman [12], a suggested value for this threshold is 10 km.
The computation of the MOID is based on the algorithm developed by Gronchi [13]. If a Keplerian propagation method is chosen, a simple MOID calculation is performed using the input data. If a more accurate propagation method is selected, then, a procedure to correctly compute this quantity is called. In the first case, the MOID is computed considering the orbital knowledge of the two objects coming from the current data and the procedure does not go any deeper. This means that the two orbits, of the primary and secondary, are considered Keplerian, resulting in a non-time-dependent MOID. In the second case, the approach presented in [7] is exploited. A first guess of the MOID is computed together with the corresponding true anomalies (the anomalies at which the two objects would be at the closest distance between their paths). The states of the two objects are propagated in correspondence of those anomalies, and the MOID is recomputed with these updated orbital parameters. The goal is to compute this quantity in the proximity of its actual location in order to exploit an updated version of states of the two objects using an accurate propagation method.
The steps to of the procedure used to correctly compute the MOID are here detailed:

- Compute the MOID and the corresponding true anomalies using the orbital elements of primary and secondary defined at the start time of the analysis.
- Compute the times of flight from the current positions to the true anomalies corresponding to the MOID
- Perform orbital propagation of the states of primary and secondary with the propagation times defined by the previous time of flight.
- Convert the propagated states into orbital elements
- Compute the MOID and the corresponding true anomalies using the updated orbital elements of primary and secondary
- Compute the times of flight from the updated positions to the updated true anomalies corresponding to the updated MOID
- Check on these times of flight: if larger than a threshold, repeat the operations.

3.1.2 One-to-one Analysis

With the use of the filtering sequence, a screening of the catalogue of secondary objects is performed and the remaining pairs are therefore analyzed in detail. The tool takes one primary and one at a time of the remaining secondaries to perform the successive operations.
The states of the two objects are propagated within the time span defined by the user, and the relative position is calculated to identify a possible TCA. This is done by exploiting the time rate of change of the relative position, which is used to identify the presence of local minima, according to the procedure presented in [11]. Three cases are possible within the time interval of the analysis:

1. The time rate of change of the relative position is only increasing: the two objects are moving away from each other and the first time instant of the time interval of the analysis is taken as TCA.
2. The time rate of change of the relative position is only decreasing: the two objects are approaching but a local minimum is not present within the time window and therefore the last time instant of the time interval of the analysis is taken as TCA.
3. The time rate of change of the relative position has a change of sign from negative to positive: within the time window there is a local minimum, and the corresponding time instant is used as first guess inside a more accurate procedure for the calculation of the TCA.

If the time interval of the analysis is sufficiently large, several local minima of the relative position could be found. In this case, the corresponding time instants are identified, and the states of the two objects are propagated at those times to calculate the relative position and its norm. The time instant corresponding to the minimum of these norms is taken as TCA and the distance between the objects is taken as MD. The other time instants and related norm of relative position are however stored for possible future purposes.
At this point, the state transition matrix is used to propagate the covariance at the TCA. Other typical quantities of a conjunction event are finally calculated and, among them, the PoC according to the method selected by the user (Chan, Patera, LAAS), and better described in Section 3.2.
The steps of the procedure are here detailed:

- Define the time vector with start time and end time given by the user.
- Perform orbital propagation of the states of primary and secondary within the time span.
- Compute the time rate of change of the relative position.
- Identify the possible TCA within each identified time window.
- Compute MD for each possible TCA and identify the minimum MD.
- Propagate the covariance to the TCA.
- Compute other conjunction quantities including PoC.

