Continuity equation-based model for the assessment of the long-term effect of mitigation and remediation measures on the space debris environment

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ABSTRACT

Excluding the untrackable debris objects, when establishing mitigation measures for the space debris problem, results in an underestimation of the actual threat. However, the inclusion of such small fragments into debris evolutionary models is an enormous challenge from a computational point of view. To address this problem, the model COMETA (Continuum Mechanics for debris EnvironmenT Analysis) has been developed. It proposes the use of continuum formulations for the description of the fragments' orbital dynamics within an elaborated probabilistic space environment propagator. As the density-based methods guarantee a computational cost that only slightly depends on the lowest fragments size considered, their use allows for the potential extension of the debris evolutionary models to any objects' dimension.

1 INTRODUCTION

The number of services provided by in-orbit satellites is massively increasing and, accordingly, our exploitation of the space environment [1]. In recent years, the space community has progressively given attention to the debris problem: the growth of artificial objects can no longer be unregulated, to avoid the collapse of such a delicate ecosystem.

In this scenario, designing tools for the estimation of the future trend of the space environment is crucial. Several debris evolutionary models have been developed over the years, with the aim of predicting the long-term evolution of the debris population and assessing the potential beneficial effect of remediation and mitigation measures. Most of them fall in the category of the semi-deterministic approaches [2]-[10], which allow modeling of arbitrarily complex dynamical systems, and thus ensure high flexibility. The bottleneck of these methods is typically the computational cost when extending the models to include sub-centimeter fragments. However, excluding the so-called untrackable debris objects limits the view we have on the actual health of the space environment [11].

The software COMETA (Continuum Mechanics for debris EnvironmenT Analysis) has been developed to answer this need. It is a novel probabilistic long-term debris environment propagator that, in the current implementation, estimates the future trend of the low-Earth orbital region under the effect of sources, i.e., launches, satellite explosions, and fragment-intact object collisions, and sinks, i.e., natural and controlled re-entry. This objective is achieved with a classification of the objects into species [12][13], which allows for the definition of mission control parameters separately for each objects' category. The main novelty of the developed software is the inclusion of density-based propagation methods [14]-[17] within a complex debris evolutionary model with the aim of extending the space environment analysis to any fragments' dimension.

2 METHOD OVERVIEW

The workflow and main components of the debris environment model COMETA are explained in this section. The classification of the whole objects population in Low-Earth Orbit (LEO) into different categories and sub-categories is first addressed in Section 2.1. The implemented launch traffic model is then presented in Section 2.2. The different approaches for the objects orbit propagation, depending on each object's category and properties, are described in Section 2.3. Finally, in Section 2.4 the considered fragments sources are presented and the way they are introduced in the simulation as a feedback effect is discussed.

2.1 Objects classification

The objects' population is primarily divided into intact objects and fragments. This division is the result of the following considerations and modelling choices:

- Fragments are assumed to be uncapable of self-fragmenting, as they do not carry any potential source of explosion.
- Intact objects are far more likely to be impacted by another object, because of their larger average dimension. Indeed, most of the fragments are in the centimeter range [18]. Thus, fragment-fragment collisions are neglected.
- When scaling down to the centimeter size (or less), the fragments population grows exponentially. Hence, a density-based description of their orbital dynamics is preferred. On the contrary, for the intact objects population an individual propagation is needed to faithfully replicate their mission profile.

Intact objects are further classified into species, i.e., payloads, rocket bodies, Mission Related Objects (MROs), large debris objects, and constellations, where each constellation is a separate species. The objects' number and properties per each modelled species at the reference epochs considered within this work, i.e., 2005, 2014, and 2022, are reported in Table 1. The Post-Mission Disposal (PMD) rates are defined based on historical values retrieved from the analysis in ESA's Space Environment Report [1]. PMD duration is set to 25 years. The objects' orbital data and physical properties are obtained from the DISCOS Database (Database and Information System Characterizing Objects in Space) [19][20].