3.2 Probability of Collision

Given the uncertainty associated to the orbital state of a tracked satellite, a stochastic description is needed to properly describe the alert level associated to a conjunction involving satellites. Thus, the PoC is a key quantity in the operational management of the CA service.
Let us consider the relative position $\boldsymbol{p}$. Associating a diameter to the involved objects geometry, through the definition of D_p and D_s for the primary and the secondary objects respectively, it is possible to define the Hard Body Radius: $HBR = (D_p + D_s)/2$. A collision occurs when, at the TCA, $|\boldsymbol{p}| \leq HBR$ and, to compute the associated PoC, two models exist, depending on the conjunction features:

- Short-term encounters model [14]: describes the conjunctions characterized by a high relative velocity between the involved objects at TCA. This model is mostly suitable for LEO encounters, and assumes constant position uncertainties throughout the conjunction as well as a deterministic description for the velocities [10].

- Long-term encounter model [15]: describes the conjunctions characterized by a small relative velocity between the involved objects. Since the objects spend significant time in close proximity, the encounter could sometimes occur several times per orbit, for several consecutive orbits. This model is mostly suitable for GEO encounters, formation flying and proximity operations.

The current work focuses on the short-term encounter model, given its advanced state of the art, whose hypotheses allow to define at TCA an encounter frame, usually called ”B-plane”. The exact definition depends on the author, but it always has the two following characteristics: it is centred on the mean position centre of gravity of one of the two objects and is orthogonal to the direction of the relative velocity.
Given the short-term model assumptions, it is possible to project the problem in the two-dimensional space, and to compute the PoC as a one-dimensional integral around the HBR contour. Equation 3 presents such an integral, in function of the infinitesimal contour dr, of the two-dimensional position $\boldsymbol{p}_{2D}$ and the related covariance $\boldsymbol{\Sigma}_{2D}$.

$$\text{PoC} = \frac{1}{(2\pi)\sqrt{\det\left(\boldsymbol{\Sigma}_{2D}\right)}} \int \exp\left(-\frac{1}{2}\,\boldsymbol{p}_{2D}^{T}\,\boldsymbol{\Sigma}_{2D}^{-1}\,\boldsymbol{p}_{2D}\right) dr \tag{3}$$

By this way the PoC can be computed as a 2D integral, which can be solved either numerically or analytically. Both of the approaches were implemented in the Suite, as follows.
The numerical approach selected is the one described in [16], and addressed as ”Patera” method: it reformulates the 2D PoC as a one-dimensional integral by writing it as a path integral over the contour of the domain of integration. This approach requires a numerical scheme to split the domain in fully non-null subdomains and it depends on the number of integration steps used. In particular, to increase the PoC numerical precision, a numerical scheme is implemented to provide the result after having reached convergence, that is when an increase in the number of steps used does not refine the result anymore.
Two analytical approaches were implemented: the method described in [14] (addressed as ”Chan” method) and, the one in in [17] (”LAAS”). Both methods rely on a series expansion of the PoC and, so, the obtained accuracy depends on the expansion order selected. It is worth to remark that this approach is fast, and possible numerical instabilities are solved in [17] through a pre-conditionner term.
Concerning the PoC computation, a final aspect is to point out. Given that the PoC value depends on the covariance associated to the orbital states of the involved objects, both the uncertainty underestimation and overestimation may significantly affect the results. Therefore, a sensitivity analysis is often conducted to assess which is the maximum PoC (”max-PoC”), by modifying the involved objects covariances through an inflating factor, usually set ranging from 0.4 to 2.5.

3.3 Collision Avoidance Maneuver

A collision avoidance maneuver can be run either in a low-thrust way [18] or in an impulsive one. The algorithm developed for the ISOC 3.0 Suite focuses on this latter application, given its operational level state of the art. Given the data about the collision event, which are read from a CDM, and other fundamental inputs, the first step is to plan the CAM by using algorithms based on a formulation that relies on Keplerian assumptions. Then, by exploiting more accurate propagation methods, a refinement procedure is launched to recalculate the conjunction quantities once the maneuver is performed.

3.3.1 CAM Planning

The planning phase of the CAM starts with the data contained in the given CDM, a range of true anomalies to be used as locations where to perform the candidate maneuvers, a desired upper limit for the PoC and a lower one for the MD to be obtained after the maneuver. Finally, the user can also require a tangential maneuver, i.e. in the direction of the velocity vector. If the thresholds are not provided by the user, reasonable suggested values are used.
To perform the calculations, the methods developed by Bombardelli and Hernando-Ayuso [19] are implemented. Bombardelli formulated a linear relation between the impulsive maneuver vector and the relative position between the primary and the secondary projected onto the conjunction plane. This linear relation allowed to develop an optimization problem in which the objective function is defined as a quadratic form, eventually reducing the problem to the solution of a simple eigenvalue problem.
Two optimization problems are defined: the first aims to maximize the MD (MMD) while the second aims to minimize the PoC (mPoC). The solution to the problem gives the direction of the impulsive maneuver vector.
In its original formulation, the solution of this problem requires to fix the magnitude of the impulsive maneuver vector to obtain its direction. In the tool, the input thresholds are used to start an optimization routine that finds the minimum impulse magnitude that minimizes the residual between the calculated quantity (MD or PoC) and the related threshold. In other words, inside the optimization routine, a first guess for the impulse magnitude is given, the eigenvalue problem is solved, the $\boldsymbol{\Delta v}$ vector of the maneuver is obtained and is used to compute the resulting PoC or MD. This quantity is then compared to the threshold to calculate the residual that has to be minimized by the optimization routine.
For the tangential case, the direction of the maneuver is obtained by simply using the flight path angle. The only unknown is the magnitude of the impulse, which is always found thanks to the optimization routine.

3.3.2 CAM Refinement

The need for a refinement procedure after the calculation of the CAM comes from the Keplerian assumptions underlying the Bombardelli formulation of the dynamics.
Consequently, a high-fidelity propagator is used to backpropagate the object's state from the TCA to the maneuvering time. The impulsive maneuver vector is added to obtain the state after the maneuver, and the state is propagated forward, from the maneuvering time to the TCA. At this point, MD and PoC are calculated.
Given that, after the maneuver, the primary is following a new trajectory, it is also interesting to calculate the new TCA. This is done by an apposite routine by using the former TCA as first guess. The state of the primary is then propagated to the new TCA to recalculate MD and PoC.
The sole purpose of this refinement part is therefore to calculate the conjunction quantities both at the former and the new TCA to then make a comparison with the one obtained in the planning phase under Keplerian assumptions. If the quantities are not close to the desired thresholds, the user can then choose to select another CAM among the candidate ones and verify if it leads to satisfying results by launching again the refinement procedure.

4 NUMERICAL ANALYSIS

The numerical validation of the ISOC Suite tool for conjunction analysis is here reported. All the simulations were implemented in MATLAB [20] and run with an Intel(R) Core(TM) i7-12700 CPU @ 2.10 GHz processor with a 16 GB RAM.

4.1 Catalogue Screening

Given the large scale of data to be used to test the catalogue screening module, its validation is carried out directly by exploiting real data, i.e. TLEs of catalogued objects downloaded from Spacetrack website [21]. In particular, the case with a single primary object, considered as an asset of interest for the user, and the entire catalogue of space objects as secondaries is presented. This scenario, which is of operational interest, represents the *one vs all* procedure, and its final output is a catalogue of potentially colliding secondary objects.
The main role of this module is to reduce the overall computational cost of the conjunction analysis process. Therefore, the emphasis is on the time spent by each filter to perform its operations and the number of discarded pairs after each step. Moreover, as explained in Section (3.1.1), each filter requires some input parameters, such as the thresholds needed to apply the filtering criteria.
The results are reported in Table 1 for an analysis performed over a time interval of one week, from 4 June 2022 to 11 June 2022, with one primary and a catalogue of 24219 secondary objects. As explained in Section 3.1, the first step consists in the pre-process phase, where the objects are propagated at the initial time.