Fragments are classified into background fragments population and simulated fragmentation debris. This distinction is introduced because a different approach is employed for their orbital propagation, as discussed in Section 2.3, and it does not translate into any different modelling assumptions.

2.2 Launch traffic model

Following the approach proposed in [4], new launches for non-constellation objects repeat the traffic pattern of the previous five years. This means that for the three considered reference epochs 2005, 2014, and 2022, the launch traffic patterns of 2000-2005, 2009-2014, and 2017-2022, respectively, are repeated every five years.

A different strategy is followed for the constellation objects in the 2022 intact objects population. Considered that a reliable prediction on the long-term deployment of new constellations is not available, future launches are only limited to the replenishment of the satellites of the already (partially) deployed constellations. In other words, when a satellite is injected in the disposal orbit, at the end of the operational phase, a new one is assumed to be instantaneously released in the constellation operational orbit. The only exception is for partially deployed constellations, for which new launches are envisioned until full deployment is achieved. The deployment rate for each partially deployed constellation is set based on the average launch rate the constellation had up to 2022.

Table 1. Intact objects' number and properties (lifetime, Collision Avoidance (COLA) capability, susceptibility to explosion, PMD rate) in 2005, 2014, 2022. Data retrieved from the DISCOS Database [20].

	# 2005, 2014, 2022			Lifetime [ys]	COLA	Explosion	PMD 2005, 2014, 2022		
Payload	1406	1861	3088	8	yes	yes	5	15	40
Rocket body	737	848	951	0	no	yes	15	25	55
MROs	88	128	230	0	no	no	/	/	/
Large debris	649	739	1519	0	no	no	/	/	/
Flock	0	0	312	3	no	no	n/a	n/a	90
Globalstar	0	0	72	15	yes	no	n/a	n/a	90
Gonets	0	0	33	5	no	no	n/a	n/a	90
Iridium	0	0	106	15	yes	no	n/a	n/a	90
OneWeb	0	0	634	10	yes	no	n/a	n/a	90
Orbcomm	0	0	60	5	no	no	n/a	n/a	90
SpaceBee	0	0	119	2	no	no	n/a	n/a	90
Starlink	0	0	4263	5	yes	no	n/a	n/a	90
Astrocast	0	0	16	5	yes	no	n/a	n/a	90
Capella	0	0	10	3	no	no	n/a	n/a	90
Kepler	0	0	20	7	no	no	n/a	n/a	90

2.3 Object orbital propagation

As already mentioned in Section 2.1, the division between intact objects and fragments allows for a separate description of their orbital evolution. Nevertheless, the same force model is considered for the propagation of every object orbit. With the objective of finding a compromise between accuracy and computational efficiency, current implementation only accounts for the effect of atmospheric drag. The superimposed King-Hele contraction method proposed in [21] is used to evaluate the long-term variation of semi-major axis and eccentricity under the effect of a solar flux-dependent atmospheric density. This latter is modelled as a time-varying sinusoidal function, whose amplitude and frequency are tuned according to the value and epoch of the solar radio flux extrema [21].

The main novelty of this work is the use of the continuum formulation for the description of the fragments' dynamics within a complex debris evolutionary model. In particular, two different continuity equation-based models are adopted: the background fragments population is propagated through the Method Of Characteristics (MOC) [22], while a Finite Volume Method (FVM) [23] is preferred for the simulated fragmentation debris. The two methods represent two different paradigms in continuum mechanics, the Lagrangian and Eulerian specification of the flow field, respectively [22]. The first is a way of looking at fluid motion where the observer follows an individual fluid parcel as it moves through space and time. On the contrary, the second places the observer in a particular location in space through which the fluid flows as time passes. The synergistic adoption of the two models aims at reducing the computational cost associated with the objects' propagation. Indeed, the MOC best works in the description of the orbital evolution of a widespread objects' population, as the initial one. The competitiveness of the FVM exponentially increases with the compactness of the debris cloud, which makes it particularly suited for the propagation of fragmentation clouds in LEO under the sole effect of atmospheric drag. Details on the implemented models can be found in [17][24].