	Threshold [km]	**Discarded pairs**	**Remaining pairs**	**Comp. time** [s]
Pre-process	-	-	24219	66.09
AP filter	30	19783	4436	0.02
T filter	30	2162	2274	1308.98
OP filter	100	619	1655	66.84

Table 1: Numerical validation results for Catalogue Screening procedure.

The results clearly highlight the importance of performing such preliminary calculations before proceeding with an accurate analysis of each single pair. This would be in fact based on the time evolution of their relative state, thus requiring a considerable expense of computational resources to perform orbital propagation. The final remaining pairs (1655) represent the 6.83% of the total (24219), which can be handled significantly better within the concluding processing stage, where, for each pair, the conjunction event is characterized by calculating the typical quantities such as TCA, MD, PoC. For the presented test case, this last phase required in fact about 3 hours of calculations, which is a very consistent part of the overall computational time required for the whole analysis.

4.2 Probability of Collision

The numerical validation of the 2D PoC methods is carried out based on the test cases provided in [14] and [22], whose features are recapped in Table 2. The algorithms performance are provided in Table 3 and compared to the reference values provided in [14] (both for Patera and Chan methods, individually) and [22] (considering the Montecarlo result with 1e+08 samples).
From Table 3 it is possible to appreciate the compliance between the results provided by the implemented PoC methods with the reference ones. In particular, for the *Alfano case 6*, it is possible to notice that the Chan method failed to compute the PoC, while the LAAS method achieved a quite

	σ_x	σ_y	R	x_m	y_m	σ_x/σ_y	R/σ_y
Chan case 1	50	25	5	10	0	2	0.2
Chan case 2	50	25	5	0	10	2	0.2
Chan case 3	75	25	5	10	0	3	0.2
Chan case 4	75	25	5	0	10	3	0.2
Chan case 5	3000	1000	10	1000	0	3	0.01
Chan case 6	3000	1000	10	0	1000	3	0.01
Chan case 7	3000	1000	10	10000	0	3	0.01
Chan case 8	3000	1000	10	0	10000	3	0.01
Chan case 9	10000	1000	10	10000	0	10	0.01
Chan case 10	10000	1000	10	0	10000	10	0.01
Chan case 11	3000	1000	50	5000	0	3	0.05
Chan case 12	3000	1000	50	0	5000	3	0.05
Alfano case 3	114.25852	1.41018	15	0.15916	-3.88721	81.02407	10.63694
Alfano case 6	1778.01770	2.20090	10	-1.2531	-2.1046	807.85864	4.54359

Table 2: Numerical validation data for PoC computation.

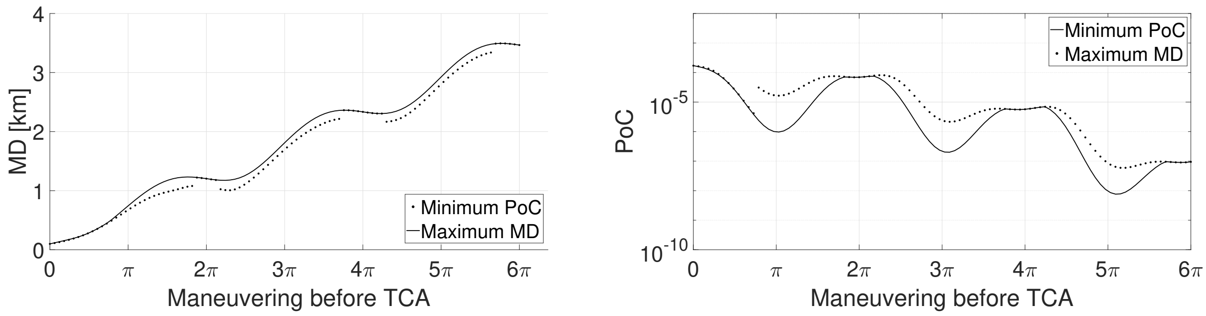
	Reference Patera	**Reference Chan**	**Patera**	**Chan**	**LAAS**
Chan case 1	9.741e-03	9.754e-03	9.742e-03	9.754e-03	9.742e-03
Chan case 2	9.181e-03	9.189e-03	9.181e-03	9.189e-03	9.181e-03
Chan case 3	6.571e-03	6.586e-03	6.571e-03	6.586e-03	6.571e-03
Chan case 4	6.125e-03	6.135e-03	6.125e-03	6.135e-03	6.125e-03
Chan case 5	1.577e-05	1.577e-05	1.577e-05	1.577e-05	1.577e-05
Chan case 6	1.011e-05	1.011e-05	1.011e-05	1.011e-05	1.011e-05
Chan case 7	6.443e-08	6.443e-08	6.443e-08	6.443e-08	6.443e-08
Chan case 8	3.219e-27	3.216e-27	3.219e-27	3.216e-27	3.219e-27
Chan case 9	3.033e-06	3.033e-06	3.033e-06	3.033e-06	3.033e-06
Chan case 10	9.656e-28	9.645e-28	9.656e-28	9.645e-28	9.656e-28
Chan case 11	1.039e-04	1.039e-04	1.039e-04	1.039e-04	1.039e-04
Chan case 12	1.564e-09	1.556e-09	1.564e-09	1.556e-09	1.564e-09
	Montecarlo		**Patera**	**Chan**	**LAAS**
Alfano case 3	1.008e-01		1.004e-01	2.445e-02	1.003e-01
Alfano case 6	4.300e-03		4.335e-03	1.081e-02	4.335e-03