2.4 Fragments sources

The developed debris evolutionary model currently accounts for two fragments sources: intact objects explosions and fragment-intact object collisions. The explosion, P_e, and collision, P_c, probabilities of every object are continuously monitored, and fragmentations are triggered in a probabilistic manner, by comparing P_e and P_c against random samples from the uniform distribution $\mathcal{U}(0,1)$.

The explosion probability is defined based on historical data of past fragmentation events. The Kaplan-Meier estimator is adopted to determine the survival functions for the objects species which are susceptible to self-fragmentation [25][26]. As reported in Table 1, for the results presented in this work, only payloads' and rocket bodies' explosions are simulated. The survival functions of these two categories, obtained from the history of fragmentations until 2023 available in the DISCOS Database [20], are depicted in Figure 1. If t is the time spent in orbit by an object with survival function $S(t)$, the cumulative probability of explosion reads as:

$$P_e(t) = 1 - S(t) \tag{1}$$

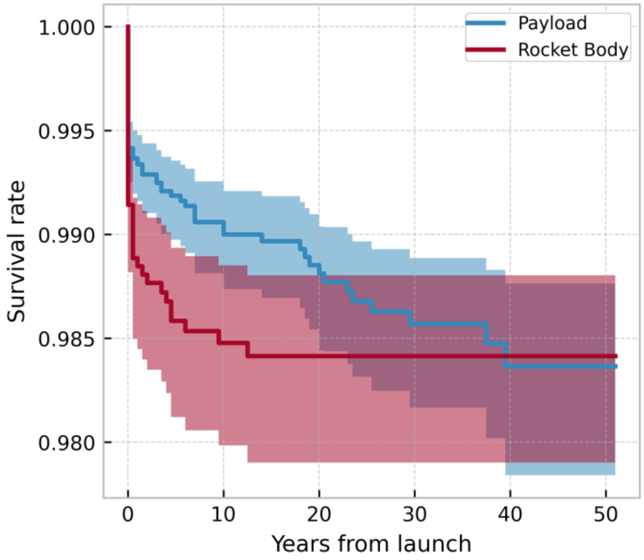
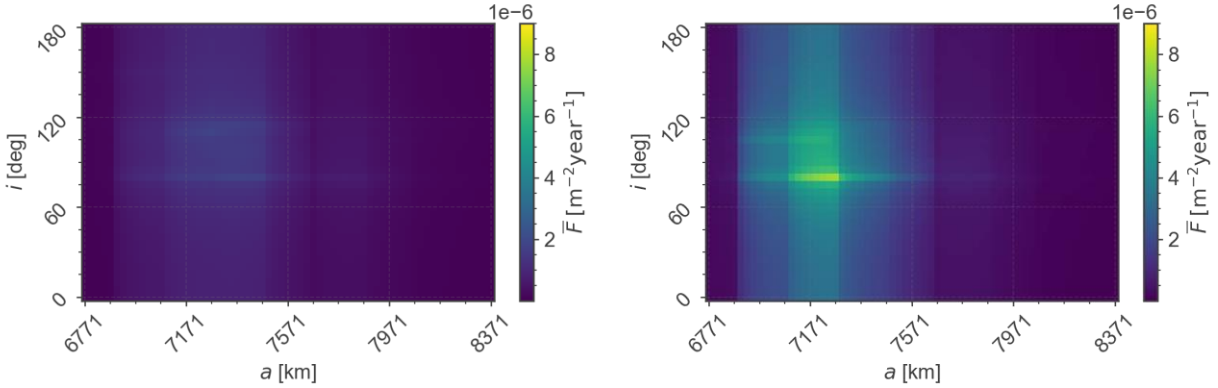


Figure 1. Survival rate as function of time elapsed since launch for payloads and rocket bodies.

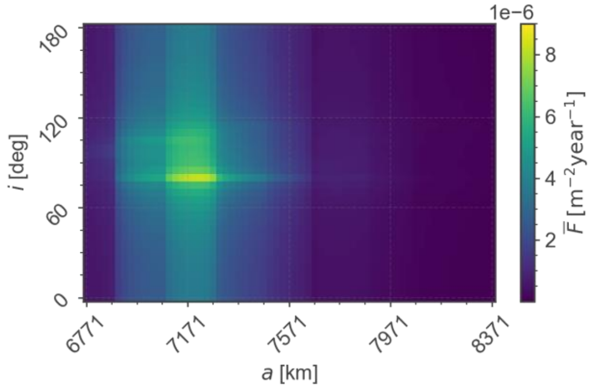
The collision probability is modelled as a Poisson distribution [27][28]. For an intact object with coordinates $(\boldsymbol{r}, \boldsymbol{v})$, the impact rate with the overall fragments population at a generic time t, $\dot{\eta}$, is computed from the average flux $\bar{F}$, i.e., the product between the fragments density and the average relative velocity over one revolution, as:

$$\dot{\eta}(\boldsymbol{r}, \boldsymbol{v}, t) = \bar{F}(\boldsymbol{r}, \boldsymbol{v}, t) A_c \tag{2}$$

with A_c intact object cross-sectional area. Note that the flux $\bar{F}$ continuously varies under the effect of atmospheric drag and of the fragments released into orbit by the simulated objects' breakup. The cumulative number of impacts is retrieved integrating Eq. (2) in time. The fragments fluxes in LEO as function of semi-major axis and inclination at the three considered reference epochs are depicted in Figure 2.



(a) Epoch: 2005 (b) Epoch: 2014



(c) Epoch: 2022

Figure 2. Fragments (> 10 cm) flux in LEO as function of semi-major axis and inclination at the three considered reference epochs.

3 SIMULATION RESULTS

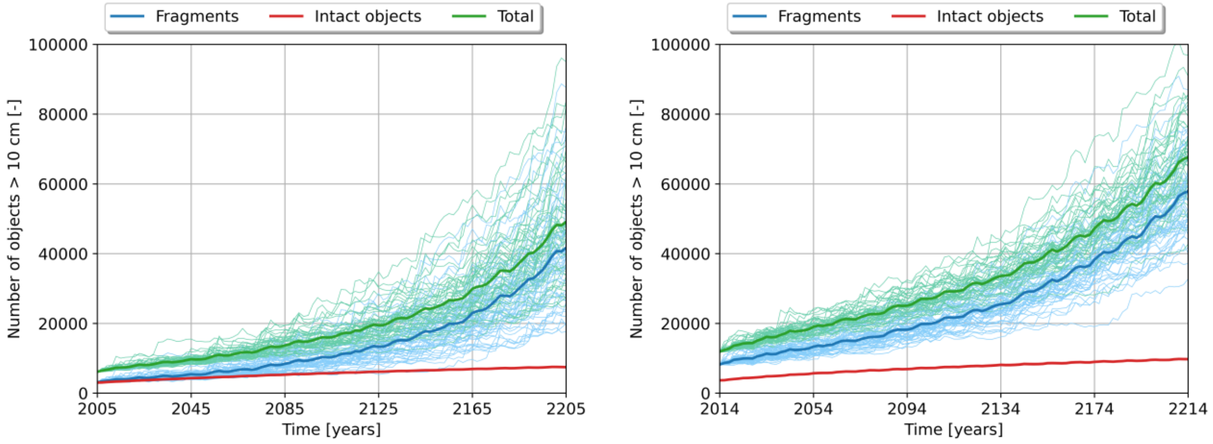
This section presents the obtained results on the long-term evolution of the objects population in 2005, 2014, and 2022 over a 200-year period. The simulations are referred to as Extrapolation in [4], as they extrapolate on the behavior of the space environment in terms of launch traffic and post-mission disposal rate relative to the considered reference epoch. For all the proposed analyses, 50 runs are performed, and the resulting estimates are eventually averaged. The simulations presented here are limited to objects larger than 10 cm, with the objective of making the software COMETA comparable to other debris evolutionary models in this preliminary phase. Future studies will apply the tool to model smaller debris objects.