Table 3: Numerical validation results for PoC computation.

accurate result. It is worth to highlight that, to achieve a correct value for the PoC, large orders should be used when computing this quantity through a series approximation such as in the Chan method. Unfortunately, large orders lead to numerical instability, which prevents the algorithm to provide the correct result. This numerical instability is remedied in LAAS method thanks to the use of the preconditionner mentioned in Section 3.2. By this way the PoC can be computed also analytically, with a remarkable computational time saving.

4.3 Collision Avoidance Maneuver

The numerical validation of the calculations performed for the CAM planning is carried out by reproducing the test case about the Iridium-Cosmos collision presented in [19].
The evolution of miss distance and probability of collision, for varying maneuvering anomaly, are



(a) Miss distance

(b) Probability of collision

Figure 2: Iridium-Cosmos conjunction

reported to compare the results given in the case of minimum collision probability and maximum miss distance, i.e. the two optimization methods presented in the reference [19].
It is possible to notice that, according to their formulation, the two methods achieve the desired result minimizing or maximizing the quantity of interest. Another remark is that larger maneuvering anomaly, which means that the maneuver is performed in large advance with respect to the assumed TCA, lead to better performance in terms of final MD or PoC. This highlights the importance of planning and then performing CAMs with reasonable advance.

5 REAL DATA

In this section, the performance of the tool is assessed based on real data.

5.1 Catalogue Screening

For the validation of the screening process, a CDM downloaded from Spacetrack [21] is considered, which contains the NORAD ID of both primary and secondary and their TCA. The TLEs of the two objects are then downloaded for an epoch prior to the given TCA, in order to put them in the screening process and verify that the tool is able to detect and correctly characterize such a possible event.
Table 4 collects the results of the operations. The time interval is set around the TCA given in the CDM, for a duration of one week. The TLEs of the two objects are given at an earlier epoch, so as a first step they are propagated to the start time of the interval. The pair passes the entire filtering sequence. It is then evaluated as a potential colliding pair and accurately analyzed by exploiting the time evolution of the relative position to calculate the TCA and other quantities. Finally, the accuracy of the calculated TCA compared with that provided by the CDM can be appreciated in the last row.

Time Interval	
2022-07-01 00:00:00	2022-07-07 00:00:00
Object ID	**Object Epoch**
50792	2022-06-29 17:48:25
50787	2022-06-29 19:30:47
TCA (calculated)	**TCA (from CDM)**
2022-07-03 04:44:25.494	2022-07-03 04:44:25.368

Table 4: Real data: catalogue screening

5.2 Probability of Collision

For the validation of the PoC methods implemented in the tool, real data are extracted from the Collision Avoidance Challenge database by ESA [23]. As in the numerical validation section, three test cases are here reported, with the data in Table 5, and the following results in Table 6.
Even in this case, the results clearly show the reliability of the implemented methods, given their accuracy compared to the reference. An interesting aspect to notice is that Patera and LAAS methods give results that are in accordance and slightly similar to the reference value than the Chan one.

	σ_x	σ_y	R	x_m	y_m	σ_x/σ_y	R/σ_y
Case 1	72.0645	26.8416	29.71	20.7120	37.8755	2.6848	1.1069
Case 2	631.2708	26.7600	23	-14.3269	345.0316	23.5901	0.8595
Case 3	2719.2000	31.3349	23	-18.6580	665.7557	86.7785	0.7340

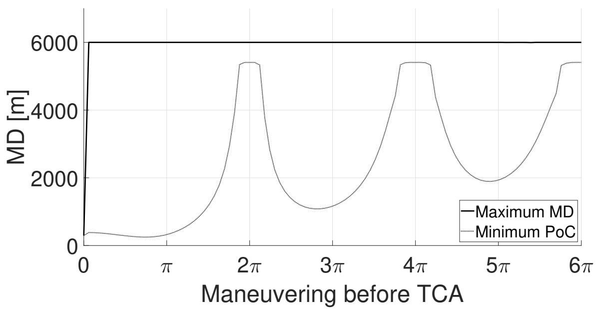
Table 5: Real data for PoC computation.

	Reference	**Patera**	**Chan**	**LAAS**
Case 1	1.360e-01	1.362e-01	1.384e-01	1.360e-01
Case 2	1.094e-02	1.096e-02	1.116e-02	1.096e-02
Case 3	2.4146-03	2.4173-03	2.5201-03	2.4173-03

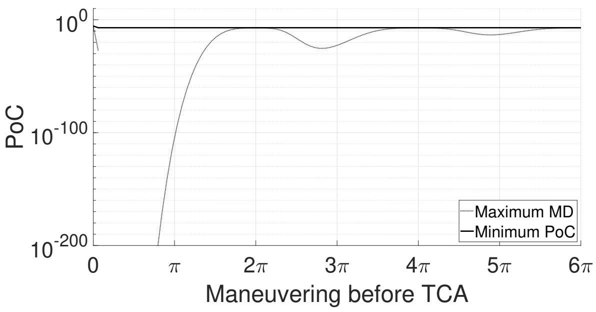
Table 6: Real data: results for PoC computation.

5.3 Collision Avoidance Maneuver

The CAM module is tested on a real CDM, which is given as input to extract mean states, covariances and other quantities of the considered pair. To start, a threshold of 6000 m for the MD and 10^{-7} for the PoC are used, which are reasonable values considering that the two objects are on GEO orbits. Based on these requirements, the impulse direction is found according to the resolution of the two optimization problems formulated by Bombardelli [19]. The impulse magnitude is instead calculated according to the optimization routine that allows to compute the minimum value for which the user threshold is satisfied. Through this feature of the tool it is therefore possible to avoid the selection of a specific value for this variable, which could be overestimated by the user.
Given that the impulse magnitude is set to be the minimum value required to meet the user's threshold, in Figure 3 it is possible to appreciate that, for both the MMD and mPoC cases, the optimized quantity is equal to the required threshold.
Figure 4 shows the values of the optimized impulse magnitude compared to the maneuvering anomaly. It can be remarked that, as expected, the earlier the maneuver is performed with respect to TCA, the less expensive it is. There is a peak in the proximity of the zero degrees because the impulse needed to satisfy the thresholds by performing a maneuver very close to the collision anomaly would be very expensive.
As explained in Section 3.3, the tool gives as output multiple candidate CAMs at different maneuvering anomalies, from which the user can then choose the preferred one. To evaluate the performance of the CAM module, the candidate maneuver with the lowest value of impulse magnitude is selected, and its characteristics are listed in Table 7. Finally, the results obtained by applying the selected CAM are listed in Table 8. In particular, the first column shows the values of MD and PoC given in the CDM, the second one gives the values that should be obtained after the application of the CAM, while the last two columns give the values recalculated according to the refinement procedure both at