Figure 3 shows the predicted evolution of the number of fragments (blue line), intact objects (red line), and whole objects population (green line) over time for the three cases. Figure 4 depicts the associated cumulative number of catastrophic collisions and explosions (blue lines) and relative cumulative number of fragments released into orbit (red lines) per objects' category. By looking at the obtained results the following considerations can be made:

- The propagation of the 2014 and 2022 LEO populations lead to a similar total number of objects after 200 years, while for the 2005 one approximately 30% less objects are predicted.
- The absolute number of objects at the end of the 200-year propagation does not picture the actual health of the environment for the three cases. By looking at the net increase of orbiting

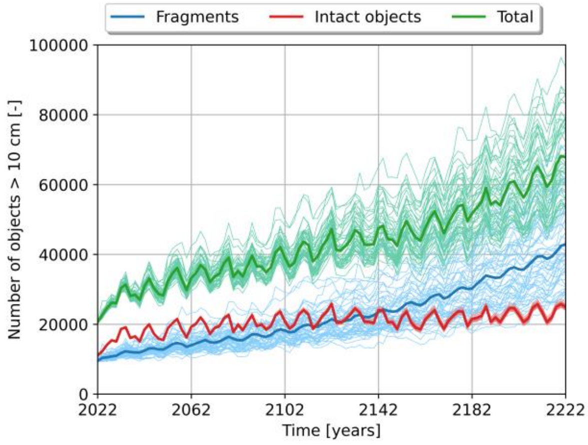
fragments one can infer that, out of the three simulation scenarios, the 2022 one is the most stable. This fact is further observable in the profiles of the cumulative number of catastrophic collisions reported in Figure 4. The exponential increase is considerably more accentuated in 2005 and 2014 compared to 2022, which testifies the dramatic beneficial effect that a higher PMD rate can provide.

- The higher number of explosion events in 2022 case is a direct consequence of the larger population of in-orbit payloads and rocket bodies, caused by the massive increase in the number of launched objects into space over the last decade [1].
- The ratio between number of fragmentation events and number of released fragments notably decreases in the extrapolation of the 2022 objects population. This behavior is caused by the lower mean size of the satellites that have been launched into space in the last years. In particular, constellation payloads are on average considerably smaller than individual satellites.
- Constellation payloads are the most likely to be impacted by a fragment in the 2022 scenario. This indicates that the more stringent PMD rate of constellations compared to individual payloads and rocket bodies does not sustain the rapid growth that this object category has been experiencing since 2017. Nevertheless, the PMD reliability of modern constellations is expected to be considerably higher than the imposed 90%.