(a) Miss distance



(b) Probability of collision

Figure 3: Real data: test case from CDM

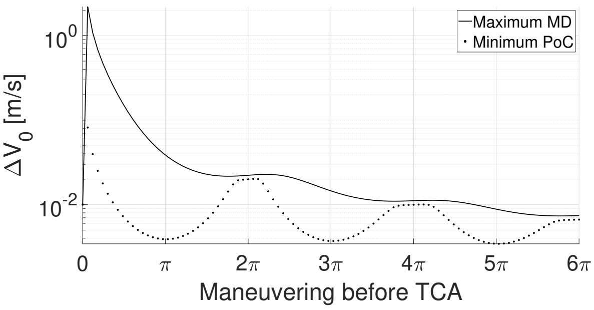


Figure 4: Impulse magnitude

the former and the new TCA. It is possible to notice that the planned values slightly differ from the refined ones, which still satisfy the thresholds. If the refinement procedure reveal values that are not compliant with the required thresholds, the user could select an alternative CAM among the candidate ones and repeat the procedure.

Collision Anomaly	Maneuvering Anomaly	Distance to Collision	Impulse Magnitude
170.96 deg	214.596 deg	2.88 orbits	7.356e-03 m/s

Table 7: Real data: selected CAM

	CDM	Planned	Refined (former TCA)	Refined (new TCA)
MD [m]	294.901	6000	6042.25	6001.4
PoC [-]	1.102e-03	3.812e-08	3.703e-08	3.704e-08

Table 8: Real data: CAM effect on MD and PoC

6 CONCLUSIONS

This paper presented the conjunction analysis process embedded in the ISOC 3.0 Suite. Potential conjunction events can be detected and accurately characterized for a given catalogue of primary objects against the entire space catalogue. The filtering sequence can be seen as the catalyst that speeds up the process of conjunction analysis. Its role is therefore central, timing being a crucial factor for these operations. Once possible colliding pairs are identified, multiple candidate CAMs can be computed such that requirements on specific quantities such as PoC and MD are satisfied. A remarkable feature of the presented tool is its versatility in handling different input types and multiple user's needs, as well as providing various outputs that could be fruitfully exploited for later analysis.

7 ACKNOWLEDGEMENT

The authors are grateful to Leonardo Company for the collaboration in the ISOC Suite realization.

REFERENCES

[1] ESA Space Debris Office, *ESA's Annual Space Environment Report*, 2022.

[2] T. Flohrer and H. Krag, "Space surveillance and tracking in ESA's SSA programme," in *7th European Conference on Space Debris*, vol. 7, 2017.

[3] European Space Surveillance and Tracking, *EUSST Service Portfolio*, 2021.

[4] M. F. Montaruli, G. Purpura, R. Cipollone, A. De Vittori, L. Facchini, P. Di Lizia, M. Massari, M. Peroni, A. Panico, A. Cecchini *et al.*, "A software suite for orbit determination in space surveillance and tracking applications," in *9th European Conference for Aerospace Sciences (EUCASS 2022)*, 2022, pp. 1–12.

[5] M. F. Montaruli, L. Facchini, P. D. Lizia, M. Massari, G. Pupillo, G. Bianchi, and G. Naldi, "Adaptive track estimation on a radar array system for space surveillance," *Acta Astronautica*, 2022. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0094576522002703

[6] R. Cipollone, M. F. Montaruli, N. Faraco, P. Di Lizia, M. Massari, A. De Vittori, M. Peroni, A. Panico, A. Cecchini *et al.*, "A re-entry analysis software module for space surveillance and tracking operations," in *INTERNATIONAL ASTRONAUTICAL CONGRESS: IAC PROCEEDINGS*, 2022, pp. 1–7.