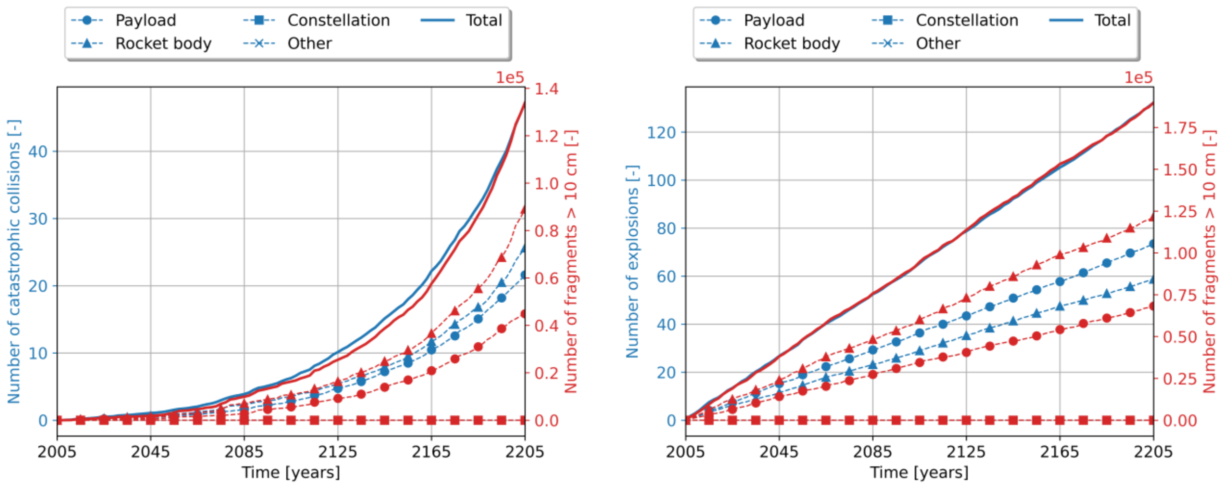
(a) Reference epoch: 2005

(b) Reference epoch: 2014

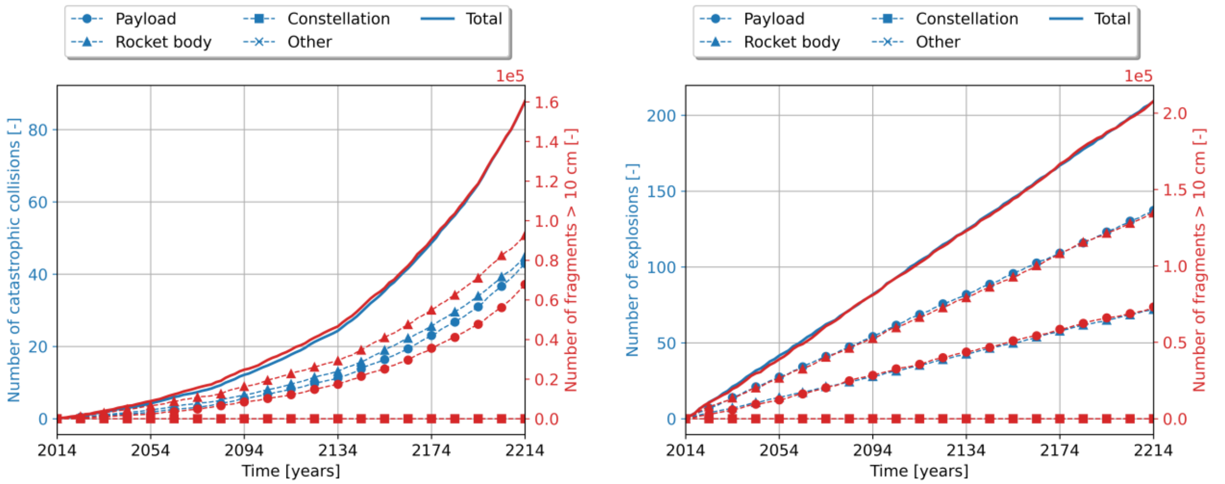


(c) Reference epoch: 2022

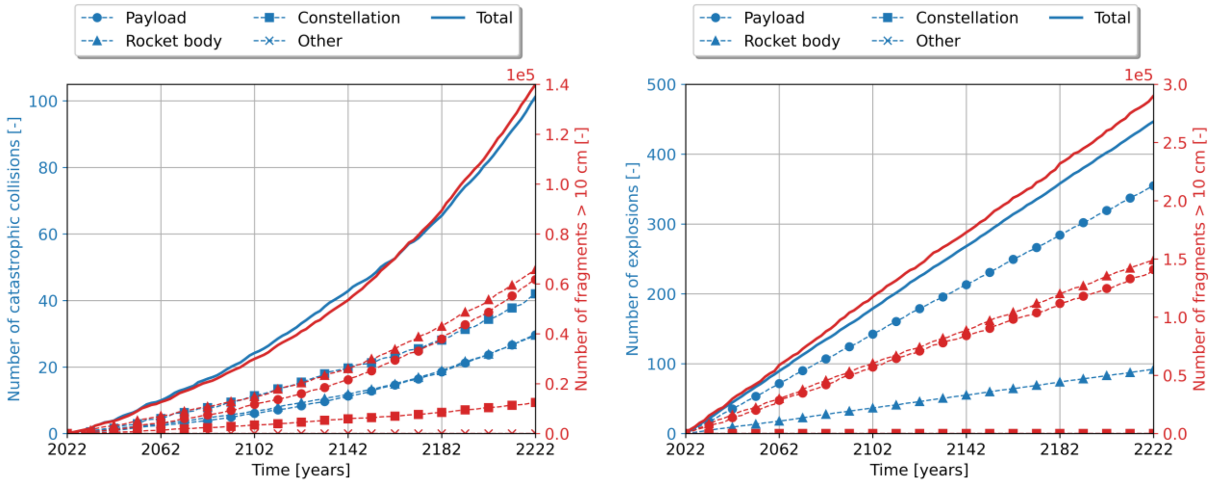
Figure 3. Number of objects > 10 cm over time, average and per single run profiles.