[7] M. F. Montaruli, P. Di Lizia, E. Cordelli, H. Ma, J. Siminski *et al.*, "A statistical approach to identify fragmentation epoch from a single fragment surveillance radar observation," in *23rd Advanced Maui Optical and Space Surveillance Technologies Conference (AMOS)*, 2022, pp. 1–12.

[8] A. Muciaccia, L. Facchini, M. F. Montaruli, G. Purpura, R. Detomaso, C. Colombo, M. Massari, P. Di Lizia, A. Di Cecco, L. Salotti *et al.*, "Observation and analysis of cosmos 1408 fragmentation," in *INTERNATIONAL ASTRONAUTICAL CONGRESS: IAC PROCEEDINGS*, 2022, pp. 1–7.

[9] Consultative Committee for Space Data Systems, *Conjunction Data Message*, 2013.

[10] M. F. Montaruli, "Collision risk assessment and collision avoidance maneuver planning," Master's thesis, Politecnico di Milano, 2019.

[11] F. R. Hoots, L. L. Crawford, and R. L. Roehrich, "An analytic method to determine future close approaches between satellites," *Journal of Celestial Mechanics and Dynamical Astronomy*, vol. 33, no. 2, pp. 143–158, June 1984.

[12] S. Alfano and D. Finkleman, "On selecting satellite conjunction filter parameters," *Acta Astronautica*, vol. 99, pp. 193–200, 2014.

[13] G. F. Gronchi, "On the Stationary Points of the Squared Distance between Two Ellipses with a Common Focus," *SIAM J. Sci. Comput.*, vol. 24, no. 1, p. 61–80, January 2002.

[14] F. K. Chan, *Spacecraft collision probability*. Aerospace Press El Segundo, CA, 2008.

[15] V. T. Coppola *et al.*, "Including velocity uncertainty in the probability of collision between space objects," in *AAS/AIAA Spaceflight Mechanics Meeting*. American Astronautical Soc, 2012, pp. 2012–2247.

[16] R. P. Patera, "Calculating collision probability for arbitrary space vehicle shapes via numerical quadrature," *Journal of Guidance, Control, and Dynamics*, vol. 28, no. 6, pp. 1326–1328, 2005. [Online]. Available: https://doi.org/10.2514/1.14526

[17] R. Serra, D. Arzelier, M. Joldes, J.-B. Lasserre, A. Rondepierre, and B. Salvy, "A Power Series Expansion based Method to compute the Probability of Collision for Short-term Space Encounters," *Archive ouverte*, March 2015.

[18] A. De Vittori, M. F. Palermo, P. D. Lizia, and R. Armellin, "Low-thrust collision avoidance maneuver optimization," *Journal of Guidance, Control, and Dynamics*, vol. 45, no. 10, pp. 1815–1829, 2022. [Online]. Available: https://doi.org/10.2514/1.G006630

[19] C. Bombardelli and J. Hernando-Ayuso, "Optimal impulsive collision avoidance in low earth orbit," *Journal of Guidance, Control, and Dynamics*, vol. 38, no. 2, pp. 217–225, 2015.

[20] MATLAB, *9.9.0.1538559 (R2020b)*. Natick, Massachusetts: The MathWorks Inc., 2020.

[21] Space-track, "Space-track website," 2022, accessed 06.06.2022, https://www.space-track.org/auth/login.

[22] S. Alfano, "Satellite conjunction Monte Carlo analysis," *Advances in the Astronautical Sciences*, vol. 134, pp. 2007–2024, January 2009.

[23] E. A. C. Team, "ESA Collision Avoidance Challenge," 2023, accessed 20.03.2023, https://kelvins.esa.int/collision-avoidance-challenge/data/.