(a) Reference epoch: 2005



(b) Reference epoch: 2014



(c) Reference epoch: 2022

Figure 4. Cumulative number of fragmentation events and fragments released into orbit, total and per objects' category profiles.

4 COMPARISON WITH ESA DELTA4 SOFTWARE

Long-term simulations of the debris environment are a complex task. Furthermore, the outcome of these analyses aims at providing scientific support to the definition of mitigation and remediation measures, which will inevitably shape the future trend of the space environment. Therefore, detailed comparisons among different debris evolutionary models are crucial to find agreement on how the space environment would react in response to countermeasures to the debris problem.

The results presented in Section 3 are here compared against the predictions provided by the ESA's DELTA4 software [3], with which calibration and validation work is on-going. The results are compared in terms of number of in-orbit objects and cumulative number of catastrophic collisions over time. The profiles for the three extrapolation scenarios relative to 2005, 2014, and 2022 are shown in Figure 5 and Figure 6, respectively.

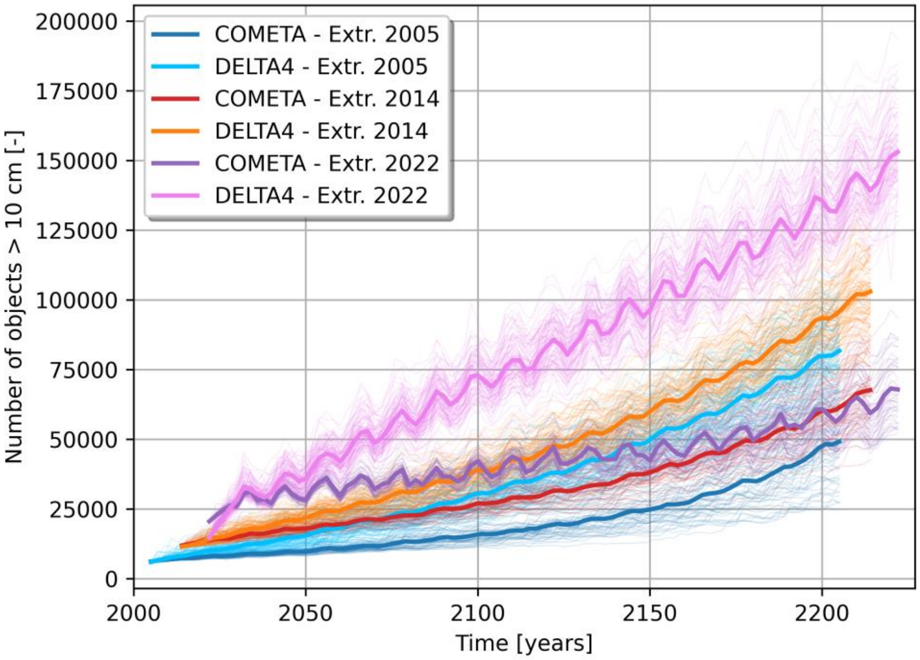


Figure 5. Number of objects > 10 cm over time, result of the extrapolation of the 2005, 2014, and 2022 reference populations. Comparison between COMETA and DELTA4 software.

As it can be inferred, the estimated profiles are notably different, with the DELTA4 software predicting a more unstable behavior of the debris environment for all the three cases. It is worth commenting that, in general, finding an agreement between different models on the absolute number of objects or catastrophic collisions over time is cumbersome, because of the different modeling assumptions adopted in the methods. Instead, it is more interesting and useful to observe the relative behavior of the three cases estimated by the two software. In this sense, COMETA and DELTA4 results are consistent if comparing the 2005 case relative to the 2014 one. In the 2022 scenario, COMETA estimates that the more stringent PMD rates effectively control the growth of space debris, if compared to the 2014 case. This result is not confirmed by the DELTA4 software, which foresees that the wider initial objects population would prevail on the improved mitigation actions. Future works will investigate the reason for this different behavior, as well as extend the comparison to other debris evolutionary models.

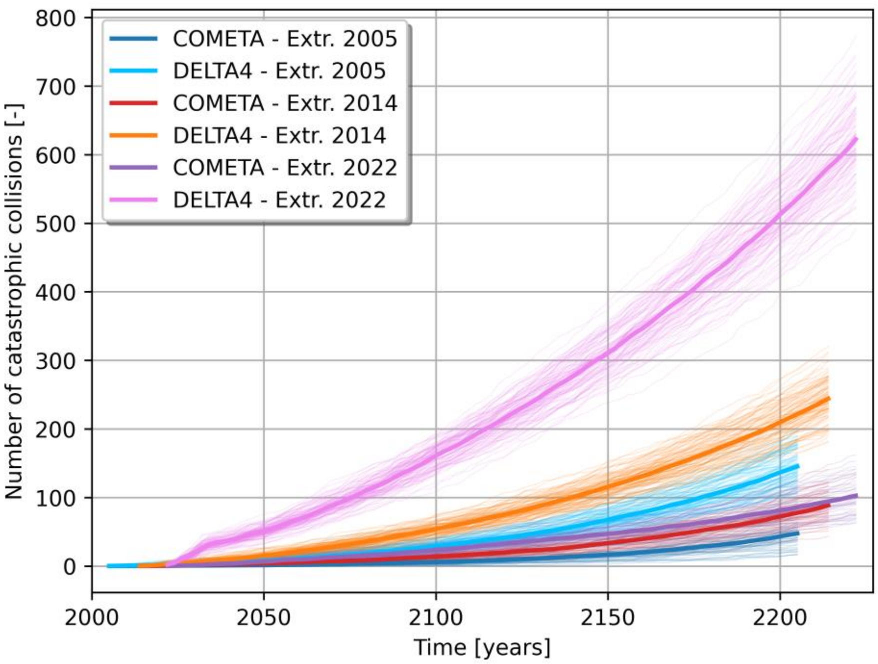


Figure 6. Cumulative number of catastrophic collisions over time, result of the extrapolation of the 2005, 2014, and 2022 reference populations. Comparison between COMETA and DELTA4 software.

5 CONCLUSIONS

The inclusion of the untrackable objects into debris evolutionary models is very computationally demanding. Several studies have highlighted how density-based propagation methods can tackle this challenge, as they abandon the individual description of each fragments' orbital evolution. In the software COMETA two continuum formulations for fragments' orbit propagation are included into a complex debris evolutionary model with the objective of its extension to any objects' dimension. Simulation results for the 2005, 2014, and 2022 reference populations were shown, and the effect on the space environment of different post-mission disposal rates was discussed. Finally, the model was compared against the predictions provided by ESA's DELTA4 software, with which a validation and calibration campaign is on-going. Common trends were identified in relation to the 2005 and 2014 extrapolation scenarios, while the results were not in agreement with respect to the 2022 case. Further analyses will be carried out to understand the reason for this different behavior.

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